Investigations on termination of equational rewriting

Isabelle Gnaedig

UNITÉ DE RECHERCHE
INRIA-LORRAINE

Institut National
de Recherche
en Informatique
et en Automatique

Domaine de Voluceau
Rocquencourt
B.P.105
78153 Le Chesnay Cedex
France
Tél.:(1) 39 63 55 11

Rapports de Recherche

N° 732

INVESTIGATIONS ON TERMINATION OF EQUATIONAL REWRITING

I. GNAEDIG

OCTOBRE 1987

Investigations on Termination of Equational Rewriting

**

Investigations sur la Terminaison de la Réécriture Equationnelle

I. Gnaedig
INRIA
Campus Scientifique BP 239
54506 Vandoeuvre

Abstract

This paper is a report of detailed investigations on the termination problem of rewriting modulo equational theories. We present here three approachs of this problem. We point out the difficulties and explain how the methods fail, giving some ideas issued from a rigorous observation of the failing processes. The first part is a new approach of the classical associative commutative (AC in short) orderings using the cooperation on abstract relations. We point out this proof method is not easy, because of an infinity of critical pairs to be considered. The second work consists in adapting the AC orderings with flattening to ensure termination modulo a large class of permutative theories. But the characterization of this class points out nothing other than the theories included in AC, for which the AC termination methods can be used. So we deduce, AC is the maximal class working with flattening. In the third chapter of this report, we propose a new AC ordering, replacing both flattening and distributing transformations by a single powerful transformation. We exhibit a counter-example to show that this method does not provide an F-compatible ordering.

Résumé

Ce rapport présente un certain nombre d'investigations détaillées sur le problème de la terminaison de la réécriture modulo une théorie équationnelle. Nous présentons ici trois approches de ce problème, en exposant les difficultés rencontrées, et en expliquant pourquoi ces méthodes tombent en échec. En première partie, nous présentons une nouvelle approche des ordres associatifs-commutatifs (AC en bref), en utilisant la coopération sur les relations abstraites. Nous montrons que cette méthode n'est pas aisée, car on est amené à considérer une infinité de paires critiques. En second lieu, nous étendons les ordres AC avec aplatissement afin d'obtenir une preuve de la terminaison modulo une large classe de théories permutatives. Mais la caractérisation de cette classe n'est en fait que celle des théories incluses dans la théorie AC, pour laquelle les méthodes déjà existantes peuvent être employées. Aussi, nous déduisons que AC est la plus grande classe compatible avec l'aplatissement. Finalement, nous proposons un nouvel ordre AC, en remplacant l'aplatissement et la distributivité par une seule opération. Un contre-exemple est exhibé pour montrer que cette méthode ne permet pas à l'ordre d'être F-compatible.

Chapter 1 AC-Termination and cooperation

ABSTRACT

We propose here a new way to prove termination of associative-commutative rewriting systems. We explain how we relax the conditions of the AC ordering SACO, using the cooperation property of relations on terms.

1. INTRODUCTION

Proving termination of AC rewriting systems is not easy. The existing tools for such proofs are recent (Bachmair Plaisted 1985),Bachmair-Dershowitz-86a).

As well as for standard rewriting, the termination of AC rewriting can be proved using reduction orderings, since they are AC-commuting (Gnaedig-Lescanne-86). These orderings are based on transformation techniques.

On another hand, a method was recently discovered, to prove termination of term rewriting systems that cannot be taken into account by classical orderings such that RPO (Bellegarde Lescanne 1986). This method is also based on transformation techniques, and the constraints on the transformation relations are easy to check. Indeed, a property similar to confluence, the cooperation, is needed, which can be proved in computing critical pairs.

Our goal is here to reconsider the ordering SACO studied in (Gnaedig Lescanne 1986) using the cooperation, to relax the conditions needed for SACO. We show thus in the same time that the method of (Bellegarde Lescanne 1986), developed for simple rewriting systems can be extended to prove termination of AC rewriting systems.

2. ADAPTING COOPERATION TO THE AC CASE

Let us recall the basis termination theorem, from which we started to construct SACO. We note F the set of operators, F_{AC} the set of AC-operators, T(F,X) the free algebra of terms, TV(F,X) the algebra of var-yadic terms.

Theorem 1 : (Jouannaud & Muñoz): Let (R,E) be an equational rewriting system defined on T(F,X). (R,E) is noetherian if there exists an E-commuting reduction ordering which orients all instances of R.

We recall then the definition of the SACO, which fulfills the conditions of the previous theorem. We denote by *,+ the unique pair of comparable AC operators, more exactly such that *>+. We write s↓ for the D-normal form of s where D is the system

$$x*(y+z) \rightarrow (x*y)+(x*z)$$
$$(x+y)*z \rightarrow (x*z)+(y*z),$$

[s] is the flattened form of s (we recall further the detailed definition of flattening).

Definition 1 : Let s and t be two terms of T(F,X). We say that s>t iff
$[s\downarrow] >_{RPO} [t\downarrow]$ or
$[s\downarrow] \simeq [t\downarrow]$ and $s \;{-}{+}{\rightarrow}_{D/AC}\; t$ by the distributivity rule $x*(y+z) \rightarrow (x*y)+(x*z)$.

Theorem 2 : Under the following conditions of precedence:
*>+
if k and k' are in F_{AC}, $k >_F k'$ => k=* and k'=+
if k is in F_{AC} and $k \neq *$, then k is minimal in F
if $* >_F f$ then f=+
the SACO is a reduction ordering, AC-commuting and stable by substitution.

Our goal is here to reconsider the previous theorem, using a different way. Indeed, the approach used to establish the SACO has been to start from a particular ordering based on a particular transformation technique (flattening and distributing transformation) to fulfill the AC-commutation, and then to prove detailed properties of this transformation ordering (F-compatibility, stability by substitution). The working mechanism is here in the opposite direction.

(Bellegarde Lescanne 1986) established a termination proving method based on transformation of terms defined by abstract relations on the term algebra. The required properties of the obtained ordering are deduced from those of the abstract relations, say the flattening relation and the RPO relation, and then we verify that our relations fulfill the previous conditions such as cooperation.

Let us recall here the definition of the cooperation of two relations.

Definition 2 : We say that $\rightarrow_S$ cooperates with $\rightarrow_T$ if and only if:
$\leftarrow^*_T o \rightarrow_S o (\rightarrow_S \cup \rightarrow_T)^* \subseteq \rightarrow^*_T o \rightarrow_S o (\rightarrow_S \cup \rightarrow_T)^* o \leftarrow^*_T$

Definition 3 : The ordering $>_{S,T}$ is defined by:
$s >_{S,T} t$ if and only if $s \rightarrow^+_T t$ or $s \rightarrow^*_T o \rightarrow_S o (\rightarrow_S \cup \rightarrow_T)^* o \leftarrow^*_T t$.

