Traffic Assignment and Gibbs-Maslov Semirings

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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N° 4809

April 2003

THÈME 4

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--4809--FR+ENG

INRIA ROCQUENCOURT

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Thème 4 — Simulation et optimisation
de systèmes complexes
Projet Metalau

Rapport de recherche n° 4809 — April 2003 — 13 pages

Abstract: The *Traffic Assignment* problem consists in determining the routes used by sets of network users taking into account the link congestions. In deterministic modelling, *Wardrop Equilibriums* are computed. They can be reduced to huge non-linear multiflow problems in the simplest cases. In stochastic modelling, *Logit Assignments* are used. They are obtained, mainly, by substituting the minplus semiring by the "Gibbs-Maslov semirings"

$$x \oplus^{\mu} y = -\frac{1}{\mu}\log(e^{-\mu x} + e^{-\mu y}) \ , \ \ x \otimes y = x + y \ ,$$

in the deterministic assignment computations.

Key-words: semiring, traffic assignment, Wardrop equilibrium, maxplus algebra, quantization, logit

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Affectation de trafic & semi-anneau de Gibbs-Maslov

Résumé : Les problèmes d'affectation de trafic consistent à déterminer les routes suivies par des usagers d'un réseau en tenant compte de la congestion des routes utilisées. Dans le cas déterministe des équilibres de Wardrop sont calculés. Ils se ramènent à la résolution de gros probèmes multiflots non linéaires. Dans le cas stochastique les affectations logit sont utilisées. Elles sont obtenues, essentiellement, en remplaçant le semi anneau minplus par le semi anneau de Gibbs-Maslov

$$x \oplus^{\mu} y = -\frac{1}{\mu}\log(e^{-\mu x} + e^{-\mu y}) \ , \ \ x \otimes y = x + y \ ,$$

dans le calcul des affectations déterministes.

Mots-clés : semi anneau, affectation de trafic, équilibre de Wardrop, algèbre maxplus, quantification, logit

1 Introduction

Given a transportation network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ and a set $\mathcal{D}$ of transportation demands from the origin $o \in \mathcal{N}$ to the destination $d \in \mathcal{N}$ the *traffic assignment* problem consists in determining the flows f_a on the arcs $a \in \mathcal{A}$ of the network when the times t_a spent on the arcs a are given functions of the flows f_a.

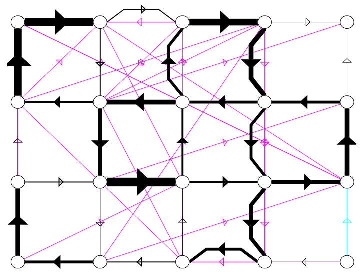

Figure 1: A Regular Small Town.

We can distinguish the deterministic case — when all the travel time are known by the users — from the stochastic cases — when the users perceive a travel time different from the actual one —. When the distribution of the error between the perceived time and the actual time satisfies a Gumbel distribution the probability that a user choose a particular route can be computed explicitly, it is a Gibbs distribution called logit in transport literature. From this distribution the arc flows can be computed using a matrix calculus which can be seen as the analogue of the smallest path computation up to the substitution of the minplus semiring by the Gibbs-Maslov semiring [15, 17]. We call Gibbs-Maslov semiring the real numbers endowed with the two following operations :

$$x \oplus^{\mu} y = -\frac{1}{\mu}\log(e^{-\mu x} + e^{-\mu y}) \ , \quad x \otimes y = x + y \ .$$

To compute effectively the flow it is useful to restrict the computation to the efficient paths [8] — which are the paths moving away from the origin and getting closer to the destination —. It is also possible, following [3], to make the computation in the general case — when we consider all the paths — but with some restrictions on the variance of the error distribution which must be small enough. We propose also to consider the case when the users choose at each crossing between routes made up of any road leaving this crossing followed by an optimal road starting at the following junction. Choosing among this set of routes by the logit method defines another stochastic assignment which does not require any restriction on the variance of the Gumbel distribution.

