Un modèle de réapprovisionnement dans un système d'assemblage avec délais de réapprovisionnement aléatoires

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Résumé : Dans ce papier nous étudions un système d'assemblage dans lequel les délais de réapprovisionnement des composants sont aléatoires. Nous supposons que la demande de produit final est déterministe et que le délai de livraison est connu. L'objectif est de trouver les instants de commande de chaque composant de façon à minimiser le coût moyen qui se compose dun coût de stockage et d'un coût de retard. L'analyse du problème est compliquée par le fait que la densité de probabilité du délai de livraison peut être différente d'une composante à l'autre. Pour contourner la difficulté, nous divisons le problème en deux parties et résolvons ces parties en utilisant les résultas proposés dans le paragraphe 3. Finalement, l'algorithme est illustré par des exemples numériques.

Mots-clés : Délais de réapprovisionnement aléatoires, recuit simulé, assemblage

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A model for ordering policy in an assembly system under lead time uncertainty

Abstract: In this paper we study a single product assembly system whose components lead-time are random. The demand of finished product is deterministic and delivery date is known. The objective is to find the optimal ordering time of each component such as to minimize the sum of average backlogging cost and inventory holding costs. An analytical study is difficult due to fact that density functions related to lead times could be different from each other. To overcome the difficulty, we divide the problem into two parts and solve both parts in accordance with the results proposed in section 3. Finally, the algorithm is illustrated by numerical examples.

Key-words: Random lead-time, assembly, simulated annealing.

1 Introduction

There are several instances when company relies on several outsourced components to assemble the required finished product. Usually the delivery times of the components depend upon the availability of stocks and reactivity of subcontractors. In other word, the lead-times of the components are random. To face this situation companies either use huge safety stocks, which is rather expensive, or utilize stochastic methods to reduce the average costs that includes inventory costs and backlogging costs.

In this paper we propose a model for assembly systems, which relies on several components whose lead times are random with known distribution. The delivery date of the finished product is known. The objective is to find the appropriate ordering times of the components in order to minimize the total expected cost. The problem is not new and several instances of this problem are available in literature. [5] consider the similar problem with random demand, but restricted to two components only. In this model author consider that components either arrive in the current period with a given probability (say β) or in the next period with probability $(1-\beta)$. This two period lead time model leads to the optimal quantity of each component to order from each suppliers. [3] proposed a single period model for two components with different density function and established the optimality conditions.
[2] proposed a multi period optimization model and established conditions for the optimal solution. In his model authors considered the same density functions for all components. [7] proposed a model for assembly planning under constant demand and random components delivery times. The authors take into account the item holding cost and the backlogging costs. This model allows to calculate the number of components of each type to be ordered at the beginning of each period and the products to be assembled are selected on the basis of a priority heuristic.
[6] and [4], defined safety stocks to overcome the randomness of lead times. [6] proposed a simulating annealing procedure to define safety stock and safety lead time. [8] found that the use of a safety lead time is more efficient than safety stock. For the exhaustive review on planning under uncertainty, readers can refer [1] and [9].

Our model is different in the sense that there is no restriction for the number of components, the lead time density function may differ from component to component and the problem is continuous instead of discrete.
This paper is organized in 5 sections. In section 2 we describe the problem, section 3 presents a solution method. In section 4 we presents a numerical example and, finally, section 5 is the conclusion.

2 Problem formulation

In this paper we consider an assembly system whose production relies on his providers for timely assembly and delivery. We assume that the final assembly is made up of n components denoted by C_i, $i \in \{1, 2, ..., n\}$. The lead-time, that is the period between the ordering time and the delivery time, of each component is stochastic and is governed by a specific probability density function, which may differ from component to component. We denote by $f(t_i - r_i)$, the probability density function of lead time for the ith component, where t_i is the time at which the component arrives at assembly site and r_i is the ordering time. Since density functions may be different from each other, then the ordering times may also be different. Furthermore, each component C_i is subject to an inventory holding cost s_i, which is applicable from the time of the arrival of the component at the assembly site to the assembly of the components. In this system we assume that the final product can only be assembled if all the ordered components are available, in other words there is no safety stock. It means that assembly-starting time (T) is the maximum among the delivery times of the components and the time Z which is the due date of final product. In other words, T is the delivery time of the component that arrives at last. Z is the time at which the assembly operation should be performed. The assembly time can be neglected. If $T < Z$, all the components should be stored for a period $Z - T$ and assembled at time Z, otherwise, the assembly operation should be performed as soon as the last component is delivered, that is at time T, and a backlogging cost applies for a period $T - Z$. The objective is to minimize the total expected inventory and backlogging costs. Figure 1 represents the assembly system in the case when the last component is available before assembly time Z.

