Approximation Algorithms for Multi-Point Relay Selection in Mobile Wireless Networks

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N° 4925

Septembre 2003

THÈME 1

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--4925--FR+ENG

INRIA ROCQUENCOURT

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Thème 1 — Réseaux et systèmes
Projet Hipercom

Rapport de recherche n° 4925 — Septembre 2003 — 20 pages

Abstract: Routing is one of the main problems for Mobile Wireless Networks. In the case of infrastructureless multihop wireless networks, the selection of Multi-Point Relays provides efficient routing schemes. As such a selection is NP-hard, an efficient heuristic has been designed and effectively implemented in protocols for Mobile Ad Hoc Networks such as the Optimized Link State Routing protocol (OLSR).

In this paper, we introduce two variants of this practical heuristic by exploiting the topological properties of the network (without assuming a knowledge of geographic positions or geometric properties). For each heuristic, we give their respective guaranteed approximation performances when compared to a solution of optimal value. We argue that the heuristics proposed are of considerable interest when other problems are considered in addition to the routing efficiency (e.g., minimum remaining bandwidth, minimum remaining energy,...).

Key-words: Approximation Algorithms, Complexity, Mobile ad hoc, MultiPoint Relays, NP-completeness, Wireless Networks.

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Algorithmes d'Approximation pour la sélection des Multi-Points Relais dans les réseaux mobiles sans fils.

Résumé : Router est l'un des problèmes les plus importants pour les réseaux mobiles sans fils. Dans le cas des réseaux sans fils multi-sauts, la sélection de Multi-Points Relais permet un routage efficace. Une telle sélection étant NP-dur, une heuristique efficace a été implémentée pour les protocoles de réseaux mobiles sans fils tel OLSR (Optimized Link State Routing protocol).

Dans ce papier, nous introduisons deux variantes de cette heuristique pratique en exploitant les propriétés topologiques du réseau (sans faire d'hypothèse sur une connaissance des positions géographiques ou des propriétés géomètriques). Pour chaque heuristique, nous donnons leur performance guarantie d'approximation par rapport à une solution de valeur optimale. Nous montrons que les heuristiques proposées ont un intérêt considérable lorsque d'autres problèmes sont à prendre en considération en même temps qu'un routage efficace (e.g., une bande passante résiduelle minimale, une énergie consommée minimale,...)

Mots-clés : Algorithmes d'Approximations, Complexité, Mobile ad hoc, MultiPoint Relais, NP-complétude, Réseaux Sans Fils.

1 Introduction.

The proliferation of wireless communicating devices has created a wealth of opportunities for the field of mobile computing. In the extreme case of self-organising networks such as Mobile Ad Hoc Networks (MANET), it is still challenging to guarantee efficient communications when mobile nodes roam at will. As the topology changes arbitrarily and rapidly, the communication protocol must obtain dynamically the adequate routing.

MANET are unlike the well-studied cellular systems that rely heavily on the robust structure of the physically connected stations. They are self organising entities that must distributedly choose how to interconnect in order to facilitate the communication within the network. This feature makes them attractive but increases the difficulty of the routing. A mobile node has to cooperate with other hosts to find routes and relay messages. As mobile nodes have a limited communication range, each message may "hop" several times from node to node before reaching its destination (i.e., multihop).

The MANET working group of the Internet Engineering Task Force (IETF)[1] is currently considering several "drafts" of developed protocols. Based on networks parameters (such as the size of the network, the technical limitations, the frequency of topological changes assumed,...) they are generally categorized either as *proactive* or as *reactive*, but *hybrid* protocols exist. Proactive protocols maintain up-to-date routing information by periodically propagating updates throughout the network: e.g., Optimized Link State Routing (OLSR) and Topology Broadcast based on Reverse-Path Forwarding (TBRPF). Reactive protocols create routes only when desired by the source node: e.g., Dynamic Source Routing (DSR) and Ad Hoc On Demand Distance Vector Routing (AODV). Hybrid protocols combine both approachs: e.g., Zone Routing Protocol (ZRP) where each node proactively only maintains routes within a local region. Link state protocols are protocols were each node has sufficient knowledge about the existing links in the network in order to compute the shortest path to any remote node.

The availability of fast, cheap and off-the-shelf based equipments that use standards such as IEEE 802.11b, Bluetooth or Hiperlan-2 has opened the possibility to interconnect easily mobile wireless nodes arbitrarily. Due to its technical specificities, 802.11b is currently widely used as MAC and physical layer below the routing MANET protocols of the IETF.

The concept of Multi-Point Relays (MPRs) was first introduced in the intra-forwarding protocol in HIPERLAN type 1 standard [11]. It was successfully extended to the case of infrastructureless multihop wireless networks such as MANET. This has been effectively implemented in proactive protocols for Mobile Ad Hoc Networks such as the Optimized Link State Routing protocol (OLSR) [1, 13] where the selection of Multi-Point Relays provides efficient routing schemes. In particular, it provides shortest-path routes for unicast, minimises the flooding of broadcast messages and reduces drastically the overhead of control traffic. Recently, it was shown that the MPRs concept can also be used effectively for reactive MANET protocols [2].

