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DELAY SYSTEM IDENTIFICATION APPLIED TO THE LONGITUDINAL FLIGHT OF AN AIRCRAFT THROUGH A VERTICAL GUST

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Abstract: This paper deals with modelling and identification of aircraft dynamic entering a vertical gust. The identification approach initiated in (Fliess 2003) falls under a prospect for identification from tests carried out in the Flight Analysis Laboratory of the DCSD of ONERA in Lille. The plane is considered into various elements which consist in the fuselage, the wing and the tail. The model incorporates delays linked to the aircraft passage through the atmospheric turbulence.

Keywords: Dynamics of flight, Delay systems, Identification

1. INTRODUCTION

Flight through atmospheric turbulence is one of the significant subjects of research in aeronautics. This topic concerns at the same time manufacturers of planes and operators of civil transport. The development of a representation of the unsteady aerodynamic phenomena as well as the establishment of command laws for gust alleviation are significant objectives to ensure safety in critical phases (takeoff and landing) but also to ensure the comfort of the passengers.

Our study relates to the mathematical representation of the behaviour of the plane in atmospheric turbulence, and to the identification of the dynamics induced by the gust. A modelling based on flight mechanics will make it possible to simulate its flight. The fast identification technique proposed in (Fliess 2003) is used here as a first step of the approach for identification based on tests carried out in the Flight Analysis Laboratory of the DCSD of ONERA in Lille. During those experiments, a model of civil aircraft equipped with an embedded instrumentation will be catapulted and will cross, during its free flight, a turbulence generated by a vertical blower.

Various approaches of the modelling of the flight of a plane in atmospheric turbulence and of the representation of the unsteady effects were considered (Van Staveren 2003), (Klein 1994). Among them, a description including terms of delays was introduced (Coton 2000), (Coton 2002), (Jauberthie 2002), in order to get a better accuracy of the representation of the effects of the penetration of the aircraft through the gust. This article falls under the continuity of the work initiated by (Coton 2000), and a more detailed attention is related to the modelling and identification of the contribution of the gust to the aerodynamic coefficients. It is known that delays create a specific difficulty for the identification (see (Richard 2003, Belkoura *et al.* 2004, Belkoura 2005)). Such distributed and delayed actions of the turbulence on the lifting surfaces is particularly considered here.

The paper is organized as follows. Section 2 introduces the approach via a delay model. Contribution of the various elements of the plane to the aerodynamic coefficients is presented in section 2.1, followed by the description of the dynamic induced by the gust in 2.2. An identification method for the gust transfer function based on simulated data is presented in 3.

2. MODELLING

The general model describing the behaviour of a plane in a longitudinal flight results from the fundamental principle of dynamics and is governed by:

$$\begin{aligned} m\dot{V} &= -mg\sin(\theta-\alpha) - \frac{1}{2}\rho S V^2 C_x, \\ mV\dot{\alpha} &= mg\cos(\theta-\alpha) + mVq - \frac{1}{2}\rho S V^2 C_z, \\ B\dot{q} &= \frac{1}{2}\rho S l V^2 C_m, \\ \dot{\theta} &= q, \end{aligned} \tag{1}$$

where V, α, q et θ represent respectively the speed, the kinematic angle of attack, the pitch rate and the pitch attitude of the plane. The constant parameters m, B, g, ρ, l and S correspond respectively to the mass of the plane, its inertia, the constant of gravity, the density of the air, a length and a surface of reference. Lastly, C_x, C_z and C_m respectively represent the aerodynamic coefficients of drag, lift and pitching moment. On the assumption of a flight with weak variation of angle of attack, the drag coefficient generally results from the lift by a relation of the form:

$$C_x = C_{x0} + C_{x1}\,C_z + C_{x2}\,C_z^2, \tag{2}$$

with known coefficients C_{xi}, and the main point of our study will be concentrated on the analysis of the lift and pitching coefficients C_z and C_m. The contribution of the gust to these global coefficients is considered separately through the relations:

$$C_z = C_z^{plane} + C_z^{gust}, \tag{3}$$
$$C_m = C_m^{plane} + C_m^{gust}. \tag{4}$$