Based on these assignments, equilibriums can be computed when the actual travel time depends on the congestion seen as a deterministic quantity deduced from the Gibbs distribution to choose a route.

All these methods are implemented in the toolbox — CiudadSim — of Scilab dedicated to traffic assignments.

2 User Equilibrium (Wardrop)

In the deterministic case, each user minimises the time spent in the network — which is supposed to be knowknown perfectly —. Based on this hypothesis an equilibrium is achieved called a Wardrop equilibrium. When this equilibrium is reached we have : *for all pair of nodes o and d, the travel times spent on all the actually used routes from o to d are the same and are smaller than the travel times of the unused routes* that is :

$$\min\{f_r, t_r - t^*_{od}\} = 0, \ \ \forall r \in R_{od}, \ \ \sum_{r\in R_{od}} f_r = d_{od} \ , \ \ \forall od \in \mathcal{C} \ , \tag{1}$$

where :

- $\mathcal{C}$ is the set of origin-destination pairs,
- R_{od} is the set of routes from o to d,
- d_{od} is the demand from o to d,
- f_r is the route flow on the route $r \in R_{od}$,
- t_r the travel time on the route $r \in R_{od}$,
- t^*_{od} the smallest travel time of the routes in R_{od}.

The Wardrop system of equations (1) is equivalent to the optimality conditions of the *nodes-arcs optimisation* problem :

$$\min_f \sum_a c_a(F_a), \ \ c_a(F_a) = \int_0^{F_a} t_a(q)\, dq, \ \ Af^{\bullet}_{od} = d_{od}, \ \ f \geq 0 \ .$$

where

- A is the $m \times n$ incidence matrix nodes-arcs associated to the network,
- f^a_{od} is the flow on the arc a of the commodity od,
- $F_a = \sum_{od} f^a_{od}$ is the total flow on the arc a,
- $t_a(q)$ denotes the time spent on the arc$a$ if the total flow on this arc is q.

The Wardrop system of equations is also the optimality conditions of the *arcs-routes optimisation* problem :

$$\min_f \sum_a c_a(F_a) \ , \ \ F_a = \sum_{r\in R^a} f_r \ , \ \ \sum_{r\in R_{od}} f_r = d_{od} \ , \ \ f \geq 0 \ ,$$

where R^a denotes the set of all the routes of the network using the arc a.

3 Algorithms

Let us give the most used first order algorithms and a very efficient second order one.

All or Nothing (AON). When the travel times do not depend on the link flows, for each pair od in $\mathcal{C}$ it is optimal to assign the flow to the smallest travel time route(s).

Frank-Wolfe algorithm (FW). The current assignment is improved according to :

- given the flows F_a, the travel times $t_a(F_a)$ are computed and based on these travel times a new AON assignment F' is computed;
- then, the new FW assignment becomes $(1-\mu)F + \mu F'$ with

$$\mu \in \arg\min_{1\geq\lambda\geq 0} \sum_a c_a \left(\lambda F_a + (1-\lambda)F'_a\right) .$$

Method of successive average (MSA). The algorithm is the same as the FW up to the following change of the value of μ which becomes $\mu = 1/n$, where n is the iteration number.

Disaggregated Simplicial Decomposition (DSD) This algorithm has been proposed by Larsson-Patriksson[18].

- At starting time, all the demand of a commodity is assigned to only one route by an AON algorithm.
- At each iteration, for all commodities, the shortest route (based on the current flow) is added to a set of memorised routes.
- The improved assignment is obtained by optimising the flow on the memorised routes by a diagonalised Newton method.

In Figure 2 we show a comparison of the speeds of the three algorithms. The plot gives the logarithm of an upper bound of the error as a function of the logarithm of the computation time for an average size (64 nodes, ~110 arcs, ~4000 commodities) regular network.

We remark that FW algorithm and MSA algorithm have almost the same speed. The DSD algorithm becomes faster when we want a good precision (smaller than 1.e-3). From this experiment, it appears that the MSA algorithm is a simple quite good algorithm when we do not need a good precision — which is always the case for transportation traffic where the data is not very accurate —.