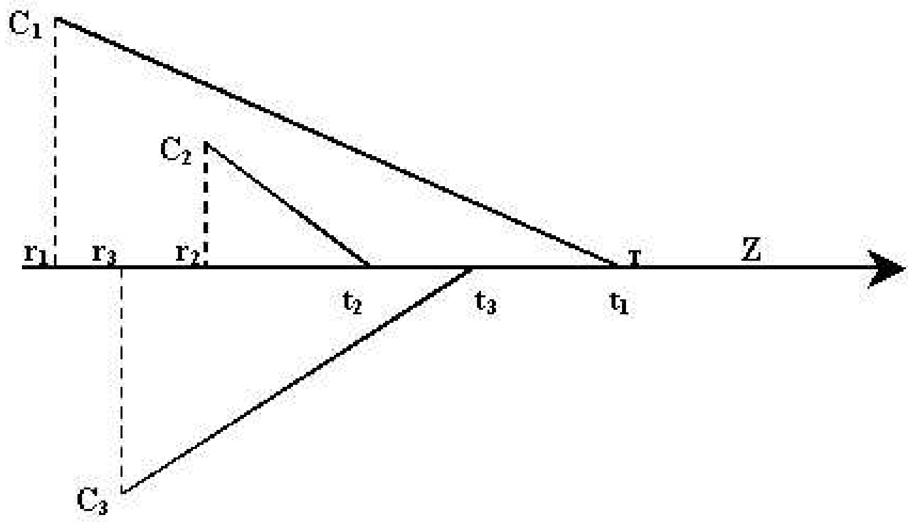

FIG. 1 – Assembly process

In this model we consider the three following costs.
(i) Inventory cost until the last component is available.
(ii) Inventory cost between times T and Z if $T < Z$.
(iii) Backlogging cost between Z and T if $Z < T$.
The probability that the last component C_i arrives at time T is the product of :
- The probabilities that components C_k, where $k \neq i$, arrive before T, that is :

$$\prod_{k=1,k\neq i}^{n} F_k(T-r_k)$$

- and, the probability that C_i arrives during the period $[T, T+dT)$, that is $f_i(T-r_i)dT$. Finally, the probability density that the last component arrives at time T knowing that this component is C_i is :

$$P_i(T) = \prod_{k=1,k\neq i}^{n} F_k(T-r_k).f_i(T-r_i). \tag{1}$$

Indeed, the last component could be anyone of the n components, and the event " the last component arrives at time T and this component is C_i", for $i = 1, 2, ..., n$ are mutually exclusive. As a consequence, the probability density that the last component arrives at time T is :

$$P(T) = \sum_{i=1}^{n} P_i(T), \tag{2}$$

or,

$$P(T) = \sum_{i=1}^{n}[\prod_{k=1,k\neq i}^{n} F_k(T-r_k).f_i(T-r_i)] \tag{3}$$

Relation (3) applies only when $T \geq Max_{j\in\{1,2,...,n\}} r_j$, otherwise $P(T) = 0$.

2.1 Average inventory cost until time T

Assume that C_i is the component that arrives during period $[T, T+dT)$. In this case, the inventory cost of the $n-1$ other components until time T is equal to :

$$I(C_i) = \sum_{k=1,k\neq i}^{n} [s_k. \int_{t_k=r_k}^{T} (T-t_k).f_k(t_k-r_k).dt_k].f_i(T-r_i)dT \tag{4}$$