We can now enunciate the termination theorem based on the cooperation concept.

Theorem 3 : (Bellegarde & Lescanne):
If $\rightarrow_S$ cooperates with $\rightarrow_T$
$\rightarrow_S \cup \rightarrow_T$ is noetherian
T is confluent
$\rightarrow_S$ and $\rightarrow_T$ are F-compatible and stable by substitution
if for all rules $l \rightarrow r$ of R, $l >_{S,T} r$ then R is noetherian.

Let us now adapt this theorem to the case of equational termination, especially the associative-commutative case, considering the conditions of Jouannaud & Muñoz' theorem.

Theorem 4 :
If $\rightarrow_S$ cooperates with T
$\rightarrow_S \cup \rightarrow_T$ is noetherian
$\rightarrow$ T is confluent
$\rightarrow_S$ and $\rightarrow_T$ are F-compatible and stable by substitution

$>_{S,T}$ is AC-commuting

then R AC-terminates if for every rule $l \rightarrow r$ of R, we have $l >_{S,T} r$.

3. DEFINING RELATIONS TO USE THE COOPERATION TERMINATION THEOREM

The problem is now to particularize the relations S and T to satisfy the conditions of the previous termination theorem. Let us recall these conditions:

$\rightarrow_S$ cooperates with $\rightarrow_T$

$\rightarrow_S \cup \rightarrow_T$ is noetherian

$\rightarrow_T$ is confluent

$\rightarrow_S$ and $\rightarrow_T$ are F-compatible and stable by substitution

$>_{S,T}$ is AC-commuting.

To have the AC-commutation of the ordering $>_{S,T}$, let us choose for T the flattening on AC operators, that can be defined by an infinite set of rewriting rules. We will see that the AC-commutation of T is sufficient to ensure the AC-commutation of $>_{S,T}$. Let us recall the definition of the flattening operation.

Definition 4 : The flattening operation is a relation on TV(F,X), defined by an infinite rewriting system FL described by the scheme of rules:

$f(y_1,..y_{i-1},f(x_1,...,x_m),y_{i+1},..y_n) \rightarrow f(y_1,..y_{i-1},x_1,...,x_m,y_{i+1},..y_n)$

for every f in F_{AC}, for every $i>0$, for every $m,n >1$.

Lemma 1 : The rewriting relation $\rightarrow_{FL}$ is AC-commuting.

Lemma 2 : The ordering $>_{S,FL}$ is AC-commuting.

We recall a property of flattening used in (Gnaedig Lescanne 1986).

Lemma 3 : FL is confluent.

Let us now choose the RPO for the relation S.

Lemma 4 : $>_{RPO} \cup \rightarrow_{FL}$ is noetherian.

Proof

The system FL is oriented by RPO since $\{y_1,\ldots y_n\} \gg_{RPO} y_1,\ldots y_n$, and RPO is well-founded. □

Lemma 5 : $>_{RPO}$ and $\rightarrow_{FL}$ are F-compatible and stable by substitution.
Proof

- $>_{RPO}$ is a simplification ordering stable by substitution
- by definition of a rewriting relation, $\rightarrow_{FL}$ is compatible and stable by substitution □

Remark : We can yet observe that the F-compatibility and the stability by substitution are immediate to prove here, since they deal with the two relations S and T separately. This approach is easier than the approach of NFLO, where the properties are to be proved on the ordering itself.

4. ENSURING THE COOPERATION

The last condition to obtain an ordering $>_{S,T}$ adapted to the AC termination proof is to verify the cooperation property on $>_{RPO}$ and $\rightarrow_{FL}$. As well as for confluence (cooperation can be viewed as a generalized confluence on two relations) cooperation can be localized under conditions on $\rightarrow_S$ and $\rightarrow_T$. The local cooperation can be established with conditions on critical pairs between S and T, when S and T are two rewriting systems.

Definition 5 : A critical pair <p,q> between a rule of S and a rule of T such that $p \leftarrow_T o \rightarrow_S q$ is cooperative if and only if $p \rightarrow_T^* o \rightarrow_S o (\rightarrow_S \cup \rightarrow_T)^* o \leftarrow_T^* q$.

Theorem 5 : (Bellegarde & Lescanne)
If $\rightarrow_S \cup \rightarrow_T$ is noetherian
$\rightarrow_T$ is confluent
$\rightarrow_T$ is regular ($V(l)=V(r)$ for every rule $l \rightarrow r$ of T) and left linear
all critical pairs between S and T are cooperative
then S cooperates with T.

We can immediately state the following obvious lemma.

Lemma 6 : FL is regular and left linear.

To use the critical pair method to establish cooperation requires to work on rewriting relations. For this reason, we will generalize the relation $\rightarrow_S$ equal to $>_{RPO}$ to any rewriting relation included in $>_{RPO}$.

Let then S be a rewriting system such that $l \rightarrow_S r \Rightarrow l >_{RPO} r$. Under this modification of S, Lemma 4 and Lemma 5 remain valid.

Theorem 6 : Let S be a rewriting system such that $s \rightarrow_S t \Rightarrow s >_{RPO} t$. If the critical pairs between S and FL are cooperative, then R is AC-terminating if for every rule $l \rightarrow_R r$, we have $l >_{S,FL} r$.

Proof

The theorem is immediately deduced from Theorem 4, using Theorem 5 and Lemmas 2, 3, 4, 5, 6. □

Let us now explain how to find the system S, on the following example. Let R be an axiomatisation of abelian groups where * is an AC symbol:

$$e*x \rightarrow x$$
$$i(x)*x \rightarrow e.$$

The terms of the two rules are in normal form for FL. By definition of the relation $>_{S,FL}$, to obtain $e*x >_{S,FL} x$, we must have:

$$e*x \rightarrow_S o(\rightarrow_S \cup \rightarrow_{FL})^* \; x.$$

Let S be the system R itself. S is oriented by the RPO, then $\rightarrow_S$ is contained in the relation $>_{RPO}$.

Let us now verify the cooperation of S and FL, in computing the critical pairs between S and FL:

$$f(y_1,..y_{i-1},f(x_1,...,x_m),y_{i+1},..y_n) \rightarrow f(y_1,..y_{i-1},x_1,...,x_m,y_{i+1},..y_n)$$

e*x → x (S1)

i(x)*x → e (S2).

A problem appears: the set of critical pairs to be computed is infinite since FL is an infinite rewrite system. Let us choose for example a simple rule in FL to compute critical pairs:

(x*y)*z → x*y*z (FL1)

Computing critical pairs between (FL1) and (S1) gives:

<p,q> = <e*y*z, y*z>

<x*e*z, x*z>.

