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Frequency Change-Point Detection in physiological Signals : an Algebraic Approach

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Abstract *This paper considers the problem of change-point detection for noisy data. Estimation of signal frequency content relies on differential algebra and non-commutative algebra together with operational calculus. We adapt this approach to the study of changes that may be observed in EEG signal dynamics during epileptic seizure and in ECG signal during the occurrence of a QRS complex. The correlation with frequency change is what this idea is based on. The interest of our estimator is firstly illustrated according to several academic examples. Then, the method is applied on real physiological signals to detect abrupt frequency changes.*

Keywords. *Abrupt change, differential algebra, non-commutative algebra, operational calculus, EEG, epileptic seizure, ECG, QRS complex.*

1 Introduction

For some years, the techniques of automatic, control, estimation together with diagnosis have always been applied more frequently in biology as well in medicine. In these fields, it is either a question of facilitating the decision-making of the specialist by highlighting particular phenomena or by assuring a control constantly.
In this paper, we are interested in the problem relying on automatic detection of changes that occur in physiological signals such as the epileptic seizure in EEG signal and the occurrence of QRS complex in ECG signal. In the analysis of epileptic seizures, it has long been noted that the frequency content often changes during seizures. 'In the most fundamental terms, electrographic seizure activity is manifested by a sequential change in frequency and amplitude that is

distinct from non-seizure activity and in most instances it is also distinct from artifacts'(Gabor *et al.* [16]). The two main aspects to consider are the following:
- the frequency aspect,
- sequential change[1].

Nowadays, the major part of the proposed solutions is inherent to representations and time-frequency studies of signal [1, 26, 27, 29]. The independent components analysis is also a recent method applied to the epileptic EEG [17, 18].
In the electrocardiogram (ECG), the most relevant tasks are the detection and the characterization of the QRS complex. As a result much information about the current state of the heart can be obtained. QRS detection is difficult not only because of the physiological variability of the QRS complexes but also due to the various types of noise that can be presented in ECG signal. Many QRS detection schemata are described in the literature [30] and are still being proposed [31].
Many practical problems arising from signal processing can be modeled with the aid of parametric models in which the parameters are subject to abrupt changes at unknown time instances. By abrupt changes, we mean changes in characteristics that occur very fast with respect to the sampling period, if not instantaneously. The detection of abrupt changes refers to tools that help us decide whether such change occurred in characteristics of considered object [2]. The detection of abrupt changes in a signal is also the subject of an abundant literature [2, 4, 5, 7, 23][2].
In addition, identifying a signal frequency in continuous time is a very hard challenge using classical linear analysis. That is why we choose to use a new tool that is based on differential algebra, non commutative algebra and Mikusiński operational calculus [24, 25]. Our algebraic approach for the abrupt frequency estimation changes had never been used in the analysis of the physiological signals. As a contribution, we suggest to improve the detection of abrupt changes in physiological signals by following-up the changes which reflect their appearance. The Work presented in this article aims at epileptic seizure detection in EEG and QRS complexes detection in ECG thanks to an estimation in continuous time of the signal frequency. Thus, a change-point or abrupt variation of the estimated frequency will translate a seizure occurrence in EEG or a QRS complex occurrence in ECG.
This paper is organized as follows: the algebraic setting for linear identifiability is tackled in the next section. In section 3, the method is tested on academic signals built from scenarios which is elaborated with an expert describing an abrupt frequency changes. Then, the method is validated on real physiological signals recorded. A brief conclusion evokes some future perspectives.

[1] These sequential changes are generally abrupt. They will be considered as change-points.
[2] These lists are not exhaustive otherwise the reader will be able to find other references there.

2 An algebraic setting for linear identifiability

In this section we recall the recent results obtained in estimation. For further details, the reader would be referred to the following articles [9, 10, 11, 12, 13, 15, 21, 22, 20].

2.1 Differential algebra

A *differential ring*[3], or more precisely, an ordinary differential ring, $\mathcal{R}$ is a commutative ring which is equipped with a single derivation, written here $\frac{d}{ds}$, i.e., a map $\mathcal{R} \to \mathcal{R}$ such that, $\forall\ a, b \in \mathcal{R}$,

- $\frac{d}{ds}(a+b) = \frac{da}{ds} + \frac{db}{ds}$,
- $\frac{d}{ds}(ab) = \frac{da}{ds}b + a\frac{db}{ds}$.

