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Ruth F. CURTAIN

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COMPARAISON DE COMPENSATEURS DE DIMENSION FINIE POUR DES SYSTEMES DE DIMENSION INFINIE

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Abstract. This paper compares five different approaches to the design of finite-dimensional controllers for linear infinite-dimensional systems. The approaches are varied and include state and frequency domain methods, exact controller designs, controller designs by approximation and robust controller designs.

Résumé. Cet article compare cinq approches différentes de conception de compensateurs de dimension finie pour des systèmes linéaires de dimension infinie. Ces approches variées, à base de représentations d'état ou fréquentielles, conduisent à des calculs de compensateurs exacts ou par approximation et à la conception de compensateurs robustes.

Keywords. Finite-dimensional controller design for infinite-dimensional systems, robust control, H^{∞} control.

Mots clef. Commande H^{∞}, commande robuste, conception de contrôleurs de dimension finie pour des systèmes en dimension infinie.

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A comparison of finite-dimensional controller designs for distributed parameter systems

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Abstract. This paper compares five different approaches to the design of finite-dimensional controllers for linear infinite-dimensional systems. The approaches are varied and include state and frequency domain methods, exact controller designs, controller designs by approximation and robust controller designs.

Keywords. finite-dimensional controller design for infinite-dimensional systems, robust control, H^∞-control.

1 Introduction and Notation

Although there has been much research on the stabilization of distributed parameter systems, the theory usually produces infinite-dimensional controllers. For example, the theory of linear quadratic control produces a state-feedback law in which both the state and the gain operator are infinite-dimensional. To produce an implementable finite-dimensional controller entails approximating both this gain operator and the state, which is a complicated numerical procedure and sometimes (in the case of unbounded inputs) difficult to justify theoretically. The continuing popularity of this approach is somewhat surprising, since there exist several exact methods of designing finite-dimensional controllers for infinite-dimensional systems. The first exact finite-dimensional compensator design was a state-space approach in 1981 (Schumacher, [63] [64]) and since then, there have been many others, (for example, Sakawa [60], Curtain [11]). More recently, various frequency domain theories for designing finite-dimensional controllers which are robust to certain types of uncertainties or which satisfy other additional performance objectives have been developed. Some of these lead directly to a finite-dimensional

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controller which is guaranteed to stabilize the infinite-dimensional system (Curtain [13], Curtain and Glover [15]), while others produce an infinite-dimensional controller (see for example, Foias and Tannenbaum [25]–[30], Flamm [21], Georgiou and Smith [31]). All of the frequency-domain design approaches involve approximating irrational transfer functions by rational ones.

In this paper we survey and compare five approaches to designing finite-dimensional controllers.

Section 2 *Approximation of infinite-dimensional deterministic LQG controllers by numerical approximation of infinite-dimensional Riccati equations.*

Section 3 *Exact state-space design of finite-dimensional compensators.*

Section 4 *Finite-dimensional compensator designs based on reduced order models.*

Section 5 *Robust controller designs with a-priori robustness bounds.*

Section 6 *Finite-dimensional H^∞-controllers for distributed systems.*

Section 7 *Concluding remarks.*

But first we introduce the notation we use and give some basic definitions of stability.

$\mathbf{H}^\infty$ The class of functions of a complex variable which are bounded and holomorphic in $Re(s) > 0$.

$\mathbf{H}^\infty_-$ The class of functions of a complex variable which are bounded and holomorphic in $Re(s) > -\varepsilon$, for some $\varepsilon > 0$.

$\mathbf{H}^\infty_- + \mathbf{R}(s)$ The class of transfer functions which are the sum of one in $\mathbf{H}^\infty_-$ and a proper rational one.

$M(Z)$ Transfer matrices of any size whose components are elements of Z.

$\mathbf{H}^\infty/\mathbf{H}^\infty$ The quotient space of $\mathbf{H}^\infty$.

We are concerned with the problem of stabilizing a plant with transfer matrix $G \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ by a controller with transfer matrix $K \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ in the following feedback configuration.

For systems described by transfer functions the appropriate concept of stability is as follows.

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Definition 1 *The feedback systems (G, K) of figure 1 with $G, K \in M(H^\infty/H^\infty)$ is said to be* input-output stable *if and only if (a) and (b) are satisfied, where*

(a) $\inf_{\mathbf{C}^+} | \det(I - GK)(s)| > 0$

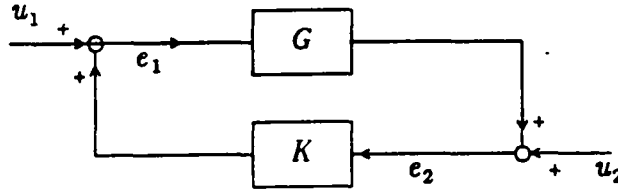*(b)* $S = (I - GK)^{-1}, \ KS, \ I - KSG \in M(H_-^\infty)$

Figure 1: Input-output stability

We remark that, unfortunately, there are many different terms describing the type of stability in Definition 1, for example, internal and external stability can both be found in the literature. Usually, our plant and controller will be restricted to the class $M(H_-^\infty + R(s))$. Condition (a) ensures that all the maps from $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ to $\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$ are well-posed and condition (b) requires that all the paths between the inputs $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ to the outputs $\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$ are stable. In fact, if (a) holds, we have

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{bmatrix} (I - GK)^{-1} & K(I - GK)^{-1} \\ (I - GK)^{-1}G & (I - GK)^{-1} \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{1}$$

Very often we are given a state-space description $\sum(A, B, C, D)$ of the plant, where Z, U and Y are separable Hilbert spaces, A is the infinitesimal generator of the strongly continuous C_0-semigroup $T(t)$ on Z, $D \in L(U, Y)$, $C \in L(Z, Y)$ and $B \in L(U, Z)$. The transfer function of this state linear system is $G(s) = D + C(sI - A)^{-1}B$. In state-space descriptions the appropriate concept of stability is exponential stability.

