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Philippe Nain, Nicolas D. Georganas, Willam J. Stewart

IRIA

CENTRE
SOPHIA ANTIPOLIS

Institut National
de Recherche
en Informatique
et en Automatique

Domaine de Voluceau
Rocquencourt
B.P.105
78153 Le Chesnay Cedex
France
Tél.(1) 39 63 55 11

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Philippe NAIN
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ANALYSIS OF A HYBRID MULTIPLE ACCESS PROTOCOL WITH FREE ACCESS OF NEW ARRIVALS DURING CONFLICT RESOLUTION

Philippe NAIN*

Nicolas D. GEORGANAS**

William J. STEWART***

RESUME

HYMAP est un protocole hybride multi-accès qui a été récemment proposé par Rios et Georganas [4]. Il combine les caractéristiques les plus intéressantes du protocole CSMA/CD et d'un protocole à jeton virtuel. Le contrôle est transféré d'un protocole à l'autre en utilisant simplement l'information circulant sur le canal de communication. Dans [4] les performances de HYMAP ont été évaluées par simulations numériques. Dans cet article, nous présentons une analyse exacte de ce protocole, basée sur la résolution d'un modèle stochastique. Un trait principal de ce modèle est qu'il autorise la génération de nouveaux messages pendant les périodes de résolution de collisions. Les outils utilisés sont à la fois probabilistes (chaines de Markov, processus régénératifs) et numériques (résolution de gros systèmes d'équations linéaires). Les performances de HYMAP (débit, délai moyen,...) peuvent alors être calculées et comparées aux performances du protocole CSMA/CD.

ABSTRACT

A Hybrid Multiple Access Protocol (HYMAP) was proposed recently (Rios and Georganas [4]), combining the best features of CSMA/CD [5] and of a conflict-free protocol. Control is tranferred from one protocol to the other according to state information sensed on the channel. HYMAP was evaluated by computer simulation. In this paper, we present an exact analysis of this hybrid protocol, based on the resolution of a stochastic model. An important feature of the model is that it permits free access of new arrivals during collision resolution periods. The tools we use are both probabilistic (Markov chains, regenerative processes) and numerical (resolution of large systems of linear equations). The basic mean performance measures (throughput, delay,...) can then be easily computed and compared to the performance of CSMA/CD.

This work was done when the second and the third author were spending their sabbatical leave from the University of Ottawa at Bull Transac (France) and from North Carolina State University at INRIA, Rennes (France), respectively.

* INRIA Centre de Sophia Antipolis, Av. E. Hughes, 06560 Valbonne, France.

** Faculty of Engineering, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5.

*** Dept. of Computer Science, North Carolina State University, Raleigh, NC 27695-8206 USA.

INTRODUCTION

This paper models and analyses the performance of HYMAP, a hybrid multiple access protocol that was recently proposed and studied by simulation (Rios and Georganas [4]). HYMAP utilizes the best features of both CSMA/CD [5, 6] and a collision-free (CF) protocol. Briefly, when the traffic load is light the protocol operates in CSMA/CD mode (except for the collision resolution procedure), whereas when a collision occurs, or at high traffic load, it uses a collision-free virtual token procedure. The protocol reverts back into CSMA/CD mode when the traffic load becomes light again.

A more detailed description of HYMAP is given below.

a) *The HYMAP protocol*

M stations (a number known of all of them) are connected on an error-free multiple-access bus. Without loss of generality, we assume that the stations are numbered from 1 to M, starting from one end of the bus. The stations initially operate on CSMA/CD mode, transmitting packets of fixed length. If a collision occurs, it is assumed that it will be detected by all stations, which then enter a wait-while-listening state (with the exception of station 1). Station 1, after sensing a collision, waits until the channel becomes idle and starts a synchronization procedure. This means sending a group of bits of duration d_b to indicate whether or not it will transmit immediately after. Station 2, sensing a positive ("P") transmission indication of station 1 will wait until the end of the packet transmission, while sensing a negative indication ("N") will consider it as a *virtual token* and will produce its own "P" message plus a packet *or* "N" message, and so on. In this fashion the virtual token is passed from station to station. In this state, previously collided packets plus any new arrivals will be transmitted. The protocol reverts back to CSMA/CD mode if M consecutive "N" messages have been sensed on the bus.

This synchronization procedure in the conflict-free state can be easily implemented by counters at each station. The protocol is completely distributed in the sense that if station 1 fails to start the synchronization process following a collision, station 2 would take that task, after a time-out and so on (see [4]).

HYMAP was shown by simulation [4] to be superior to other well known protocols such as GBRAM (Gold and Franta [2]), SDAM (Li, Hughes and Greenberg [3]) and of course CSMA/CD [5, 6], for both data packets and voice packets.

b) *Methodology and Outline of the Analysis*

The analysis follows the approach of Tobagi and Hunt [6], in defining, in section II, an *embedded homogeneous Markov chain* — with finite state space — describing the evolution of the number of backlogged stations. The transition matrix of the chain is composed of two terms, one corresponding to the CSMA/CD mode of the protocol and the other to the CF period. The stationary probability distribution of the channel backlog at the observation instants can then be obtained by solving — numerically — the standard invariant measure equation for Markov chains.

This result, together with the properties resulting from the *theory of regenerative processes*, enable us to derive, in section III, an explicit expression for the *stationary channel throughput*. However, this formula contains $2(M+1)$ unknown constants, which have to be computed. Roughly speaking, these numbers refer to the mean duration of a CF period and to the mean number of packets transmitted in this period. We show, in section 4, that these unkown quantities can be determined by solving two systems of linear equations of dimension $M^2 \times M^2$. Using a similar method, the *mean delay* is investigated in section 5. Numerical results showing, in particular, the *throughput-delay* performance of HYMAP and comparison of this protocol with CSMA/CD are presented in section VI.

II. THE MODEL

Our model is similar to that defined by Tobagi and Hunt [6]. The channel is assumed to be slotted along the time axis, the slot size being equal to the *end-to-end* propagation delay. Without loss of generality (see remark 5.2), we suppose that $d_b = 1$, namely the emission of a synchronization message in the CF period takes exactly one slot. The packet size is equal to T time slots and the collision detect time in CSMA/CD mode is equal to γ time slots. All stations are synchronized and start their transmission at the beginning of a slot. Note that because of the end-to-end propagation delay, the total transmission time of a packet (resp. collision detect time) is $T+1$ (resp. $\gamma+1$) time slots — see [6] and figure 1.

At the beginning of a slot, a station can be in one of the following two states: (1) *thinking* if it does not have any information to send (packet or "P/N" message), and (2) *backlogged* if it has a packet and/or a "P/N" message to transmit.

A station in the thinking state is allowed to generate a new packet in the current slot with the probability σ. A backlogged station remains in that state until it completes successful transmission of its packet, at which time it switches to the thinking state. A backlogged station cannot generate a new packet (i.e., a station cannot store more than one packet at a time). In the CSMA/CD mode the rescheduling delay of a backlogged packet is assumed to be geometrically distributed with mean $1/\nu$ slots. In other words, a backlogged user (only in the CSMA/CD mode) will sense the channel and, if free, will transmit in the current slot with the probability ν.

The following notation will be used throughout this paper: (see figure 1)

- t_e is the beginning of the e-th idle-period, $e = 1, 2, \ldots$;
- I is the duration of an idle period;
- CF* refers to the period of the CF mode where the M negative consecutive messages are sent;
- θ (resp. η) denotes the duration of (resp. the number of transmitted packets in) the period $(t_e + I + \gamma + 2,\ t_{e+1} - 1)$, given that there was a collision in the time interval $(t_e + I,\ t_e + I + 1)$.

Let $N(t_e)$ be the number of backlogged stations at time t_e. $(N(t_e))_{e\in\mathbf{N}}$ is a *homogeneous Markov chain* with state space $\{0, 1, \ldots, M\}$. From the model assumptions, it is readily seen that this Markov chain is *irreducible, aperiodic* and *positive recurrent*. Denote $\mathbf{P}$ the corresponding transition matrix.

Let $\Pi = (\Pi_i)_{0\le i\le M}$ be the *unique* probability measure satisfying the equation $\Pi = \Pi\,\mathbf{P}$. Π is the *stationary distribution* of the Markov chain $(N(t_e))_{e\in\mathbf{N}}$ [1, p. 153].