2.1 *Contribution of the elements of the plane to the aerodynamic coefficients*

Within the framework of the longitudinal flight, the plane is split up into three elements which consist in the fuselage, the wing and the horizontal tail. Expressions of the aerodynamic lift coefficients (gust not considered) are obtained from integration, along each element, of local angles of attack, respectively weighted by the section variation of the fuselage (denoted $\frac{dS}{dx}$) in the case of the fuselage, and by local coefficients depending on the surface of reference S, the local chord $c(y)$ and the lift gradient $C_{z\alpha}(y)$ of the surface considered (these coefficients are noted $C_{z\alpha}^{wing}(y)$ for the wing and $C_{z\alpha}^{ht}(y)$ for the horizontal tail). A similar method is followed for the pitching moment coefficients where preceding weightings are multiplied by $\frac{x}{l}$ with x the X-coordinate of the element at the point of integration and l a length of reference. These various operations reveal a behaviour closely connected to the state in the form (see e.g (Etkin 1995))

$$C_z^{plane} = C_{z0} + C_{z1}\,\alpha + C_{z2}\,q/V, \tag{5}$$
$$C_m^{plane} = C_{m0} + C_{m1}\,\alpha + C_{m2}\,q/V. \tag{6}$$

2.2 *Contribution of the gust*

In order to obtain an accurate description of the the aerodynamic coefficients induced by the gust, terms of delays are introduced as shown in Figure 1.

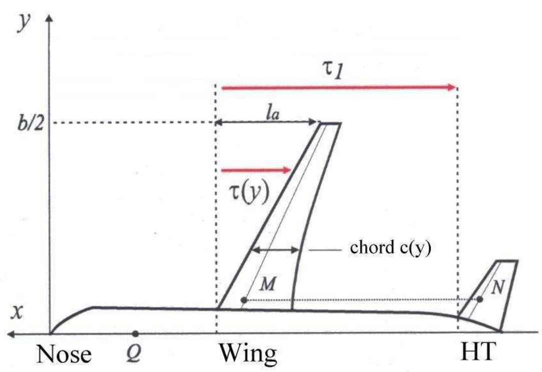

Fig. 1. Description of the delays in the modelling.

A distributed delay $\tau(y)$ is introduced in order to take the sweepback wing into account, while the delay τ_1 corresponds to the delayed effect of the gust on the horizontal tail. The fuselage effect is assumed negligible and we consider separately the wing and horizontal tail contributions to the aerodynamic coefficients. We therefore write, with obvious notations,

$$C_z^{gust} = \zeta_z^{wing} + \zeta_z^{ht}, \tag{7}$$
$$C_m^{gust} = \zeta_m^{wing} + \zeta_m^{ht}. \tag{8}$$

2.2.1. *Contribution on the wing*

The method adopted in this section consists in considering the distributed action of the gust on the wing induced by its sweep. In a matter of space saving, only lift is analyzed and relations relating to pitching moment are deduced as shown in section 2.1, by replacing the gradient $C_{z\alpha}^{wing}(y)$ by $\frac{x}{l}C_{z\alpha}^{wing}(y)$ with x the X-coordinate of the point considered and l the length of reference.

The modelling of the unsteady effects induced on a airfoil by its penetration through a vertical gust was introduced by Küssner (Kussner 1932) and Theodorsen (Theodorsen 1935). Within this framework, the contribution of the gust to the lift on the wing is expressed through the relation:

$$\zeta_z^{wing} = \int_{-b/2}^{b/2} C_{z\alpha}^{wing}(y)\left[k(t,y) * \frac{w(t,y)}{V(t)}\right] dy \tag{9}$$

where b is the span of the wing, $*$ denotes the convolution product, $w(t,y)$ is the local gust value, and $k(t,y)$ is the Küssner function. The latter may be regarded as the impulse response of a transfer function $K(s,y)$ which depends on the chord $c(y)$ of the wing (see figure 1), the average velocity V_m and a dominant mode a according to the relation:

$$2K(s,y) = 1 + \frac{aV_m/c(y)}{s + aV_m/c(y)} \tag{10}$$

The distributed delay $\tau(y) = \frac{2l_a}{V_m b}\,y = \tau\,y$ (see figure 1) describing the sweepback wing effects is particularly considered. The approximation of the speed $V(t)$ by its average value V_m results in a input $w(t,y) = w(t-\tau\,y)$ and allows us to formulate the gust contribution to the lift coefficient of the wing as:

$$V_m\,\zeta_z^{wing}(s) = (\Phi^w(s) + \Phi_z^w(s))\;w(s), \tag{11}$$

where

$$\Phi^w(s) = \int_0^{b/2} C_{z\alpha}^{wing}(y)\,e^{-\tau.ys}\;dy, \tag{12}$$

and

$$\Phi_z^w(s) = \int_0^{b/2} C_{z\alpha}^{wing}(y)\,\frac{e^{-\tau.ys}}{1+\frac{c(y)}{aV_m}s}\;dy. \tag{13}$$

The main difficulty lies in the fact that $\Phi_z^w(s)$ does not admit an explicit primitive. Even in the restrictive case where the sweepback wing is neglected and the local coefficient $C_{z\alpha}^{wing}(y)$ is assumed constant, integration of (13) results in a transfer function of the form $\log(\frac{1+\lambda_1\,s}{1+\lambda_2\,s})$ not easily workable in identification or control perspectives. Two successive approximations are then carried out.

(a) The gust is supposed to be relatively smooth in comparison to the dimensions of the wing so the distributed delay is approached by $e^{-\tau.ys} \simeq 1 - \tau\,y\,s$.
(b) The Laplace transform of the impulse response $\Phi_z^w(t)$ is also approximated by a Pade's development of order 1/1 with a corrected static gain.

Let us note that although approximation (a) is getting worst as one moves away from the fuselage, it should be attenuated by the decrease of the local chord $c(y)$ and the lift gradient $C_{z\alpha}(y)$, like generally confirmed by wind tunnel tests. The correction listed in (b) is due to the development in $1/s$ (and not in s) for which we wish to preserve the static gain $\Phi_z^w(0) = \Gamma^w$. The obtained approximations are therefore given by:

$$\Phi^w(s) \simeq \Gamma^w - s\Gamma_1, \tag{14}$$

with the constant terms $\Gamma^w = \int_0^{b/2} C_{z\alpha}^{wing}(y)\,dy$ and $\Gamma_1 = \int_0^{b/2} C_{z\alpha}^{wing}(y)\,\tau\,y\,dy$ and

$$\Phi_z^w(s) \simeq \left(\Gamma^w - \frac{\alpha_0 s}{s + \alpha_2/\alpha_1}\right), \tag{15}$$

where

$$\alpha_i = \int_0^{b/2}\left(\frac{aV_m}{c(y)}\right)^i\left(1 + \frac{2a\,l_a}{b\,c(y)}\,y\right)C_{z\alpha}^{wing}(y)\,dy. \tag{16}$$

Figure 2 represents in simulation the theoretical (Eq.13) and approached (Eq.15) responses of the transfer $\Phi_z^w(s)$ to a crenel gust. The data correspond to the model and the experiments under consideration at the Flight Analysis Laboratory of the DCSD of ONERA: $b = 2.23\,\mathtt{m}$, $V_m = 23.25\,\mathtt{m/s}$, $\tau = 0.023\mathtt{s}$.

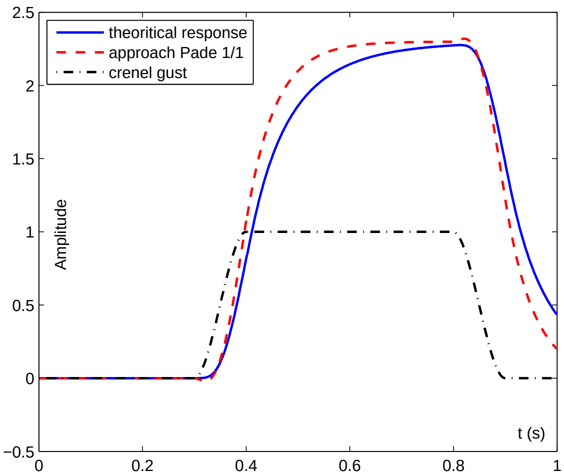

Fig. 2. Theoretical and approached responses of the transfer $\Phi_z^w(s)$ to a crenel gust.