Consider the integral (5) :

$$J(C_k) = \int_{t_k=r_k}^{T} (T-t_k).f_k(t_k-r_k)dt_k \tag{5}$$

Integrating (5) by parts, we get :

$$J(C_k) = \int_{t_k=r_k}^{T} F_k(t_k - r_k)dt_k \tag{6}$$

Thus, relation (4) can be rewritten as :

$$I(C_i) = \sum_{k=1,k\neq i}^{n} [s_k \int_{t_k=r_k}^{T} F_k(t_k - r_k)dt_k].f_i(T - r_i) \tag{7}$$

Since the events corresponding to the arrival of different components at time T are mutually exclusive, and that T evolves from $R = Max_{j=1,2,...,n} r_j$ to $+\infty$, the average inventory cost until the last component is available is given by the following relation (8) :

$$I_1 = \sum_{i=1}^{n} (\int_{R}^{+\infty} \{ \sum_{k=1,k\neq i}^{n} [s_k. \int_{t_k=r_k}^{T} F_k(t_k - r_k).dt_k].f_i(T - r_i)\}.dT) \tag{8}$$

where $R = Max\{r_j\}_{j\in\{1,2,...,n\}}$. Note that I_1 is a function of the ordering time $r_1, r_2, ..., r_n$ and not of Z.

2.2 Inventory cost between T and Z

It is the inventory cost for keeping in stock the components from time T to the delivery time Z when $Z > T$, i.e. for a period of $Z - T$. Since this cost is applicable to all components, the inventory cost per unit time is $\sum_{k=1}^{n} s_k$. Now, the inventory cost for a period of $Z - T$ is $I = (\sum_{k=1}^{n} s_k)(Z - T)$, and therefore the expected inventory cost is :

$$I_2 = (\sum_{k=1}^{n} s_k) \int_{T=R}^{Z} (Z - T).P(T).dT \tag{9}$$

where $R = Max\{r_j\}_{j\in\{1,2,...,n\}}$ and the $P(T)$ is given by (3).

2.3 Average backlogging cost

The backlogging cost is applicable only if the final product (assembly) is not ready by the delivery date Z. In this case it appears for a period of $T - Z$. Let h be the backlogging cost per time unit, then the backlogging cost is $h.(T - Z)$ and therefore the expected backlogging cost is :

$$B = h. \int_{T=Z}^{+\infty} (T - Z).P(T).dT \tag{10}$$

Finally, the total expected cost $S = I_1 + I_2 + B$ is the summation of all the three costs.

3 Problem analysis

The objective is to minimize the total cost S. We can see that the costs I_2 and B are functions of Z and the cost I_1 is constant w.r.t. Z. In other words we can represent the costs I_2 and B as a function of Z if all $r_1, r_2, ..., r_n$ are fixed :

$$W(Z) = \sum_{k=1}^{n} s_k \int_{T=R}^{Z} (Z-T).P(T).dT + h \int_{T=Z}^{+\infty} (T-Z).P(T).dT \quad (11)$$

3.1 A particular case

In this section, we assume that the lead-time density functions are as follows :

$$f_i(x_i) = \left\{ \begin{array}{cc} >0 \quad if & a_i \leq x_i \leq b_i \\ 0 & otherwise \end{array} \right\} \quad (12)$$

In this case, the expression (8) can be rewritten as follows :

$$I_1 = \sum_{i=1}^{n} (\int_{R_s}^{r_i+b_i} \{ \sum_{k=1,k\neq i}^{n} [s_k. \int_{t_k=r_k+a_k}^{T} F_k(t_k-r_k).dt_k].f_i(T-r_i)\}.dT) \quad (13)$$

Where $R_s = Max\{r_j+a_j\}_{j\in\{1,2,...,n\}}$ The following results hold good for this particular case :

Result 1
The average inventory cost S is minimum for some ordering time that verify $Z-b_i \leq r_i \leq Z-a_i$, $i=1,2,...,n$.
Proof
a. Assume that for the $p-th$ component $r_p > Z-a_p$. In this case relation (13) becomes :

$$I_{1,Z<r_p+a_p} = \sum_{i=1,i\neq p}^{n} (\int_{R_s}^{r_i+b_i} \{ \sum_{k=1,k\neq i}^{n} [s_k. \int_{t_k=r_k+a_k}^{T} F_k(t_k-r_k).dt_k].f_i(T-r_i)\}.dT)$$

$$+ \int_{R_s}^{r_p+b_p} \{ \sum_{k=1,k\neq p}^{n} [s_k. \int_{t_k=r_k+a_k}^{T} F_k(t_k-r_k).dt_k].f_i(T-r_p)\}.dT$$