Definition 2 $\sum(A, B, C, D)$ *is* exponentially stable *if $T(t)$ satisfies $\|T(t)\|_{L(Z)} \leq Me^{-\alpha t}$ for certain constants $M, \alpha > 0$.*

$\sum(A, B, C, D)$ *is* exponentially stabilizable *if there exists $F \in L(Z, U)$ such that $A + BF$ generates an exponentially stable C_0-semigroup.*

$\sum(A, B, C, D)$ *is* exponentially detectable *if there exists $G \in L(Y, Z)$ such that $A + GC$ generates an exponentially stable C_0-semigroup.*

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In general, input-output stability is weaker than exponential stability, but for certain systems they are equivalent.

Theorem 3 *(Jacobson and Nett [40]). Suppose that $\sum(A, B, C, D)$ is exponentially stabilizable and detectable and that $U = \mathbf{R}^m, y = \mathbf{R}^p$. Then $G(s) = D + C(sI - A)^{-1}B \in M(\mathbf{H}_\infty^\infty)$ if and only if A generates an exponentially stable C_0-semigroup.*

The state-space approach to stabilizing a state linear system $\sum(A, B, C, D)$ is to find a dynamic controller described as a state linear system $\sum(F, G, N, M)$ on the Hilbert space W, i.e.

$$\dot{w} = Fw + Gy$$
$$u = Nw + My \tag{2}$$

such that the closed loop system operator A

$$A_{cl} = \begin{pmatrix} A + BMC & BN \\ GC & F \end{pmatrix} \tag{3}$$

generates an exponentially stable C_0-semigroup on $Z \oplus W$. It is desirable that the state-space of the compensator be finite-dimensional. However, the most well-known state-space design, the so-called *LQG* design produces an infinite-dimensional compensator.

The linear quadratic (*LQG*) control design is based on the following theoretical results for Riccati equations.

Theorem 4 *(Curtain and Pritchard [16]). Suppose that $\sum(A, L, C, \phi)$ and $\sum(A, B, M, \varphi)$ are exponentially stabilizable and detectable state linear systems. Then there exist unique solutions to the following Riccati equations in the class of self-adjoint, nonnegative operators $Q, P \in L(Z)$*

$$A^*Qx + QAx - QBR^{-1}B^*Qx + M^*MX = 0, \tag{4}$$

where $x \in \mathbf{D}(A)$ and $R, R^{-1} \in L(Z)$.

$$APz + PA^*z - PC^*S^{-1}CPz + LL^*z = 0, \tag{5}$$

where $z \in \mathbf{D}(A)$ and $S, S^{-1} \in L(Z)$. Moreover, $A - BR^{-1}B^*Q$ and $A - PC^*S^{-1}C$ generate exponentially stable semigroups.

Just as for finite-dimensional systems, it can be readily verified that an infinite-dimensional *LQG* controller exists.

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Theorem 5 *Suppose that* $\sum(A, B, M, \phi)$ *and* $\sum(A, L, C, \phi)$ *are exponentially stabilizable and detectable state-linear systems. Then the following dynamic controller exponentially stabilizes the state linear system* $\sum(A, B, C, \phi)$.

$$\dot{w} = (A - PC^*S^{-1}C)w + PCS^{-1}y + Bu$$
$$u = -R^{-1}B^*Qw \tag{6}$$

where Q *and* P *are the unique solutions to (4) and (5) respectively.*

Notice that the transfer function of the LQG controller is

$$K(s) = -R^{-1}B^*Q(sI - A + PC^*S^{-1}C + BR^{-1}B^*Q^{-1})PCS^{-1}, \tag{7}$$

and the closed loop operator is given by

$$A_{cl} = \begin{bmatrix} A & -R^{-1}B^*Q \\ A - PC^*S^{-1}C - BR^{-1}B^*Q & PCS^{-1}C \end{bmatrix}. \tag{8}$$

It can be shown that A_{cl} is similar to the operator $\begin{pmatrix} A - PC^*S^{-1}C & 0 \\ BR^{-1}B^*Q & A - BR^{-1}B^*Q \end{pmatrix}$

and so the closed loop system is exponentially stable. If $G(s) = C(sI - A)^{-1}B$ and $K(s)$ is given by (7), then K also stabilizes G in the input-output sense of Definition 1. Theorems 4 and 5 hold for infinite-dimensional input and output spaces, but in the applications these will be finite-dimensional. In this situation, the assumption of exponential stabilizability on detectability implies that the spectrum of the system operator A can be decomposed into a stable part in $Res < -\varepsilon$ for some $\varepsilon > 0$ and an unstable part in $Res \geq 0$ comprising at most finitely many unstable eigenvalues (Jacobson and Nett [40]). This result depends on the bounded nature of B and C and need not hold if B and C are very unbounded.

Natural generalizations of Theorem 4 and 5 to the case of unbounded B and C exist (see Lasiecka and Triggiani [43] for references); we have only quoted the results for bounded B and C to keep the exposition uncluttered.