4 Stochastic Assignment

The main drawback with the Wardrop equilibrium point of view is that each traveller is supposed to have a perfect information on the whole network. In more realistic formulations

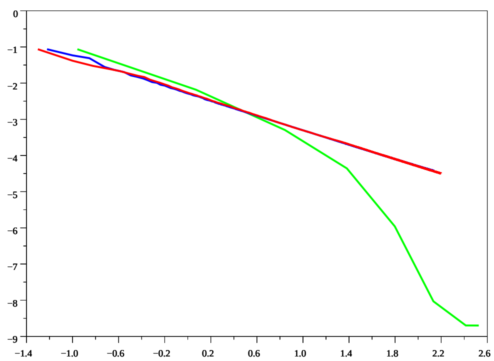

Figure 2: Precision as a function of time for FW algorithm (red), MSA algorithm(blue) and DSD algorithm (green).

the user minimises his perceived travel time. This perceived travel time is defined as the sum of the effective travel time and a random error :

$$T_a(f_a, \omega) = t_a(f_a) + E_a(\omega), \; \forall a \in \mathcal{A} ,$$

where f_a — the flow on the arc a — is considered as a macroscopic deterministic variable.

When the probabilistic distribution of the errors is known we can define stochastic equilibriums. In the *Probit Assignment* problem the errors E_a are supposed to be centred Gaussian random variables. But, because $\mathcal{N}\{E_a > e\}$ for $\mathcal{N}$ a Gaussian law, is not known explicitly the computation of probit assignment is difficult and is done by Monte Carlo methods.

When the error on the perceived times of the routes (not the arcs) are independent centred and Gumbel [1] distributed with the same variance, the probability to choose a route $r \in R_{od}$ can be computed explicitly. It is given by

$$\mathbb{P}\{r \in R_{od}\} = \frac{e^{-\mu t_r}}{\sum_{r \in R_{od}} e^{-\mu t_r}} , \tag{2}$$

where t_r denotes the time spent on the route r. This assignment is called *Logit Assignment.*

The only interest of the Gumbel distribution comes from the facility to compute the probability of the maximum of two independent random variables and from its shape, close to the normal distribution.

[1] $\mathcal{G}\{E_r < x\} = e^{-e^{-\mu x - \eta}}$ where η is the Euler constant, the variance of $\mathcal{G}$ is $\pi^2/(6\mu^2)$. The max of independent Gumbel random variable with the same variances is still Gumble with the same variance.

5 Gibbs-Maslov Semirings

For all μ, we denote by $\mathbb{R}_\mu$ the set $\mathbb{R} \cup \{+\infty\}$ endowed with the two operations $\oplus^\mu$ and $\otimes$ defined by

$$x \oplus^\mu y = -\frac{1}{\mu} \log(e^{-\mu x} + e^{-\mu y}) , \quad x \otimes y = x + y .$$

It is a semiring with zero element $\varepsilon = +\infty$ and unity $e = 0$ that we call Gibbs-Maslov semiring. Then, denoting $\mathbb{R}_\infty = \lim_{\mu \to +\infty} \mathbb{R}_\mu$, we have $\mathbb{R}_\infty = \mathbb{R}_{\min}$ with $\oplus^\infty = \min$.

We can endow a finite set $\Omega = \{\omega_1, \cdots, \omega_n\}$ with a normalised cost function $c : \Omega \mapsto \mathbb{R}_\mu$ with $c(\Omega) = \bigoplus_i^\mu c(\omega_i) = e$. Then we have $\sum_i e^{-\mu c(\omega_i)} = 1$ and $e^{-\mu c(\omega_i)}$ is a Gibbs probability distribution.

This remark shows that to a Gibbs distribution on a finite set we can associate a linear structure with its matrix calculus. For matrices A and B we have $A \otimes^\mu B = -1/\mu \log(e^A e^B)$, where log and exp have to be applied element-wise.