Value adopted for a mode is $a = 0.13$ (Fung 1969), the chord $c(y)$ comes from extrapolated geometrical data of the scale model and the curve $C_{z\alpha}^{wing}(y)$

results from experiments carried out in wind tunnel. This result consolidates the approximation carried out, although a more accurate approximation is possible by complexifying the model.

2.2.2. *Contribution on the horizontal tail* Although the method is similar to that used for the wing, reduced dimensions of the tail allow a simpler description of the dynamics induced by the gust. In particular, the chord $c(y)$ is estimated at its average value and the sweepback tail is neglected. This yields the delayed transfer function:

$$V_m\,\zeta_z^{ht}(s) = \Phi_z^{ht}(s)\,w(s), \tag{17}$$

$$\Phi_z^{ht}(s) = \Gamma^{ht}\;\frac{s + 2aV_m/c_m}{s + aV_m/c_m}\;e^{-\tau_1 s}. \tag{18}$$

2.2.3. *First order approach* In order to evaluate the performances of our identification technique, we consider, in a first step, a first order approximation of relation (11) for the contribution on the wing. This expression can be reformulate, considering $K(s) = \frac{1}{ks+1}$ and using approximation (a), as :

$$V_m\,\zeta_z^{wing}(s) = \left(\frac{-s2\Gamma_1/k + 2\Gamma^w/k}{s + 1/k}\right) w(s) \tag{19}$$

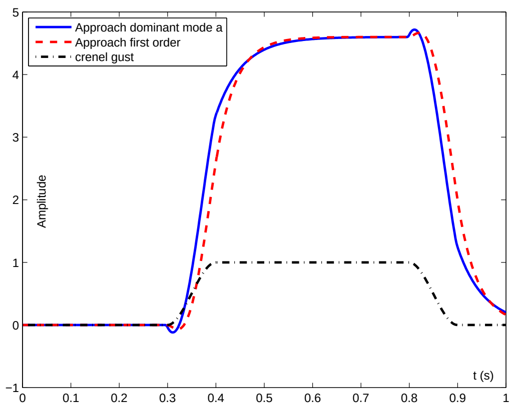

Fig. 3. Responses of $V_m\,\zeta_z^{wing}$ to a crenel gust.

The figure above compares the responses of $V_m\,\zeta_z^{wing}$ to a crenel gust considering the dominant mode a (Eq.11) and the first order (Eq.19). It clearly confirms the approximation carried out.

3. IDENTIFICATION

During the tests in the Flight Analysis Laboratory of the DCSD of ONERA, the state and state derivative in equation (1) are available using on the one hand, embedded gyrometers, accelerometers and Kalman-Rauch filtering, and on the other hand, an optical trajectory system based on two sets of ten video cameras spaced along the facility. Moerover, by means of wind tunnel experiments, the aerodynamic coefficients associated to the elements of the plane are also estimated. Therefore, the gust aerodynamic coefficients are available.

To avoid repetition, only the transfer function of lift coefficient C_z^{gust} is considered for the identification, and by virtue of the developments of the previous section, it admits an expression of the form:

$$y = \left[\frac{k_{w0} + k_{w1}s}{1 + \tau_w s} + \frac{k_{t0} + k_{t1}s}{1 + \tau_t s}\,e^{-\tau s}\right] w \tag{20}$$

where for ease of notations we have denoted:

$$\begin{aligned}
&y = C_z^{gust} = C_z - C_{z0} - C_{z1}\,\alpha - C_{z2}\,q/V,\\
&k_{w0} = 2\Gamma^w/V_m, \quad k_{w1} = -2\Gamma_1/V_m,\\
&k_{t0} = 2\Gamma^{ht}/V_m, \quad k_{t1} = \Gamma^{ht}c_m/(aV_m^2),\\
&\tau_w = k, \quad \tau_t = c_m/(aV_m).
\end{aligned}$$

The main difficulty in this problem lies in the presence of the unknown delay τ. We shall focus in this section on the identification of the parameters τ, τ_w, and τ_t from step responses. As we shall see, the remaining coefficients need not be known and could be identified in a second step. The approach used here is based on the work initiated in (Fliess 2003) and extended in our paper to parameters and delay identification.