PROPOSITION 2.1. — *The transition matrix* $\mathbf{P}$ *is given by*

$$\mathbf{P} = \mathbf{S}\,\mathbf{Q}^{T+1}\,\mathbf{J} + \mathbf{F}\,\mathbf{D}\,\mathbf{Q} \tag{2.1}$$

where

$$\mathbf{S} := (s_{i,k})_{0\le i,k\le M} \quad \mathbf{J} := (j_{i,k})_{0\le i,k\le M} \quad \mathbf{Q} := (q_{i,k})_{0\le i,k\le M}$$
$$\mathbf{F} := (f_{i,k})_{0\le i,k\le M} \quad \mathbf{D} := (\delta_{i,k})_{0\le i,k\le M} \text{ with}$$

$s_{i,k}$:=*P*($N(t_e+I) = k$ *and the transmission is successful* | $N(t_e+I-1) = i$),
$f_{i,k}$:=*P*($N(t_e+I) = k$ *and the transmission is unsuccessful* | $N(t_e+I-1) = i$),
$\delta_{i,k}$:=δ_k:=*P*(k *packets are generated during CF**).

The matrices **S, Q, J, F** *are given in* [6].

The proof is straightforward. Note that the first term in (2.1) corresponds to the CSMA/CD mode of the protocol (see [6, formula (6)]) and the second term corresponds to the CF period.

The elements of the unknown matrix **D** are provided by the following result.

PROPOSITION 2.2. — *The δ_j's (cf. proposition 2.1) are given by the following relations*

$$\delta_0 = (1-\sigma)^{\frac{M(M-1)}{2}},$$

$$\delta_j = (1-\sigma)^{M(\frac{M-1}{2}-j)} \sum_{E_j} (1-\sigma)^{\sum_{k=1}^{j} i_k} \prod_{k=1}^{j} \left(1-(1-\sigma)^{M-i_k}\right), \quad 1 \le j \le M-1, \tag{2.2}$$

$$\delta_M = 0,$$

where

$$E_j := \Big\{(i_1,\ldots,i_j) \in \{1,\ldots,M-1\} \text{ such that } i_1 < i_2 < \cdots < i_j\Big\}.$$

PROOF. Without loss of generality, let us assume that the first of the M messages ("N" messages) transmitted in CF* has been emitted by station 1. It readily follows that

$$\text{P(station } i \text{ generates a new packet during CF}^*) = 1-(1-\sigma)^{M-i}, \quad 1 \le i \le M. \tag{2.3}$$

From (2.3) we deduce that $\delta_M = 0$, since station M cannot generate a message in CF*.

On the other hand, we clearly have that, cf. (2.3),

$$\begin{aligned}\delta_0 &= \prod_{i=1}^{M-1} (1-\sigma)^{M-i} \\ &= (1-\sigma)^{\frac{M(M-1)}{2}}.\end{aligned}$$

Let us now examine the remaining cases $1 \le j \le M-1$. We have

$$\begin{aligned}\delta_j &= \sum_{E_j} P(i_1,\ldots,i_j \text{ generate new packets}, i_{j+1},\ldots,i_{M-1} \text{ do not generate new packets}), \\ &= \sum_{E_j} P(i_1,\ldots,i_j \text{ generate new packets})\, P(i_{j+1},\ldots,i_{M-1} \text{ do not generate new packets}), \\ &= \sum_{E_j} \prod_{k=1}^{j} \left(1-(1-\sigma)^{M-i_k}\right) \prod_{k=j+1}^{M-1} (1-\sigma)^{M-i_k}, \text{ from (2.3)}, \\ &= \sum_{E_j} (1-\sigma)^{M(M-j-1)-\sum_{k=j+1}^{M-1} i_k} \prod_{k=1}^{j} \left(1-(1-\sigma)^{M-i_k}\right).\end{aligned} \tag{2.4}$$

Combining now the relation $\sum_{k=1}^{j} i_k + \sum_{k=j+1}^{M-1} i_k = \frac{(M-1)M}{2}$ together with (2.4), we get (2.2). ∎

The numerical computation of the δ_j's (an explicit calculation seems to be hopeless) requires some precautions. Indeed, for fixed j with $1 \leq j \leq M-1$, the cardinal of E_j is $\binom{M-1}{j}$. For instance, if $M = 50$ (i.e., 50 stations) and $j = 25$, then $\binom{M-1}{j} \sim 6.10^{13}$!... To overcome this numerical difficulty, we propose in appendix A a convolution type algorithm performing the computation of *all* the δ_j's in $0(M^2)$ operations.

Propositions (2.1) and (2.2) allow the computation of the transition matrix $\mathbf{P}$. The invariant measure Π can then be obtained by solving the following system of linear equations:

$$\Pi = \Pi\,\mathbf{P};$$

$$\Pi.\mathbf{1} = 1,$$

where $\mathbf{1}$ denotes the unit vector.

III. THROUGHPUT ANALYSIS

Let $Z(t)$ be the number of backlogged stations at the beginning of the t-th slot. $\{Z(t),\, t \geq 1\}$ is a *semi-regenerative* process associated with the *Markov renewal* process $\{(N(t_e), t_e),\, t_e = 1, 2, ...\}$ [1, pp. 343-350].

Let S be the stationary throughput of the channel, i.e., S is defined as the fraction of channel time occupied by valid transmissions. A standard result from the renewal theory tells us that S is obtained as the ratio of the time the channel is carrying a successful transmission during a cycle (i.e., during (t_e, t_{e+1})) averaged over all cycles, to the average cycle length. Therefore we have

$$S = \frac{\sum_{i=0}^{M} \Pi_i \left[T\, P_s(i) + (1 - P_s(i))\; T\, R_i\right]}{\sum_{i=0}^{M} \Pi_i \left[I_i + 1 + P_s(i)\, T + (1 - P_s(i))\, (\gamma + L_i + 1)\right]}, \tag{3.1}$$

where, given that $N(t_e) = i$,

- $P_s(i)$ is the probability of a successful transmission during a cycle;
- I_i is the average idle period duration (i.e., $(t_e, t_e + I)$);
- R_i is the average number of packets transmitted in a CF period;
- L_i is the average duration of a CF period.

All the above quantities are defined at steady state. $P_s(i)$ and I_i are given in [6, formula (12)]. R_i and L_i, $i = 0, 1, \ldots, M$, are *unknown* numbers.

The remainder of this section as well as the next section are devoted to the determination of these quantities. We first investigate (cf. lemma 3.1) the connection between the state of the system at time t_e and its state at time $t_e + I + \gamma + 2$ (i.e., one slot after the beginning of a CF period, cf. figure 1).

Define the four following $(M+1)$-vectors

$$\mathbf{L}^{(0)} := \begin{pmatrix} L_0^{(0)} \\ \vdots \\ L_M^{(0)} \end{pmatrix} \qquad \mathbf{L}^{(1)} := \begin{pmatrix} L_0^{(1)} \\ \vdots \\ L_M^{(1)} \end{pmatrix} \qquad \mathbf{R}^{(0)} := \begin{pmatrix} R_0^{(0)} \\ \vdots \\ R_M^{(0)} \end{pmatrix} \qquad \mathbf{R}^{(1)} := \begin{pmatrix} R_0^{(1)} \\ \vdots \\ R_M^{(1)} \end{pmatrix}$$

where

$$\begin{cases} L_0^{(0)} = L_1^{(0)} = L_M^{(0)} = 0; \\ L_k^{(0)} := E\{\theta \mid \text{station 1 is idle at } t_e + I + \gamma + 2, N(t_e + I + \gamma + 2) = k\}, \quad \text{for } 2 \le k \le M-1, \end{cases}$$

$$\begin{cases} L_0^{(1)} = L_1^{(1)} = 0; \\ L_k^{(1)} := E\{\theta \mid \text{station 1 is non-idle at } t_e + I + \gamma + 2, N(t_e + I + \gamma + 2) = k\}, \quad \text{for } 2 \le k \le M. \end{cases}$$

For $0 \le k \le M$, $R_k^{(0)}$ and $R_k^{(1)}$ are obtained from $L_k^{(0)}$ and $L_k^{(1)}$ respectively, by replacing θ by η (η and θ have been defined in section II).

REMARK 3.1. — It is to be noted that station 1 is in the same state (i.e., idle or not) at time $t_e + I + \gamma + 1$ (i.e., at the beginning of the CF period) and at time $t_e + I + \gamma + 2$ (i.e., one slot later). Indeed, if station 1 has a packet to transmit at time $t_e + I + \gamma + 1$, then it has still a packet to transmit at time $t_e + I + \gamma + 2$. On the other hand, if station 1 is idle at time $t_e + I + \gamma + 1$, then it is still idle one slot later, since a station cannot generate a new packet while sending a "P/N" message (here "N" message).