[1] IETF is a large open international community of network designers, operators, vendors, and researchers concerned with the evolution of the Internet architecture and the smooth operation of the Internet. http://www.ietf.org/overview

In this paper, we consider the problem of selecting effective Multi-Point Relay nodes and we offer two variants of the original heuristic by exploiting the topological properties of the network (without assuming a knowledge of geographic positions or geometric properties). We argue that the heuristics proposed are of considerable interest when other problems are considered in addition to the routing efficiency (e.g., minimum remaining bandwidth, minimum remaining energy,...).

For completeness, we first recall some of the known complexity results in Section 2. We also introduce some notations and properties of interest for the other sections. In Sections 3 and 4, we introduce two different heuristics. For each variant, we give its respective guaranteed approximation performances compared to an optimal algorithm.

2 MPRs Selection and Complexity.

To introduce and illustrate the importance of the Multi-Point Relay concept, we briefly present its use in the OLSR protocol for MANET. We limit our presentation of OLSR to the basic algorithmic point-of-view for the approximation performances, readers interested in further details should read the latest protocol version [1]. Again, MPRs are of interest for many other applications (*e.g.,* [2, 11]).

The goal of Multi-Point Relays is to reduce the flooding of broadcast packets in the network by minimizing the duplicate retransmissions locally. Each node selects a subset of neighbors called Multi-Point Relays (MPRs) to retransmit broadcast packets. This allows neighbors which are not in the MPR set to read the message without retransmitting it, this prevents the flooding of the network (*i.e.,* the so-called *broadcast storm*). Of course, each node must select an MPR set among its neighbors that guarantees that all two-hop away nodes will get the packets, *i.e.,* all two-hop away nodes must be a neighbor of a node in the MPR set (see Figure 1, where MPR nodes are in red/grid).

In the OLSR protocol, each node periodically broadcast the information about its immediate neighbors which have selected it as an MPR. Upon receipt of this information, each node calculates and updates its routes to each destination (*i.e.,* the sequence of hops through the successive MPRs from source to destination). Notice that the neighbor discovery overhead is unchanged and its locality makes easy to implement distributedly in an efficient way. The MPR flooding still follows a simple rule: a node retransmits a broadcast packet if and only if it was received the first time from a node for which it is an MPR.

The main gain obtain by introducing MPRs is that: the smaller the MPRs set, the smaller the number of packet retransmissions.

For example, in Figure 1 only five out of the nine neighbors may retransmit the packets, and this MPR set is of minimum size.

Several important properties can be proved about this scheme [11]. In particular, the use of MPRs (instead of all the neighbours) does not destroy the connectivity properties of

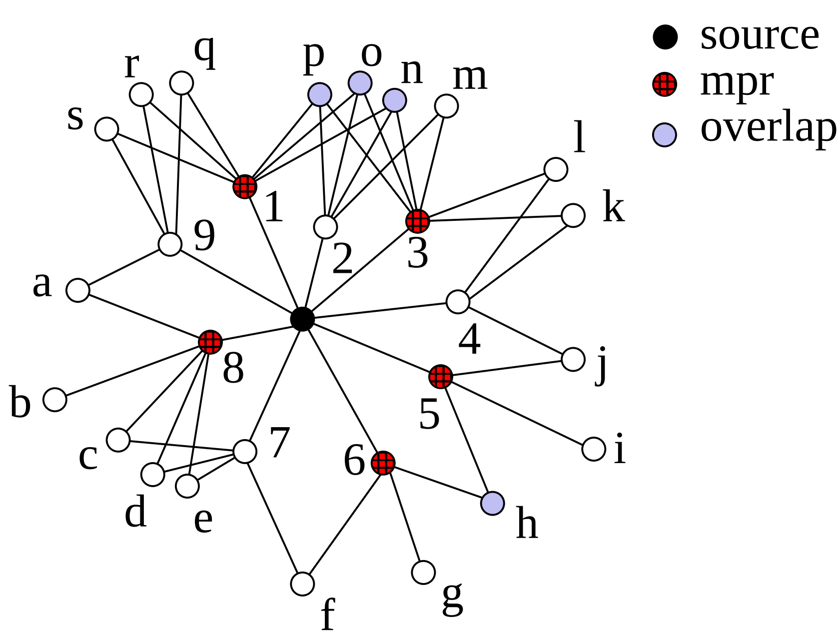

Figure 1: MPRs Selection.

the network and MPRs provide shortest-path routes for unicast with respect to the original graph.

Several polynomial-time algorithms were proposed to select an MPRs set of minimal cardinality at each node as the network topology can be arbitrary. However it was proven that the selection of a minimum size MPRs set is NP-complete (e.g., see [13, 16]) by reduction to the Minimum Domination Set problem (e.g., [9]). In fact, it is easy to see that this MPR problem is essentially the same as the Minimum Set Cover problem, well-known to be NP-complete (e.g., [9]).

After introducing our notations, we provide the alternative NP-completeness proof.

Notations. We introduce several notations. (For other basic graph-theoretical definitions we refer the reader to Diestel [7].)

Formally we can define the wireless network as a bidirectional undirected graph $G(V, E)$. Nodes i and j share a bidirectionnal link (i, j) (*i.e.,* an edge) if and only if nodes i and j hear each other and can communicate. Note that this assumption is not a convenient modeling simplification as, technically, bidirectionnal links are used to achieve unicast transmission with 802.11b.