2 Approximation of infinite-dimensional deterministic LQG controllers: Numerical approximation of infinite-dimensional Riccati equations

One of the early successes of infinite-dimensional systems theory was the solution of the linear quadratic LQ control problem (Lions [45] and Bensoussan et al [5]) and

this topic dominated the literature for decades. In particular, the case of boundary control for p.d.e. systems and delayed control for delay systems presented difficult mathematical problems and was the subject of many treatises. As stated in Theorem 4, the solution to the linear quadratic control problem yields a state-feedback controller $u = -BR^{-1}B^*Qz$, where Q is the solution of the infinite-dimensional Riccati equation (4). The numerical approximation of infinite-dimensional Riccati equations like (4) has been the subject of many papers, most of which assume bounded input operators. A comprehensive survey of infinite-dimensional Riccati equations for p.d.e. systems, including those with unbounded input operators can be found in Lasiecka and Triggiani [43]. Although there are fewer results on numerical approximations with unbounded inputs, the underlying idea is the same. One approximates the underlying system operator A (p.d.e. or delay type) by a suitable finite-dimensional scheme and one gives sufficient conditions on the approximations to obtain strong convergence of the finite-rank approximating solutions Q_n to the infinite-dimensional operator solution Q of the Riccati equation (4). Although there exist general sufficient conditions to ensure convergence (see Kappel and Salamon [41], Itô [38]), the sharpest convergence results are obtained by specializing to the particular type of system involved. Lasiecka and Triggiani [43] distinguish between two classes of p.d.e. systems.

The first class corresponds to analytic semigroups such as those arising in parabolic p.d.e's and the numerical theory for this class provides optimal convergence results under mild assumptions on the approximations; this allows for all spline, modal or spectral approximations, for example. As well as obtaining the strong convergence of the finite-rank approximating solutions Q_n to the operator solution Q of the Riccati equation (1.4), and the strong convergence of the approximating feedback gain $B_n^*Q_n$ to the optimal infinite-dimensional gain B^*Q, one can prove that $\exp(A - BB_n^*Q_n)t$ converges strongly to $\exp(A - BB^*Q)t$ as $n \to \infty$, where by $\exp(At)$ we mean the C_0-semigroup generated by A. This last property is crucial, because it shows that the approximating state-feedback controller will stabilize the original infinite-dimensional system for sufficiently large n, and this is what one would implement in practice.

In the case of the second class, which includes hyperbolic p.d.e.'s, and delay equations, the numerical theory is less satisfactory. While it is possible to give sufficient conditions under which Q_n converge strongly to Q as $n \to \infty$ and $B_n^*Q_n$ converge strongly to B^*Q as $n \to \infty$, it is more difficult to verify that natural approximating schemes actually satisfy these conditions. Moreover, it is not clear whether or not the approximating state-feedback controller will stabilize the original infinite-dimensional system even for sufficiently large n. In the case of delay equations, the situation is better than in the hyperbolic case, since the sufficient conditions on the approximations are more readily verified (see Kappel and Salamon [41], Itô [38]). However, the sufficient conditions are more restrictive than in the parabolic case (see Burns et al [9]).

Of course, even if one has a sequence of finite-rank gains which converge strongly to the optimal gain, implemention of the LQ control law requires knowledge of the whole state. It is impossible to measure an infinite-dimensional state and so it is necessary to develop a controller design based on a finite-dimensional measurement. While there is a stochastic LQG theory (Curtain and Pritchard [16]), most attempts at designing finite-dimensional compensators have been deterministic, and there have not been very many of these either. The classic approach is to combine a LQ control law with a state-observer - a deterministic LQG design approach (see Theorem 5). In principle, this reduces to two Riccati approximation schemes: one for the control Riccati equation (4) and the other for the "filter" Riccati equation (5). This was the approach taken in Itô [38] where sufficient conditions were given for the convergence of the finite-dimensional approximating compensators to the infinite-dimensional one. Moreover, he showed that one could always design such an approximating finite-dimensional compensator which produces an exponentially stable closed loop operator (8). The assumptions were that A, B, C have compact resolvent, B and C be bounded and that the approximations satisfy certain uniform stabilizability and detectability assumptions. They cover parabolic and retarded systems with Galerkin type approximation schemes and so represent an improvement on earlier results in Balas [1]. An earlier detailed analysis of a specific application of the LQG design, to flexible systems in Gibson and Adamian [33] reveals that if one wishes to do a realistic engineering filter design, it might be difficult to satisfy all of the assumptions one needs to guarantee convergence. In this example, a physically meaningful choice of the state noise weights L in (5) did not satisfy their theoretical assumptions and so they could not prove convergence. However, their numerical results indicated that things did converge nicely.

More recently, a new approach to the convergence of approximations was taken in Morris [49]. Using coprime representations of systems, she was able to prove convergence of the approximating systems in the graph topology. However, this is a weaker form of convergence and only implies that the optimal controller stabilizes the approximations in an input-output sense (see Definition 1).

Summarizing, we can make the following general comments on finite-dimensional compensator design via approximation of infinite-dimensional, deterministic LQG compensators (6). At least for the case of bounded control and measurement, there is a fairly complete theory of convergence of the approximations and sufficient knowledge about the appropriate choice of the approximations for parabolic, hyperbolic and delay systems. Care often needs to be exercised in the choice of the weights and the approximations to satisfy the theoretical conditions for specific applications, but even if not all the theoretical conditions are satisfied, in practice, the LQG design usually leads to a finite-dimensional controller which stabilizes the original infinite-dimensional system. This notwithstanding, this design method is not recommended for nonexperts (see Burns et al [9]) and research is still contin-

uing into important properties of the approximations, such as the stabilizability and detectability properties of the approximations (see Burns and Peichl [10]) and convergence properties of approximationa of Riccati equations for hyperbolic p.d.e's (see Lasiecka and Triaggiani [43]). Moreover, most of the theory on finite-dimensional LQG compensator design has been restricted to the case of bounded B and C operators (see Itô [38]) and further research on the case of unbounded B and C operators is needed. Proving that the Riccati equations can be approximated is not sufficient; one needs to prove that there exists a finite-dimensional approximating controller which stabilizes the original infinite-dimensional system. It is surprising how much research has been done on approximating the solutions of LQ Riccati equations and how little has been done on the practical question of a finite-dimensional LQG compensator design.