A *differential field*, or more precisely, an ordinary differential field, is a differential ring which is a field[4]. A *constant* $c \in \mathcal{R}$ is such that $\frac{d}{ds}(c) = 0$. A differential ring (resp. field) of constants is a differential ring (resp. field) whose elements are constants. A differential field *extension* $\mathcal{L}/\mathcal{R}$ is given by two differential fields $\mathcal{R}$, $\mathcal{L}$, such that:

- $\mathcal{R} \subseteq \mathcal{L}$,
- the restriction to $\mathcal{R}$ of the derivation of $\mathcal{L}$ is the restriction of $\mathcal{R}$.

Note that $\mathcal{R}\langle S\rangle$, $S \subset \mathcal{L}$, the differential subfield of $\mathfrak{L}$ generated by $\mathcal{R}$ and S. Assume that $\mathcal{L}/\mathcal{R}$ finitely generated, i.e., $\mathcal{L} = \mathcal{R}\langle S\rangle$, where S is finite. An element $\xi \in \mathcal{L}$ is said to be *differentially algebraic* over $\mathcal{R}$ if, and only if, ξ satisfies an algebraic differential equation $P(\xi, \ldots, \xi^{(n)}) = 0$, where P is polynomial over $\mathcal{R}$ in $n+1$ indeterminate. The extension $\mathcal{L}/\mathcal{R}$ is said to be *differentially algebraic* if, and only if, any element of $\mathcal{L}$ is differentially algebraic over $\mathcal{R}$. The following result plays an important role: the extension $\mathcal{L}/\mathcal{R}$ is differentially algebraic if, and only if, its transcendence degree is finite [22].
An element of $\mathcal{L}$ which is not differentially algebraic over $\mathcal{R}$ is said to be *differentially transcendental*. A differentially transcendental extension $\mathcal{L}/\mathcal{R}$ is an extension which is not differentially algebraic. A set $\{\xi_\iota \in \mathcal{L} \mid \iota \in I\}$ is said to be *differentially algebraically independent* over $\mathcal{R}$ if, and only if, no trivial relation exists over $\mathcal{R}$: $Q(\xi_\iota^{(\nu_\iota)}) = 0$, where Q is a polynomial over $\mathcal{R}$, imply: $Q \equiv 0$. An independent set which is maximal with respect to inclusion is called a *differential transcendence basis*. The cardinalities, i.e., the numbers of elements, to such bases are equal. This cardinality is the *differential transcendence degree* of the extension $\mathcal{L}/\mathcal{R}$. Note that this degree is 0 if, and only if, $\mathcal{L}/\mathcal{R}$ is differentially algebraic.

[3] See [6, 19] for more details
[4] see [6, 19] for more details. All fields are assumed to be of characteristic 0.

2.2 Linear identifiability

Let k_0 be a given ground field, which is assumed to be a differential field of constants. Let k be a finite algebraic extension of $k_0(\theta)$ where $\theta = (\theta_1, \cdots, \theta_r)$ is a finite set of *unknown parameters*. Thus, the transcendence degree of the extension k/k_0 is $\leq r$. Moreover, we give to k a canonical structure of the differential field of constants. Let $K/k(s)$ be a finitely generated differentially algebraic extension. A signal x is an element ok K.

Remark 1. The Mikusiński $\mathcal{M}$, generated by Mikusiński's operators [24, 25, 28] is a differential field with respect to the derivation $\frac{d}{ds}$ which corresponds to multiplication by $-t$. Its subfield of constants is $\mathbb{C}$.

The set of linear differential operators of the form $\sum_{fini} a_\alpha \frac{d^\alpha}{ds^\alpha}$, $a_\alpha \in k_0(s)$, is a commutative ring, principal left and right; it is written $k_0(s)[\frac{d}{ds}]$. A differential operator is said to be proper (resp. strictly proper) if, and only if, the coefficients are proper (resp. strictly proper) rational functions (we remind that rational functions are said to be (strictly) proper if, and only if, the degree of numerator is (strictly) less than the degree of denominator). It is said to be polynomial in $\frac{1}{s}$ if, and only if, $a_\alpha \in k_0[\frac{1}{s}]$.

The parameters θ are said to be *linearly identifiable*[5] with respect to $x \in K$ if, and only if,

$$P \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_r \end{pmatrix} = Q \tag{1}$$

where the entries of the $r \times r$ square matrix P, and the $r \times 1$ vector Q, belong to $\text{span}_{k_0(s)[\frac{d}{ds}]}(1, x)$, and det(P)$\neq 0$.