Let $N(u)$ be the neighbors of node u. Let $N^2(u)$ denote the two-hop neighbors of u (the nodes neighbors of the neighbors of u which are not already neighbors of u). Let Δ denote the maximum degree of a node in the graph (*i.e.,* $\Delta = \max_{u \in V} |N(u)|$).

Let $d_u^+(v) = |\{w \in N(v) | v \in N(u) \text{ and } w \in N^2(u)\}|$, that is the number of neighbors of a neighbor v of u that are two-hop away from u. Conversely, let $d_u^-(w) = |\{v \in N(w) | v \in N(u) \text{ and } w \in N^2(u)\}|$, that is the number of neighbors of a two-hop neighbor w of u that are also neighbors of u. Let Δ_u^+ denote $\max_{v \in N(u)} d_u^+(v)$ and Δ_u^- denote $\max_{w \in N^2(u)} d_u^-(w)$. An example is given in Figure 2 where $\Delta_u^+ = 6$ and $\Delta_u^- = 3$.

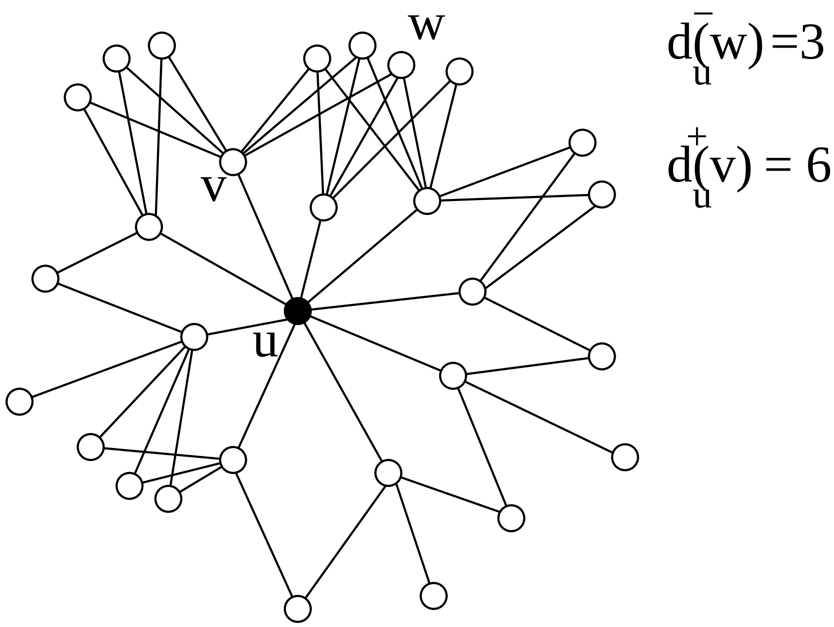

Figure 2: Out-Degree $d_u^+(v)$ and In-Degree $d_u^-(w)$.

Using our notations, an MPR set of a node u is a subset $MPR(u)$ of $N(u)$ such that:

$$\forall w \in N^2(u), \exists v \in MPR(u) \text{ such that } w \in N(v).$$

Let $MPR^*(u)$ denote an MPRs set of minimum cardinality for a node u.

Definition 1 *Minimum Multi-Point Relay (MPR) is defined as:*
Instance: *A network G (defined as a graph $G(V, E)$), a node u of $V(G)$ and an integer B.*
Question: *Is there a Multi-Point Relay $MPR(u)$ set of u of size less than B?*

We now formally define the Minimum Set Cover problem.

Definition 2 *Minimum Set Cover (SC) is defined as:*
Instance: *A Collection C of subsets of a finite set S and an integer B.*
Question: *Is there a set cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C', such that $|C'| \leq B$?*

It is now easy to observe the similarity of these two problems. Let $S = N^2(u)$. By assigning a subset in C with each node of $N(u)$, we define for each $v \in N(u)$ the subset $S_v = \{w \mid w \in N^2(u) \text{ and } w \in N(v)\}$ and the collection $C = \{\cup_{v \in N(u)} S_v\}$. The MPR problem is hence equivalent to the SC problem, as there is no real polynomial reduction required.

Two approximations results of interest are known for the SC problem:

1. The SC problem is approximable within $1 + \ln |S|$ [12]. Unfortunately, it is also known that it is not approximable within $(1 - \varepsilon) \ln |S|$ for any $\varepsilon > 0$, unless NP $\subset \text{DTIME}(n^{\log \log n})$ [8].

2. When the size of each subset of C is bounded by a constant Δ independent of the size of the input, it is approximable within $H(\Delta) = \sum_{j=1}^{\Delta}(1/j)$ (the Harmonic function) [12]. This ratio is slightly less than $1 + \ln \Delta$ (as $H(\Delta) < 1 + \ln \Delta < H(\Delta) + \frac{1}{2}$).

Such bounds are achievable by running an algorithm that follows a simple degree-greedy strategy algorithm. In particular, the $H(\Delta)$ approximation ratio is tight as there exist graphs (with sufficiently large $|S|$) for which such an algorithm will attain such ratio [12].

Such bounds are achieved by the MPR original algorithm [13, 16] currently used in the OLSR protocol implementation. Using our notations, we recall this algorithm in Figure 3 where $MPR^+(u)$ denotes the MPRs set returned by the algorithm.