LEMMA 3.1.

$$\mathbf{L} = \mathbf{G}\,\mathbf{F}\,\mathbf{Q}^{\gamma+1}\left[(\mathbf{I}-\mathbf{U})\,\mathbf{Q}\,\mathbf{L}^{(0)} + \mathbf{U}\,\mathbf{Q}\,\mathbf{L}^{(1)} + \mathbf{1}\right], \tag{3.2}$$

$$\mathbf{R} = \mathbf{G}\,\mathbf{F}\,\mathbf{Q}^{\gamma+1}\left[(\mathbf{I}-\mathbf{U})\,\mathbf{Q}\,\mathbf{R}^{(0)} + \mathbf{U}\,\mathbf{Q}\,\mathbf{R}^{(1)}\right], \tag{3.3}$$

where

$$\mathbf{L} := \begin{pmatrix} L_0 \\ \vdots \\ L_M \end{pmatrix}, \qquad \mathbf{R} := \begin{pmatrix} R_0 \\ \vdots \\ R_M \end{pmatrix}.$$

$\mathbf{U}:=(u_{i,j})_{0\le i,j\le M}$ *and* $\mathbf{G}:=(g_{i,j})_{0\le i,j\le M}$ *are diagonal matrices, where*

$$g_{i,i} := (1 - P_s(i))^{-1},$$

and

$$u_{i,i} := \begin{cases} 0, & \text{if } i = 1; \\ i/M, & \text{otherwise.} \end{cases}$$

$\mathbf{I}$ *is the identity matrix of dimension* $(M+1)\times(M+1)$. *The* L_i*'s and* R_i*'s have been previously defined.*

PROOF. First, let us define for $2 \le i \le M$,

$$H_i := E(\Delta \mid N(t_e + I + \gamma + 1) = i), \tag{3.4}$$

where Δ is the duration of a CF period, i.e., $\Delta = (t_{e+1} - 1) - (t_e + I + \gamma + 1)$, cf. figure 1. (Note that $\Delta = \theta + 1$.)

H_i is therefore the average duration of a CF period, given that i stations have a packet to transmit at the beginning of this period (i.e., at time $t_e + I + \gamma + 1$). Note that clearly i must be greater or equal to 2, since there was a collision in the time slot $(t_e + I, t_e + I + 1)$.

From (3.4), it is easily seen, together with the definitions of $\mathbf{Q}$, $\mathbf{F}$ and $\mathbf{G}$, cf. section II, that

$$\mathbf{L} = \mathbf{G}\,\mathbf{F}\,\mathbf{Q}^{\gamma+1}\,\mathbf{H}, \tag{3.5}$$

where

$$\mathbf{H} := \begin{pmatrix} 0 \\ 0 \\ H_2 \\ \vdots \\ H_M \end{pmatrix}.$$

It is to be noted that the first two components of vector $\mathbf{H}$ can be chosen arbitrarily (here we took 0), since the elements of the first two columns of matrix $\mathbf{G}\,\mathbf{F}\,\mathbf{Q}^{\gamma+1}$ are all equal to zero.

Also define,

$$\begin{cases} H_i^0 := E\left(\Delta \,|\, \text{station 1 is idle at } t_e + I + \gamma + 1 \text{ and } N\left(t_e + I + \gamma + 1\right) = i\right), \\ \text{for } 2 \le i \le M-1; \\ H_M^0 := 0; \\ H_i^1 := E\left(\Delta \,|\, \text{station 1 is non-idle at } t_e + I + \gamma + 1 \text{ and } N\left(t_e + I + \gamma + 1\right) = i\right), \\ \text{for } 2 \le i \le M. \end{cases} \tag{3.6}$$

Since all stations are statistically identical, we get from (3.4) and (3.6) that

$$H_i = \left(\frac{M-i}{M}\right) H_i^0 + \left(\frac{i}{M}\right) H_i^1. \tag{3.7}$$

On the other hand, it is easily seen from (3.6) and the definitions of $\mathbf{L}^{(0)}$, $\mathbf{L}^{(1)}$, $\mathbf{Q}$, as well as from remark 3.1, that

$$\begin{aligned} H_i^0 &= 1 + \sum_{k=i}^{M-1} L_k^{(0)}\, q_{k,i}, \qquad 2 \le i \le M-1, \\ H_i^1 &= 1 + \sum_{k=i}^{M} L_k^{(1)}\, q_{k,i}, \qquad 2 \le i \le M. \end{aligned} \tag{3.8}$$

Rewriting (3.8) under a matrix form and using (3.7), (3.5) we obtain (3.2). Relation (3.3) can be obtained in a similar way. ∎

The above lemma shows that the determination of the two sought vectors $\mathbf{L}$ and $\mathbf{R}$ turns out to be equivalent to the determination of the four vectors $\mathbf{L}^{(i)}$, $\mathbf{R}^{(i)}$, $i = 0, 1$.

The calculation of these four vectors is the object of the next section.

IV. COMPUTATION OF VECTORS $\mathbf{L}^{(0)}$, $\mathbf{L}^{(1)}$, $\mathbb{R}^{(0)}$, $\mathbb{R}^{(1)}$

Let $(j; m, l)$ with $(j, m) \in \{1, 2, \ldots, M\}^2$, $l \in \{1, 2, \ldots, M-1\}$ be the state of the system *just after* the emission of a "P/N" message by station j (this time will be denoted by t_j^*) where:

- m indicates the number of *consecutive* "N" messages which have been transmitted at time t_j^* if $1 \leq m \leq M-1$;
- $m = M$ means that station j has just sent a "P" message, i.e., station j will transmit a packet during the period $(t_j^*, t_{j+1}^* - 1)$ (here $t_{j+1}^* - 1 = t_j^* + T + 1$);
- l gives the number of backlogged stations at time t_j^*, non included station j.

Also define $\mathcal{F}$ the *final state*, i.e., when M consecutive "N" synchronization messages have been emitted in a CF period. State $\mathcal{F}$ is reached by the system at time $t_{e+1} - 1$, cf. figure 1.

a) *Computation of vectors* $\mathbf{L}^{(0)}$ *and* $\mathbf{L}^{(1)}$

Define $X(j; m, l)$ the average time for reaching state $\mathcal{F}$, starting from the initial state $(j; m, l)$. We immediately observe that the $X(j; m; l)$'s and the components of the sought vectors $\mathbf{L}^{(0)}$ and $\mathbf{L}^{(1)}$ are connected through the following relations:

$$\begin{cases} L_k^{(0)} = X(1; 1, k), & \text{for } 2 \leq k \leq M-1; \\ L_k^{(1)} = X(1; M, k-1), & \text{for } 2 \leq k \leq M. \end{cases} \tag{4.1}$$

Let us come back to the $X(j; m, l)$'s. They must satisfy the following linear relations:

$$X(j; m, l) = 1 + \sum_{l', m'} X(j+1; m', l')\, P(\,(j; m, l) \to (j+1; m', l')\,), \tag{4.2}$$

for $1 \leq m \leq M-1$, $0 \leq l \leq M-1$;

$$X(j; M, l) = 1 + (1+T) + \sum_{l', m'} X(j+1; m', l')\, P(\,(j; M, l) \to (j+1; m', l')\,), \tag{4.3}$$

for $0 \leq l \leq M-1$, where j is taken mod $M+1$.

As expected, we can observe from (4.2) and proposition B.1 (appendix B) that $X(j; M-1, 0) = 1$.

The derivation of equations (4.2), (4.3) is quite straightforward. It suffices to notice that only states $(j+1; \bullet, \bullet)$ can be reached from states $(j; \bullet, \bullet)$, and then to distinguish the type of "P/N" message sent by station j during the time interval $\left(t_j^*, t_j^* + 1\right)$ (if it is a "N" message, then we get (4.2), otherwise we get (4.3)).

For fixed j, $1 \leq j \leq M$, let us arrange states $(j; m, l)$ as follows:

$$(j; 1, 0)$$

$$\vdots$$

$$(j; 1, M-1)$$

$$(j;2,0)$$
$$\vdots$$
$$(j;2,M-1)$$
$$\vdots$$
$$(j;M,0)$$
$$\vdots$$
$$(j;M,M-1)$$

Note that state $(j;m,M-l)$ is in position $M(m-1)+l+1$.