To a $n \times n$ matrix C in $\mathcal{M}_n(\mathcal{K})$ with entries in the semiring $\mathcal{K}$, we associate a *precedence graph* $\mathcal{G}(C) = (\mathcal{N}, \mathcal{A})$ with nodes $\mathcal{N} = \{1, 2, \cdots, n\}$, and arcs $\mathcal{A} = \{a = pq \mid p, q \in \mathcal{N},\ C_{pq} \neq \varepsilon\}$.

The *weight* of a path π, denoted $\pi(C)$, is the $\otimes$-product of the weights of its arcs. For example we have $pqs(C) = C_{pq} \otimes C_{qs}$.

The set of all the paths with ends od and length l is denoted $\mathcal{R}^l_{od}$. Then, $\mathcal{R}^*_{od}$ is the set of the all paths with ends od and $\mathcal{R}^*$ the set of all the paths.

$$\mathcal{R}^* \stackrel{\text{def}}{=} \bigcup_{l=0}^{\infty} \mathcal{R}^l, \ \ \rho \subset \mathcal{R}^*, \ \ \rho(C) \stackrel{\text{def}}{=} \bigoplus_{\pi \in \rho} \pi(C) .$$

We define the *star operation* by $C^* \stackrel{\text{def}}{=} \bigoplus_{i=0}^\infty C^i$.
We have $\mathcal{R}^l_{od}(C) = C^l_{od}$, $\mathcal{R}^*_{od}(C) = C^*_{od}$.

6 Dial Logit Assignment

To compute a logit assignment when the average time does not depend on the flow, Dial [8] suggests to consider only *efficient paths* $r = op_1p_2 \cdots d$ that are paths such that the travel times to the destination $t_{p_i \cdots d}$ are decreasing with i and travel time from the origin $t_{o \cdots p_i}$ are increasing with i.

Then denoting : – $\mathcal{G}^{od} = (\mathcal{N}, \mathcal{A}^{od})$ the new graph obtained by eliminating the arcs preventing the increasing and decreasing properties from happening, – $\mathcal{O}^{od}$ [resp. $\mathcal{D}^{od}$] the correspondent incidence nodes arc-origins [resp. nodes arcs-destination] matrix, we can define the *transition weight matrix :*

$$W^{od} = \mathcal{O}^{od} T (\mathcal{D}^{od})' , \ \ T = \operatorname{diag}(e^{-\mu t_a}) , \ \ W^{od}_{ij} = T_{aa}, \ \forall a = ij \in \mathcal{A} .$$

It exists a node numbering such that the matrices W^{od} are strictly upper diagonal and therefore they are nilpotent. Thus $(W^{od})^*$ exists where $N^* = \sum_{i=0}^{\infty} N^i$. Then the logit assignment on efficient paths is given by :

$$F_{ij} = \sum_{od \in \mathcal{D}} d_{od} (W^{od})^*_{oi} W^{od}_{ij} (W^{od})^*_{jd} / (W^{od})^*_{od} \,. \tag{3}$$

Indeed :

- $(W^{od})^*_{od}$ is the total weight of all the efficient paths from o to d,
- $(W^{od})^*_{pi} W^{od}_{ij} (W^{od})^*_{jd}$ is the total weight of the efficient paths that use the arc ij,
- $(W^{od})^*_{oi} W^{od}_{ij} (W^{od})^*_{jd} / (W^{od})^*_{od}$ is the probability for a path from o to d to use the arc ij for the logit distribution on the efficient paths.

That is, for each pair od, we consider the set of routes $\mathcal{R}_{od}$ — with weights defined by $c(r) = t_r / \bigoplus_r^{\mu} t_r$ — and compute $\mathcal{R}^{ij}_{od}(c) = c(\{r \in \mathcal{R}_{od} \mid ij \in r\})$ using the matrix product of the corresponding Gibbs-Maslov semiring — the computation being done in the standard algebra —. This a quantization of the AON algorithm.