2.3 Linear estimators

Let $N/k_0(s)$ be a differential field extension such that K and N are linearly disjoint over $k_0(s)$. A noise n, or a perturbation, is an element of N. It is said to be structured if, and only if, it is annihilated by $\pi \in k_0(s)[\frac{d}{ds}]$, $\pi \notin k_0(s)$: $\pi n = 0, \pi \neq 0$. If not, the noise is said to be unstructured. A signal with an additive noise is a sum $x+w$, where $x \in K$ and $w \in N$ a noise. Let $y = (y_1, \ldots, y_\kappa)$, where $y_\imath = x_\imath + w_\imath$ be a finite set of such noisy signal depending upon the parameters θ. If the parameters θ are linearly identifiable, then (1) becomes

$$P \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_r \end{pmatrix} = Q + Q' \tag{2}$$

where the matrices P and Q are obtained from (1) by substituting y to x. The entries of the $(r \times 1)$-vector Q' belong to $\text{span}_{k'(s)[\frac{d}{ds}]}(w)$, where k' is the quotient field of $k \otimes_{k_o} k_1$, and $w = (w_1, \ldots, w_\kappa)$.

[5] This definition is borrowed from [9, 10, 11, 12].

Remark 2. In practice we will assume that an unstructured perturbation corresponds to a rapidly oscillating time-function, i.e., a high frequency signal, which may be attenuated by a low pass filter.

Assume that the components of w are structured. Multiplying both sides of (2) by $\triangle \in k_0(s)[\frac{d}{ds}]$, leads to the following estimator:

$$\triangle P \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_r \end{pmatrix} = \triangle Q \tag{3}$$

This result derives from the fact that $k_0(s)[\frac{d}{ds}]$ is the left ideal annihilator's entries of Q'. Equation (3), which is independent of the noises, is called a *linear estimator* of unknown parameters if, and only if, $det(\Delta P) \neq 0$. The estimator is said to be proper (resp. strictly proper) if, and only if, the entries of $\triangle P$ and $\triangle Q$ are (resp. strictly) proper differential operators. Multiplying both sides of (3) by a suitable proper element of $k_0(s)$ yields the

Proposition 1. *Any linear estimator may be replaced by a proper (resp. strictly proper) one.*

2.4 Simple example of estimation

In order to explain the used approach, let us start with an example with a first order linear input-output system :

$$\dot{y}(t) = ay(t) + n(t) \tag{4}$$

for $t \geq 0$, where a is an unknown parameter to estimate and $n(t)$ a noise corruption. This noise may be decomposed as : $n(t) = n_0(t) + \gamma$, i.e., the sum of a constant γ representing its mean (average) value and zero-mean term $n_0(t)$. Translated into the operational domain, this differential equation reads as:

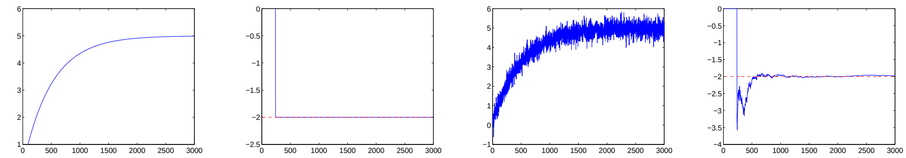
$$sy(s) = ay(s) + y(0) + \frac{\gamma}{s} + n_0(s) \tag{5}$$

where, due to the output noise, the initial condition $y(0)$ is not necessarily well known [8]. The constant γ is considered as undesired perturbation, like the initial condition $y(0)$. Note that these perturbations are easily annihilated by multiplying both sides of (5) by s and after 2 derivations with respect to s. This amounts to applying the linear differential operator:

$$\varPi = \frac{d^2}{ds^2}s = s\frac{d^2}{ds^2} + 2\frac{d}{ds} \tag{6}$$

to both sides of (5). The resulting equation, given by :

$$\left[s\frac{d^2y}{ds^2} + 2\frac{dy}{ds}\right]a = s^2\frac{d^2y}{ds^2} + 4s\frac{dy}{ds} + 2y \tag{7}$$



(a) noisy free signal y *(b)* $a_e(t)$ (–) a (- -) *(c)* noisy signal y, SNR=25dB *(d)* $a_e(t)$ (–) a (- -), SNR=25dB

Figure1: First order system with online identification

shows that the constant γ and the initial condition $y(0)$ will have no effect on the estimation result.

Remark 3. Recall that : multiplication by s in operational domain, in turn, corresponds to derivation in time domain.