Except for the initial phase, this algorithm is similar to the one presented in [12]. The first phase is added to adequately use the fact that, whatever the strategy chosen, one-hop nodes that are the sole "cover" of two-hop nodes must be included in the MPRs set. For example, in Figure 1, nodes b, g and i have only one neighbor among the neighbors of the source; hence these respective neighbors - nodes 5, 6 and 8 - must be included in the MPR set. This slight modification yields better-in-practice solutions without weakening the approximation bound [13, 16].

In the OLSR wireless context, it implies that there exist a heuristic that selects an MPR set with the respective performance approximation ratio $MPR^+(u)/MPR^*(u)$:

1. $1 + \ln |S| = 1 + \ln |N^2(u)|$ in the first case and,

2. $1 + \ln \Delta_u^+$ in the second case, when Δ_u^+ (the maximum number of two-hop nodes that each one-hop neighbors of u may cover) is bounded by a constant independent of the size of the network.

It should be noted that it makes sense to assume that, whatever the wireless technology used, the number of nodes each node can "communicate" with is upper bounded by a constant independent of the size of the input. Although the maximum degree of a wireless node can

```
MPR_OLSR Selection(u)
```

$MPR^+(u) = \emptyset$
$S = N^2(u)$
$M = N(u)$
for all $v \in N(u)$ do $d(v) = d_u^+(v)$ endo
for all $w \in N^2(u)$ do $d(w) = d_u^-(w)$ endo

for all $w \in N^2(u)$ with $d(w) = 1$ do
    $S = S - \{w\}$
    $MPR^+(u) = MPR^+(u) \cup \{v | v \in N(u)$ and $v \in N(w)\}$
    $M = M - \{v\}$
    for all w' such that $w' \in N^2(u)$ and $w' \in N(v)$ do
        $S = S - \{w'\}$
        for all v' such that $v' \in N(u)$ and $v' \in N(w')$ do
            $d(v') = d(v') - 1$
        endo
    endo
endo

While $S \neq \emptyset$ do
    pick $v \in M$ with $d(v) = \max_{v' \in M} d(v')$
    $MPR^+(u) = MPR^+(u) \cup \{v\}$
    $M = M - \{v\}$
    for all w' such that $w' \in N^2(u)$ and $w' \in N(v)$ do
        $S = S - \{w'\}$
        for all v' such that $v' \in N(u)$ and $v' \in N(w')$ do
            $d(v') = d(v') - 1$
        endo
    endo
endo

Figure 3: MPR Selection Heuristic used in OLSR: MPR^+.

be fairly large (*i.e.,* with the current technology, Δ possibly reaching theoretically thousands and few dozens in practice), the approximation factors will remain small ($1 + \ln \Delta$ will reach 8 or 3, respectively). However, the important fact is that this value will remain a constant independent of the size of the network.

We will exploit such an important topological property in our heuristics in the next sections.

3 Alternate Bounded Degree MPR Selections.

In addition to routing, mobile networking differs substantially from traditional networking in many areas such as: scalability (the number of mobile nodes may be huge and continuously varies as some appear or disappear at any time); connectivity (availability and bandwidth); security (increase in physical threats); autonomy (limited power); collisions. For these reasons, it is important to develop routing algorithms that take as much consideration as possible of such difficulties.

It is clear now that each of these difficulties, even singled out, generate NP-hard problems in wireless environments, while they are polynomially-solvable in wired networks (*e.g.,* bandwidth reservation for Quality of Service [10], minimum-energy broadcast [4],...).

The simpler heuristic to solve several problems at once is to consider them one after the other. For example, in the last phase of the algorithm for the MPR selection: while picking nodes to be included in the MPR set, one may choose to distinguish nodes with the same (remaining) degree by adding a secondary criteria (*e.g.,* , maximum available bandwidth, minimum energy consumed,...). However it is clear that this method explicitly gives a priority to the routing problem compared to the other problem at hand.

One must keep in mind that we are concerned with wireless networks and these often offer some particular topologies as, due to the radio nature of their communications, they are often depend on specific physical constraints (to guarantee connectedness for example).

In this section, we exploit properties that are widely available for any wireless networks, such as the maximum degree of each node. We distinguish each heuristic methodologies and its respective complexity. One of most common model used to represent such specific topologies is the geometric model and its different variants, *e.g.,* unit-disk graphs. We also make a simple remark in respect to this last model in subsection 3.3.

3.1 Weighted set cover

A popular method to tackle several problems at once is to combine all desired components into weights that represents the trade-offs that seem appropriate. For example, to minimise the occupied bandwidth at each node at any time while maintaining relatively short paths, the MPRs selection can be done by assigning weights $w(i)$ at each node i that correspond to the remaining bandwidth available at each node. In this case, a variant of the original MPR algorithm presented in Figure 3 could be to modify the last phase to pick the node v which maximized its cover of nodes with remaining bandwidth available.

The weighted version of the MPR^+ problem is also equivalent to the weighted version of the SC problem where the objective is to minimize the sum of the weights in a set cover. It suffices to change the maximum criteria to $\frac{d_i}{w_i}$ and to update the weight in each node accordingly. It is also approximable within $1 + \ln|S|$ and within $1 + \ln|\Delta|$ when the size of each subset of C is bounded by a constant Δ independent of the size of the input [6].