Since $r_p > Z-a_p$, we have $Z < r_p+a_p < r_p+b_p$. Thus $I_{1,Z<r_p+a_p}$ can be written as :

$$= \sum_{i=1,i\neq p}^{n} (\int_{R_s}^{r_i+b_i} \{ \sum_{k=1,k\neq i}^{n} [s_k. \int_{t_k=r_k+a_k}^{T} F_k(t_k-r_k).dt_k].f_i(T-r_i)\}.dT)$$

$$+ \int_{R_s}^{Z} \{ \sum_{k=1,k\neq i}^{n} [s_k. \int_{t_k=r_k+a_k}^{T} F_k(t_k-r_k).dt_k].f_i(T-r_p)\}.dT)$$

$$+\int_{Z}^{r_p+b_p}\{\sum_{k=1,k\neq p}^{n}[s_k.\int_{t_k=r_k+a_k}^{T}F_k(t_k-r_k).dt_k].f_p(T-r_p)\}.dT)$$

Finally, :

$$I_{1,Z<r_p+a_p}=I_{1,Z=a_p+r_p}+\int_{Z}^{r_p+b_p}\{\sum_{k=1,k\neq p}^{n}[s_k.\int_{t_k=r_k+a_k}^{T}F_k(t_k-r_k).dt_k].f_p(T-r_p)\}.dT \tag{14}$$

Equation (14) shows that inventory cost in the case when $r_p + a_p > Z$ is higher than in the case when $r_p + a_p = Z$. This completes the first part of the proof.

b. Now, assume that for the $p-th$ component $r_p < Z - b_p$. In this case relation (13) becomes :

$$I_{1,Z>r_p+b_p}=\sum_{i=1}^{n}[\int_{Max\ (r_j+a_j)}^{r_i+b_i}\{\sum_{k=1,k\neq i,p}^{n}(s_k.\int_{t_k=r_k+a_k}^{T}F_k(t_k-r_k).dt_k)$$

$$+s_p.\int_{t_p=r_p+a_p}^{T}F_p(t_p-r_p).dt_p\}.f_i(T-r_i).dT]$$

Since, $r_p + b_p < Z$ we can write,

$$=\sum_{i=1}^{n}[\int_{Max\ (r_j+a_j)}^{r_i+b_i}\{\sum_{k=1,k\neq i,p}^{n}(s_k.\int_{t_k=r_k+a_k}^{T}F_k(t_k-r_k).dt_k)+s_p.\int_{t_p=r_p+a_p}^{Z-b_p}F_p(t_p-r_p).dt_p$$

$$+s_p.\int_{t_p=Z-b_p}^{T}F_p(t_p-r_p).dt_p\}.f_i(T-r_i).dT]$$

$$=\sum_{i=1}^{n}[\int_{Max\ (r_j+a_j)}^{r_i+b_i}\{\sum_{k=1,k\neq i,p}^{n}(s_k.\int_{t_k=r_k+a_k}^{T}F_k(t_k-r_k).dt_k)+s_p.\int_{t_p=Z-b_p}^{T}F_p(t_p-r_p).dt_p$$

$$+s_p.\int_{t_p=r_p+a_p}^{Z-b_p}F_p(t_p-r_p).dt_p\}.f_i(T-r_i).dT]$$

Finally, :

$$I_{1,Z<r_p+b_p} = I_{1,Z=r_p-b_p} + \sum_{i=1}^{n} [\int_{Max\ (r_j+a_j)}^{r_i+b_i} \{ \sum_{k=1,k\neq i,p}^{n} [s_p . \int_{t_p=r_p+a_p}^{Z-b_p} F_p(t_p - r_p).dt_p].f_i(T-r_i)\}.dT) \tag{15}$$

Expression (15) shows that the average inventory cost is less when the $p-th$ component is ordered at $Z-b_p$ than if it is ordered at $r_p < Z-b_p$. This completes the proof. ■