3.1 *Mathematical framework*

Functions are considered through the distributions they define and are therefore indefinitely differentiable. If y is a continuous function except at a point a with a finite jump σ_a, its derivative dy/dt writes

$$dy/dt = \dot{y} + \sigma_a\,\delta_a \tag{21}$$

where $\dot{y}$ is the distribution defined from the usual derivative of y. Derivation, integration and translation can be formed from the convolution products

$$\dot{y} = \delta^{(1)}{*}y, \quad \int y = H{*}y, \quad y(t-\tau) = \delta_\tau{*}y \tag{22}$$

where $\delta^{(1)}$ is the derivative of the Dirac distribution, and H denotes the Heaviside function. With a slight abuse of notations, we shall write $H^k y$ the iterated integration of y and more generally T^k the iterated convolution product of order k. A distribution is said to be of order r if it acts continuously on C^r-functions but not on C^{r-1}-functions. Measures and functions are of order 0.

The multiplication of two distributions (say α and T) always make sense when one of the two terms is a smooth function. Particularly, when T is a Dirac derivative of order n, on has:

$$\alpha\,\delta^{(n)} = \sum_{k=0}^{n} (-1)^{(n-k)} C_n^k\,\alpha^{(n-k)}\,\delta^{(k)}, \qquad (23)$$

The next Theorem is the key result from which most of the parameters (including the delays) can be identified from step input responses.

Theorem 1. (Schwartz 1966) If a distribution T has a compact support K and is of order m (necessarily finite), $\alpha T = 0$ whenever α and its derivatives of order $\leq m$ vanish on K.

The following examples illustrate this statement in case α is a polynomial and T a singular distribution. Note that, in forming the product $\alpha\,T$, the delay τ involved in the argument $T(t-\tau)$ now appears also as a coefficient.

$$t\,\delta = 0, \quad (1-e^{-\gamma t})\,\delta = 0,$$
$$(1-e^{-\gamma(t-\tau)})\,\delta_\tau = 0. \qquad (24)$$

We shall make use of another property involving both multiplication with $e^{-\gamma t}$ and the convolution product, in case one of the two distributions (S or T) has a compact support.

$$e^{-\gamma t}\,(S * T) = e^{-\gamma t}\,S * e^{-\gamma t}\,T. \qquad (25)$$

With $S = \delta^{(p)}$ and $T = y$ this equation allows us to transform terms of the form $e^{-\gamma t}\,y^{(p)}$ into linear combinations of derivatives of products $e^{-\gamma t}\,y$. Denoting $z = e^{-\gamma t}\,y$, one has for example,

$$e^{-\gamma t}\,y^{(2)} = \gamma^2\,z + 2\gamma\,z^{(1)} + z^{(2)}. \qquad (26)$$

Note that integrating twice this expression by considering $H^2 e^{-\gamma t}\,y^{(2)}$ results in nothing but the integration by parts with available data z.

3.2 *Application to the gust transfer identification*

With a step gust $w = w_0 H$, a first order derivation of the differential equation induced by (20) results in a relation of the form

$$a_2\,y^{(3)} + a_1\,y^{(2)} + y^{(1)} =$$
$$\varphi_0 + \beta_0\delta + \beta_1\delta^{(1)} + \beta_2\delta_\tau + \beta_3\delta_\tau^{(1)} \qquad (27)$$

where $a_2 = \tau_t\tau_w$, $a_1 = \tau_t + \tau_w$, φ_0 (of order 2 and support $\{0\}$) contains the initial condition terms (discontinuities of y at $t = 0$) that naturally appear in the distributional framework, and β_i are combinations of the unknown parameters.

By virtue of Theorem 1, the right hand side of equation (27) can be cancelled by means of a multiplication with a function α such that $\alpha^{(k)}(0) = \alpha^{(k)}(\tau) = 0$ for $k = 0, 1, 2$. The choice of the function

$$\alpha(t) = (1-e^{-\gamma t})^3(1-e^{-\gamma(t-\tau)})^3 \qquad (28)$$

results in

$$(1-e)^3(1-\lambda\,e)^3\,(a_2\,y^{(3)} + a_1\,y^{(2)} + y^{(1)}) = 0, \quad (29)$$

where for ease of notations we denoted $e = e^{-\gamma t}$ and $\lambda = e^{\gamma\tau}$. Note that the latter equation no longer requires the knowledge of the gains k_{w0}, k_{w1}, k_{t0} and k_{t1}. As an equality of singular distributions, this relation doesn't make sense for any t (otherwise we would have $\tau = t$). However, $k \geq 1$ successive integrations (or a convolution with H^k) result in functions equality from which the delay τ and the coefficients τ_w, τ_t become available. More precisely, equation (29) combined with integrations leads to the following formulation:

$$(A_0 + \lambda\,A_1 + \lambda^2\,A_2 + \lambda^3\,A_3)\begin{pmatrix} a_2 \\ a_1 \\ 1 \end{pmatrix} = 0, \quad (30)$$

where the matrices entries described below are realized by means of integration by part formula described previously.