Using the above arrangement between states $(j;m,l)$, we define for fixed j the following transition matrix:

$$\Big(\mathrm{P}((j;m,l)\rightarrow(j+1;m',l'))\Big)_{(l,m),(l',m')}.$$

The components of this matrix of dimension $M^2\times M^2$ are explicitly given in appendix B. Its components *do not depend on* j (which is rather obvious), and the matrix will be denoted by $\mathbf{A}$.

These results allow us to rewrite (4.2)-(4.3) under the following matrix form:

$$\begin{pmatrix} \mathbf{I} & -\mathbf{A} & & & \\ & \mathbf{I} & -\mathbf{A} & & \\ & & \ddots & \ddots & \\ & & & \mathbf{I} & -\mathbf{A} \\ -\mathbf{A} & & & & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{X}_1\\ \mathbf{X}_2\\ \vdots\\ \mathbf{X}_{M-1}\\ \mathbf{X}_M\end{pmatrix}=\begin{pmatrix}\mathbf{B}\\ \mathbf{B}\\ \vdots\\ \mathbf{B}\\ \mathbf{B}\end{pmatrix}, \tag{4.4}$$

where

$$\mathbf{X}_j:=\begin{pmatrix}\mathbf{X}(j;1,0)\\ \vdots\\ \mathbf{X}(j;1,M-1)\\ \vdots\\ \mathbf{X}(j;M,1)\\ \vdots\\ \mathbf{X}(j;M,M-1)\end{pmatrix},\qquad \mathbf{B}:=\begin{pmatrix}1\\ 1\\ \vdots\\ 1\\ 2+T\\ \vdots\\ 2+T\end{pmatrix}.$$

The dimension of the matrix in the left-hand side of (4.4) is $M^3\times M^3$ (all the missing elements are equal to zero). The $\mathbf{X}_j$'s are vectors of dimension M^2.

$\mathbf{B}$ is a vector of dimension M^2 where components number 1 to $(M-1)\,M$ are equal to 1, and where components number $(M-1)\,M+1$ to M^2 are equal to $2+T$.

We observe that a solution of (4.4) is

$$\mathbf{X}_1=\mathbf{X}_2=\cdots=\mathbf{X}_M:=\mathbf{X},$$

where $\mathbf{X}$ is given by

$$\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}. \tag{4.5}$$

This is the *only solution*, since there necessarily exists one and only one solution to (4.4) from model assumptions.

Having computed the vector $\mathbf{X}$, the vector $\mathbf{L}^{(0)}$ is given by the components number 3 to M of $\mathbf{X}$, and the vector $\mathbf{L}^{(1)}$ is given by the components number $(M-1)\,M+2$ to M^2. This follows from (4.1).

b) *Computation of vectors* $\mathbf{R}^{(0)}$ *and* $\mathbf{R}^{(1)}$

Let $Y(j;m,l)$ be the mean number of packets transmitted between state $(j;m,l)$ and the final state $\mathcal{F}$. Similarly to (4.2)-(4.3), we can derive the following relations:

$$Y(j;m,l) = \sum_{l',m'} Y(j+1;m',l')\,P(\,(j;m,l) \rightarrow (j+1;m',l')\,)\,, \tag{4.6}$$

for $1 \leq m \leq M-1$, $0 \leq l \leq M-1$;

$$Y(j;M,l) = 1 + \sum_{l',m'} Y(j+1;m',l')\,P(\,(j;M,l) \rightarrow (j+1;m',l')\,), \tag{4.7}$$

for $0 \leq l \leq M-1$.

By analogy with case a) (see equations (4.2), (4.3)), it is readily seen from (4.6), (4.7) that

$$\mathbf{Y} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{C}, \tag{4.8}$$

where

$$\mathbf{Y} := (Y(j;m,l))_{(l,m)}$$

and

$$\mathbf{C} := \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \underline{1} \\ \vdots \\ 1 \end{pmatrix}. \tag{4.9}$$

$\mathbf{C}$ is a vector of dimension M^2 where only the last M components are non null and equal to 1.

The sought vectors $\mathbf{R}^{(0)}$ and $\mathbf{R}^{(1)}$ are then obtained as the components number 3 to M and $(M-1)M+2$ to M^2 respectively, of the vector $\mathbf{Y}$.

V. DELAY ANALYSIS

Let $\bar{N}$ be the average channel backlog. Similarly to the computation of the throughput in section III, $\bar{N}$ is obtained as the ratio of the expected sum of backlogs over all slots in a cycle averaged over all cycles, to the average cycle length. Therefore we have:

$$\bar{N} = \frac{\sum_{i=0}^{M} \Pi_i \left[i\, I_i \, + \, A(i) \right]}{\sum_{i=0}^{M} \Pi_i \left[I_i + 1 + P_s(i)\, T + (1 - P_s(i))\, (\gamma + L_i + 1) \right]}, \tag{5.1}$$

where $A(i)$ is the expected sum of backlogs over all slots in a busy period, given that $N(t_e) = i$. I_i, $P_s(i)$ and L_i are known numbers, defined in section III.

For fixed i, $i = 0, 1, \ldots, M$, $A(i)$ is given by the following formula

$$\begin{aligned} A(i) := & \sum_{k=i}^{M} k \left[\mathbf{S} \sum_{l=0}^{T} \mathbf{Q}^l \, + \, \mathbf{F} \sum_{l=0}^{\gamma+1} \mathbf{Q}^l \right]_{i,k} \\ & + \sum_{k=i}^{M} B(k) \, \left[\mathbf{F}\, \mathbf{Q}^{\gamma+2} \right]_{i,k} \, + \, (1 - P_s(i)) \sum_{k=0}^{M-1} k\, \delta_k, \end{aligned} \tag{5.2}$$

where $B(k)$ is the expected sum of backlogs over the period $[t_e + I + \gamma + 2,\, t_{e+1} - 1[$ given that $N(t_e + I + \gamma + 2) = k$.

Formula (5.2) has the following interpretation:

- the first term in the right-hand side of (5.2) is obtained in direct analogy with the corresponding result in [6, formula (14)]. It gives the expected sum of backlogged stations over the period $[t_e,\, t_e + I + \gamma + 2[$, cf. figure 1;
- the second term determines the expected sum of backlogged stations over the period $[t_e + I + \gamma + 2,\, t_{e+1} - 1[$;
- the third term gives the expected sum of backlogged stations in the time slot $[t_{e+1} - 1, t_{e+1}[$.

In order to compute the unknown quantities $B(0), B(1), \ldots, B(M)$ involved in (5.2), let us define $Z(j; m, l)$ the expected sum of backlogs over the period $[t_j^*,\, t_{e+1} - 1[$, for $1 \le j \le M$, $1 \le m \le M$, $0 \le l \le M - 1$.

Similarly to the analysis developped in section III, it is easily seen that

$$B(k) = \left(\frac{M-k}{M} \right) Z(1; 1, k) \, + \, \left(\frac{k}{M} \right) Z(1; M, k-1), \tag{5.3}$$

for $0 \le k \le M$.

Computation of vector $\mathbf{Z} := (Z(j; m, l))_{(m,l)}$

The components of vector $\mathbf{Z}$ must satisfy the following linear relations:

$$Z(j; m, l) = l + \sum_{(l', m')} P((j; m, l) \to (j+1; m', l'))\, Z(j+1; m', l'), \tag{5.4}$$

for $1 \le m \le M - 1$, $0 \le l \le M - 1$;

$$Z(j;M,l) = l+1+\sum_{(l',m')} P((j;M,l)\to(j+1;m',l'))\big[Z(j+1;m',l') + \mathbf{1}_{\{m'\neq M\}}\,\alpha(l+1,l'+1) + \mathbf{1}_{\{m'=M\}}\,\alpha(l+1,l')\big], \tag{5.5}$$

for $0 \le l \le M-1$, where

$$\alpha(I,J) := \frac{1}{|E_{I,J}|} \sum_{\substack{I\le i_1\le\cdots\le i_T\\ i_T-1\le i_{T+1}\le J}} i_1 + i_2 + \cdots + i_{T+1}, \tag{5.6}$$

with

$$E_{I,J} := \Big\{ i_1, i_2, \ldots, i_{T+1} \text{ such that } I \le i_1 \le i_2 \le \cdots \le i_T,\ i_T - 1 \le i_{T+1} \le J \Big\}.$$

$|\mathcal{E}|$ denotes the cardinal of any set $\mathcal{E}$.