It is important to note that, in both cases, the approximation ratio is not related to the weight function used. In the wireless case at hand, this means that the approximation ratio is only related to $N^2(u)$ and Δ_u^+ (the maximum degree d^+ of the neighbors of the source towards the two-hop nodes).

3.2 In-degree greedy set cover

The original MPR selection complexity mainly depends of the out-degree of the neighbour nodes v of the source u, *i.e.,* the maximum possible value of $d_u^+(v)$, if this value is bounded. Due to their intrinsic connectedness, wireless networks may be highly clustered. In this case, one may observe that conversely, Δ_u^-, the maximum value of the in-degree $d_u^-(w)$ of the two-hop nodes $w \in N^2(u)$, is likely to be an even smaller constant.

This observation allows us to introduce a variant of the original MPR selection heuristic in Figure 4 where $MPR^-(u)$ denotes the MPRs set returned by the algorithm. We highlighted the changes compared with the original algorithm by numbering the lines modified or introduced.

We give the approximation performance of this new algorithm.

Theorem 3 *When Δ_u^- is bounded by a constant independent of the size of the input, the MPR Selection algorithm using in-degree presented in Figure 4 guarantees an approximation ratio to the optimal size of the MPR set of Δ_u^- for a source node u.*

Proof. The algorithm introduced here follows the same line as the algorithm presented in [3], hence the same bound and the same proof apply. As for OLSR, the major difference is the introduction of an initial phase that includes immediately the one-hop nodes that are the sole neighbor of a two-hop node to the optimal size of the EC MPR set of neighbor into the MPR set. This yields better-in-practice solutions without weakening the approximation bound. ■

A weighted version of this algorithm where each node i is assigned a weight $weight(i)$ is also described for completeness in Figure 5. We highlighted the changes compared with the original algorithm by numbering the lines modified or introduced. As described in our algorithm it is possible to make the weight assigned to each node a specific value depending of the source u. One must keep in mind that the MPR selection must be implemented distributedly and only local information, within the two-hop vicinity, of node u may be available.

Again this algorithm has the same approximation ratio which is independent of the weight function used. Its proof follows the same lines than in [3].

Theorem 4 *When Δ_u^- is bounded by a constant independent of the size of the input, the MPR Selection Algorithm using weighted in-degree presented in Figure 4 guarantees an approximation ratio to the optimal weight of the MPR set of Δ_u^- for a source node u.*

It is clear that if Δ_u^- is large or larger than $1 + \ln \Delta_u^+$, this algorithm is inferior to the original MPR algorithm implemented in the OLSR protocol. However, as both heuristics have a small polynomial complexity, it is reasonable to run both in order to choose the best MPR set.

```
MPR_InDegree Selection(u)

    MPR^-(u) = ∅
    S = N^2(u)
    M = N(u)
    for all v ∈ N(u) do d(v) = d_u^+(v) endo
    for all w ∈ N^2(u) do d(w) = d_u^-(w) endo

    for all w ∈ N^2(u) with d(w) = 1 do
        S = S − {w}
        MPR^-(u) = MPR^-(u) ∪ {v|v ∈ N(u) and v ∈ N(w)}
        M = M − {v}
        for all w' such that w' ∈ N^2(u) and w' ∈ N(v) do
            S = S − {w'}
            for all v' such that v' ∈ N(u) and v' ∈ N(w') do
                d(v') = d(v') − 1
            endo
        endo
    endo

    While S ≠ ∅ do
1       pick w ∈ S
2       pick v ∈ M such that v ∈ N(w) and d(v) = min_{v'∈M} d(v')
        MPR^-(u) = MPR^-(u) ∪ {v}
        M = M − {v}
        for all w' such that w' ∈ N^2(u) and w' ∈ N(v) do
            S = S − {w'}
            for all v' such that v' ∈ N(u) and v' ∈ N(w') do
                d(v') = d(v') − 1
            endo
        endo
    endo
```

Figure 4: MPR Selection Algorithm using in-degree: MPR^-.

`MPR_WeightedInDegree Selection(u)`

$MPR^-(u) = \emptyset$
$S = N^2(u)$
$M = N(u)$
for all $v \in N(u)$ do $d(v) = d_u^+(v)$ endo
1 for all i do $rweight(i) = weight_u(i)$ endo
for all $w \in N^2(u)$ do $d(w) = d_u^-(w)$ endo

for all $w \in N^2(u)$ with $d(w) = 1$ do
    $S = S - \{w\}$
    $MPR^-(u) = MPR^-(u) \cup \{v | v \in N(u) \text{ and } v \in N(w)\}$
    $M = M - \{v\}$
    for all w' such that $w' \in N^2(u)$ and $w' \in N(v)$ do
        $S = S - \{w'\}$
        for all v' such that $v' \in N(u)$ and $v' \in N(w')$ do
            $d(v') = d(v') - 1$
2           $rweight(v') = rweight(v') - rweight_{(}w')$
        endo
    endo
endo

While $S \neq \emptyset$ do
    pick $w \in S$
3   pick $v \in M$ such that $v \in N(w)$ and $rweight_u(v) = \min_{v' \in M} rweight_u(v')$
    $MPR^-(u) = MPR^-(u) \cup \{v\}$
    $M = M - \{v\}$
    for all w' such that $w' \in N^2(u)$ and $w' \in N(v)$ do
        $S = S - \{w'\}$
        for all v' such that $v' \in N(u)$ and $v' \in N(w')$ do
            $d(v') = d(v') - 1$
4           $rweight_u(v') = rweight_u(v') - rweight_u(v)$
        endo
    endo
endo

Figure 5: MPR Selection Algorithm using weighted in-degree: MPR_w^-.