3.2 General result

Result 2

A necessary and sufficient condition for the average cost $W(Z)$ to be minimum is that the probability of arrival of the components by time Z is a constant function $\frac{h}{\sum_{k=1}^{n} s_k + h}$ of the backlogging and inventory costs, or in other words :

$$\prod_{k=1}^{n} F_k(Z - r_k) = \frac{h}{\sum_{k=1}^{n} s_k + h} \leq 1$$

Proof

The function $W(Z)$ is a convex continuously differentiable function w.r.t. Z on $]R, +\infty)$(see appendix). To obtain the minimum optimum value of $W(Z)$, we derive equation (11) w.r.t. Z and set the derivative equal to zero.

$$\frac{dW(Z)}{dZ} = \sum_{k=1}^{n} s_k \int_{T=R}^{Z} P(T).dT - h \int_{Z}^{\infty} P(T).dT$$

where $R = Max\{r_j\}_{j=1,2,\dots,n}$.

Now, by setting $W^{'}(Z) = 0$, we get,

$$\sum_{k=1}^{n} s_k \int_{T=R}^{Z} P(T).dT = h \int_{Z}^{\infty} P(T).dT$$

or,

$$\int_{Z}^{\infty} P(T).dT = \frac{\sum_{k=1}^{n} s_k}{\sum_{k=1}^{n} s_k + h} \tag{16}$$

Further, from the equation (3) we get,

$$\int_{T=Z}^{\infty} P(T).dT = \int_{T=Z}^{\infty} \sum_{i=1}^{n} [\prod_{k=1,k\neq i}^{n} F_k(T - r_k).f_i(T - r_i)]dT$$

The function to integrate is the derivative of $\prod_{k=1}^{n} F_k(Z - r_k)$. Thus,

$$\int_{T=Z}^{\infty} P(T).dT = 1 - \prod_{k=1}^{n} F_k(Z - r_k) \tag{17}$$

and from equation (16) and (17) we deduce,

$$\prod_{k=1}^{n} F_k(Z - r_k) = \frac{h}{\sum_{k=1}^{n} s_k + h} \leq 1 \tag{18}$$

This completes the proof. ■

From the equation (18), we can see that, for a given Z, an infinite number of $r_i's$ exists that satisfy equation 18 and for all those values the minimum of $W(Z)$ is known. Since our objective is to minimize $S = I_1 + W(Z)$, and since I_1 is a function of $r_1, r_2, ..., r_n$, we have to choose r_i in such a manner that it not only minimizes I_1 but also satisfies equation (18). The previous result suggest to use an algorithm that decreases I_1 step by step making sure that relation (18) is satisfied at each step. It is why we present a simulated annealing algorithm applied to I_1 and such that the computation of the neighbors solution is constrained by relation (18) : this algorithm will be developped in the next section. Note that since $\prod_{k=1}^{n} F_k(Z - r_k) \leq 1$, it is always possible to define ordering times $r_i \in [Z - b_k, Z - a_k]$ that satisfy (18).

In general approach, when density functions are different, we use simulating annealing to modify the ordering time in order to reduce I_1 and use Monte Carlo simulation method to evaluate the inventory cost I_1. The algorithm is presented in section 3.4.

3.3 The case of identical components

The problem will be quite simple if all the components have the same density function and follow the uniform density. In this case ordering point will be the same for all components. Consider the uniform density function $f(t - r)$ defined in $[a, b]$.

$$f(t-r) = \left\{ \begin{array}{ccc} \frac{1}{b-a} & if & t \in [r+a, r+b] \\ 0 & & otherwise \end{array} \right\} \tag{19}$$