The critical pairs don't cooperate since p not$\rightarrow_S$ q and p not$\rightarrow_{FL}$ q. This second problem is due to the fact that the simplification rules that could simplify the critical pairs don't adapt to the varyadic case. Indeed, we would like to use a rule such that e*x*y → x*y to reduce the first critical pair.

The problem is the same if we try to compute the critical pairs between (FL1) and (S2):

<p,q> = <i(x)*x*z, e*z>.

This critical pair would cooperate if we could have the rule i(x)*x*z → e*z in S.

May be the solution is to consider a set of metarules instead of the set of rules S. Each metarule would express in the varyadic term algebra each rule of S. For example, to the inverse rule (S2) of S would correspond the metarule:

i(x)*x*...*z → e*...*z

where the arity of * can be any number greater than 1. Remark that the system FL itself can be viewed as a metarule.

5. Conclusion

This method for proving AC termination would involve the choice of a system S for each system R to be proved terminating (one can simply choose R itself if R is included in the RPO) . But difficult proofs of properties such as F-compatibility, stability by instanciation are here replaced by verifying the cooperation of S and FL. Introducing metarules to deal with the infinite system FL could provide an easy termination proof.

References

Bachmair-Dershowitz-86a.

L. Bachmair and N. Dershowitz, "Commutation, Transformation, and Termination," in *Proc. 8th Conf. on Automated Deduction*, Lecture Notes in Computer Science, vol. 230, Springer Verlag, Oxford (England), 1986.

(Bachmair Plaisted 1985).

L. Bachmair and D. Plaisted, "Associative Path Orderings," in *Proc. 1st Conference on Rewriting Techniques and Applications*, Lecture Notes in Computer Science, vol. 202, pp. 241-254, Springer Verlag, Dijon (France), 1985.

(Bellegarde Lescanne 1986).

F. Bellegarde and P. Lescanne, "Termination Proofs Based On Transformation Techniques," *Submitted*, 1986.

Gnaedig-Lescanne-86.

I. Gnaedig and P. Lescanne, "Proving Termination of Associative Rewriting Systems by Rewriting," in *Proc. 8th Conf. on Automated Deduction*, Lecture Notes in Computer Science, Springer Verlag, Oxford (England), 1986.

Chapter 2
Permutative theories, flattening and termination

Abstract

Starting from transformation ordering techniques, we show how flattening provides a termination proof for rewriting in a large class of permutative theories like associativity, left-commutativity and Lie-brackets.

1 Introduction

Rewriting applications are constantly gaining importance in the area of automated deduction. Especially, equational rewriting must be investigated since it covers more general problems than simple rewriting. In the simple case, many techniques were established to ensure the crucial property of termination.

However, in the equational case, the problem is much more difficult. Recent works have proposed to extend the simple rewriting termination proofs to the equational case [7]. But until now, an implementable particular ordering has been provided only for the associative-commutative (AC in short) case [5,1].

In this paper, we propose a termination ordering for a large class of permutative theories. For that, we point out how the flattening transformation, already used in the AC case, can be generalized to the permutative case. Conditions on permutative axioms are given to define the maximal class of permutative theories, for which the flattening relation can give a termination ordering.

We see that these conditions are nothing more than the permutative theories which are transformed in the permutation congruence by flattening. Starting from the transformations methods of Bellegarde Lescanne based on the nice property of cooperation of the transformation (which is here the flattening) with another relation (which may be an ordering relation like RPO), we show how an implementable reduction ordering can be obtained, that is P-commuting with any theory P of the class mentioned above.

In Section 2, the general termination method based on cooperation is presented. In Section 3, conditions on permutative axioms are given to define the maximal class of permutative theories transformed in the permutation congruence by flattening. Section 4 establishes how the transformation ordering based on flattening and RPO is a reduction ordering commuting with any permutative theory defined in Section 3.

2 A transformation ordering based on cooperation

The reader is supposed to know the basic term rewriting notations. Here only the main tools necessary for this paper are given. For a more comprehensive survey of rewriting termination proofs, see [4]. In the following, we present all necessary results concerning transformation orderings, which are the basis of our approach.

T(F,X) is the free term algebra where F denotes a set of operator symbols and X a set of variables. Let s(u) be the operator of the term s at occurrence u, s/u the subterm of s at occurrence u, s[u←t] the term obtained in replacing s/u in s by the term t. Let us write #(x,s) the number of occurrences of x in the term s. Relations on terms are defined, which are not only rewriting relations but abstract relations denoted by $\rightarrow$. The notation $\rightarrow^{-1}$ or $\leftarrow$ denotes the inverse of the relation $\rightarrow$. $\rightarrow^{*}$ and $\rightarrow^{+}$ respectively denote the transitive closure and the strict transitive closure of $\rightarrow$. $\rightarrow_{R1} \cdot \rightarrow_{R2}$ is the composition of two relations $\rightarrow_{R2}$ and $\rightarrow_{R1}$, and $\rightarrow_{R1} \subseteq \rightarrow_{R2}$ if $\{ (x,y) / x \rightarrow_{R1} y \} \subseteq \{ (x,y) / x \rightarrow_{R2} y \}$.

A relation $\rightarrow$ is terminating or noetherian if any sequences of terms such as $t_1 \rightarrow t_2 \ldots \rightarrow t_n$ is finite. The relation $\rightarrow$ is confluent if $\leftarrow^{*} \cdot \rightarrow^{*} \subseteq \rightarrow^{*} \cdot \leftarrow^{*}$. If $\rightarrow_R$ is confluent and noetherian, each term t of T(F,X) has an unique irreducible form $t\downarrow_R$ called normal form.

A rewriting relation $\rightarrow_R$ is a particular relation on terms, which can be defined by a set of rules R. The rules are ordered pairs of terms written l→r, such that V(r) ⊆ V(l) (V(t) being the set of variables in t).

We now recall the notions of transformation, cooperation of relations, and of termination ordering issued from these concepts. Furthermore, the termination ordering proposed for simple rewriting is adapted to equational rewriting.

Let $\rightarrow_S$ and $\rightarrow_T$ be two relations on terms.

Definition: [Bachmair & Dershowitz]
$\Rightarrow_{S,T}$ is the relation $\rightarrow_T^{*} \cdot \rightarrow_S \cdot (\rightarrow_S \cup \rightarrow_T)^{*} \cdot \leftarrow_T^{*}$.

Definition: [Bellegarde & Lescanne]
$\rightarrow_S$ cooperates with $\rightarrow_T$ if and only if $\leftarrow_T^{*} \cdot \rightarrow_S \cdot (\rightarrow_S \cup \rightarrow_T)^{*} \subseteq \Rightarrow_{S,T}$.

Let us recall the termination theorem using the previous relation on terms.