The derivation of equations (5.4), (5.5) is quite obvious by noting that $\alpha(I,J)$ simply expresses the expected sum of backlogged stations over the period $[t_j^*+1,\, t_{j+1}^*[$ given that $N(t_j^*) = I$, $N(t_{j+1}^*) = J$ and given that station j has sent a "P" message in the time slot $(t_j^*-1,\, t_j^*)$.

Then, similarly to section IV, it can be shown that

$$\mathbf{Z} = (\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{E}, \tag{5.7}$$

where $\mathbf{E}$ is a vector of size M^2 given by

$$\mathbf{E} := \begin{pmatrix} 0 \\ 1 \\ \vdots \\ M-1 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ M-1 \\ \beta(0) \\ \beta(1) \\ \vdots \\ \beta(M-1) \end{pmatrix}, \tag{5.8}$$

with

$$\beta(l) := l+1+\sum_{l',m'} P((j;M,l)\to(j+1;m',l'))\,\big[\mathbf{1}_{\{m'\neq M\}}\,\alpha(l+1,l'+1) + \mathbf{1}_{\{m'=M\}}\,\alpha(l+1,l')\big], \tag{5.9}$$

for $0 \le l \le M-1$.

Combining now (5.1), (5.2), (5.3) and (5.7) we get $\bar{N}$.

Finally, by Little's formula, the average packet delay D — normalized to T — is given by

$$D = \frac{\bar{N}}{S}. \tag{5.10}$$

REMARK 5.1. — A numerical procedure is given in appendix C for computing the $\alpha(I, J)$'s involved in (5.9).

REMARK 5.2. — Let σ' be the probability that a station in the thinking state generates a new packet while a station is sending a "P/N" synchronization message in the CF period (note that $\sigma' = \sigma$ if $d_b = 1$). Then, the analysis developped in the case $d_b = 1$ remains valid if $d_b \neq 1$, up to the following obvious modifications:

i) σ must be replaced by σ' in proposition 2.2, in appendix A and in formulas (B.1), (B.2) and (B.5)-(B.8) of appendix B;

ii) in section IV, the vector $\mathbf{B}$ becomes

$$\mathbf{B} := \begin{pmatrix} d_b \\ d_b \\ \vdots \\ d_b \\ d_b + T + 1 \\ \vdots \\ d_b + T + 1 \end{pmatrix};$$

iii) in appendix B, p must be replaced by $p := 1 - (1-\sigma)^{T+1}(1-\sigma')$.

VI. NUMERICAL RESULTS AND DISCUSSION

Numerical results were obtained for $M = 20$ (i.e., 20 stations), $T = 100$ (i.e., packet size = 100 slots), $\gamma = 2$ (i.e., duration of the detect time =2 slots) and $d_b = 1$ (i.e., length of a synchronization message = 1 slot).

Figures 2 and 3 display the *throughput-delay* curves for both HYMAP and CSMA/CD protocols, for fixed ν.

Figure 4 gives the percentage of time HYMAP operates in the Collision-Free mode versus the throughput, for fixed values of ν.

The numerical computations were performed on a Honeywell Bull DPS 68 computer (about 1 Mflops). For given values of σ and ν, the computation of the *throughput* S and of the *mean delay* D requires, in particular, the resolution of the linear systems (4.5), (4.8) and (5.7) (see sections IV and V). Here, $\mathbf{I} - \mathbf{A}$ is a $M^2 \times M^2$ matrix, which contains approximatively M^3 nonzero elements. For instance, if $M = 20$ then $\mathbf{I} - \mathbf{A}$ contains 8551 nonzero elements.

For fixed values of σ and ν, these three linear systems were solved by using a standard iterative method (SOR method, see [7, p. 73]), where only the nonzero elements of the matrix $\mathbf{I} - \mathbf{A}$ were stored in the computer. Each iterative procedure was stopped as soon as the relative error between two consecutive steps became smaller than 10^{-6}. The corresponding CPU time needed for computing S *and* D was approximatively 200 seconds (s) if $S \sim 0.2$, 300 s if $S \sim 0.4$, 500 s if $S \sim 0.7$, 1400 s if $S \sim 0.9$ and 1900 s if $S \sim 0.97$ (the maximum achievable throughput we obtained). We observed that this CPU time was insensitive to the value of ν, which is not surprising since equations (4.5), (4.8) and (5.7) do not depend on ν.

These results confirm the results obtained by computer simulation in [4], namely, the performance of HYMAP are always better than the performance of CSMA/CD. When the system is lightly loaded (say $S \leq 0.5$ if $\nu = 0.02$ or $\nu = 0.05$ — figure 2 — and $S \leq 0.3$ if $\nu = 0.15$ or $\nu = 0.25$ - figure 3) then the improvement over CSMA/CD is only marginal, since in that case HYMAP spends more than 90% of its time in CSMA/CD mode (figure 4).

However, when the load increases, as expected the performance of HYMAP (throughput and mean delay) are always better than the performance of CSMA/CD, particularly when ν is large (figure 2). This is clearly explained by the fact that at heavy load, HYMAP almost behaves as a token protocol (when $S \leq 0.9$ then HYMAP spends more than 80% of its time in CF mode, if ν is large, cf. figure 4), which is known to be the best protocol in that case. We can also conclude from figure 4 that HYMAP operates mostly in the CSMA/CD mode, the break-even point being obtained for S around 0.8.

VII. CONCLUSIONS

A probabilistic model of HYMAP, a hybrid multiple access protocol combining the best features of CSMA/CD and of a conflict-free protocol [4], was proposed. An exact analysis of this model was done. Explicit formulas were obtained for the throughput and the mean delay of HYMAP. The unknown quantities contained in these formulas and related to the fact that HYMAP permits free access of new arrival during collision resolution, were computed through an efficient numerical scheme, involving, in particular, the resolution of large systems of linear equations. It was shown that the performance of HYMAP were always better than the performance of CSMA/CD. We characterized this improvement in term of the achievable channel capacity and of the packet delay at a given utilization for fixed values of the average retransmission delay.

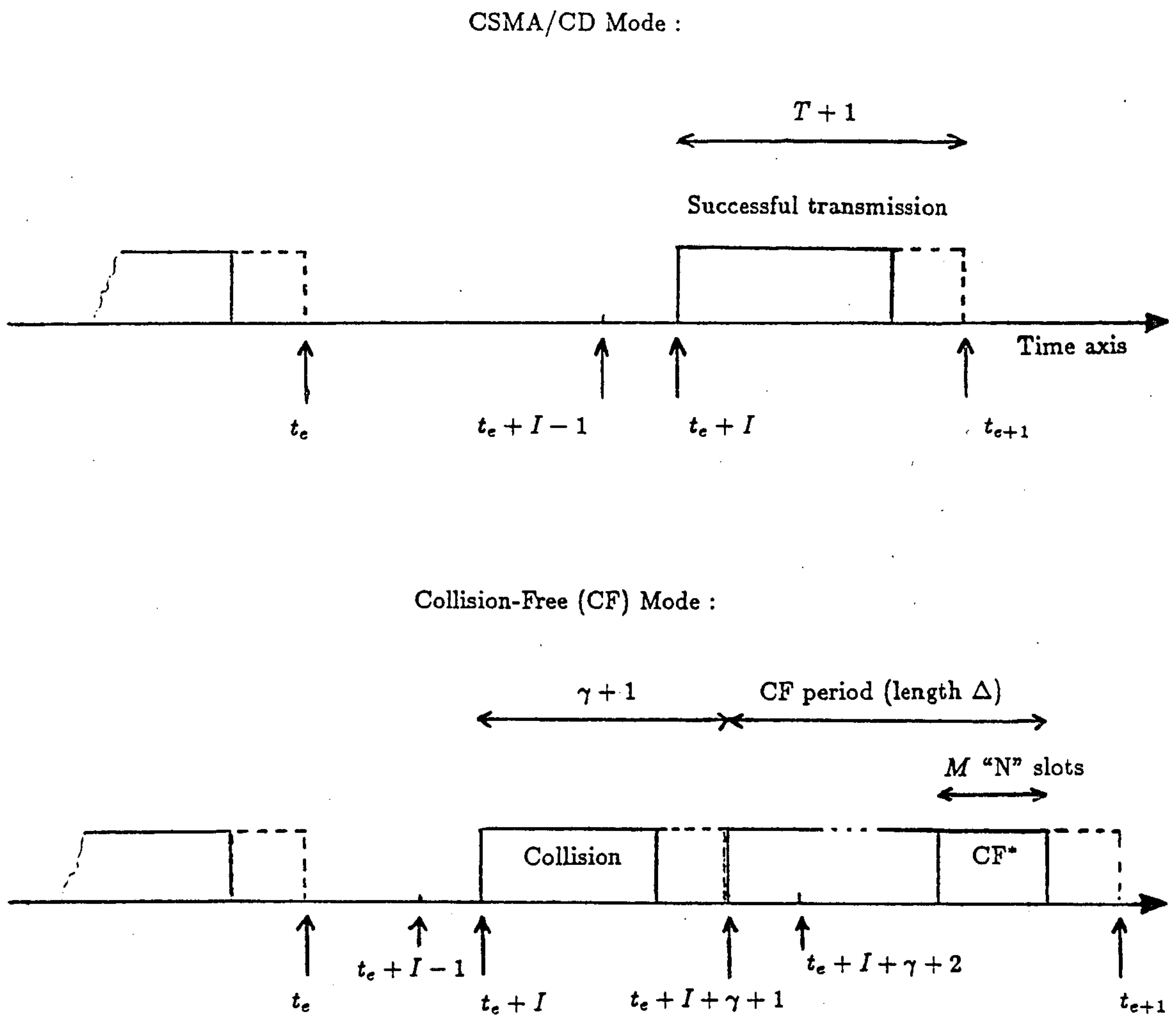

Figure 1.

