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Summary. In this paper, we intend to show that the flocking phenomenon observed in many animal species behaviors, may be modeled as a synchronization process occurring within entity states. Although flocking has been widely studied and simulated in Swarm Intelligence, few works mention synchronization as a key aspect of the problem and model it properly. This paper proposes a modeling in terms of a reactive multi-agent system composed of interacting logistic agents moving in an environment. This specific MAS called Logistic MAS (LMAS) takes actually inspiration from the coupled map lattice field, which provides also many tools to analyse convergence and stability of the system. We develop our approach in both theoretical and applied way to demonstrate its relevance.

1 Introduction

One of the most famous paradigm in swarm intelligence [1] and one of the archetype of self-organization is the flocking phenomenon. The first flocking algorithm invented by Reynolds [6] is based on three main deterministic behavioral rules: maintain a minimum distance from others, match velocities with others in its neighborhood, and move toward the perceived center of mass in its neighborhood. This algorithm description derives from biological observations. The particle swarm optimization field has taken advantage of the self-organization mechanisms designed in this latter way to provide high performance algorithms for solving optimization problems. The challenging aim of swarm intelligence consists therefore in finding and modeling these self-organization mechanisms so as to improve the understanding and performance of swarm algorithms.
Contrary to previous approaches, our approach is a theoretical one, due to the origin of the model. Our main working hypothesis is that the flocking phenomenon may be described and analyzed in terms of synchronization processes, taking inspiration from the study of ensembles of coupled chaotic elements in nonlinear sciences. More precisely, our research refers to the particular phenomenology of nonlinear Coupled Map Lattices (CML), notably

the ones involving logistic maps. In this paper, we propose a derived model which is a reactive Multi-Agent System (MAS) called Logistic Multi-Agent System (LMAS) composed of logistic agents. On a mathematical viewpoint, LMAS is close to a specific CML instance called the CML gas [7]. We intend nevertheless to make clear the differences between both in this paper. The LMAS will be firstly described in section 2. We then show how to perform flocking simulations with LMAS in section 3, whose results will be analysed and discussed regarding CML theory in section 4.

2 From Coupled Map Lattice to Logistic MAS...

2.1 Overview of the CML approach

A CML is a discrete time and space computation model in which cell states take their values in a continuous domain. Let us focus on a mean-field approximation instance of the CML class: the globally coupled map lattice (GCM) designed by K. Kaneko [4] to study spatiotemporal chaos phenomena. A GCM based on the local map f, can be expressed by:

$$x_i(t+1) = f\left((1-\epsilon)x_i(t) + \frac{\epsilon}{N}\sum_{j=1}^{N} x_j(t)\right) \tag{1}$$

where $x_i(t)$ is the state variable of the cell on site i at time t, ϵ is the diffusive coupling coefficient, N is the total site number in the lattice. This CML has been widely studied when f was the well-known logistic map defined on the interval $[0,1]$ by the following recurrent equation:
$x(n+1) = f(x(n)) = 4\ a\ x(n)(1-x(n)) = f^{n+1}(x(0))$
This system displays full synchronization, due to the diffusive and contracting coupling, that is a global stable state where all lattice cells have the same x value, which occurs in this symmetric coupling case exactly for $\epsilon > \epsilon_*$, with the following threshold definition: $\epsilon_* = 1 - \exp(-\lambda)$, where λ is the regular Lyapunov exponent of the map f [4]. In the case of the above logistic map, with $a = 1$ (namely the chaotic phase), λ equals $\ln 2$ and consequently $\epsilon_* = \frac{1}{2}$. When coupling is not symmetric anymore, or when coupling becomes local or random, full synchronization turns into many different synchronization clusters according to the chosen map. A conjecture established in [3] postulates indeed that the chaotic map has to hold some periodic stable windows within its chaotic domains, so as to make partial synchronization occur. The logistic map belongs precisely to this class of maps, whereas the tent map for example does not. The next section presents briefly the CML gas model and introduces the LMAS model with its specificities.