Theorem 1: [Bellegarde & Lescanne]
If $\rightarrow_S$ cooperates with $\rightarrow_T$, $\rightarrow_S \cup \rightarrow_T$ is noetherian, T is confluent, then $\Rightarrow_{S,T}$ is a well-founded ordering. Moreover if $\rightarrow_S$ and $\rightarrow_T$ are F-compatible and stable by substitution, then $\Rightarrow_{S,T}$ is compatible and stable by substitution.

The Manna & Ness theorem allows to apply the previous ordering to the termination proof of rewriting systems.

Corollary 0: [Bellegarde & Lescanne]
With conditions of Theorem 1, if for all rules $l \rightarrow r$ of R, we have $l \Rightarrow_{S,T} r$, then R is noetherian.

An important result by Jouannaud & Munoz [7] extends the Manna & Ness theorem to the equational case. Let E be an equational theory defined on T(F,X).

Theorem: [Jouannaud & Munoz]
Let $>$ be a reduction ordering (i.e. a F-compatible well-founded ordering) stable by substitution and E-commuting (i.e. such that $s' =_E s > t =_E t' \Rightarrow s' > t'$). Then if for all rules $l \rightarrow r$ of R, $l > r$, R is E-terminating.

Theorem 2 gives an E-termination proof with the ordering $\Rightarrow_{S,T}$. It is an obvious consequence of Corollary 0 and Jouannaud & Munoz Theorem.

Theorem 2:
If $\rightarrow_S$ cooperates with $\rightarrow_T$, $\rightarrow_S \cup \rightarrow_T$ is noetherian, $\rightarrow_T$ is confluent, $\Rightarrow_{S,T}$ is E-commuting, $\rightarrow_S, \rightarrow_T$ are F-compatible and stable by substitution, then R is E-terminating if for every rule $l \rightarrow r$ of R, we have $l \Rightarrow_{S,T} r$.

As for confluence, the cooperation can be localized under the well-foundedness of $\rightarrow_S \cup \rightarrow_T$ and the confluence of $\rightarrow_T$. The local relation is expressed by:

Definition:
$\rightarrow_S$ locally cooperates with $\rightarrow_T$ if and only if $\leftarrow_T . \rightarrow_S \subseteq \Rightarrow_{S,T}$.

Therefore, in Theorem 2, cooperation can be relaxed into local cooperation.

3 Permutative theories and flattening

3.1 PER-condition

The definition of permutative theories was introduced by Lankford and Ballantyne, when giving a decision procedure for equational theories with permutative axioms [9].

Definition:
If $\#(x, s) = \#(x, t)$ for all symbols $x \in F \cup X$, then the equation $s = t$ is called a permutation equation.

Definition:
An equational theory E is called permutative if each equation of E is a permutation equation.

To define the behavior of permutation with flattening, we now introduce the notion of transposition which can be seen as an elementary permutation. More exactly, this notion of transposition will allows us to define the maximal class of permutative theories transformed in the permutation congruence by flattening.

Definition 1:
Let s be a term of T(F,X). A transposition equation is a permutation equation s=t defined by a pair of occurrences (u,v) of s called permutative occurrences such that:

(1) arity(s(u)) = arity(s(v)) and t(w)=s(w) for every occurrence w $\neq$ u,v, and t(u)=s(v) and t(v)=s(u)
or
(2) u and v are disjoint and t = (s[v←s/u])[u←s/v]

Intuitively, (1) means that two operators are permuted in the term to be transformed whereas (2) means that two whole subterms are permuted.

Property:
Every permutative theory P is included in or equal to a theory induced by a finite set S of transposition equations. The set of permutative equations inducing P is said to be decomposed in the set S of transposition equations.

For example the associative axiom x*(y*z) = (x*y)*z is a permutative axiom. The following decomposition in transposition axioms can be done:

$$x * (y * z) = (y * z) * x$$
$$(y * z) * x = (x * z) * y$$
$$(x * z) * y = (x * y) * z$$

Remark the equational theories induced by the two previous sets of axioms are not equivalent: the associative theory is strictly included in the theory induced by the three axioms above since x*(y*z) = (y*z)*x doesn't implies x*(y*z) = (x*y)*z.

Permutative theories include some usual equational theories we will present now. The most usual is the AC theory. The AC termination has recently been investigated and some methods exist now to prove rewriting termination in this theory. Here, we are interested by other theories that have not yet termination proofs.

Let us introduce the left [respectively right] commutative theory defined by an axiom Cl [respectively Cr] of the form:

$$x * (y * z) = y * (x * z)$$

[respectively

$$(x*y)*z = (x*z)*y]$$

The problem of Cr-unification has been studied by Jeanrond [6] and C.Kirchner [8]. This axiom is used in many algebraic operations like real division. Note that the theory defined by the two axioms of commutativity and left-commutativity is equivalent to the AC theory.

The associative theory defined by the unique axiom:

$$x*(y*z) = (x*y)*z$$

or the Lie bracket theory defined by:

$$(x*y)*z = z*(y*x)$$

or the theory defined by

$$(x*y)*(z*t) = (x*z)*(y*t)$$
$$x*y = y*x.$$

are other examples of permutative theories.

Since unification has been studied for many of these theories, it is interesting to provide a corresponding termination proof. Indeed, we would like to be able to compute with E-rewriting and with simple rewriting as well. Our aim is to provide termination proofs as general as possible. We will then propose an ordering, valid for a very large class of equational permutative theories called C(PER). This ordering is based on flattening and C(PER) will be the biggest class of permutative theories transformed in the permutation congruence by flattening. The following PER-condition is useful to define the class C(PER).

Let $<_{lex}$ be the lexicographical ordering on sequences of integers. Let max(u,v) be the biggest of the two occurrences u and v, and min(u,v) the smallest (for the ordering $<_{lex}$).