CSMA/CD and Collision-Free Periods.

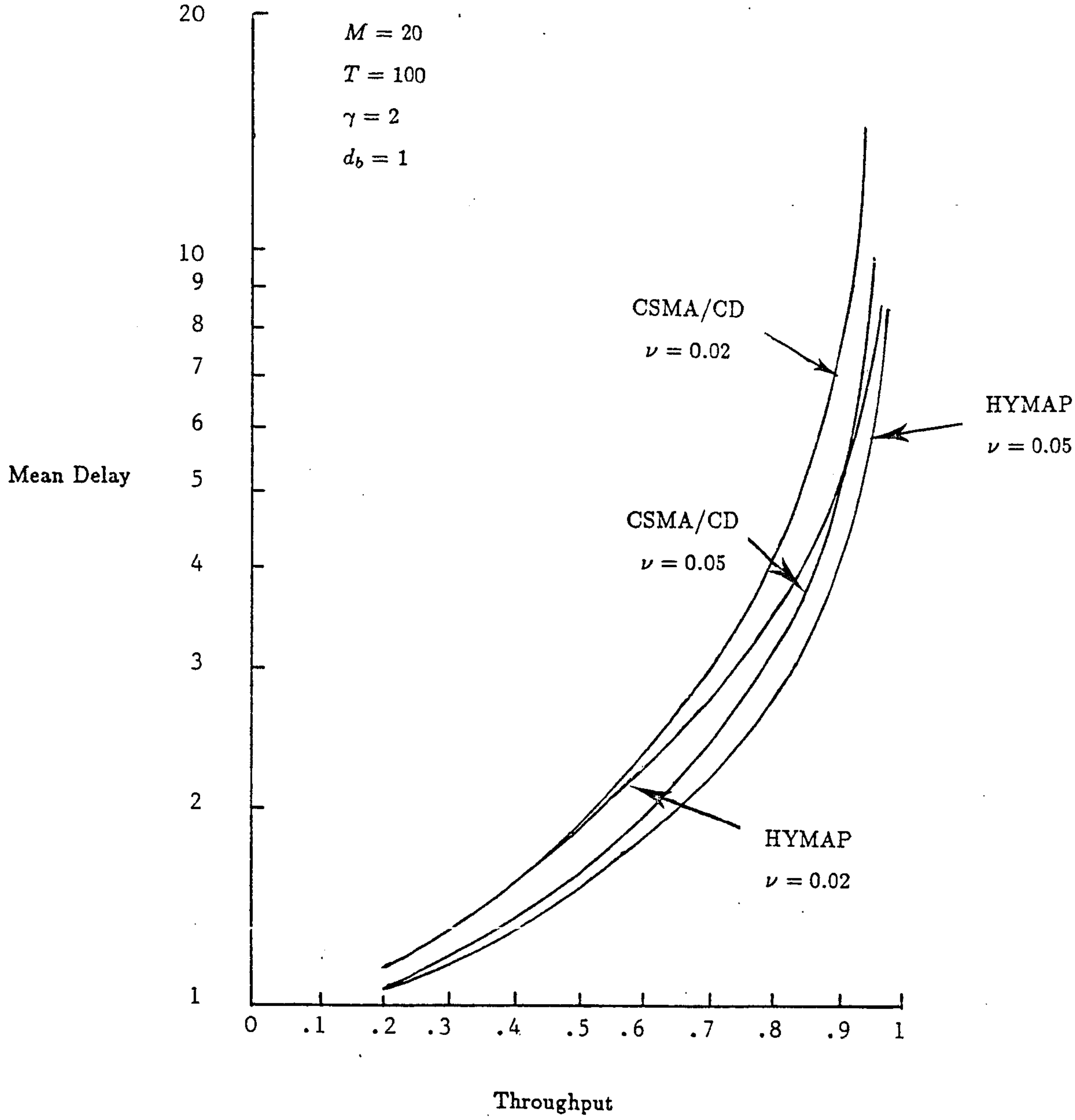

Figure 2.

Mean Delay Vs. Throughput at Fixed ν.

(large mean retransmission delay)

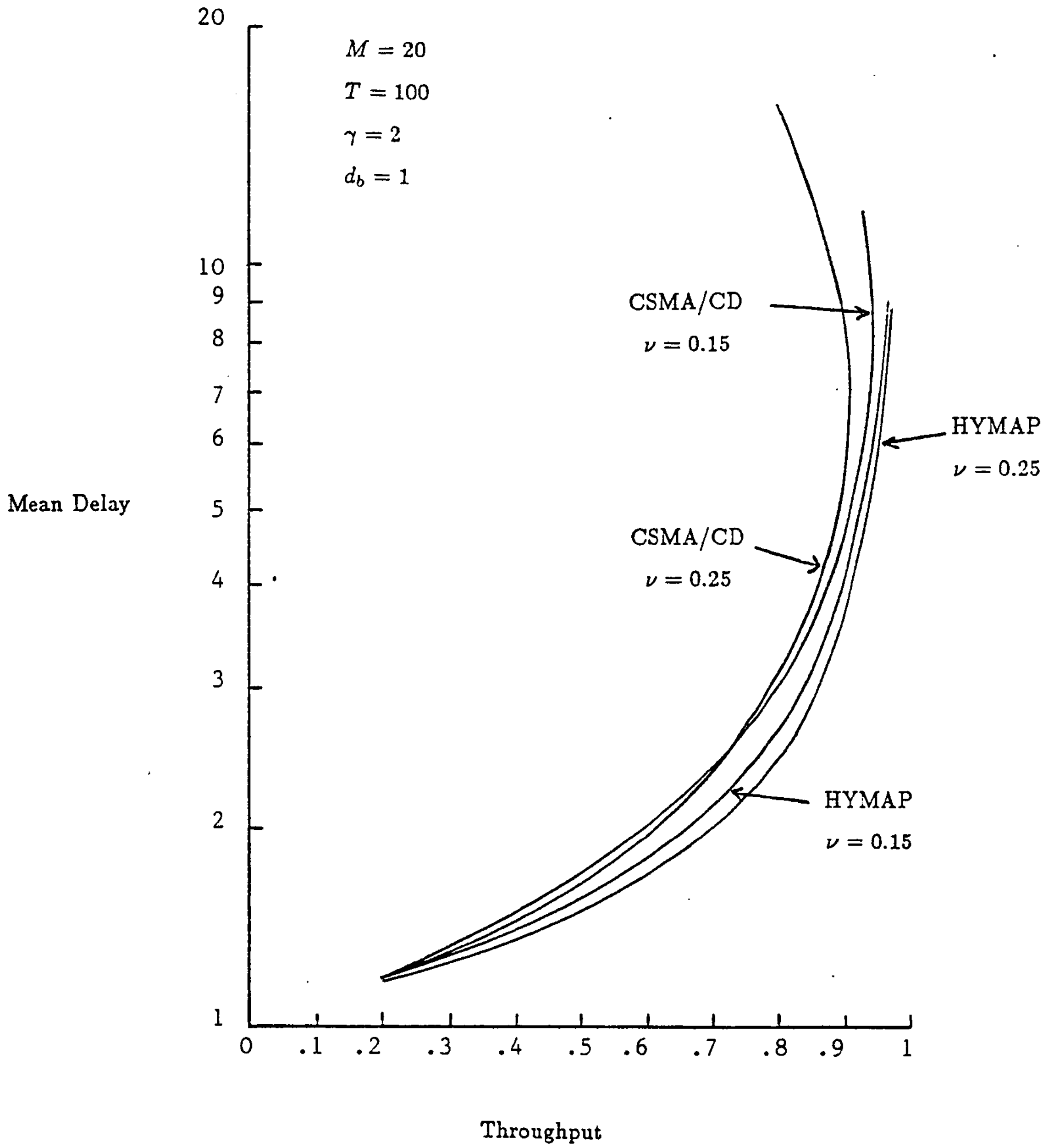

Figure 3.