3 Exact state-space design of finite-dimensional compensators

Surprisingly enough, it took a long time to realize that it was possible to directly design a finite-dimensional compensator for an infinite-dimensional system without approximating anything. The first design in Schumacher [63], [64] was a geometric approach and involved very simple numerical calculations. The assumptions were basically that the inputs and output operators B and C were bounded, the system operator A should generate a C_0-semigroup, $\sum(A, B, C)$ should be stabilizable and detectable and the eigenvectors of A should span the state-space. It was this last assumption which was restrictive. While most p.d.e. systems will satisfy this assumptions, many retarded systems will not. The finite-dimensional compensator designs in Sakawa [60] and Curtain [11] are similar to each other and use a type of modal approach in designing the compensator; they also need the assumption the the eigenvectors span the state-space. A completely different approach was taken in Bernstein and Hyland [4] who presented a theory for designing a fixed order, finite-dimensional compensator. They do not need the assumptions on the eigenvectors, but they do need to assume the existence of solutions of nonlinear "optimal projection equations" and they need to solve these numerically. This is a nontrivial task, in contrast to the other three design approaches which can be carried out in using PCMATLAB routines. Finally, we mention that the LQG finite-dimensional compensator design by Itô [38] we have already discussed in Section 2 could in some sense be considered as an exact state-space compensator design.

4 Finite-dimensional compensator designs based on reduced order models

Basic engineering methodology is to design a compensator based on an approximate model of the true system: the reduced order model. In our application the true system is infinite-dimensional, the approximation is finite-dimensional and the compensator will be finite-dimensional. However, we apply the finite-dimensional controller to the infinite-dimensional system, hoping that it will stabilize, although we are only guaranteed that it will stabilize the reduced order model. Usually it works fine, but it can happen that it fails to stabilize the original infinite-dimensional systems. This phenomenon was demonstrated in Balas [2] and was christened as the "spillover" problem. In Balas [1], [3] he suggested various choices of reduced modal or Galerkin approximations of the infinite-dimensional state-space and a list of sufficient conditions which were rather complicated to check. An elegant existence result for finite-dimensional stabilizing compensators for a class of irrational transfer functions based on model reduction was proved in Nett [50], [51]. In Bontsema and Curtain [7] an explanation of and a remedy for the spillover problem were given in terms of robustness properties of the plant. In fact, it is a simple corollary of vthe theory of additively robust controllers which we will discuss in Section 5. The following lemma is an easy consequence of the theory of robust stabilization under additive perturbations which we shall discuss in more detail in Section 5.

Lemma 6 *Suppose the plant G is strictly proper and has a decomposition $G = G_u + G_s$, where $G_s \in M(\mathrm{H}^\infty_-)$ and G_u is a strictly proper, rational transfer matrix with all its poles in $Re(s) > 0$. Let $G_f = G_u + G_a$, where G_a is an $\mathbf{L}_\infty$-approximation for G_s. If K_f stabilizes the reduced order model G_f, then K_f also stabilizes G provided that*

$$\|G - G_f\|_\infty = \|G_a - G_s\|_\infty < \|K_f(I - G_fK_f)^{-1}\|_\infty^{-1}$$

Furthermore, a stabilizing finite-dimensional compensator K_f for G_f and G exists provided that

$$\|G - G_f\|_\infty < \sigma_{min}(G_u^\dagger).$$

($\sigma_{min}(G_u^\dagger)$ is the smallest Hankel singular value of the stable rational transfer function $G_u^\dagger(s) = G_u^\dagger(-s)$. It gives an upper bound to the $\mathbf{L}_\infty$-approximation error.)

The class of infinite-dimensional systems covered by this lemma is very large. It includes systems with a state-space representation $\sum(A, B, C, D)$ in which B and C may even be unbounded. The system should be "well-posed" in the sense that it should have well-defined controllability, observability and input-output maps,

but these are very mild requirements (see Curtain and Weiss [18]. The essential restriction is that the operator A should have only finitely many unstable eigenvalues, but, as remarked after Theorem 1.5, this is also necessary for the LQG theory in the case of bounded, finite-rank inputs and outputs. In addition, this class includes systems which do not have a nice state-space representation and for which Riccati equations are not well-posed. For example, it includes the important class of systems of the type $G(s) = e^{-\alpha s} G_f(s)$, where $G_f(s)$ is a proper, rational transfer function (see Curtain [14] for more examples). Lemma 6 suggests the following

Reduced order model design algorithm 1

Step 1 Find a reduced order model G_f so that $\|G_s - G_f\|_\infty < \varepsilon$.

Step 2 Design your favorite compensator K_f (e.g.LQG) for G_f and calculate its robustness margin $\|K_f(I - G_f K_f)^{-1}\|_\infty^{-1}$.

Step 3 Check whether $\|G_s - G_f\|_\infty < \|K_f(I - G_f K_f)^{-1}\|_\infty^{-1}$. If not, go to step 1 and obtain a more accurate reduced-order model G_f and repeat the loop.

Step 4 If the inequality checks out, K_f stabilizes G.

The only numerical approximation occurs in step 1 and there are many excellent techniques for these approximations; see for example Glover, Curtain and Partington [34], Glover, Lam and Partington [35], [36], Gu, Khargonekar, Lee [37], Zwart, Curtain, Partington and Glover [70] and Partington, Glover, Zwart and Curtain [56]. This is a numerically simpler design approach than the one approximating solutions of infinite-dimensional Riccati equations discussed in Section 2. In addition, the number $\sigma_{min}(G_u^\dagger)$ gives a good indication of the order of the reduced-order model required. The $\mathbf{L}_\infty$–approximation error should be less than $\sigma_{min}(G_u^\dagger)$. A small value of $\sigma_{min}(G_u^\dagger)$ indicates, a priori, that a low order reduced order model could lead to spillover problems.