Definition: The PER-condition

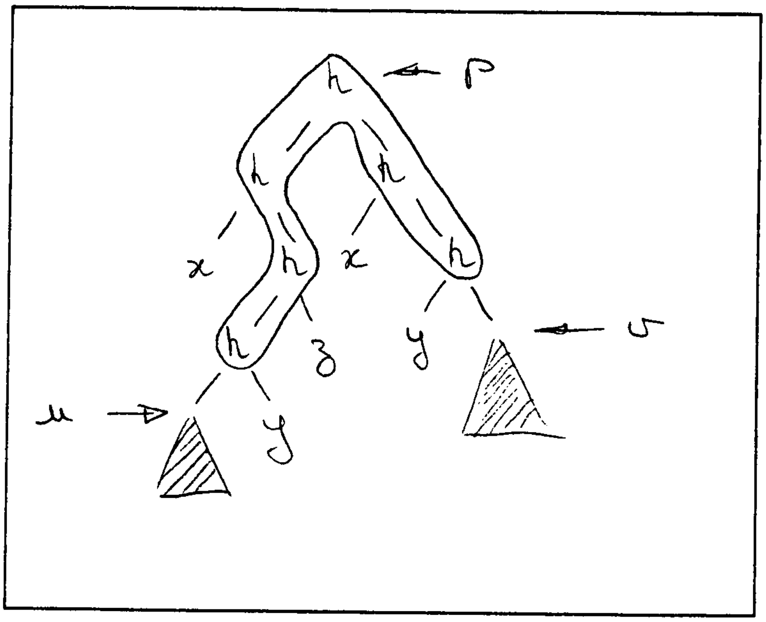
A permutation equation e satisfies the PER-condition if and only if for every transposition equation s=t decomposing e, for the pair (u,v) of permutative occurrences of s=t, we have:

if s=t is defined by (1) in Definition 1, then s(u)=s(v)
else
s(w) = t(w) = h for every occurrence w such that w $>_{lex}$ p, w is a prefix occurrence of u or v, and p is the biggest prefix occurrence common to u and v such that t(p)=h=s(p).

Definition 0:
The operator h is called a permutative operator of the equation s=t.

Note that the notion of permutative operator is allowed by the PER-condition. For example, by the first case of this condition, we cannot define the permutative operator for f(h(x))=h(f(x)). The first case of the PER-condition forbids a such permutation.

Practically, the second case of the PER-condition means that the operators have to be equal on the two path in s starting from the biggest common occurrence to u and v and ending respectively in u and v. Let the following figure illustrate this condition.



Let us note for example that the PER-condition forbids axioms like f(h(x))=h(f(x)) or like f(g(x,y),(h(a,b))=f(g(x,a),(h(y,b)).

C(PER) is the set of permutative theories defined by equations satisfying the PER-condition.

As we will see later, C(PER) is the largest set of permutative theories flattening can work on. It is easy to see that the previous examples of permutative theories are in C(PER), since the terms have a single operator in each equation.

3.2 Choosing flattening for the transformation T

Let us consider the ordering $\Rightarrow_{S,T}$. Now, we will propose particular relations for $\rightarrow_S$ and $\rightarrow_T$ that gives an ordering for permutative theories PER of C(PER). A first property required by the ordering $\Rightarrow_{S,T}$ is the

PER-commutation. To enforce this property one introduces a common structure for all PER-equal terms. Therefore, introducing a relation transforming two PER-equal terms into the same structure, we choose for T the flattening transformation (already used in a particular theory of PER: the AC theory) [5,1].

We have to specify on which operator we flatten the terms. For a given permutative theory, i.e. for a given set of PER-axioms, we introduce the set of permutative operators following Definition 0 and the set of corresponding flattening rules (following the permutative operators).

For example, the equation g(f(f(x,y),(f(a,b)),a)=g(f(f(x,a),f(y,b)),a) introduces the permutative operator f.

In the following, + is an arbitrary permutative operator. Thus, + means "for each permutative operator". Flattening is formally defined by a rewriting relation $\rightarrow_A$ often written A without ambiguity.

Definition:
The flattening relation A is defined by the following infinite rewriting system:

$$+(x_1, \dots x_{i-1}, +(y_1, \dots y_n), x_{i+1} \dots x_m) \rightarrow +(x_1, \dots, x_{i-1}, y_1, \dots y_n, x_{i+1}, \dots x_m)$$

for all i>0, for all m and n>1, for every permutative operator + of P.

On the previous example, the permutative operator f introduces the flattening rules:

$$f(x_1, \dots x_{i-1}, f(y_1, \dots y_n), x_{i+1} \dots x_m) \rightarrow f(x_1, \dots, x_{i-1}, y_1, \dots y_n, x_{i+1}, \dots x_m)$$

for all i>0, for all m and n>1.

In spite of the infinity of rules, the relation A is decidable, since we apply to a term only a finite number of rules. As any rewriting relation, A is stable by instantiation and F-compatible, Moreover, A is confluent and A is noetherian since $A \subseteq >_{RPO}$. The relation A projects the terms of T(F,X) on the varyadic term algebra TV(F,X). Thus the second relation S has to be defined on TV(F,X). The definition of a permutative theory has to be extended to TV(F,X). Let us note A-nf the set of all elements of TV(F,X) in A-normal form.

Thus all PER-equal flattened terms are permutatively equivalent (in the sense of the permutation congruence $\cong$). Let us recall the definition of the permutation congruence.

Definition:
The permutation congruence $\cong$ on TV(F,X) is defined by: s=f(...si...) $\cong$ t=g(...tj...) iff
(1) f=g and
(2) there exists a permutation p of [1, n=arity(f)] such that si $\cong$ $t_{p(i)}$.

Let us now formalize the property of C(PER) induced by flattening.

Property 0:

Two PER-equal flattened terms are $\cong$-equal and therefore: $=_{PER} \cdot \downarrow_A = \downarrow_A \cdot \cong$ and $(\downarrow_A)^{-1} \cdot =_{PER} = \cong \cdot (\downarrow_A)^{-1}$.

We can see now why the PER-condition is necessary. On the first hand, we have seen above that we are unable to define the permutative operator without the PER-condition. On the second hand, without the PER-condition, two PER-equal flattened terms are not necessary $\cong$-equal. For example, with f(f(x,y),g(a,b)) $=_{PER}$ f(f(x,a),g(y,b)) but f(x,y,g(a,b)) and f(x,a,g(y,b)) are not $\cong$-equal.

Proof of Property 0:

Let PER be induced by a set of transpositions. For every transposition s=t defining PER, if (u,v) is the pair of permutative occurrences of s=t, if p is the biggest prefix occurrence common to u and v such that t(p)=s(p)=h, then we have: s(w)=t(w)=h for every occurrence w such that w $>_{lex}$ p, and w is a prefix occurrence of u or v.

Let A be the flattening following h. Then $s\downarrow_A$ has s/u and s/v as subterms at occurrence p.i and p.j respectively (i,j are integers such that p.i is a prefix occurrence of u, p.j is a prefix occurrence of v). On another hand, $t\downarrow_A$ has s/v and s/u as subterms at occurrence p.i and p.j respectively. Then $s\downarrow_A \cong t\downarrow_A$.

Therefore, if A is now the flattening following all permutative operators of PER, then $s =_{PER} t \Rightarrow s\downarrow_A \cong t\downarrow_A$.

Elsewhere, for a theory PER $\subseteq$ PER' such that PER' is induced by a set of transposition equations, then $s =_{PER'} t \Rightarrow s\downarrow_A \cong t\downarrow_A$. Since PER $\subseteq$ PER', then $s =_{PER} t \Rightarrow s =_{PER'} t \Rightarrow s\downarrow_A \cong t\downarrow_A$. ◆

Note that the permutation congruence is a particular case of permutative theories, satisfying the PER-condition.