Corollary 5 *When* Δ_u^- *and* Δ_u^+ *are bounded by constants independent of the size of the input, an MPR Selection Algorithm using weights for a source node u guarantees an approximation ratio to the optimal weight of the MPR set of u of:* $\min(\Delta_u^-, 1 + \ln \Delta_u^+)$.

In section 4, we consider alternate solutions that also take advantage of the limited degree of nodes two-hop away from the source.

3.3 Geometric Model

In this paper, we do not consider a knowledge of geographic positions or geometric properties such as a unit-disk graph model (*i.e.,* where an edge exists between two nodes if and only of their euclidean distance is less or equal to a given constant). Polynomial-time algorithms that achieve a constant approximation-factor exist for such models (*e.g.,* [5]). However, we remark that, in such a model, the original MPR^+ selection algorithm is likely to perform better than the MPR selection using in-degree. Indeed, by comparing Δ_u^+ and Δ_u^- (as defined in Corollary 5) in the unit-disk graph, it is easy to see that the area covered by a neighbor of the source can be less than twice the area that a two-hop node may cover within the neighboring of the source.

Such an extreme case occurs when a two-hop node is as close as possible from the source's range as depicted in Figure 6 where the source node u is in the top unit-disk and a two-hop node w is in the bottom unit-disk (and the A region). On one hand, the maximum area including the source's neighbors that w can cover corresponds to the region B represented by vertical lines, and the number of nodes in the area B will define the maximum in-degree $d_u^-(w)$ of a two-hop nodes. On the other hand, a source's neighbor may cover a region as large as the A region (the grey region), and the number of nodes in the area A will define the maximum out-degree $d_u^+(v)$ of a neighbor v of the source in $N^2(u)$. Let us define D as a whole unit-disk area. Note that $A + B$ covers exactly a whole unit-disk area D. By defining in the figure an area C (represented by the area with horizontal lines) and standard geometry, it is easy to see that $D = 3(B - C)$ and $A = 2B - 3C$. Thus, in the extreme case $B > \frac{1}{2}A$, the minimum ratio in $\min(\Delta_u^-, 1 + \ln \Delta_u^+)$ is likely to be $(1 + \ln \Delta_u^+)$ if nodes are uniformly distributed.

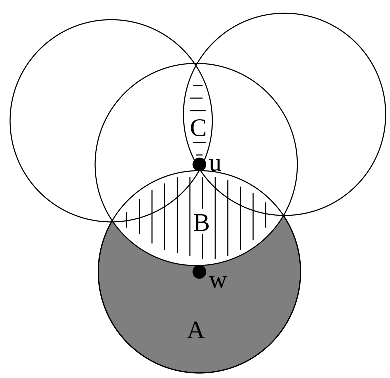

Figure 6: Maximum cover for two-hop nodes.

4 MPR Selection with Minimum Overlapping.

The quality of the weighted solutions obtained by the previous algorithms will depend considerably on the adequacy of the weight function used to combined the required features for each problems. A trade-off compromising too much for each problem may generate inadequate solutions for all problems. Assigning weights that genuinely represent the trade-off of the combined problems to solve is not always easy. In particular, when considering two problems, one objective is to be maximised whilst the other is to be minimised.

Another possibility is to modify the MPRs selection objective altogether by considering the "quality" of the solution instead of its simplest "quantity". Indeed, as defined the MPR problem aims at reducing the cardinality of MPR set without considering its topology. The topology of the MPR set, *i.e.,* the way the MPR nodes are positioned from one another will have a real impact on the behaviour of the routing while considering other problems, such as collision avoidance or available remaining bandwidth.

For example, the principal role of the MPRs during the broadcast phase is to forward packets effectively with reduced duplication in order to limit the traffic and risk of collisions. In Figure 1, the size of MPRs set is minimum, however nodes 1 and 3 both cover nodes n, o and p. Such overlap will be detrimental to some applications as knowns problems such as the so-called *Hidden Terminal* may occur and packets may be lost.

We are proposing that a desired feature of the MPR set should be that the MPR nodes are as evenly spread as possible around the source to reduce the overlapping of cover by MPR nodes.

Ideally, one would like to obtain small MPR set for which MPR nodes "cover" disjoint sets of two-hop away nodes. Of course, this may be impossible to achieve if two neighbors of the source are each the sole cover of a respective two-hop node and overlap in their covers. For example, in Figure 1, nodes 4 and 5 both cover node h but both need to be included in the MPR set as they both are sole neighbor of a two-hop node, nodes g and i respectively. In this case, the overlap cannot be avoided and the two nodes must be included in the MPR set.