In this case equation (8) and equation (18) take the following form :

$$I_1 = n.(n-1)s \int_{r+a}^{r+b} \int_{r+a}^{T} [F(t-r).dt] f(T-r).dT \tag{20}$$

or,

$$I_1 = n.(n-1).s.(b-a)/6 \tag{21}$$

Equation (21) shows that in the case of same density function, I_1 do not depend upon ordering time r. Thus the optimal value of r can be computed directly from Equation (18) as follows :

$$F(Z-r) = (\frac{h}{n.s+h})^{\frac{1}{n}} \tag{22}$$

But,

$$F(Z-r) = \frac{Z-r-a}{b-a}, r \in [Z-b, Z-a]$$

This relation leads to :

$$r = Z - a - (b-a)(\frac{h}{n.s+a})^{\frac{1}{n}}$$

3.4 The general algorithm

To start with, we compute a set of $r_1, r_2, ..., r_n$ values that satisfy equation (18) and then calculate the cost corresponding to this set using Monte Carlo simulation. For simulated annealing, we initialize the starting temperature T_0, freezing temperature T_f and current temperature T_c. We also define M the maximum number of iterations required to change the temperature. For the solution, we define the best solution by S^* and corresponding set of ordering times by R^*. Similarly we define the current solution by S_c and corresponding set of ordering times by R_c. Now the algorithm goes as follows :

1. Compute I_1 using Monte Carlo simulation and set,
   (a) $S^* = S_c = I_1$.
   (b) $T_c = T_0$.
2. Define R_c^1 that satisfy 18 in the neighborhood of R_c.
3. Compute I_1 for R_c^1. Let denote this value by S_c^1.
4. Compute $\delta = S_c - S_c^1$.
5. If $\delta < 0$, set $S_c = S_c^1$, otherwise set $S_c = S_c^1$ with probability $exp(-\delta/T_c)$.
6. If $S_c^1 < S^*$, set $S^* = S_c^1$.
7. $Count = Count + 1$.
8. If (Count=M), set $T_c = \alpha * T_c$ and $Count = 0$.
9. If $T_c < T_f$, stop otherwise go to 2.

4 Numerical illustration

In this section we present two examples. In both the examples we consider three components and uniform probability densities for all the components. In the first example, we present ten experiments with different backlogging costs. Table 2 shows the ordering time of components in all the ten cases. In the second example we use fix backlogging cost $h = 50$

and uniform density function with same parameters for all components. Other parameters remain the same. This assumption allows us to compute optimal solution easily and to evaluate the algorithm. For all the examples we use $Z = 300$.

TAB. 1 – Parameters

Component	Inventory cost per unit	Uniform density parameter
1	20	$[100, 150]$
2	15	$[50, 170]$
3	10	$[50, 100]$

TAB. 2 – Ordering time for different backlogging costs

BL	50	45	40	35	30	25	20	15	10	5
1	159.08	159.82	161.06	161.99	163.11	164.03	166.15	168.41	171.62	176.69
2	152.96	154.61	156.6	158.90	161.58	164.71	168.99	174.41	182.02	194.31
3	210.23	210.87	211.21	212.14	213.26	215.07	216.35	218.61	221.77	226.89

BL : Backlogging costs.
In table 2, we can see that as the backlogging cost decreases, the ordering times of components approach to Z. Example 2 :

$$f(x_i) = \left\{ \begin{array}{cc} \frac{1}{170-100} & if \quad x_i \in [100, 170] \\ 0 & otherwise \end{array} \right\} \tag{23}$$

Inventory cost $s_i = 20$, where $i = 1, 2, 3$
In this case, the ordering time of all components will be the same. The optimal ordering time in this case is 146.178 and ordering times obtained by applying the algorithm are :
$r_1 = 146.369$
$r_2 = 143.601$
$r_3 = 148.481$

5 Conclusion

In this paper we present a method to determine the ordering times for each component in an assembly system in order to minimize the sum of inventory holding and backlogging costs. The cost function of this problem is complicated due to several different density functions, which make this problem complicated to analyze analytically. We divided the ordering problem into two instances. The objective is to minimize the cost related to both instances. The property of optimal solution is proposed for the second instance. Simulated annealing is used to explore the neighborhood solution to minimize the first instance, keeping in mind that new solution must satisfy the optimal condition of second instance.

Appendix

$W(Z)$ is a convex function.
Proof
Differenciation of $W(Z)$ w.r.t. Z gives

$$\frac{dW(Z)}{dZ} = \sum_{k=1}^{n} s_k \int_{T=R}^{Z} P(T).dT - h \int_{Z}^{\infty} P(T).dT$$

where $R = Max(r_j)_{j=1,2,\ldots n}$.