2.2 Towards the logistic MAS

Some variations on the CML model are useful to our case study of flocking, notably the globally randomly coupled map lattice [5], which can be considered as an approximate situation of moving cells, and mostly the CML gas [7] proposed by Shibata and Kaneko more recently. This CML provides cells free to move on the lattice with only local couplings. Authors kept therefore an abstract formulation close to CML and studied a specific case with a gradient derived force responsible of the cell moving behavior. However, this modeling in our opinion do not lead to a flocking simulation. Although we agree with the latter rooting principle, we design a complete MAS using logistic maps as decision functions in the following way:

- Coupling and control parameters become local internal variables to give autonomy and adaptation capabilities to the agents. The agent internal state s is therefore a tuple of the following variables (here scalars but might be vectors as well): $s = \langle x, a, \epsilon \rangle$. Let D denote the interval $[0, 1] \subset \mathbb{R}$.
  - $x \in D$ is the decision variable for the agent to perform some actions
  - $a \in D$ is the control variable governing the logistic decision map
  - $\epsilon \in D$ is the coupling variable with neighboring agents
- The open aspect of the system is made clear by a distinction between agents and their environment, and increases the importance of the adaptation processes and the flows of exchanged data through perceptions and actions of agents. Let $s_i(t)$ be the state of agent i ($1 \leq i \leq N_a$), and $\sigma(t)$ the state of the environment at time t.
- A logistic map governs the agent decision making (this is the reason why we call it a logistic agent), and includes agent perceptions from $\sigma(t)$. The agent state transition is expressed in the equation system:

$$\begin{cases} a(t+1) = F_a\left(\sigma(t)\right) \\ \epsilon(t+1) = F_\epsilon\left(\sigma(t)\right) \\ x(t+1) = F_x\left(x(t), \sigma(t)\right) \end{cases} \quad (2)$$

  F_x is a compound of several operators: $F_x\left(x, \sigma\right) = f_a\left(I_\epsilon(x, p(\sigma))\right)$ where f_a is the logistic map with control parameter $a(t+1)$, I_ϵ a coupling operator with parameter $\epsilon(t+1)$, and p a perception function for the x component.

We think therefore that LMAS provides a more appropriate semantics for swarm intelligence than CML gas, by distinguishing clearly what is the environment from what are the agents and dividing agent processes in three parts, namely the perception-decision-action loop. We specify in the following section how LMAS may implement a basic flocking instance.

3 Flocks modeling with LMAS

In our context, flocking means the way a population of situated agents gets self-organized into groups of similar moving behaviors.

3.1 A basic flocking model with LMAS

- The state of the agent i at time t turns to the tuple: $s(t) = \langle x_i(t), a_i, \epsilon_0 \rangle$ a does not depend on time, but only on the considered agent and ϵ_0 is a uniformly constant factor, which may be considered here as an intrinsic characteristic of the whole population.
- Environment and fields : the environment is a 2D discrete torus composed of $N_e \times N_e$ sites, with two fields denoted X and N. The field X stores the cumulated x variables of each agent on a given site k at time t, that is $X_k(t) = \sum_{j \, \in \, k} x_j(t)$, and $N_k(t)$ is the number of agents located on the same site k at time t.
- The perception function p_i^x of an agent i corresponds to the mean of the X field over the agent neighborhood denoted $V(i)$:

$$p_i^x(t) = \frac{\sum_{k \in V_i} X_k(t)}{\sum_{k \in V_i} N_k(t)} \tag{3}$$

  It is easy to verify that $p_i^x(t)$ always belongs to $[0, 1]$.
- The decision is achieved through the internal state update in (2).
- Moving and updating actions: the updated $x(t+1)$ indicates the new direction of the velocity, its magnitude remaining a constant here. By doing this, we reduce the move in a 2D-space to a 1D-problem, contrary to the CML gas implementation [7]. If k' denotes the agent new site location, an agent i updates the fields on k' according to the two formulas: $N_{k'}(t+1) = N_{k'}(t+1) + 1$ and $X_{k'}(t+1) = X_{k'}(t+1) + x_i(t+1)$