The choice of the control design for the reduced order model G_f is open, provided only that its robustness margin is greater than the model error (see step 3). In particular, one could use an LQG design to produce K_f and so obtain an alternative LQG design which produces a finite-dimensional controller which is guaranteed to stabilize the original infinite-dimensional system. This design is numerically simpler than the one discussed in Section 2, as it bypasses the problems of conditions on the state-space approximations.

Finally, a word about exponential stability of the closed loop system and stability in the sense of Lemma 2: in general, this type of input-output stability is weaker than exponential stability. However, for large classes of state-space realizations it will imply exponential stability of the closed loop system , not only these

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with bounded input and output operators (see Theorem 3), but several classes with unbounded input and output operators as well(see Curtain [12], Yamamoto [66], Logemann [47].

5 Robust controller designs with a-priori robustness bounds

Since the infinite-dimensional model is, at best, an approximate one, it is highly desirable that the controllers will also stabilize perturbations of the nominal model, the *robustness property*. Depending on the class of perturbations chosen and the topology chosen to measure the size of the perturbations, one obtains different definitions and measurements of robustness. A minimal requirement is that of robustness with respect to the graph topology, (Vidyasagar [65], chapter 7), but this says little about the size and nature of the perturbations which retain stability. Although deterministic LQG controllers usually give good results, it is known that even for finite-dimensional systems they are not always robust to small perturbations. It is well-known that certain stabilizing control laws for flexible beam models are destabilized by small delays (see Datko et al [19] Curtain [14], Section 8). Numerical results carried out on an example using the design in Curtain [11] discussed in Section 3 indicated that little robustness is guaranteed by this method either. On the other hand, it is clear that Lemma 6 is a type of robustness result. It says that there is robustness with respect to stable additive perturbations in the plant and it also gives an indication of the robustness tolerance in $\|K_f(I - G_fK_f)^{-1}\|_\infty^{-1}$.

In this section, we discuss two simple finite-dimensional robust controller designs for infinite-dimensional systems with finitely many unstable poles, which guarantee a degree of robustness and give information about the size and nature of perturbations which retain stability.

The first is that of *robust stabilization of a plant G under additive perturbations* Δ.

G and Δ are assumed to satisfy the following assumptions

(5.1) G is regarded as the nominal plant and is assumed to be a strictly proper transfer matrix in $M(\mathbf{H}^\infty + \mathbf{R}(s)$ and moreover, it is assumed to have no poles on the imaginary axis.

(5.2) The permissible perturbations Δ are also in $M(\mathbf{H}^\infty + \mathbf{R}(s))$ and they are such that G and $\Delta + G$ have the same number of unstable poles in $Re(s) \geq 0$.

Definition 7 *The feedback system* (G, K) *of Figure 1 with* $G, K \in M(\mathbf{H}^\infty + \mathbf{R}(s))$ *is said to be additively robustly stable with robustness margin* ε *if the feedback*
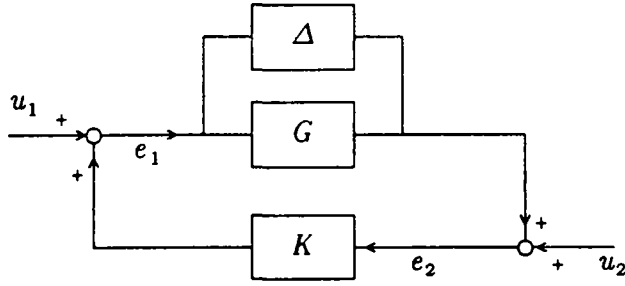
Figure 2: Additive robust stability

system of Figure 2 is input-output stable for all G and Δ satisfying (5.1) and (5.2) and $\|\Delta\|_\infty < \varepsilon$.

The assumptions (5.1) and (5.2) are needed to apply a Nyquist argument and the following result from Logemann [46] is a generalization of first version for infinite-dimensional systems in Curtain and Glover [15].

Theorem 8 *If $G \in M(\mathbf{H}_-^\infty + \mathbf{R}(s))$ and (5.1) is satisfied, given an $\varepsilon > 0$ there exists a compensator K which stabilizes $G + \Delta$ for all Δ satisfying (5.2) and $\|\Delta\| < \varepsilon$ if and only if*

$$\sigma_{min}(G_u^\dagger) \geq \varepsilon \tag{9}$$

where $\sigma_{min}(G_u^\dagger)$ is as in Lemma 6.

In addition to this existence result, there are also explicit formulas for a controller K which achieves the maximum robustness margin $\sigma_{min}(G_u^\dagger)$. These depend on the stable part of G, G_s, and so this K is *infinite-dimensional*, which is undesirable for applications. However, is one replaces G_s by a finite-dimensional $\mathbf{L}_\infty$-approximation, G_s^k, say, then one can design a finite-dimensional controller (of the order of k plus the McMillan degree of G_u) which has a robustness margin of at least $\sigma_{min}(G_u^\dagger) - \|G_s - G_s^k\|_\infty$. So in combination with a theory for good $\mathbf{L}_\infty$-approximations, we have a practical design technique for additively robust finite-dimensional controllers for a large class of infinite-dimensional systems.

Reduced-order model design algorithm 2

Step 1 Calculate $\sigma_{min}(G_u^\dagger)$

Step 2 Approximate G by $G_s^k + G_u$ so that $\|G_s^k - G_s\|_\infty << \sigma_{min}(G_u^\dagger)$.

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Step 3 Find the finite-dimensional additively robustly stabilizing compensator K_f which stabilizes $G_s^k + G_u$.

