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An Optimal Smart Market for the Pricing of Telecommunications Services

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Abstract: In [7], Mackie-Mason and Varian suggested the creation of a *Smart market* for the pricing of Internet services, based on the idea of having a Vickrey-type auction, at the packet level, at each node of the network. While this idea has attracted the attention of numerous researchers, in telecommunications and in economics, no optimization model has to date been proposed that captures this behavior. In this paper, we present such a mathematical model. We provide techniques for solving the model through an equivalent linear transformation, and present some numerical experience on sample networks.

Key-words: Generalized Vickrey Auction, Congestion pricing, Internet economics, Optimization model

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Un *Smart Market* Optimal pour la Tarification des Services de Télécommunications

Résumé : Dans [7], Mackie-Mason et Varian ont suggeré la création d'un *Smart market* pour la tarification des services de l'Internet, basé sur le principe d'une enchère de Vickrey, au niveau packet, à chaque noeud du réseau. Tandis que cette ideé a intéressé de nombreux chercheurs, en télécom et en économie, il n'y a pas eu jusqu'à présent, un modèle d'optimisation qui quantifie ce phénomène. Dans ce papier, on présente un tel modèle. On définit des techniques pour résoudre le modèle à partir d'une transformation linéaire équivalente, ainsi que quelques expériences numériques.

Mots-clés : Tarification de la congestion, Enchère de Vickrey Généralisée, Internet, Optimisation

1 Introduction

The internet uses today a service model called "best effort", that is, the network allocates bandwidth among all the instantaneous users as best it can, and attempts to serve all of them without making any explicit commitment as to routing or any other service quality. Indeed, some the traffic may be dropped. When congestion occurs, the users are expected to detect this, and slow down their transmission rates, so that overall, the collective sending rate is reduced towards the capacity of the network.

There are two general ways to regulate usage of the network during congestion. The practical one is to use technical mechanisms (such as the existing TCP congestion control) to adjust flows and thereby influence client behavior. An alternative method is to use pricing mechanisms to charge the user directly so as to influence behavior through economical means.

As a starting point, it is useful to look at the approaches proposed for control of usage and explicit allocation of resources among users in time of overload. The ideas of these approaches is as follows: each user uses the internet if his marginal benefit is greater then his marginal cost. Therefore, increasing the marginal cost, by including congestion costs, will lead to a reduction of the usage level.

1.1 Standard priority scheduling

An accepted idea for the allocation of bandwith among users is to create service classes of different priorities to serve users with different needs. The definition of priority is that, if requests of different priority arrive at a router at the same time, the higher priority request always departs first. This has the effect of shifting delay from the higher priority request to the lower priority request. When the network is congested, this delay could be significant, whereas, when the network is uncongested, priority scheduling would have no perceptible effect.

There are two ways to set the price for each level of priority. Static pricing is independent of the state of congestion of the network, while dynamic pricing modulates prices as a function of the traffic levels, number of incoming requests, or delays. Practically, for dynamic pricing to be used, a user would have to consult an online database in order to obtain the unit cost and estimated duration for using each service.

1.2 The Smart Market concept revisited

The underlying idea is that, since much of the time the network is uncongested, congestion pricing in this case should not be in effect. When the network is congested, currently, requests are queued and delayed, in a FIFO manner. Mackie-Mason and Varian [7], in a thought-provoking paper, suggested instead that requests be prioritized based on the value that the user puts on getting the request through quickly, and that an auction would then ensue to chose those requests that would pass. To do this, each user assigns to his request a bid measuring his willingness to pay for immediate servicing. Then, at congested routers, requests are prioritized based on bids. In order to make the scheme incentive, users are not charged the price they bid, but rather are charged the bid of the lowest request that is admitted to the network. Indeed, this is an approximation of the Second Price, or Vickrey, auction, in which case the price paid is that of *the highest non-accepted bid.* Therefore, it is easy to see that in the limit, as the number of bids increases, the Smart Market concept, as suggested by Mackie-Mason and Varian, converges to a Vickrey auction.