$$A_0(i,j) = +1H^{i+3}(1-e)^3 y^{(4-j)}$$
$$A_1(i,j) = -3H^{i+3}e(1-e)^3 y^{(4-j)}$$
$$A_2(i,j) = +3H^{i+3}e^2(1-e)^3 y^{(4-j)}$$
$$A_3(i,j) = -1H^{i+3}e^3(1-e)^3 y^{(4-j)}$$

The identification problem is therefore transformed into a generalized eigenvalue problem (30) for which the delay is deduced from one of the eigenvalues (i.e. $\tau = \log(\lambda)/\gamma$), while the coefficients a_2 and a_1 are obtained from the corresponding normalized eigenvector.

Figure 4 shows in simulation the step response of transfer to be identified with the numerical values $\tau_w = 0.6$, $k_{w0} = 2$, $k_{w1} = 0.5$, $\tau_t = 0.4$, $k_{t0} = 0.7$, $k_{t1} = 0.1$, $\tau = 0.5$ and a time scale of $1/10$.

Fig. 4. Step response of the gust transfer

Figure 5 shows the generalized eigenvalues of (30) (and more precisely $\log(\lambda)/\gamma$) solved using the `polyeig` Matlab function and the parameter (in (28)) $\gamma = 0.2$. The result clearly shows one constant eigenvalue corresponding to the unknown delay τ. This value appears after a transitory phase for which the delay is not identifiable.

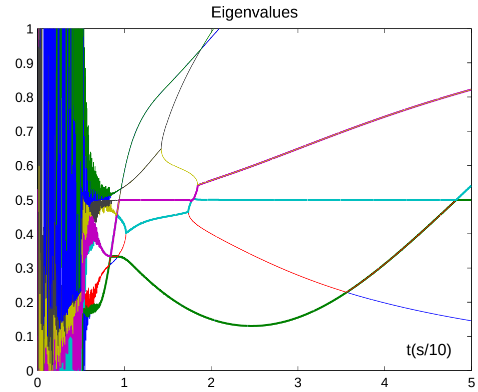

Fig. 5. Eigenvalues of problem (30)

Figure 6 shows the second and third component of the corresponding normalized eigenvector. These clearly converge to the desired values $a_2 = \tau_w \tau_t$ and $a_1 = \tau_w + \tau_t$. Moreover, and although these coefficients are not identifiable for $t < \tau$, they seem to converge in a first step to the pair (a_2, a_1) for which $\tau_w = 0$. Note also that a singularity occurred for $t \simeq 1$s.

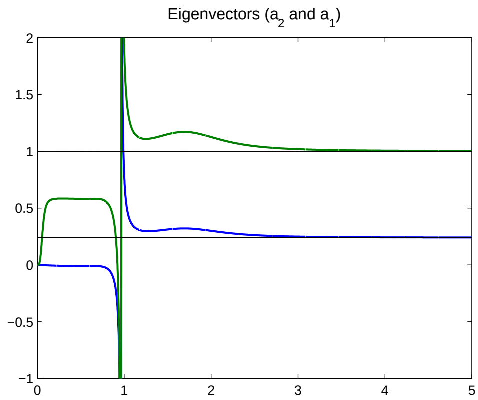

Fig. 6. Eigenvector of the constant eigenvalue

4. CONCLUSION AND PERSPECTIVES

This paper illustrated new identification and modelling approaches and a first step towards the identification of the aerodynamics induced by a gust in the longitudinal flight of an aircraft. Robustness issues, higher order of the unsteady aerodynamics, fuselage effects as well as additional delays taking into account the deflection term in the expression of the angle of attack of the horizontal tail are under investigation.

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