Step 4 K_f additively robustly stabilizes G with a robustness margin of at least

$$\sigma_{min}(G_u^\dagger) - \|G_s^k - G_s\|_\infty.$$

The same remarks made in section 4 concerning the $\mathbf{L}_\infty$-approximation techniques and the class of systems covered apply here too. The advantage of this approach over the reduced order model design approach 1 is that we can give an a priori robustness margin of $\sigma_{min}(G_u^\dagger) - \|G_s^k - G_s\|_\infty$ and design a finite-dimensional controller to achieve this. Applications of this technique to design robustly stabilizing finite-dimensional controllers for a class of flexible systems can be found in Bontsema [6].

The second robust controller design is applicable to an even wider class of systems, including those with infinitely many unstable poles.

Robustness stabilization under normalized coprime factor perturbations

The plant G and the controller are allowed to be in $M(\mathbf{H}^\infty/\mathbf{H}^\infty)$; the elements of the transfer matrix are quotients of elements in $\mathbf{H}^\infty$. In addition, it is assumed that G has a normalized coprime factorization.

Definition 9 *If $G \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$, then $\tilde{M}^{-1}\tilde{N}$ is called a left-coprime factorization of G if and only if*

(i) $\tilde{M}$ and $\tilde{N} \in M(\mathbf{H}^\infty)$

(ii) $G = \tilde{M}^{-1}\tilde{N}$

(iii) $\tilde{M}$ and $\tilde{N}$ are left coprime in the sense that there exist X and Y in $M(\mathbf{H}^\infty)$ such that

$$\tilde{N}Y - \tilde{M}X = I \tag{10}$$

$\tilde{M}^{-1}\tilde{N}$ *is called a* normalized left coprime factorization of G *if it is a left coprime factorization and*

$$\tilde{N}(j\omega)\tilde{N}^t(-j\omega) + \tilde{M}(j\omega)\tilde{M}^t(-j\omega) = I \text{ for all real } \omega \tag{11}$$

The definition of a right-coprime factorization is analogous.

It is known that $G \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ has a normalized left-coprime factorization if and only if G is stabilizable in the sense of Definition 1 which can be extended in the obvious way to cover G, $K \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$. (see Smith [61].
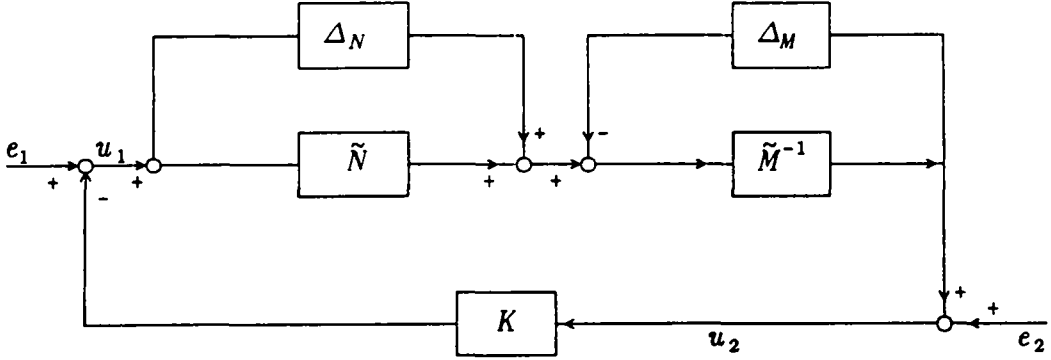
Figure 3: Left coprime factor perturbations

We consider the robustness of stability with respect to perturbations of the form

$$G_\Delta = (\tilde{M} + \Delta_N)^{-1}(\tilde{N} + \Delta_N) \tag{12}$$

where $G = \tilde{M}^{-1}\tilde{N}$ is a left-coprime factorization of G and $\Delta_M, \Delta_N \in M(\mathbf{H}^\infty)$.

The robust design objective is to find a feedback controller $K \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ which stabilizes not only the nominal system G, but also the family of perturbed systems defined by

$$\mathcal{G}_\epsilon = \{G_\Delta \in M(\mathbf{H}^\infty/\mathbf{H}^\infty) : G_\Delta = (\tilde{M} + \Delta_M)^{-1}(\tilde{N} + \Delta_N)$$
$$\text{such that } \Delta_M, \Delta_N \in M(\mathbf{H}^\infty) \tag{13}$$
$$\text{and } \|[\Delta_M, \Delta_N]\|_\infty < \epsilon\}(\text{see Figure 3}).$$

This leads to the following definition of factor robust stability.

Definition 10 *Suppose that the system $G \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ has the normalized left-coprime factorization of Definition 9. Then the feedback system $(\tilde{M}, \tilde{N}, K, \epsilon)$ of Figure 3 is factor robustly stable if and only if (G_Δ, K) is input-output stable for all $G_\Delta \in \mathcal{G}_\epsilon$. If there exists a K such that $(\tilde{M}, \tilde{N}, K, \epsilon)$ is factor robustly stable, then G is said to be factor robustly stabilizable with factor robustness margin ϵ.*

This problem has an elegant solution.