The most obvious disadvantage of the Smart Market and of dynamic pricing, in general, is that in the real-word, the computational burden of updating prices can be quite high. In the Smart Market, the efficient price is determined by comparing a list of user bids to the available capacity and determining the cut-off price. Since requests arrive not at once but over time, it would thus be necessary to clear the market periodically based on a time-slice of the current bids. The efficiency of this scheme, then, depends on how costly it is to clear the market and on how persistent the periods of congestion are.

Further, the concept of the Smart Market, as proposed by Mackie-Mason and Varian, is practicable only the case of one resource; as proposed, it cannot be applied optimally to a network, or subnetwork, for several reasons. The first reason is that there has been no optimization model of resource allocation over a network that captures this behavior. This issue is the primary focus of this paper.

Other issues persist and we have dealt with these by making certain assumptions on the objectives of the optimization and its characteristics. These are detailed below.

Assumption 1 The complete network topology, as well as the list of requests and the quantity of resource desired for each, are given.

Note that the demands for service are considered to be known in advance; that is, the optimization described in this paper falls into the category of static pricing and allocation.

Assumption 2 The bid level of each request and the paths available for each request are given.

The bid level is taken to be the amount that a user is willing to pay for the completion of the entire request, as opposed to a per-node amount. In general, one cannot know the bids at each router since each user offers a bid for transmission of his message, although he does not know the route that will be used.

Assumption 2 is a standard one, though it is clearly restrictive if one considers the Internet as a whole.

Assumption 3 Either a request can be transmitted in its entirety or it not is accepted at all.

Definition 1 *Congestion* occurs on a link of the network when it is impossible to route an another request on that link.

A precise and unambiguous definition of congestion in this context is not obvious, since flows are by nature integer-valued. The usual way to impose a capacity constraint in the optimization setting is to require that the amount of flow on a link be less than or equal to a fixed capacity. With general integer-valued flows, a link may not be at capacity but it may nonetheless be impossible to route further flow on the link. In standard resource allocation models, this has no consequence. However, in the Smart Market context, the auction is triggered *only when congestion is reached.* Therefore, it is crucial to be able to indicate that no further flow can traverse a link, *even if the link flow is not at capacity*; otherwise, it could happen that the auction is never triggered, whereas the network is indeed congested. For this reason, we have defined congestion on a link to be precisely that point at which *no further request can be routed on the link.* Note that this definition involves not only the physical capacity of the link, but information on the flows that are able to use the link.

Assumption 4 The Smart Market auction pricing mechanism is triggered only when congestion is reached, where congestion is defined as in Definition 1.

Assumption 5 The objective of the pricing policy can be defined as a weighted average of the revenue and the number of requests routed.

Assumption 5 permits a purely revenue-maximizing objective to be pursued, or a system-optimal flow to be sought, or any efficient combination of the two.

We suppose that a good resource allocation policy would allow new users to use the network without reducing network revenue. Note that purely revenue-maximizing behavior would suffer the drawback of enducing congestion on the network, since the auction is triggered only when congestion occurs. We have avoided this perverse phenomenon by including in the multi-objective function a term that maximizes the number of requests satisfied. Another way to avoid the congestion-enducing phenomenon is to allow the auction to be triggered before congestion is reached. Revenue maximization could then be achieved by selecting the most profitable requests.

Example 1 *Following is an illustration of the Smart Market auction pricing scheme in a very simple network. We use the notation $\varphi^r(p,x)$ where p represents the bid price of request r and x the amount of bandwidth requested. For example, the first request $\varphi^1(5,1)$ represents a unit request to be routed from node 1 to node 6 with a bid price of 5. The arc $(4,5)$ has a capacity of 2 units, while the other arcs have infinite capacity.*

The demands are then given by $\varphi^1(5,1)$, $\varphi^2(3,1)$, and $\varphi^3(7,1)$, and the network is provided in Figure 1.

Due to the topology of the network and to the limited capacity of the arc $(4,5)$, we cannot route more than 2 requests. The question is then which requests should be accepted, in order to maximize both revenue and the number of requests transmitted, according to some convex combination of these two (sometimes contradictory) objectives. The response in this example is to choose the two best requests in terms of profit. The two accepted are therefore the first and the third requests.