As second relation $\rightarrow_S$, often written S, it is natural to introduce an ordering that is not sensitive to the permutation congruence (formally that is $\cong$-commuting): we can start from the RPO of Dershowitz [3] on TV(F,X).

3.3 S is a relation contained in RPO

Following the hypothesis of Theorem 2, S has to be a relation in TV(F,X) such that $S \cup A$ is noetherian: it is sufficient that S $\subseteq$ RPO. Moreover, S has to cooperate with A. Let us note that RPO does not cooperate with A. For example:

- $+(+(x,y),z) >_{RPO} +(f(x,y),z)$ with $+ > f$ and $+(x,y,z)$ is not $>_{RPO} +(f(x,y),z)$
- $+(+(a,x),y) >_{RPO} +(a,a,x)$ and $+(a,x,y)$ is not $>_{RPO} +(a,a,x)$

Intuitively, S is the RPO without these pairs. This is achieved by adding to the definition of RPO a case f=g=+ that requires $s{\downarrow}_A >_{RPO} t{\downarrow}_A$.

Definition:
Let > be a precedence in F. The relation S on T(F,X) is defined by $s=f(\bar{s}) >_S t=g(\bar{t})$ iff:

(1) there exists $si >_S t$ or $si \cong t$ for an i or

(2) f>g and $s >_S ti$ for each i or

(3) $f=g \neq +$ and $\bar{s} >>_S \bar{t}$

(4) $f=g=+$ and $\bar{s} >>_S \bar{t}$ and $s{\downarrow}_A >_{RPO} t{\downarrow}_A$.

$>>_S$ denotes the multiset extension of $>_S$.

Notations:
If $\bar{u}$ = {u1,...un}, $\bar{v}$ = {v1,...vn} are multisets, then the multiset {u1,...un,v1,...vn} is denoted by $(\bar{u},\bar{v})$.
We write $s >_S \bar{u}$ for $s >_S ui$ for all i.

4 A cooperating reduction ordering

Many results are direct consequences of the definition of $>_S$, and will be used in the proof of cooperation of A and S.

Property 1:
$(S \vee \cong)$ is a quasi-ordering and S is an $\cong$-commuting ordering.

Property 2:
$S \subset RPO$ then $S \cup A$ is noetherian.

Property 3:
S = RPO in A-nf.

Property 4:
S has the subterm property which means $s >_S si$ for every subterm si of s.

Property 5:
S satisfies the deletion property i.e. f(...s...) $>_S$ f(......) for every f in F.

Let us now state the main property of cooperation of A with S.

Property 6:
S cooperates with A i.e. $(A^{-1})^* . S . (S \cup A)^* \Rightarrow \Rightarrow_{S,A}$.

Sketch of proof:

As we have noticed in Section 2, it is sufficient to prove the local cooperation i.e. $A^{-1} . S \subseteq \Rightarrow_{S,A}$, since S∪A is noetherian and A is confluent.

We have only to show that $A^{-1} . S \subseteq A^* . S . (A^{-1})^*$ since $A^{-1} . S . (A^{-1})^* \subseteq A^* . S . (A \cup S)^* . (A^{-1})^* = \Rightarrow_{S,A}$.

Let us have s and t such that $s >_S t$ and $s \rightarrow_A y$. We have to prove that y $A^* . >_S . (A^{-1})^*$ t. The proof is made by subterm induction following the cases of definition of $s >_S t$. ◆

As consequence of the cooperation of S with T, we get the following property:

Property 7:
$\Rightarrow_{S,A} = \downarrow_A . >_{RPO} . (\downarrow_A)^{-1}$.

Proof:
We use here the following result given in [2]: if $\Rightarrow_{S,T}$ is a well-founded ordering, then $\Rightarrow_{S,T} = \downarrow_T . S . (S \cup T)^* . (\downarrow_T)^{-1}$. By Theorem 1, $\Rightarrow_{S,A}$ is noetherian, then $\Rightarrow_{S,A} = \downarrow_A . S . (A \cup S)^* . (\downarrow_A)^{-1}$.

- $\Rightarrow_{S,A} \subseteq \downarrow_A . >_{RPO} . (\downarrow_A)^{-1}$ since A⊆RPO and S⊆RPO.
- $\downarrow_A . S . (\downarrow_A)^{-1} = \downarrow_A . >_{RPO} . (\downarrow_A)^{-1}$ since S=RPO in A-fn. Moreover, $\downarrow_A . S . (\downarrow_A)^{-1} \subseteq \downarrow_A . S . (A \cup S)^* . (\downarrow_A)^{-1} = \Rightarrow_{S,A}$. Then $\downarrow_A . >_{RPO} . (\downarrow_A)^{-1} \subseteq \Rightarrow_{S,A}$. ◆

Property 8:
S is F-compatible i.e. sSt ⇒ f(...s...) S f(...t...) for all f in F, if the precedence satisfies the minimality conditions.

Minimality conditions:

- for all symbol f with arity(f)>0, then f>+ or f is not comparable with +

- if a is a constant with $+ > a$, then all constants are minimal and equivalent.

Without these conditions, S is not F-compatible. For example, $+(x,y)$ S $f(x,y)$ for $+ > f$, and $+(+(x,y),z)$ is not S $+(f(x,y),z)$ since $+(+(x,y),z)\downarrow_A = +(x,y,z)$ is not $>_{RPO}$ $+(f(x,y),z)\downarrow_A = +(f(x,y),z)$.

Proof of Property 8:

We have to prove that sSt $\Rightarrow$ f(...s...) S f(...t...). By property of the multiset extension, we have (...s...) SS (...t...).

(1) if $f \neq +$, f(...s...) S f(...t...) by definition of S.

(2) if $f = +$, we have to prove that $(+(...s...))\downarrow_A >_{RPO} (+(...t...))\downarrow_A$. We have $s\downarrow_A >_{RPO} t\downarrow_A$ since $S \subseteq \downarrow_A . >_{RPO} . (\downarrow_A)^{-1}$. We can distinguish the following cases:

Let $+(------)$ be a simplified notation for $(+(......))\downarrow_A$.