Mean Delay Vs. Throughput at Fixed ν.

(small mean retransmission delay)

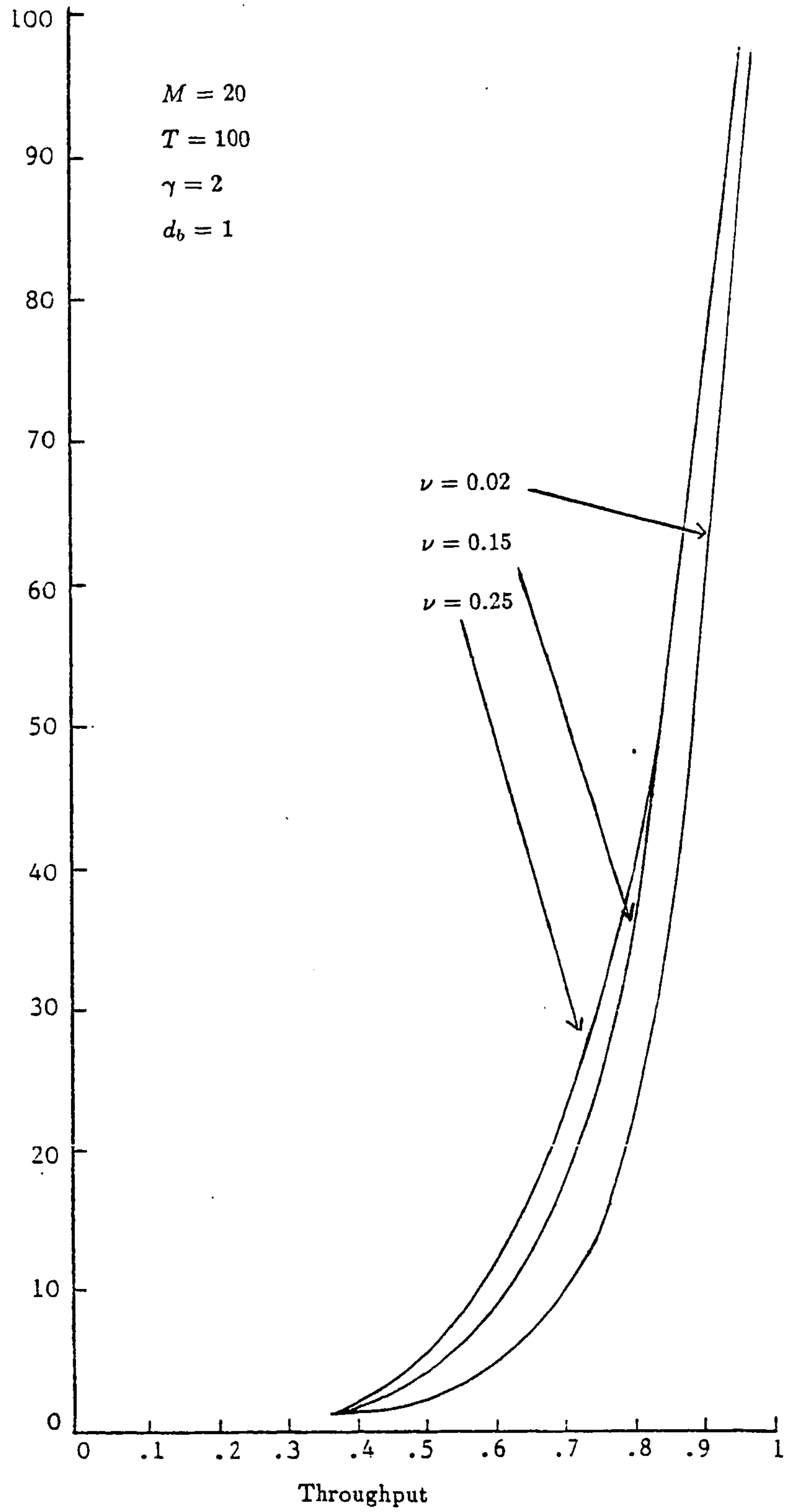

Figure 4.

Percentage of Time Spent by Hymap
in CF Mode Vs. Throughput.

APPENDIX A

Computational algorithm for the elements of matrix D

For fixed j, $1 \le j \le M-1$, let us rewrite δ_j as follows, cf. (2.2):

$$\begin{aligned}\delta_j =& c_j \sum_{\substack{i_1<\cdots<i_j \\ i_1,\cdots,i_j\in\{1,\ldots,M-1\}}} \prod_{k=1}^{j} \Big((1-\sigma)^{i_k} - (1-\sigma)^M\Big), \\ =& c_j \sum_{i_1=1}^{M-j} \Big((1-\sigma)^{i_1} - (1-\sigma)^M\Big) \sum_{i_2=i_1+1}^{M-(j-1)} \Big((1-\sigma)^{i_2} - (1-\sigma)^M\Big) \sum_{i_3=i_2+1}^{M-(j-2)} \cdots \\ & \cdots \sum_{i_j=i_{j-1}+1}^{M-1} \Big((1-\sigma)^{i_j} - (1-\sigma)^M\Big), \end{aligned} \qquad (A.1)$$

where

$$c_j := (1-\sigma)^{M\left(\frac{M-1}{2}-j\right)}.$$

For $1 \le j \le M$ and $1 \le k \le M-j$, define

$$\begin{aligned} p_j(k) :=& \sum_{i_1=M-j-k+1}^{M-j} \Big((1-\sigma)^{i_1} - (1-\sigma)^M\Big) \sum_{i_2=i_1+1}^{M-(j-1)} \Big((1-\sigma)^{i_2} - (1-\sigma)^M\Big) \sum_{i_3=i_2+1}^{M-(j-2)} \cdots \\ & \cdots \sum_{i_j=i_{j-1}+1}^{M-1} \Big((1-\sigma)^{i_j} - (1-\sigma)^M\Big). \end{aligned} \qquad (A.2)$$

For fixed j, $1 \le j \le M-1$, we can observe that the $p_j(k)$'s satisfy the following recurrence relations:

$$\begin{aligned} & p_0(k) := 1, \qquad \text{for } 1 \le k \le M-1, \\ & p_j(1) := \Big((1-\sigma)^{M-j} - (1-\sigma)^M\Big)\, p_{j-1}(1), \\ & p_j(k) := \Big((1-\sigma)^{M-j-k+1} - (1-\sigma)^M\Big)\, p_{j-1}(k) + p_j(k-1), \quad \text{for } 2 \le k \le M-j. \end{aligned} \qquad (A.3)$$

From (A.2) it is seen that

$$\delta_j = c_j\, p_j(M-j), \qquad \text{for any } j \text{ with } 1 \le j \le M-1. \qquad (A.4)$$

We then deduce from (A.3) a convolution type algorithm (i.e., $p_j(k)$ only depends on $p_{j-1}(k)$ and $p_j(k-1)$) allowing the computation of *all* the δ_j's in $0(M^2)$ operations.

APPENDIX B

Computation of the transition matrix $\mathbf{A}:=(P((j;\bullet,\bullet)\to(j+1;\bullet,\bullet)))$

Define

$$q := 1-(1-\sigma)^{T+1},$$
$$p := 1-(1-\sigma)^{T+2}.$$

PROPOSITION B.1. — *For fixed j, $1\le j\le M$ with j mod $M+1$, then*

- *for $1\le l\le M$, $1\le m\le M-2$, $0\le l'\le l-1$*

$$P((j;m,M-l)\to(j+1;m+1,M-l+l'))=\frac{l-1}{M-1}\binom{l-1}{l'}\sigma^{l'}(1-\sigma)^{l-l'-1};\qquad (B.1)$$

- *for $1\le l\le M$, $1\le m\le M-1$, $-1\le l'\le l-1$*

$$P((j;m,M-l)\to(j+1;M,M-l+l'))=\frac{M-l}{M-1}\binom{l}{l'+1}\sigma^{l'+1}(1-\sigma)^{l-l'-1};\qquad (B.2)$$

(B.1), (B.2) give all the non-null transitions between states $(j;m,M-l)$ and states $(j+1;\bullet,\bullet)$ if $1\le l\le M$ and $1\le m\le M-1$.