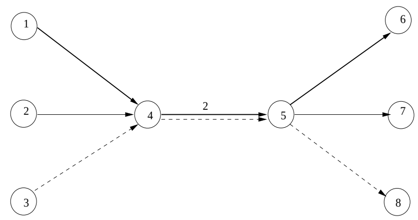

Figure 1: Example of Smart Market network pricing

This paper is organized as follows. Our optimization model of the Smart Market in which requests are singly-routed is formulated in section 2. In the third section, we generalize the model to solve the case of several possible paths per request. Some computational results are reported and discussed in section 4.

2 The optimization model of a Smart Market

The challenge in developing an optimization model for the Smart Market auction and resource allocation is that the price on each link of the network is not known *a priori*, as is the case in all traditional resource allocation models. Indeed, the prices on the links are not fixed constants, but depend on the messages that are routed on the links. The auction takes place only across those requests, and the final outcome of the auction depends on the allocation obtained, which in turn depends on the capacities of the links. In other words, the price charged on each link of the network is, in the Smart Market setting, an *output* of the model, rather than an input.

Furthermore, unlike in some other work in telecommunications pricing, Lagrange multipliers cannot be used to obtain the Smart Market prices. On the one hand, the multipliers will be zero unless capacity is *reached exactly*; following Definition 1, we explained why this condition will not be sufficient to trigger the Smart market. On the other hand, the ordering of requests in terms of their contribution to overall revenue, as required by the Smart market, cannot be taken into account in a usual multi-commodity flow model that would give rise to the Lagrange multipliers.

2.1 Notation

Before describing how the calculation of the Smart Market prices is accomplished, we introduce the following notation necessary for developing the mathematical model.

$G(N, E)$: Directed graph

N: The set of nodes: routing centers,

E: The set of arcs in the graph,

m: The cardinality of the arc set $|E|$,

$u(i,j)$: An arc representing a physical link between nodes i and j,

K: The set of requests: each request is represented by a communication node pair, $k := (i,j) \in N \times N$,

d^k: The amount of bandwidth requested by user $k \in K$,

P^k: User k's bid; that is, the maximum price that the user is willing to pay for the transmission of request $k \in K$,

$O(k)$: The origin node of request k,

$D(k)$: The destination node of request k,

F: The 0-1 indicator matrix of possible paths for each request, where $F_u^k = 1$ if the k^{th} request can use arc $u \in E$ on its pre-determined path, and 0 otherwise,

C_{ij}: The capacity of the arc $u = (i,j)$.

2.2 The mathematical model

The Smart Market auction and resource allocation problem is then defined as follows: Given a graph $G(N,E)$ and a set of requests, we seek to maximize simultaneously the profit obtained from the requests routed and the number of requests routed, while not exceeding the capacities on the telecommunication links. The decision variables are:

$$y^k = \begin{cases} 1 & \text{if the request } k \text{ is routed,} \\ 0 & \text{otherwise,} \end{cases}$$

and

$q_u =$ the price determined by the Smart Market auction on the arc $u \in \mathbf{E}$.

The mathematical model is then given by

[Problem P]

$$\max_{q,Y} (1-\alpha) \sum_{k \in \mathbf{K}} \sum_{u \in \mathbf{E}} d^k q_u F_u^k Y^K + \alpha \sum_{k \in \mathbf{K}} Y^k \tag{1}$$

Subject to

$$\begin{aligned} \sum_{k \in \mathbf{K}} F_u^k Y^k d^k &\leq C_u, \quad u \in \mathbf{E}, && (2) \\ \sum_{u \in \mathbf{E}} q_u F_u^k Y^k d^k &\leq P^k, \quad k \in \mathbf{K}, && (3) \\ P^o \sum_{k \in \mathbf{K}} F_u^k (1 - Y^k) &\geq q_u, \quad u \in \mathbf{E}, && (4) \\ q_u &\geq 0, \quad u \in \mathbf{E}, && (5) \\ Y^k &\in \{0,1\}, \ k \in \mathbf{K}, && (6) \end{aligned}$$

where $\alpha \in (0,1)$ is a weight that arbitrates between revenue maximization and the (possibly non profitable objective of) maximizing the number of messages routed. The constant P^o is defined in section II.G.