6.1 Sheffi Improvement

A first improvement, given in [19], consists in considering as efficient paths the paths which have an increasing time from the origin and not restricting them to have also a decreasing time to the destination. With this new definition of efficient paths we build new graphs $\mathcal{G}^o = (\mathcal{N}, \mathcal{A}^o)$, incidence matrices $\mathcal{O}^o$, $\mathcal{D}^o$ transition weight matrix W^o. The corresponding flow has a formula similar to (3). It can also be written :

$$F_{ij} = \sum_{o \in \mathcal{N}} (W^o)^*_{oi} W^o_{ij} (W^o)^*_{j.} D^o \tag{4}$$

with

$$D^o_d = \begin{cases} d_{od} / (W^o)^*_{od} & \text{if } od \in \mathcal{D} \\ 0 & \text{else} \end{cases} \,.$$

By this choice the complexity is reduced of one order in the number of nodes.

6.2 Bell Improvement

Bell [3] propose to consider all the paths, not only efficient paths, by remarking that we have not to enumerate all the paths but to be able to compute the star of the weight matrix. But $W^* = (I - W)^{-1}$ is well defined as soon as the spectral radius of W, denoted $\rho(W)$, is smaller than 1. Denoting by W the transition weight matrix for the original graph $\mathcal{G}$, the flows are given by :

$$F_{ij} = \sum_{o\in\mathcal{N}} (I-W)^{-1}_{oi} W_{ij} (I-W)^{-1}_{j.} D^o \tag{5}$$

with

$$D^o_d = \begin{cases} d_{od}/(I-W)^{-1}_{od} & \text{if } od \in \mathcal{D} \\ 0 & \text{else} \end{cases} .$$

With this point of view we must choose μ large enough : such that $\rho(W) < 1$ (this is always possible — if there is no circuit with zero travel time — because the entries of W are positive and go to zero when μ goes to $+\infty$).

7 Markov Logit

If we want to consider all the paths (not only the efficient ones), in the case when the hypothesis about the spectral radius of the weight matrix is not satisfied, we can adopt a little different point of view. We can build a Markov chain on $\mathcal{G}$, depending on a parameter μ, the trajectories of which converge to the minimal time trajectories when μ goes to $+\infty$. For that purpose, let us consider the transition matrices

$$M^d_{ij} = e^{-\mu\tau_{ij}} / \sum_j e^{-\mu\tau_{ij}} \;, \quad \tau_{ij} = t_{ij} + t^*_{jd} - t^*_{id} \;,$$

where t^*_{id} denotes the minimal time needed to go from i to d. They give the logit probability to deviate from the optimal trajectory. Clearly when μ grows the probabilities to charge the optimal deterministic trajectories increase.

Therefore, we consider a multidecision logit where at each node p_i we have to choose between trajectories composed of an arbitrary admissible link $p_i p_{i+1}$ from this node p_i followed by an optimal route from p_{i+1} to the destination d.

Based on this type of decision the arc flows can be computed explicitly. We have

$$F_{ij} = \sum_d D_d (I - M^{\natural d})^{-1} M^{\flat d} M^d_{ij} \;, \tag{6}$$

where

- D_d is the row-vector of the demand from any node to d,
- $M^{\natural d}$ denotes the matrix obtained by skipping column and row d in matrix M^d,
- $M^{\flat d}$ the column-vector obtained by extracting column d of M^d and skipping entry d.

8 Logit Equilibrium

In the previous sections about stochastic assignments we have supposed implicitly that the travel time does not depend on the flow. If it is not the case we have to solve an implicit equation. For example in the third case, given the function $t_a(F_a)$ for $a \in \mathcal{A}$ we have to solve the equilibrium :

$$F = \mathcal{F}(F) \tag{7}$$

with

$$[\mathcal{F}(F)]_{ij} = \sum_{o \in \mathcal{N}} (I - W(F))^{-1}_{oi} W(F)_{ij} (I - W(F))^{-1}_{j.} D^o(F) \ ,$$

where $W(F)$ is the weight transition matrix which depend on the flows.