- $s = g(\bar{s})$, $t = h(\bar{t})$, $g \neq +$ and $h \neq +$:
  $(+(...s...))\downarrow_A = +(---s\downarrow_A---)$ and
  $(+(...t...))\downarrow_A = +(---t\downarrow_A---)$. By definition of RPO, we have
  $(+(...s...))\downarrow_A >_{RPO} (+(...t...))\downarrow_A$.
- $s = +(\bar{s})$, $t = +(\bar{t})$:
  then $s\downarrow_A = +(\bar{u})$, $t\downarrow_A = +(\bar{v})$ and the ui and vi are not rooted in +. Moreover, since $s\downarrow_A >_{RPO} t\downarrow_A$, we have $(\bar{u}) >>_{RPO} (\bar{v})$ and
  $(+(...s...))\downarrow_A = +(---\bar{u}---)$ and
  $(+(...t...))\downarrow_A = +(---\bar{v}---)$. By definition of RPO, we have
  $(+(...s...))\downarrow_A >_{RPO} (+(...t...))\downarrow_A$.
- $s = f(\bar{s})$, $t = +(\bar{t})$ and $f \neq +$:
  then $s\downarrow_A = f(\bar{u})$, $t\downarrow_A = +(\bar{v})$ and the vi are not rooted in +. Moreover, since $s\downarrow_A >_{RPO} t\downarrow_A$, we have $s\downarrow_A >_{RPO} \bar{v}$. Then
  $(+(...s...))\downarrow_A = +(---s\downarrow_A---)$
  $(+(...t...))\downarrow_A = +(---\bar{v}---)$. By definition of RPO, we have
  $(+(...s...))\downarrow_A >_{RPO} (+(...t...))\downarrow_A$.
- $s = +(\bar{s})$, $t = f(\bar{t})$ and $f \neq +$:
  then $s\downarrow_A = +(\bar{u})$, $t\downarrow_A = f(\bar{v})$ and the ui are not rooted in +. Moreover, since $s\downarrow_A >_{RPO} t\downarrow_A$, by the minimality conditions, there exists ui $>_{RPO}$ or $\cong t\downarrow_A$. Then
  $(+(...s...))\downarrow_A = +(---\bar{u}---)$,
  $(+(...t...))\downarrow_A = +(---t\downarrow_A---)$. By definition of RPO, we have
  $(+(...s...))\downarrow_A >_{RPO} (+(...t...))\downarrow_A$. ◆

Let us now prove that S is stable by instantiation.

Property 9:

S is stable by instantiation, i.e. sSt $\Rightarrow$ σ(s) S $\sigma(t)$ for all substitution σ, even if the precedence doesn't satisfy the minimality conditions.

Sketch of proof:
The proof is made by subterm induction, distinguishing the cases of the definition of S.

Let us prove finally that $\Rightarrow_{S,A}$ is PER-commuting, and more, that $\Rightarrow_{S,A}$ is PER-complete.

Property 10:
$\Rightarrow_{S,A}$ is PER-complete i.e. $=_{PER} \cdot \Rightarrow_{S,A} \cdot =_{PER} \;=\; \Rightarrow_{S,A}$.

Proof:
$=_{PER} \cdot \Rightarrow_{S,A} \cdot =_{PER}$
$= =_{PER} \cdot \downarrow_A \cdot >_{RPO} \cdot (\downarrow_A)^{-1} \cdot =_{PER}$
$= \downarrow_A \cdot \cong \cdot >_{RPO} \cdot \cong \cdot (\downarrow_A)^{-1}$ by Property 0
$= \downarrow_A \cdot >_{RPO} \cdot (\downarrow_A)^{-1}$ (since the RPO is $\cong$-complete)
$= \Rightarrow_{S,A}$. ◆

The main termination theorem can now be given. It is a consequence of Theorem 2.

Theorem 3:
Provided the minimality conditions of the precedence, a rewriting system R is PER-terminating if for all rule l→ of R, we have l $\Rightarrow_{S,A}$ r,

Note that $\Rightarrow_{S,A}$ is easily decidable since, by Property 7, $\Rightarrow_{S,A} \;=\; \downarrow_A \cdot >_{RPO} \cdot (\downarrow_A)^{-1}$.

Let us suggest an extension of this work. If the minimality conditions of the precedence are not valid, the ordering is no more F-compatible. For example, let * and + two permutative operators such that *>+. We have x*y $\Rightarrow_{S,T}$ x+y but (x*y)*z not $\Rightarrow_{S,T}$ (x+y)*z since x*y*z not $\Rightarrow_{S,T}$ (x+y)*z. Like in the AC case [5,1], we have to introduce an additional transformation in A like distributivity of * over +. We have then to prove that S cooperates with the new relation A and that $\Rightarrow_{S,T}$ is PER-commuting.

Important remark:
The characterization of the PER equational theories seems to be nothing other that a characterization of the equational theories included in the AC theory. Since we have tools to prove AC termination, these tools can be used here to prove P-termination.

Proof:
The AC termination implies the PER-termination. Indeed, suppose there exists an infinite chain $\rightarrow \cdot =_{PER} \cdot \rightarrow \ldots =_{PER} \cdot \rightarrow \ldots$ The infinite chain $\rightarrow \cdot =_{AC} \cdot \rightarrow \ldots =_{AC} \cdot \rightarrow \ldots$ can be deduced since PER $\subseteq$ AC.

5 Conclusion

In this paper, a generalization of the use of flattening transformation was proposed, to ensure termination of rewriting in a large class of permutative theories. Conditions were pointed out to define the maximal class of permutative theories, flattening can work with. But these conditions seem to be a characterization of the theories included in AC and so becomes obvious the fact that flattening provides an ordering for the PER theories. This work however leads us to an interesting property: the AC theory is the maximal theory working with flattening: flattening is a bijection between the AC classes and the $\cong$-classes, whereas it is a surjection between PER-classes and $\cong$ classes. It would be interesting to find a bijective transformation for a given PER theory, in order to obtain an ordering more precise than the AC ordering. Indeed an AC class can contain several PER-classes. Then two terms of different PER-classes can be equivalent for the AC ordering, and become comparable with a finer ordering.

References

[1] L. Bachmair, D. Plaisted, "Associative Path Orderings", 1985, Proc. 1st Conference on Rewriting Techniques and Applications, Springer Verlag, Lecture Notes in Computer Science, Dijon (France)

[2] F. Bellegarde, P. Lescanne, "Transformation Ordering", 1986, Report CRIN 86−R−092

[3] Nachum Dershowitz, "Orderings for Term−Rewriting Systems", Theoretical Computer Science, 1982

[4] N. Dershowitz "Termination", May 1985, Proc. 1rst Conf. Rewriting Techniques and Applications, 180−224, Springer Verlag, Lecture Notes in Computer Science, Dijon (France)

[5] I. Gnaedig, P. Lescanne, "Proving Termination of Associative Rewriting Systems by Rewriting", 1986, Springer Verlag, Lecture Notes in Computer Science, Proc. 8th Conf. on Automated Deduction, Oxford (England)