2.3 The objective function

The objective of the auction, (1), is not pure revenue maximization. Rather, the objective is to simultaneously maximize a weighted combination of the revenue generated by transmitting the requests and the number of requests routed, where the weight $\alpha \in (0,1)$. The first term states that the revenue is given by the quantity routed by each user, d^k, times the decision variable Y^k, $k \in \mathbf{K}$, times the decision variable giving the Smart Market auction price on each link, q_u, $u \in \mathbf{E}$ resulting from the auction process. Recall that the matrix F_u^k selects those links on the pre-determined route for request, k. The second term of the objective, (1), simply maximizes the number of requests routed, given by the sum of the decision variables Y^k, $k \in \mathbf{K}$. It is important to note that the objective function is nonlinear, due to the first term, as both q and Y are decision variables

There is a clear advantage in transforming the model to an equivalent linear program, when possible. Following is a transformation of the nonlinear objective of (1) into an equivalent linear objective with additional linear constraints.

2.4 Linearization of the objective function

Let

$$Z_u^k = d^k q_u F_u^k Y^K \tag{7}$$

be the price that the k^{th} request, $k \in \mathbf{K}$, pays for using the link $u \in \mathbf{E}$.

Then the first term of (1) can be equivalently expressed as a maximization of the weighted sum of all Z_u^k, $u \in \mathbf{E}$, $k \in \mathbf{K}$, that is,

$$(1-\alpha) \sum_{k \in \mathbf{K}} \sum_{u \in \mathbf{E}} Z_u^k \tag{8}$$

along with the following two additional constraints:

$$Z_u^k \leq P^k Y^k, \quad k \in \mathbf{K}, \quad u \in \mathbf{E} \tag{9}$$

and

$$Z_u^k \leq d^k q_u F_u^k, \quad k \in \mathbf{K}, \quad u \in \mathbf{E}. \tag{10}$$

The first constraint, (9), ensures that, if request k is not routed, i.e. $Y^k = 0$, then Z_u^k is also equal to zero. If $Y^k = 1$ then the constraint is not binding.

The second constraint, (10), along with the fact that the sum of the Z_u^k is maximized, ensures that, if $Y^k = 1$, then $Z_u^k = d^k q_u F_u^k$. Indeed, since the sum over all new variables Z_u^k is maximized, when each Z_u^k is not constrained to be null by (9), it will be precisely equal to the left-hand side of (10). Together, constraints (9) and (10) guarantee that $Z_u^k = d^k q_u F_u^k Y^k$.

2.5 Capacity constraint

The constraint (2) ensures that the sum over all requests that are accepted and traverse a given link, $u \in \mathbf{E}$, multiplied by the size of each request, does not exceed the capacity of each link, C_u, for all $u \in \mathbf{E}$.

2.6 Smart Market auction price constraint

The constraint (3) captures the behavior of the Smart Market auction in determining the price that will be set on each link as a function of the requests that will traverse it. A first requirement is that the cost of routing a request be equal or less than the maximum price that the user is willing to pay. That is,

$$\sum_{u \in \mathbf{E}} q_u F_u^k Y^K d^k \leq P^k, \quad k \in \mathbf{K}. \tag{11}$$

The right-hand side of (11) is the bid price of user k and the left-hand side is what user k will actually pay for having his message traverse all the arcs on its pre-determined path.

The terms on the left-hand side of (11) are the individual contributions to network revenue, and since their sum is maximized, this has the effect of forcing the left-hand side as close to P^k as possible, for all $k \in \mathbf{K}$. Since, further, this must hold on each arc for every request k, the value of q_u that will satisfy all the constraints (including the capacity constraints) will be the request having the minimum bid that can traverse the link, where the accepted requests will be those having the highest bid prices.

Nonetheless, this constraint, as was the original objective function, is nonlinear. In order to render the model tractable, we transform it into an equivalent linear constraint, as follows.

2.7 Linearization of the auction price constraint

Let

$$P^o := \max_k \frac{P^k}{d^k} \tag{12}$$

be an upper bound on the Smart Market price of all arcs, q_u, $u \in \mathbf{E}$. Similarly, define

$$d^o := \max_k d^k \tag{13}$$

to be an upper bound on the amount of bandwidth requested, and

$$C^o := mP^o d^o. \tag{14}$$

The constant C^o is an upper bound on the actual price paid by each user $k \in \mathbf{K}$.