Moreover the logit assignment is the only distribution satisfying the efficiency principle [9] that is : *an event of flows on the available road having a smaller average time than another one has a larger probability to appear.*

Logit assignment admits the variational formulation [10] :

$$\min_f \sum_a c_a(F_a) + \frac{1}{\mu} \sum_r f_r \log f_r \ , \tag{8}$$

$$F_a = \sum_{r \in R^a} f_r \, , \quad \sum_{r \in R_{od}} f_r = d_{od} \, , \quad f \geq 0 \, .$$

As soon as there is a circuit in the graph, it exists a pair *od* such that the route number in R_{od} is infinite and therefore it may be difficult to compute the logit equilibrium (8) with this method (because we have to compute infinite sums to evaluate the criterion).

The MSA algorithm

$$F^{n+1} = \frac{n}{n+1} F^n + \frac{1}{n} \mathcal{F}(F^n) \ ,$$

can be used to solve (7). The convergence of this method can be derived from the variational formulation. Indeed $\mathcal{F}(F^n)$ is the solution of

$$\min_f \sum_r f_r \log(f_r) + \sum_a t_a(F^n_a)(F_a - F^n_a) \ ,$$

which is a partially linearised version of problem (8). Then using convexity properties it can be shown that $\mathcal{F}(F^n) - F^n$ is a descent direction of the problem (8).

The divergent series method is used because it is difficult to compute the value of the criterion (the route number can be infinite) and linear search implies the effective computation of the criterion, moreover we have seen that FW is not better than MSA for deterministic assignments.

9 Scilab Traffic Toolbox

A toolbox in Scilab [12] dedicated to traffic assignments called CiudadSim [14] has been developed. In Figure 3 we show (left) the Markov logit equilibrium for a regular town, when $\theta = 60$. There is a unique demand in grey (diagonal arc). The arc width is the level of congestion. The right figure shows the difference between the stochastic and the Wardrop assignments.

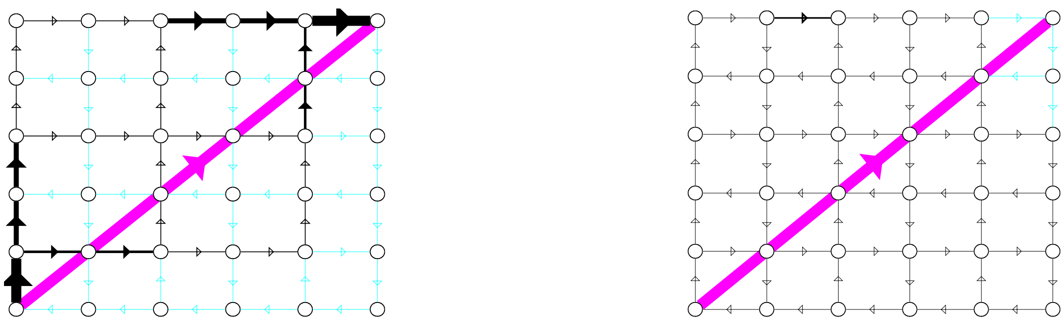

Figure 3: A stochastic assignment ($\theta = 60$) and the difference between the logit and Wardrop equilibriums.

This toolbox has the following functionality.

- A Scilab data structure*NetList* plays the role of mini geographic data base. It is editable by the graphic interface Scigraph of Scilab. At the end of the study, it contains also the assignment result.
- A Scilab function *TrafficAssig* computes the assignment using one of the current implemented algorithm which are AON, incremental assignment, Frank-Wolfe, DSD, Capres, MSA, Logit and Probit. The four different kinds of logit assignments are available. We can choose the travel time as a function of the flow among a palette of standard functions.
- The function *TrafficExample* provides some standard network examples as (Sioux Falls, Steenbrick). It can generate also random or regular networks.
- Using the Scilab function *ShowNet* the results can be visualised.

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http://www.inria.fr
ISSN 0249-6399