- *for $2\le l\le M$, $-1\le l'\le l-1$*

$$P((j;M,M-l)\to(j+1;M,M-l+l'))=a(l,l');\qquad (B.3)$$

- *for $2\le l\le M$, $0\le l'\le l-1$*

$$P((j;M,M-l)\to(j+1;1,M-l+l'))=b(l,l');\qquad (B.4)$$

$$P((j;M,M-1)\to(j+1;M,M-1))=\sigma;\qquad (B.5)$$

$$P((j;M,M-1)\to(j+1;M,M-2))=1-\sigma,\qquad (B.6)$$

where

$$a(l,l') := \frac{M-l}{M-1}p^{l'}(1-p)^{l-l'-2}\left[(1-\sigma)p\binom{l-1}{l'+1}\mathbf{1}_{\{-1\le l'\le l-2\}}+\sigma(1-p)\binom{l-1}{l'}\mathbf{1}_{\{0\le l'\le l-1\}}\right]$$
$$+q\frac{l-1}{M-1}p^{l'-1}(1-p)^{l-l'-2}\left[(1-\sigma)p\binom{l-2}{l'}\mathbf{1}_{\{0\le l'\le l-2\}}+\sigma(1-p)\binom{l-2}{l'-1}\mathbf{1}_{\{1\le l'\le l-1\}}\right];\qquad (B.7)$$

$$b(l,l') := \frac{l-1}{M-1}(1-q)\,p^{l'-1}(1-p)^{l-l'-2} \times \left[(1-\sigma)\,p\binom{l-2}{l'}\mathbf{1}_{\{0\le l'\le l-2\}} + \sigma(1-p)\binom{l-2}{l'-1}\mathbf{1}_{\{1\le l'\le l-1\}}\right]. \qquad (B.8)$$

(B.3)-(B.6) give all non-null transitions between states $(j; M, M-l)$ and states $(j+1; \bullet, \circ)$ if $1 \le l \le M$.

PROOF. Recall the definition of t_j^* (see section IV). We first prove (B.1). The transition

$$(j; m, M-l) \to (j+1; m+1, M-l+l')$$

with $1 \le m \le M-2$, $1 \le l \le M$ and $0 \le l' \le l-1$ occurs *if* station $j+1$ is idle at time t_j^* (probability $\frac{l-1}{M-1}$) *and if* l' stations among the remaining $(l-1)$ idle stations at time t_j^* generate a new packet in the time interval (t_j^*, t_{j+1}^*) (note that here $t_{j+1}^* = t_j^* + 1$). This last event occurs with probability $\binom{l-1}{l'}\sigma^{l'}(1-\sigma)^{l-l'-1}$.

Multiplying the two above probabilities — since the corresponding events are clearly independent — we get relation (B.1). The proof of (B.2) is similar.

Let us now consider (B.3). We distinguish the following two cases.

Case 1: station $j+1$ is backlogged at time t_j^* (probability $\frac{M-l}{M-1}$). Then, two cases must be examined:

i) station j *does not generate* a new packet during the emission of the "P" message by station $j+1$ *and* $(l'+1)$ new packets are generated (among the $l-1$ idle stations at time t_j^*) in the time interval (t_j^*, t_{j+1}^*) (note that here $t_{j+1}^* = t_j^* + T + 2$). The joint probability of these two independent events is:

$$(1-\sigma)\binom{l-1}{l'+1}p^{l'+1}(1-p)^{l-l'-2}. \qquad (B.9)$$

ii) station j *generates* a new packet while station $j+1$ is sending its "P" message *and* l' new packets are generated (among the $l-1$ idle stations at time t_j^*) in the time interval (t_j^*, t_{j+1}^*). The joint probability of these two independent events is:

$$\sigma\binom{l-1}{l'}p^{l'}(1-p)^{l-l'-1}. \qquad (B.10)$$

Summing now (B.9) and (B.10) and multiplying the result by $\frac{M-l}{M-1}$, we obtain the first part of the right-hand side of (B.7).

Case 2: station $j+1$ is idle at time t_j^* (probability $\frac{l-1}{M-1}$) and it generates a new packet in the time interval $(t_j^*, t_{j+1}^* - 1)$, i.e., during the emission of a packet by station j. The probability of this event is q. As before, two cases have to be considered:

i) station j *does not generate* a new packet during $(t_{j+1}^* - 1, t_{j+1}^*)$ (i.e., while station $j+1$ is sending its "P" message) *and* l' new packets are generated (among the $(l-2)$ remaining idle stations at time t_j^*) in the time interval (t_j^*, t_{j+1}^*). This event occurs with the probability:

$$(1-\sigma)\binom{l-2}{l'}p^{l'}(1-p)^{l-l'-2}. \qquad (B.11)$$

ii) station j *generates* a new packet in the time interval (t_{j+1}^*-1, t_{j+1}^*) *and* $l'-1$ new packets are generated (among the $(l-2)$ idle stations at time t_j^*) in the time interval (t_j^*, t_{j+1}^*). This event occurs with the probability:

$$\sigma\binom{l-2}{l'-1}p^{l'-1}(1-p)^{l-l'-1}. \qquad (B.12)$$

Summing now (B.11) and (B.12) and multiplying the result by $q\,\frac{l-1}{M-1}$, we obtain the second part of the right-hand side of (B.7).

The derivation of (B.4) is quite similar and relations (B.5), (B.6) are obvious from model assumptions. ■

APPENDIX C

Computational algorithm for the $\alpha(I, J)$'s

For fixed I, J we have, cf (5.3),

$$\alpha(I,J) = \left(\sum_{\substack{I \le i_1 \le \cdots \le i_T \\ i_T - 1 \le i_{T+1} \le J}} 1 \right)^{-1} \left(\sum_{\substack{I \le i_1 \le \cdots \le i_T \\ i_T - 1 \le i_{T+1} \le J}} i_1 + i_2 + \cdots + i_{T+1} \right). \qquad (C.1)$$

Define

$$c(k,l) := \sum_{i_1 = J+1-l}^{J+1} \sum_{i_2 = i_1}^{J+1} \cdots \sum_{i_k = i_{k-1}}^{J+1} 1, \qquad (C.2)$$

and

$$p(k,l) := \sum_{i_1 = J+1-l}^{J+1} \sum_{i_2 = i_1}^{J+1} \cdots \sum_{i_k = i_{k-1}}^{J+1} (i_1 + i_2 + \cdots + i_k). \qquad (C.3)$$

From (C.1), (C.2) and (C.3) we see that

$$\sum_{\substack{I \le i_1 \le \cdots \le i_T \\ i_T - 1 \le i_{T+1} \le J}} 1 = c(T+1, J+1-I), \qquad (C.4)$$

and

$$\sum_{\substack{I \le i_1 \le \cdots \le i_T \\ i_T - 1 \le i_{T+1} \le J}} i_1 + i_2 + \cdots + i_{T+1} = \sum_{I \le i_1 \le \cdots \le i_{T+1} \le J+1} i_1 + i_2 + \cdots + i_{T+1} - 1,$$

$$= -c(T+1, J+1-I) + p(T+1, J+1-I). \qquad (C.5)$$

Combining (C.1), (C.2) and (C.5), we get that

$$\alpha(I,J) = -1 + p(T+1, J+1-I)/c(T+1, J+1-I). \qquad (C.6)$$

The numbers $c(T+1, J+1-I)$ and $p(T+1, J+1-I)$ involved in (C.6) are determined as follows. We first observe that the $c(k,l)$'s and $p(k,l)$'s satisfy the following recurrence relations :

- for $2 \le k \le T+1$

$$p(k,0) = k\,(J+1),$$

$$c(k,0) = 1;$$

• for $1 \le l \le J+1-I$

$$p(1,l) = [(J+1)(J+2) - (J-l)(J-l+1)]/2,$$
$$c(1,l) = l+1;$$

• for $2 \le k \le T+1$ and $1 \le l \le J+1-I$

$$p(k,l) = p(k,l-1) + p(k-1,l) + (J+1-l)\,c(k-1,l),$$
$$c(k,l) = c(k,l-1) + c(k-1,l).$$

From these relations, we then deduce a numerical procedure for the computation of $c(T+1, J+1-I)$ and $p(T+1, J+1-I)$.

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