Then, using the new constants P^o and C^o, it is possible to reformulate (11) as follows:

$$\sum_{u \in \mathbf{E}} F_u^k d^k q_u \leq P^k Y^k + C^o(1 - Y^k), \quad k \in \mathbf{K}, \tag{15}$$

where the right-hand side of (15) says that, if request k is not routed, i.e. $Y^k = 0$, then the price paid by user k is bounded by C^o, clearly not a binding constraint. On the other hand, if request k is routed, then the price paid by k is bounded by its bid price. Since this must hold on each arc for every request that traverses it, we obtain once again the desired outcome of the Smart Market auction.

2.8 Auction trigger constraint

Following Definition 1, we suppose that, if an arc is uncongested, then the auction price for usage should be zero. Note that this clearly does not preclude the presence of other fixed, or non-auction prices, on the link or for the entire route.

The presence of congestion on a link, as described after Definition 1, cannot be detected by summing the flow on the link and comparing it to the link capacity. Indeed, this traditional definition of capacity does not permit triggering the Smart Market auction.

Instead, we say that a link $u \in \mathbf{E}$ is congested if some message that *would* traverse link u, according to its pre-determined route from the routing matrix F, does not do so. Following this reasoning, we can write that link $u \in \mathbf{E}$ is congested if there exists a $k \in \mathbf{K}$ such that

$$F_u^k(1 - Y^k) = 1, \tag{16}$$

and therefore that link u is *not congested* if all messages that could be routed through link u are indeed routed, that is

$$\sum_{k \in \mathbf{K}} F_u^k(1 - Y^k) = 0. \tag{17}$$

Equation (17) allows us to write the constraint that states that the Smart Market auction on a link $u \in \mathbf{E}$ is only triggered when there is congestion on the link, as in (4). Indeed, the Smart Market price on the link q_u is constrained to be zero in the event that the link is uncongested.

2.9 The linearized Smart Market model

We summarize in this subsection the linearized Smart Market auction and allocation model. [Problem LP]

$$\max_{q,Z,Y} (1 - \alpha) \sum_{k \in \mathbf{K}} \sum_{u \in \mathbf{E}} Z_u^k + \alpha \sum_{k \in \mathbf{K}} Y^k \tag{18}$$

Subject to

$$\sum_{k \in \mathbf{K}} F_u^k Y^K d^k \leq C_u, \; u \in \mathbf{E}, \tag{19}$$

$$\sum_{u \in \mathbf{E}} F_u^k q_u d^k \leq P^o Y^k + C^o(1 - Y^k), k \in \mathbf{K} \tag{20}$$

$$q_u \leq P^o \sum_{k \in \mathbf{K}} F_u^k(1 - Y^k), u \in \mathbf{E} \tag{21}$$

$$Z_u^k - P^k Y^k \leq 0, \quad k \in \mathbf{K}, u \in \mathbf{E}, \tag{22}$$

$$Z_u^k - d^k q_u F_u^k \leq 0, \quad k \in \mathbf{K}, u \in \mathbf{E} \tag{23}$$

$$q_u \geq 0, \quad u \in \mathbf{E}, \tag{24}$$

$$Y^k \in \{0, 1\}, \quad k \in \mathbf{K}, \tag{25}$$

$$Z_u^k \geq 0, \quad u \in \mathbf{E}, k \in \mathbf{K}. \tag{26}$$

2.10 Effect of the parameter α

The following example illustrates the influence of the parameter α on the result of the Smart Market optimization.

Example 2 *Consider the following one-arc network with three requests from A to B:* $\varphi^1(5,1)$, $\varphi^2(15,2)$ *and* $\varphi^3(2,1)$.

As in the first example, the first element in the ordered pair represents the bid price associated with transmitting the message, and the second is the bandwidth requested.