Theorem 11 *(Georgiou and Smith [31]). If $G \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ has a normalized left-coprime factorization, then $(\tilde{M}, \tilde{N}, K, \epsilon)$ is factor robustly stable if and only if*

$$\epsilon \leq (1 - \|[\tilde{N}, \tilde{M}]\|_H)^{1/2}$$

where $\| \cdot \|_H$ represents the Hankel norm of the system $[\tilde{M}, \tilde{N}]$. The maximum factor robustness margin is given by

$$\epsilon_{max} = (1 - \|[\tilde{N}, \tilde{M}]\|_H)^{1/2}.$$

14

In general, explicit formulas for normalized coprime factorizations and hence for the optimal controllers are not available. In Georgiou and Smith [32] they illustrate a technique for obtaining a closed form formula for the optimal controller for single-input single-output systems of the type $e^{-hs}g(s)$, where $g(s)$ is rational. Moreover, for proper rational transfer functions and a large class of infinite-dimensional systems they are given in terms of the solutions of algebraic Riccati equations (see MacFarlane and Glover [48], Curtain [13], Curtain and Van Keulen [17]). In these cases explicit formulas for robustly stabilizing controllers are also known. While it is nice to have an elegant mathematical solution for infinite-dimensional systems in terms of algebraic Riccati equations, these are not always a good basis for designing finite-dimensional robustly stabilizing controllers. An alternative is for example the following approximation results from Bontsema and Curtain [8] for plants in $G \in M(\mathbf{H}^{\infty}_{-} + \mathbf{R}(\mathbf{s}))$.

Lemma 12 *Suppose that $G \in M(\mathbf{H}^{\infty}_{-} + \mathbf{R}(s))$ has the decomposition $G = G_f + G_s$, where $G_s \in M(\mathbf{H}^{\infty}_{-})$ and G_f is a rational transfer function (not necessarily stable). G_f is a reduced order model for G such that $\|G - G_f\|_\infty < \mu < \varepsilon_{max}$. If K_f stabilizes G_f with a factor robustness margin of $\varepsilon > \mu$, it stabilizes G with the factor robustness margin of at least $\varepsilon - \mu$*

This yields the following algorithm for designing a robust finite-dimensional compensator.

Reduced order model design algorithm 3

Step 1 Find a reduced-order model G_f for G with an $\mathbf{L}_\infty$–error $\|G - G_f\|_\infty < \mu$.

Step 2 Obtain a minimum realisation for G_f and calculate the maximal factor robustness margin for G_f (This involves solving finite-dimensional algebraic Riccati equations), say ε_{max}.

Step 3 Compare ε_{max} and μ. If $\varepsilon_{max} - \mu$ is acceptable as a robustness margin, go to step 4. Otherwise, return to step 1 and find a reduced-order model with a smaller $\mathbf{L}_\infty$–error.

Step 4 Find a factor robustly stabilizing compensator K_f for G_f with robustness margin $\varepsilon >> \mu$.

Step 5 K_f factor robustly stabilizes G with robustness margin $\varepsilon - \mu$.

This is analogous to the reduced-order model design 2 for additive perturbations. It is applicable to a wider class of systems, since (5.1) and (5.2) do not need to be satisfied here. In particular, G can have finitely many poles on the imaginary axis and G and G_Δ do not need to have the same number of unstable poles.

It includes systems with an exponentially stabilizable and detectable state-space realization $\sum(A,B,C,D)$, where B and C are allowed to be unbounded. For examples of systems in $M(\mathbf{H}_-^\infty + \mathbf{R}(s))$ (see Curtain [14]).

The design approach 3 was applied to a flexible beam model with parameter uncertainty in Bontsema and Curtain [8]. This study was a follow-up of that on additively robustly stabilizing compensators in Bontsema [6]. Both the additive and the factor theories allow one to measure the perturbations allowed (in the $\mathbf{L}_\infty$-norm and in the gap metric) and this was done for parameter variations in a flexible beam model. The reduced order model design approach 3 allowed the largest range of parameter variations in the model, both in the theoretically guaranteed robustness margin and in the actual robustness margin. This is to be expected, as it admits a larger class of perturbations; in particular, G and G_Δ do not need to have the same number of unstable poles. This very same reduced order model design algorithm 3 together with the same flexible example (both from Bontsema and Curtain [8] were given in Zhu [68] to illustrate his new results on convergence of approximations in the gap topology. An approximation approach designed specifically for delay systems is given in Partington and Glover [57] and Partington [58].

Finally, we remark that both robust controller designs in this section can be modified to allow for frequncy weighting. The idea is to replace the plant G by the frequency weighted plant $G_w = W_1 G W_2$ for suitable proper and invertible rational weights W_1 and W_2 and to design a robust controller K_W for G_W. $W_1 . K_W W_2$ is then a robust controller for G (see Curtain and Glover [15] and MacFarlane and Glover [48] for details).

6 Finite-dimensional $\mathbf{H}^\infty$–controllers for distributed systems

Although there has been considerable interest in $\mathbf{H}^\infty$-control design for distributed systems (Flamm and Mitter [23], Foias, Tannenbaum and Zames [29],[30], Foias and Tannenbaum, [25]–[27], Flamm [21], Flamm and Yang, [22], Ozbay et al [53], Ozbay and Tannenbaum [54],[55], Khargonekhar et al [42]), the $\mathbf{H}^\infty$–controllers are infinite-dimensional and often improper as well. While one can always approximate in an ad-hoc way as in Lenz et al [44], it is clearly desirable to develop a theory for finite-dimensional controller designs which are guaranteed to achieve similar objectives. That this is not necessarily achievable by approximating the plants or the controllers is convincingly illustrated in Smith, [62]. There he gives examples of $\mathbf{H}^\infty$–control problems which are not continuous to perturbations in the plant.