[6] H. J. Jeanrond, "Deciding Unique Termination of Permutative Rewriting Systems: Choose Your Term Algebra Carefully", 1980, Proceedings 5th international Conference on Automated Deduction, Les Arcs (Savoie, France), Lecture Notes in Computer Science, Springer Verlag

[7] J.P. Jouannaud, M. Munoz, "Termination of a set of rules modulo a set of equations", 1984, Proceedings 7th Conference on Automated Deduction, Napa Valley (California, USA), Lecture Notes in Computer Science, Springer Verlag, 175−193

[8] C. Kirchner, "Méthodes et outils de conception systématique d'algorithmes d'unification dans les théories équationnelles", Thèse de doctorat d'Etat, Université de Nancy I, 1985

[9] D.S. Lankford, A.M. Ballantyne, "Decision Procedures for Simple Equational Theories With Permutative Axioms: Complete Sets of Permutative Reductions", Report Atp−37, Dept. of Mathematics And Computer Science U.Texas, Austin April 1977

Chapter 3
A new transformation for proving AC termination

Abstract

A new ordering for proving AC termination is proposed, based on a new transformation method. This ordering try to replace both transformations of flattening and distributivity, the existing orderings are based on. We show why this method fails.

Until now, the orderings for proving AC termination are based on a double transformation expressed by rewriting rules: flattening terms on their AC operators, and distributing an AC operator on an operator comparable with the AC operators for the precedence [4,1].

The NFLO ordering [4] consists in transforming the terms to be compared with the previous methods, then in comparing the resulting terms with the RPO, largely used in termination proofs of standard rewriting systems.

Recall that flattening allows the ordering to be AC−commuting (the terms are transformed into the representative of their AC equivalence class). Moreover the distributivity permits the ordering to be F−compatible. Indeed, with flattening alone, we have $x * y >_{RPO} x + y$ if $* > +$ but $(x * y) * z$ *not* $>$ $(x + y) * z$ since (x*y)*z flattens into x*y*z and $x * y * z$ *not* $>_{RPO}$ $(x + y) * z$. If however the second term is transformed by distributivity of * on +, we obtain $x * y * z >_{RPO} (x * z) + (y * z)$.

We propose here a much simpler method, replacing flattening by a "weaker" transformation, conserving the original depth of the term; we will so avoid to use the distributive transformation. Let us introduce our method on an example.

Example1:
The term (x*y)*z is transformed into *(*(x,y,z)). This transformation respects the compatibility of the ordering on the previous example. Indeed, we have $*(\,*(x,y,z)\,) >_{RPO} *(\,+(x,y),z)$.

Remark:
The same symbol appears twice in the transformed terms with a different arity: we work on the varyadic term algebra TV(F,X).

The new transformation is named "decanting transformation".

Definition1:
The decanting transformation is formalized by an infinite rewrite system of the form:

$$f(x_1, \ldots x_{i-1}, f(y_1, \ldots y_n), x_{i+1} \ldots x_m) \rightarrow f(\,f(x_1, \ldots, x_{i-1}, y_1, \ldots y_n, x_{i+1}, \ldots x_m)\,)$$

for $i >= 2$ or $n >= 1$, for every AC operator f of F.

Decanting appears as a bijection from T(F,X)/AC to TV(F,X)/≅, as well as flattening. This transformation seems to have a dual behavior in comparison with flattening. Instead of minimality of the AC operators (for flattening), the maximality is needed here. Indeed, if x*y > f(x,y) with *>f, then f((x*y),z) not> f(f(x,y,z)): the ordering is not F-compatible. But with the precedence f>*, we obtain f(x,y) >x*y and f(f(x,y,z)) > f((x*y),z).

Our aim is to provide a termination proof for AC rewriting systems. Let us recall basic definitions. Let S and T be two abstract relations on terms.

Definition: [Bachmair & Dershowitz]
$\Rightarrow_{S,T}$ is the relation $\rightarrow_T^* . \rightarrow_S . (\rightarrow_S \cup \rightarrow_T)^* . \leftarrow_T^*$.

Definition: [Bellegarde & Lescanne]
$\rightarrow_S$ cooperates with $\rightarrow_T$ if and only if $\leftarrow_T^* . \rightarrow_S . (\rightarrow_S \cup \rightarrow_T)^* \subseteq \Rightarrow_{S,T}$.

Now the termination theorem of Gnaedig & Bellegarde (see Chapter 2) can be given.

Theorem:
If $\rightarrow_S$ cooperates with $\rightarrow_T$, $\rightarrow_S \cup \rightarrow_T$ is noetherian, $\rightarrow_T$ is confluent, $\Rightarrow_{S,T}$ is E-commuting, $\rightarrow_S, \rightarrow_T$ are F-compatible and stable by substitution, then R is E-terminating if for every rule l→r of R, we have $l \Rightarrow_{S,T} r$.

Let us choose the decanting transformation for T and the RPO for S. The proofs of the following properties are made rigorously in the same way that Bellegarde"s ones in [2].

The relation T is confluent and terminating. Since T is a rewriting relation, it is F-compatible and stable by substitution, like S which is the RPO. Moreover, $S \cup T$ is well-founded and the relation $\Rightarrow_{S,T}$ is AC-commuting.

Unfortunately, $\Rightarrow_{S,T}$ is not F-compatible. Let us look at the following counter-example.

We have f((x+y),z) $\Rightarrow_{S,T}$ (x+y) but f((x+y),z)+a not $\Rightarrow_{S,T}$ (x+y)+a since (x+y)+a $\rightarrow_T$ +(+(x,y,a)) and f((x+y),z)+a not $>_{RPO}$ +(+(x,y,a)). Indeed {f((x+y),z), a} not $\gg_{RPO}$ {+(x,y,a)}.

Let us try to avoid this critical case in suppressing the following clause in the definition of S: s $>_S$ t if there exists si ≅ t and top(si)=top(t)=+.

This new definition of S is unfortunately not valid since it suppresses the subterm property that is essential in the proof of cooperation of S with T: indeed, f((x+y),z) not $>_S$ x+y. The decanting transformation seems then not provide a valid ordering for AC termination proofs.

References

[1] L. Bachmair, D. Plaisted, "Associative Path Orderings", 1985, Proc. 1st Conference on Rewriting Techniques and Applications, Springer Verlag, Lecture Notes in Computer Science, Dijon (France)

[2] F.Bellegarde "Terminaison AC et cooperation" Internal Report CRIN To appear

[3] F.Bellegarde P.Lescanne "Transformation Ordering", CAAP 1987

[4] I. Gnaedig, P. Lescanne, "Proving Termination of Associative Rewriting Systems by Rewriting", 1986, Springer Verlag, Lecture Notes in Computer Science, Proc. 8th Conf. on Automated Deduction, Oxford (England)

Imprimé en France

par

l'Institut National de Recherche en Informatique et en Automatique