The capacity on arc (A,B) *is infinite. Now, depending on the level of* α, *clearly one*

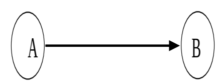

Figure 2: Influence of α

or the other of the two terms in the objective function will be favored. What we illustrate through this example is that there is a cutoff value for α *beyond which we can be certain that all requests that can be routed, given capacity limits, will indeed be routed.*

Letting $\alpha = 0.5$, *we obtain* $Z_{0.5}(1,1,0) = 0.5(15+2) = 8.5$, *and* $Z_{0.5}(1,1,1)) = 0.5(8+3) = 5.5$, *where* $Z_\alpha(Y_1,Y_2,Y_3)$ *is the value of the objective function. Thus, since* $Z_{0.5}(1,1,0) > Z_{0.5}(1,1,1))$ *we observe that routing requests* 1 *and* 2 *only,* $(Y_1 = Y_2 = 1)$, *gives a larger objective value than routing all three requests.*

Letting now $\alpha = 0.95$, *we have that* $Z_{0.95}(1,1,0) = 2.85$ *whereas* $Z_{0.95}(1,1,1) = 3.25$, *and therefore the optimal routing in this case would include all three requests. Whether or not this is the desired objective will depend of course upon the nature of the application...*

2.11 Early-trigger Smart Market

A disadvantage of the policy of Smart Market concept as proposed by Mackie-Mason and Varian [7] is the creation of congestion. This can be explained by the fact that, since the auction should be triggered on a link only when congestion is reached, total revenue will be higher the greater the number of links experiencing congestion.

An Early-trigger Smart Market could however avoid this perverse phenomenon, and thereby allow revenue maximization, without necessarily inducing congestion. This alternative model requires only a minor modification to the Smart Market optimization model presented previously. In particular, it is sufficient to eliminate the auction trigger constraints (4). The Smart Market auction would then ensue on each link regardless of the congestion level on the link, and the requests allowed to pass would depend only on the objective of revenue maximization.

3 Several paths per commodity

In this section, we generalize the model of [Problem P] and [Problem LP] by allowing each request to be routed on a single path selected from a given set of possible paths.

The following additional or modified notation is necessary:

F: The 0-1 indicator matrix of possible paths for each request,

$F_u^{k,j} = 1$ if the j^{th} path of the k^{th} request traverses the link $u \in \mathbf{E}$, and

J: The set of possible paths for each request,

The decision variables are analogous to those of the first model:

$$y^{k,j} = \begin{cases} 1 & \text{if the request } k \text{ is routed by the path } j, \\ 0 & \text{otherwise,} \end{cases}$$

and

q_u: The price fixed by the network manager on the arc $u \in \mathbf{E}$.

Then, the analogous mathematical model for the Smart Market auction and resource allocation with multiple possible paths is given by

[Problem P2]

$$\max_{q,Y} (1-\alpha) \sum_{u\in\mathbf{E}} q_u \sum_{k\in\mathbf{K}} \sum_{j\in\mathbf{J}} d^k q_u F_u^{k,j} Y^{k,j} + \alpha \sum_{k\in\mathbf{K}} \sum_{j\in\mathbf{J}} Y^{k,j} \tag{27}$$

Subject to

$$\sum_{k\in\mathbf{K}} \sum_{j\in\mathbf{J}} F_u^{k,j} Y^{k,j} d^k \le C_u, \quad u \in \mathbf{E}, \tag{28}$$

$$\sum_{u\in\mathbf{E}} q_u F_u^{k,j} Y^{k,j} d^k \le P^k, \quad k \in \mathbf{K}, \quad j \in \mathbf{J} \tag{29}$$

$$q_u \le P^o \sum_{k\in\mathbf{K}} \sum_{j\in\mathbf{J}} F_u^{k,j} (1 - \sum_{j\in\mathbf{J}} Y^{k,j}), \quad u \in \mathbf{E}, \tag{30}$$

$$\sum_{j\in\mathbf{J}} Y^{k,j} \le 1, k \in \mathbf{K}, \tag{31}$$

$$q_u \ge 0, \quad u \in \mathbf{E}, \tag{32}$$

$$Y^{k,j} \in \{0,1\}, \quad k \in \mathbf{K}, \quad j \in \mathbf{J}, \tag{33}$$

where $\alpha \in (0,1)$ is as before. The new constraint (31) states that only one of the multiple paths is to be used by each message.