In particular, the problem of weighted sensitivity minimization studied in [21]–[23], [29], [30] mentioned above is not always continuous to perturbations in the

plant. For a stable plant this reduces to the following H^∞-minimization problem

$$\min_{Q \in M(\mathbf{H}^\infty)} \|W_1(I - GQ)\|_\infty = \mu(G), \qquad (14)$$

where $G \in M\mathbf{H}^\infty)$ is the plant, W_1 is a rational transfer function in $M(\mathbf{H}^\infty)$ and $K = Q(I - GQ)^{-1}$ is the stabilizing controller achieving (6.1). In general, it is not true that if $G_n \in M(\mathbf{H}^\infty)$ is an approximating sequence to G in the H^∞-norm ($\|G - G_n\|_\infty \to 0$ as $n \to \infty$), then $\mu(G_n) \to \mu(G)$. Even if $\mu(G_n) \to \mu(G)$, the corresponding stabilizing controllers $K_n = Q_n(I - G_nQ_n)^{-1}$ need not converge to $K = Q(I - GQ)^{-1}$. Consequently, it is not clear that K_n will stabilize G for sufficiently large n.

In Section 5 we have discussed two special examples of H^∞-control problems which do have all these nice properties and more. However, for other H^∞-problems this remains an open problem. In Engelbert and Smith [20] they give sufficient conditions under which the 4-block problem

$$\mu(G, W_i) = \inf_{stab\ K} \left\| \begin{pmatrix} W_2 \\ W_3K \end{pmatrix} (I - GK)^{-1}(W_1GW_4)\right\|_\infty \qquad (15)$$

will be continuous with respect to variations in the weights W_i and in the plant.

Theorem 13 *Suppose that $G \in M(\mathbf{H}^\infty/\mathbf{H}^\infty)$, W_i and $W_i^{-1} \in M(\mathbf{H}^\infty)$ and let G_n be a sequence of perturbed plants in $M(\mathbf{H}^\infty/\mathbf{H}^\infty)$ such that $\lim_{n \to \infty} \delta(G_n, G) = 0$ (convergence in the gap metric). Let W_i^n be sequences such that $\|W_i^n - W_i\|_\infty \to 0$ as $n \to \infty$. Then $\lim_{n \to \infty} \mu(G_n, W_{i,n}) = \mu(G, W_i)$.*

The gap metric is closely related to robustness concepts and an excellent analysis of this is contained in Zhu [67]. Although it can be calculated for finite-dimensional systems, for infinite-dimensional systems it is difficult and the following result from Zhu [68] is particularly useful (see also Zhu [69]).

Lemma 14 *Suppose that G and $G_n \in M(\mathbf{H}^\infty_- + \mathbf{R}(s))$ and that they have no poles on the imaginary axis. If G and G_n have the same number of unstable poles for sufficiently large n, then $\|G - G_n\|_\infty \to 0$ as $n \to \infty$ if and only if $\delta(G, G_n) \to 0$ as $n \to \infty$.*

These results are important steps in the development of an approximation theory, but as already pointed out in Smith [62], one also needs sufficient conditions for continuity with respect to the controller. Otherwise, applying a reduced order model approach to H^∞-control design along the lines advocated in Section 5 will not work. An important step in this direction can be found in Ozbay [52] There he considers a H^∞ sensitivity minimization problem for single-input, single-output plants and obtains a finite dimensional controller by approximating the optimal

one in the gap metric. In addition, he deduces an explicit bound for the achievable performance. A more direct approach to finite-dimensional compensator design is taken in Rodriguez and Dahleh [59]. They consider the SISO sensitivity problem: $\mu(G, W_i)$ in (6.1) with $W_1 = I$, $W_3 = W_4 = 0$, W_2 an outer, proper stable rational transfer function and $G \in \mathbf{H}^\infty$. They define approximants W_2^n to W_2 and G_n to G so that $\mu(G_n, W_i^n) \to \mu(G, W_i)$ as $n \to \infty$. Moreover, they give conditions under which the finite-dimensional controllers designed for G_n will also stabilize G. They give a similar treatment of the SISO mixed-sensitivity problem.

It is interesting to note that in this approach they approximate the inner and outer factors of the plant separately. Finding inner and outer factors for irrational transfer functions is possible, but not trivial numerically (see Flamm, Yang, Ran and Klipec [24]).

7 Conclusions

We have discussed five different approaches to designing finite-dimensional compensators for infinite-dimensional systems. The first two approaches are state-space ones. The infinite-dimensional LQG approach in Section 2 finds an infinite-dimensional compensator first and then approximates to obtain a finite-dimensional compensator, whereas the state-space approaches in Section 3 lead in one step to a finite-dimensional compensator. Although the LQG approach usually works well in practice, the theory for the case of unbounded input and output operators still needs to be developed further.

The remaining approaches are frequency domain ones and follow the philosophy of approximating the irrational transfer function first by a reduced-order model and then designing a finite-dimensional compensator for it. Coupled with good $\mathbf{L}_\infty$-approximation techniques, these offer computational simplicity. The class of systems covered by the frequency domain approaches is much wider than that covered by state-space techniques. It includes those systems with an exponentially stabilizable and detectable state-space realizator $\sum(A, B, C, D)$, where B and C may be unbounded and also systems of the form $e^{-s}G(s)$, where $G(s)$ is rational. The reduced order model design approach 1 discussed in Section 4 has long been standard engineering practice. It is very flexible and allows one to use any finite-dimensional controller design on the reduced order model provided that the $\mathbf{L}_\infty$-model error os less than the robustness margin. In particular, it gives an alternative approach to LQG controller design. The two robust controller designs discussed in Section 5 have the extra advantage of guaranteeing robustness to uncertainties in the model. Furthermore, by introducing suitable weights other performance criteria can be met.

Finally, in Section 6 we have touched on some very recent developments in $\mathbf{H}^\infty$-controller design for infinite-dimensional systems. For general $\mathbf{H}^\infty$-control

problems it is not clear that designing a controller for a reduced order model will produce a good controller for the infinite-dimensional system. Partial results on the convergence of the performance criteria of the approximating systems exist, but the question of convergence of the associated controllers is still open, except for very special cases. This is an important area for future research.

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