Consider the $W(Z)$ when $h = 0$. In this case the function has a zero value at $Z = R$ and tends to infinity as Z tends to infinity (see figure(2)). In contrast to inventory holding cost, backlogging cost is maximum at $Z = R$ and tends to zero as Z tends to infinity (see figure (3)). The cost shape in general case is presented in figure (4), where we can see that the slope of $W(Z)$ is negative at $Z = R$ and then become zero at some point and after it becomes positive. The slope of the curves depends upon the distribution of density function $P(T)$.

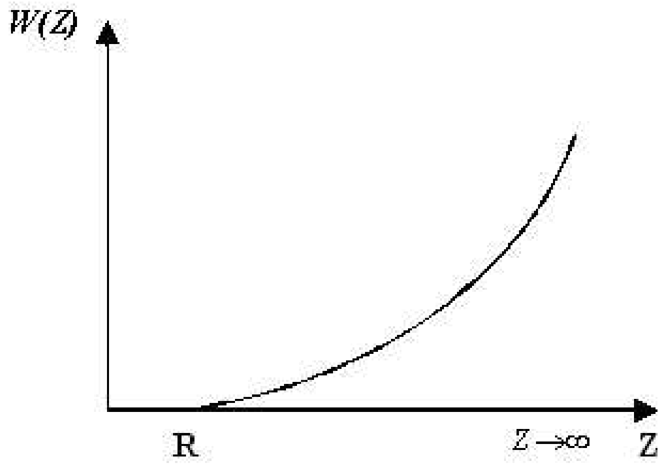

FIG. 2 – Inventory cost w.r.t. Z

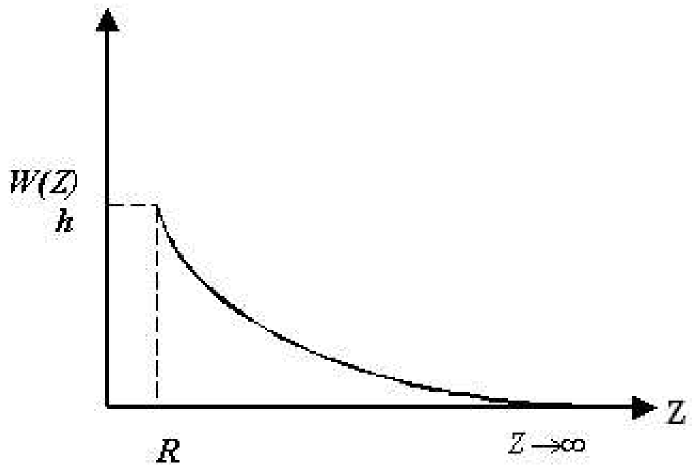

FIG. 3 – Backlogging cost w.r.t. Z

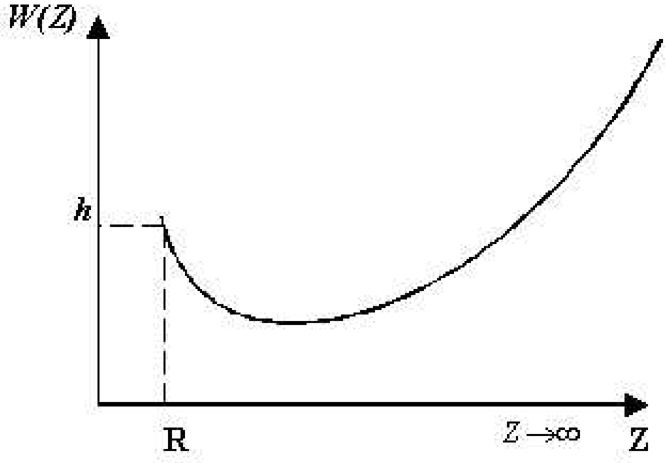

FIG. 4 – W(Z) in general case w.r.t. Z

Furthermore, we can see mathematically that the second derivative of $W(Z)$ w.r.t. Z is positive, i.e.

$$\frac{d^2W}{dZ^2} = (\sum_{k=1}^{n} s_k + h).P(Z)$$

Since, $P(Z)$ is always greater than zero under the assumption $Z \geq R$, $W''(Z)$ is always positive. It is also the sufficient condition for any function to be convex. This completes the proof. ■

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