As before, the objective function (27) can be linearized, whence the following linearized multiple-paths model is obtained:

[Problem LP2]

$$\max_{q,Y} (1-\alpha) \sum_{u \in \mathbf{E}} \sum_{k \in \mathbf{K}} \sum_{j \in \mathbf{J}} Z_u^{k,j} + \alpha \sum_{k \in \mathbf{K}} \sum_{j \in \mathbf{J}} Y^{k,j} \tag{34}$$

Subject to

$$\sum_{k \in \mathbf{K}} \sum_{j \in \mathbf{J}} F_u^{k,j} Y^{k,j} d^k \le C_u, \quad u \in \mathbf{E} \tag{35}$$

$$\sum_{u \in \mathbf{E}} F_u^{k,j} q_u \le P^k Y^{k,j} + C^o(1 - Y^{k,j}), k \in \mathbf{K},\ j \in \mathbf{J}, \tag{36}$$

$$q_u \le P^o \sum_{k \in \mathbf{K}} \sum_{j \in \mathbf{J}} F_u^{k,j} (1 - \sum_{j \in \mathbf{J}} Y^{k,j}) \quad u \in \mathbf{E}, \tag{37}$$

$$\sum_{j \in \mathbf{J}} Y^{k,j} \le 1,\ k \in \mathbf{K}, \tag{38}$$

$$Z_u^{k,j} - P^k Y^{k,j} \le 0, k \in \mathbf{K},\ j \in \mathbf{J}, u \in \mathbf{E}, \tag{39}$$

$$Z_u^{k,j} - d^k q_u F_u^{k,j} \le 0, k \in \mathbf{K},\ j \in \mathbf{J},\ u \in \mathbf{E}, \tag{40}$$

$$q_u \ge 0, \quad u \in \mathbf{E}, \tag{41}$$

$$Y^{k,j} \in \{0,1\}, \quad k \in \mathbf{K}, \quad j \in \mathbf{J}, \tag{42}$$

$$Z_u^{k,j} \ge 0, k \in \mathbf{K},\ j \in \mathbf{J},\ u \in \mathbf{E}. \tag{43}$$

4 Numerical results

In this section, we solve the model on an 11-node and 24-arc network, given in Figure 3. The results in this section were obtained by using the mixed-integer solver of CPLEX 6.0. The numerical experiment aims to evaluate the implication of varying α on the somewhat more complicated example than that of the previous section.

The table below shows the number of requests routed out of 16 requests submitted and the revenue generated therein. We note that, as expected, when α increases, the number of requests routed increases, but since the revenue term is progressively weighted less strongly, total revenue decreases. It is also possible to vary α to obtain the full set of Pareto-optimal allocations.

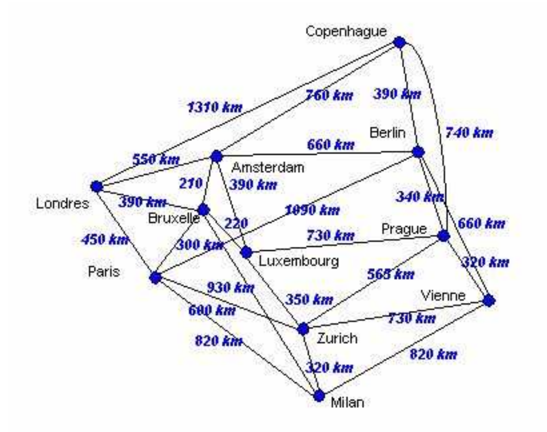

Figure 3: A European network.

α	Revenue	Number of requests accepted
0.5	170	10
0.9	154	11
0.99	97	12

5 Conclusions

We have presented an optimization model that solves a static instance of the Smart Market auction and resource allocation, quantifying through a mathematical program the idea that was put forth in Mackie-Mason and Varian [7]. Several issues, however, remain.

The first issue is that of distributed network computation. Clearly, to be applicable to the Internet, a distributed model of the Smart Market auction and allocation must be available. Contrary to work in traditional resource allocation models, in which some separability can be obtained through Lagrange relaxation of capacity constraints, in the Smart Market auction, the user bid prices are defined over the entire path. Decomposing the auction over nodes or links of the network, therefore, would require some means of decomposing the total bid price on the other bids present at that node.

Other issues include the ability to perform such computations sufficiently rapidly, and being able to refresh the state of the network preiodically as some requests are removed from the auctions.

These issues will most likely need to be evaluated through simulation, and could make use of this Smart Market optimization model at each time-slice of the simulation.

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