This unfortunate case may involve an arbitrarily large number of nodes (up to the maximum degree of the nodes). Hence, it is pointless to expect to reduce the maximum number of overlapping per node. However, it is possible to limit the impact of the overall overlapping according to a given source, *i.e.,* the least overall overlapping of the MPR set. Using the same topology as Figure 1, one could choose a MPR set that reduces the number of nodes that "overlap" from 4 to 3, by increasing slightly the size of the MPR set (see Figure 7).

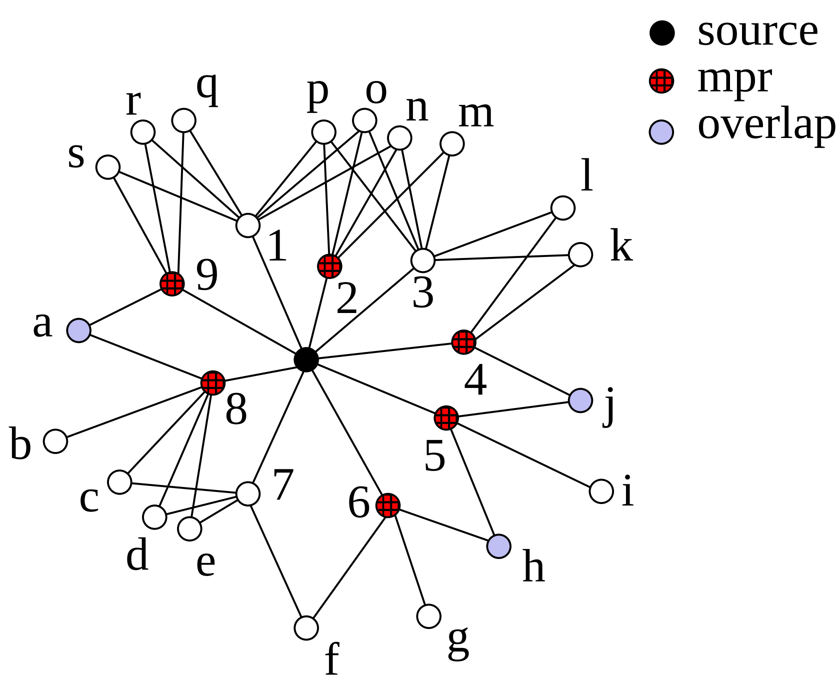

Figure 7: MPRs Selection with minimum overlapping.

This minimization problem, known as the Minimum Exact Cover, is NP-complete and is defined formally as follows.

Definition 6 *Minimum Exact Cover (EC) is defined as:*
Instance: *A Collection C of subsets of a finite set S and an integer B.*
Question: *Is there an exact cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C', such that*

$$\sum_{c \in C'} |c| \leq B?$$

This is approximable within $1 + \ln |S|$ [12] but as hard to approximate as SC [14]. The only difference between SC and EC is the definition of the objective function.

Again the same approximation ratio is achievable by the weighted version of this problem (where the objective is to minimize the sum of the weights in the set cover and where the weights are counted as many time as that they are covered).

Our proposal is not a slight formal modification. Although related, the two problems EC and SC may diverge. The best approximation heuristic known for SC (as is the MPR selection algorithm presented in Figure 3) may obtain an optimal solution for a given instance of the SC problem while obtaining an extremely poor solution for the same given instance for the EC problem: the approximation factor could be as bad as $O(|S|)$.

Fortunately a heuristic with good approximation ratio can be designed. The idea of the algorithm is to limit the overlapping by greedily selecting nodes that minimize their respective ratio of the number of already covered nodes over the number of uncovered nodes. In fact, besides the greedy strategy, the algorithm is similar to the MPR selection algorithm presented in Figure 3).

For completeness, we present the specific heuristic for the MPRs selection with reduced overlap in Figure 8 where $d^u(v)$ and $d^c(v)$ represent the current numbers of two-hop nodes uncovered and covered respectively. We highlighted the changes compared with the original algorithm by numbering the lines modified or introduced.

Following the lines of the approximation bounds known for the EC problem introduced by Johnson [12], we can immediately deduced the two following theorems.

Theorem 7 *The MPR Selection Algorithm for limited overlapping presented in Figure 8 guarantees an approximation ratio to the optimal size of the EC MPR set of* $1 + \ln |N^2(u)|$ *for a source node u.*

Theorem 8 *When* Δ_u^+ *is bounded by a constant independent of the size of the input, the MPR Selection Algorithm for limited cover overlapping presented in Figure 8 guarantees an approximation ratio of* $1 + \ln \Delta_u^+$ *for a source node u.*

Proof. The algorithm introduced here follows the same line as the algorithm presented in Section 6 in [12], hence the same bound and the same proof apply. As for OLSR, the major difference is the introduction of an initial phase that includes immediately the one-hop nodes that are the sole neighbor of a two-hop neighbor into the MPR set. This yields better-in-practice solutions without weakening the approximation bound. ■

Similarly, in each case, it is known that one can build a graph for which such a bound is attainable [12].