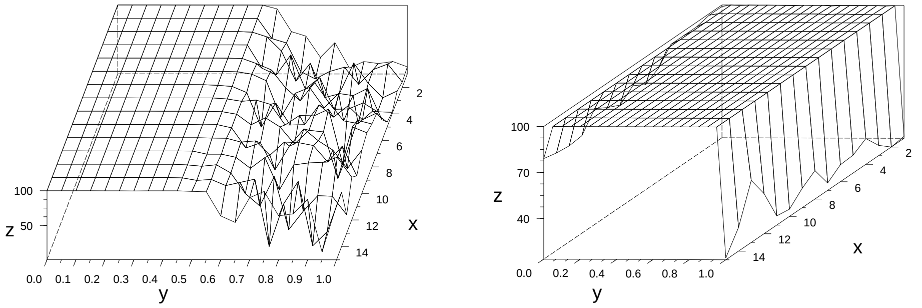
The final master transition equation for the x component is summarized in the following expression:

$$x_i(t+1) = f_a \left((1 - \epsilon_0) x_i(t) + \frac{\epsilon_0}{\sum_{k \in V_i} N_k(t)} \sum_{k \in V_i} X_k(t) \right) \tag{4}$$

which is similar to the CML master equation (1).

3.2 Simulations and results

Two specific initial configurations have been explored. In both cases, $N = 100$ agents evolve in a torus environment with 30×30 sites. The case 1 consists in a population of fully chaotic agents ($a = 1$ for all). The case 2 considers a population of agents whose variable a is uniformly distributed over $[0, 1]$.



(a) case 1: fully chaotic agents

(b) case 2: uniform control distribution

Fig. 1. $\bar{N}_c$ computation with precision 10^{-10}; $N_a = 100$ agents; radius of neighborhood on x-axis, ϵ_0 on y-axis, $\bar{N}_c$ on z-axis

The interesting point lies then in the emergence of clusters of stable synchronization, despite of the chaotic behaviors. A cluster is defined recursively: by default each agent corresponds to one cluster at the beginning of the recursion; then an agent i belongs to a given cluster, if there is another agent j in its neighborhood belonging to the same cluster, and if $|x_i - x_j| < \delta$. Let us mention that the precision δ equals 10^{-10} in the current analysis. Let $N_c(t)$ denote this number of clusters at time t and let $\bar{N}_c$ be the average of $N_c(t)$ over 500 consecutive time steps after the transient period $t > 2500$. We define $\bar{N}_c$ as a measure to analyse the global behavior: the charts on fig.(1) show the variations of $\bar{N}_c$ according to ϵ and to the neighborhood radius, whose maximum is the half of the the environment size. Smaller is the $\bar{N}_c$ value, more complete is the synchronization in the system.
Considering the issues of these simulations, the first case we observe on (1(a)), fits well with the theoretical law for the synchronization transition threshold $\epsilon_* = \frac{1}{2}$, whereas the second case does not provide any synchronization at this precision level, except when $\epsilon_0 \to 1$. In this latter case, the large randomly distribution of the internal control variables does not favor synchronization, but clusters of synchronization appears as we increase the precision threshold. In the first case, full synchronization occurs rarely due to the local and non stationary coupling connexions, which may assimilated as a randomly coupling situation. Manrubia and Mikhailov proposed a mathematical formulation of this problem in [5]. The second case corresponds in return to a more realistic simulation of flocking than the first one.

4 Discussion and future prospects

This paper has presented a way for modeling the swarm phenomenon of flocking, by means of a logistic multi-agent system derived from the logistic CML class of models. We have shown in this way that self-organization in the system is caused by a state synchronization process from the inside of agents. We have also verified some theoretical results established for CML models with LMAS in the flocking case: with fully chaotic agents, the synchronization transition occurs for $\epsilon = \frac{1}{2}$, whatever the radius of perception might be, except zero. This approach totally differs from the existing algorithms on flocking, but makes clearer the origin of the self-organization process. The limitation of the study lies currently in the lack of computed stability criteria like Lyapunov exponents associated to forming clusters. The precision in number computation has also to be enhanced to confirm the observed results.The logistic MAS appears nevertheless to be a promising modeling tool for swarm intelligence and we consider it as semantically more appropriate to this field than CML gas for example. In addition we have shown in [2] that LMAS can also simulate ant colony foraging behavior. In this latter case, synchronization reveals to be achieved through individual perception of a pheromone field, which modifies directly the internal control variable of each agent. We hope therefore that LMAS will allow us to unify these swarm approaches, so as to improve their understanding.

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