`MPR_EC Selection(u)`

$MPR(u) = \emptyset$
$S = N^2(u)$
$M = N(u)$
1 for all $v \in N(u)$ do $d^u(v) = d_u^+(v)$ endo
2 for all $v \in N(u)$ do $d^c(v) = 0$ endo
for all $w \in N^2(u)$ do $d(w) = d_u^-(w)$ endo

for all $w \in N^2(u)$ with $d(w) = 1$ do
  $S = S - \{w\}$
  $MPR(u) = MPR(u) \cup \{v | v \in N(u) \text{ and } v \in N(w)\}$
  $M = M - \{v\}$
  for all w' such that $w' \in N^2(u)$ and $w' \in N(v)$ do
    $S = S - \{w'\}$
    for all v' such that $v' \in N(u)$ and $v' \in N(w')$ do
3     $d^c(v') = d^c(v') + 1$
4     $d^u(v') = d^u(v') - 1$
    endo
  endo
endo

While $S \neq \emptyset$ do
5   pick $v \in M$ with $\frac{d^c(v)}{d^u(v)} = \min_{v' \in M} \frac{d^c(v')}{d^u(v')}$
  $MPR(u) = MPR(u) \cup \{v\}$
  $M = M - \{v\}$
  for all w' such that $w' \in N^2(u)$ and $w' \in N(v)$ do
    $S = S - \{w'\}$
    for all v' such that $v' \in N(u)$ and $v' \in N(w')$ do
6     $d_c(v') = d^c(v') + 1$
7     $d_u(v') = d^u(v') - 1$
    endo
  endo
endo

Figure 8: MPR Selection Algorithm for limited overlapping.

5 Conclusion.

In this paper, we have recalled some important properties of the known approximation algorithms for computing a MPR set of minimum cardinality (when no knowledge of the geographic locations or of geometric properties are assumed). We have introduced two variants that may be of interest when other optimizations are considered concurrently. All these variants extend easily to a weighted version and have provable approximation performances that solely depend on the maximum degree of the nodes in the network.

Experimental simulations are currently in progress to characterise the performances of each algorithm in usual MANET contexts.

References

[1] C. Adjih, T. Clausen, P. Jacquet, A. Laouiti, P. Minet, P. Mühlethaler, A. Qayyum, L. Viennot et al., *Optimized Link State Routing Protocol*, http://www.ietf.org/html.charters/manet-charter.html

[2] G. Allard, P. Jacquet and L. Viennot, *Ad hoc routing protocols with multipoint relaying*, 5eme Rencontres Francophones sur les aspects Algorithmiques des Telecommunications, (AlgoTel'2003), Banyuls-sur-mer, France, http://dept-info.labri.u-bordeaux.fr/algotel03/

[3] R. Bar-Yehuda, and S. Even, *A linear-time approximation algorithm for the weighted vertex cover problem*, Journal of Algorithms, 2 (1981), pp. 198–203.

[4] M. Cagalj, J.-P. Hubaux and C. Enz, *Minimum-energy broadcast in all-wireless networks: NP-completeness and distribution issues*, Proceedings of the Eighth ACM International Conference on Mobile Networking and Computing, Atlanta, September 2002 (Mobicom 2002).

[5] G. Calinescu, I. Mandoiu, P.-J. Wan and A. Zelikovsky, *Selecting Forwarding Neighbors in Wireless Ad Hoc Networks*, Proceedings of the Fifth International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications (DIALM 2001), ACM Press, pp. 34–43.

[6] V. Chvátal, *A greedy-heuristic for the set-covering problem*, Mathematics of Operations Research, 4(3) (1979), pp. 233–235.

[7] R. Diestel, *Graph Theory*, Springer-Verlag, New York, 2000.

[8] U. Feige, *A threshold of* $\ln n$ *for approximating set cover*, Journal of the ACM, 45 (1998), pp. 634–652.

[9] M.R. Garey and D.S. Johnson, *Computers and Intractability*, W. H. Freeman and Company, San Francisco, California, 1979.

[10] L. Georgiadis, P. Jacquet and B. Mans, *Bandwidth Reservation in Multihop Wireless Networks: Complexity and Mechanisms*, INRIA-RR-4876, July 2003.

[11] P. Jacquet, P. Minet, P. Mühlethaler and N. Rivierre, *Increasing Reliability in cable-free radio lans: low level forwarding in HiperLan*, Wireless Personal Communications, 4(1) (1997), pp. 65–80.

[12] D. S. Johnson, *Approximation algorithms for combinatorial problems*, Journal of Computer and System Sciences, 9 (1974), pp. 256–278.

[13] A. Laouiti, A. Qayyum and L. Viennot, *Multipoint Relaying: An Efficient Technique for Flooding in Mobile Wireless Networks*, 35th Annual Hawaii International Conference on System Sciences (HICSS'2001).

[14] C. Lund and M. Yannakakis, *On the hardness of approximating minimization problems*, Journal of the ACM, 41 (1994), pp. 960–981.

[15] C. H. Papadimitriou and M. Yannakakis, *Optimization, approximation and complexity classes*, Journal of Computer and System Sciences, 43(3) (1991), pp. 425–440.

[16] L. Viennot, *Complexity Results on Election of Multipoint Relays in Wireless Networks*, INRIA-RR-3584, 1998.

[17] P.-J. Wan, G. Calinescu, X.-Y. Li and O. Frieder, *Minimum-Energy Broadcasting in Static Ad Hoc Wireless Networks* Wireless Networks, 8(6) (2002), pp. 607–617.

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ISSN 0249-6399