Implementing the linear estimator (7) in its present form is therefore not convenient : derivation amplifies the high frequency components and consequently, the noise contribution. A simple solution is obtained by making the estimator (strictly) proper. So, it satisfies to multiply both sides of (7) by s^ν, where ν is an integer greater than or equal to the highest power of s in (7). Here, the operators deduced from (7) are strictly proper for $\nu \geq 3$. Thus, we obtain the following estimator:

$$\left[s^{-2}\frac{d^2y}{ds^2} + 2s^{-3}\frac{dy}{ds}\right] a = s^{-1}\frac{d^2y}{ds^2} + 4s^{-2}\frac{dy}{ds} + 2s^{-3}y \tag{8}$$

To obtain numerical estimate of the parameter a, it needs to express equation (8) in time domain, using the classical rules of operational calculus. Recall that:

- the derivation $\frac{d^\alpha}{ds^\alpha}$ with respect to s, $\alpha \geq 0$, translates into the multiplication by $(-1)^\alpha t^\alpha$,
- $\frac{1}{s^\alpha}$ is replaced by the α^{th} iterated time integration

$$\int_0^t \int_0^{t_{\alpha-1}} \cdots \int_0^{t_1} x(\tau) d\tau dt_1 \cdots dt_{\alpha-1}$$

- which, thanks to Cauchy rule, equals

$$\frac{1}{(\alpha-1)!}\int_0^t (t-\tau)^{\alpha-1} x(\tau) d\tau$$

- the unstructured noises are viewed as highly fluctuating phenomena. They are attenuated by the iterated time integrals, which are simple examples of low pass filters[6].

[6] In [14], the reader will find all the theoretical justifications.

Finally, the estimation of the parameter a is given, as time-function by the explicit formula:

$$a_e(t) = \frac{\displaystyle\int_0^t \left[\tau^2 - 4(t-\tau)\tau + (t-\tau)^2\right] y(\tau)d\tau}{\displaystyle\int_0^t \left[(t-\tau)\tau^2 - (t-\tau)^2\tau)\right] y(\tau)d\tau} \tag{9}$$

In Both figures 1-(b) and (d), the evaluation of the estimator is shown. The abscissa axis is graduated in number of samples and the initial condition $y(0) = 0.2$. In the free-noise case ($n_0(t) = 0$), the estimation is practically perfect. It remains very good in the noisy case. The purpose of this paper is not to compare these results to those using classical techniques (the interested reader can always be referred to the publications previously mentioned) nevertheless, we insist, as well, on the poverty of the excitation signal.

3 Application to physiological signals

In order to fit our approach to physiological signals, we choose to represent these signals according to a very simple model:

$$y(t) = sin(\varphi_0 + \varphi_1 t) + n(t) \tag{10}$$

where φ_1 is the angular frequency of the signal (proportional to the frequency) and φ_0 the phase, $n(t)$ an unstructured perturbation. The use of such a model is justified because it doesn't represent the signal only during a very short lapse of time. The variation of the frequency content is translated by the variation of φ_1. Therefore, we are interested in estimating it quickly and online. The algebraic techniques are all indicated. Keeping the continuous-time nature of the signal we readily observe that the noise-free signal $x(t) = y(t) - n(t)$ satisfies the following linear differential equation with constant coefficient:

$$\ddot{x}(t) + \varphi_1^2 x(t) = 0 \tag{11}$$

In translating this equation to operational domain, we obtain :

$$s^2 x(s) - sx(0) - \dot{x}(0) + \varphi_1^2 x(s) = 0 \tag{12}$$

or also in calculating initial condition

$$(s^2 + \varphi_1^2)\, x(s) = \varphi_1 \cos\varphi_0 + s\, sin\, \varphi_0 \tag{13}$$

the parameter φ_1^2 is then linearly identifiable[7] and its estimation is now described.

[7] Let's note that to estimate φ_1^2 rather than φ_1 is enough to translate the frequency change-point.

Taking derivative, twice, with respect to s permits to ignore structured perturbations, here are the initial conditions

$$2x + 4s\frac{dx}{ds} + (s^2 + \varphi_1^2)\,\frac{d^2x}{ds^2} = 0 \tag{14}$$

After that, we multiply both sides by s^{-2} to avoid derivations with respect to time (positive power of s)

$$2s^{-3}x + 4s^{-2}\frac{dx}{ds} + (s^{-1} + s^{-3}\varphi_1^2)\,\frac{d^2x}{ds^2} = 0 \tag{15}$$

The well-known rule of operational calculus yields to the following on-line estimator of φ_1^2, i.e., a time-domain representation with no derivative but only integrations with respect to time:

$$\varphi_{1e}^2(t) = -2\,\frac{\displaystyle\int_{t-T}^{t}((T-\tau)^2 - 4(T-\tau)\tau + \tau^2)x(\tau)d\tau}{\displaystyle\int_{t-T}^{t}(T-\tau)^2\tau^2x(\tau)d\tau} \tag{16}$$

Let us note that the multiple integrals are transformed in simple integrals with the help of Cauchy formula. Besides, T indicates the size of sliding estimation window.

3.1 Academic example

We focus our attention on two types of test signals corrupted by a gaussian white noise. The noise level, measured by the signal to noise ratio in dB, i.e, $SNR = 10\lg_{10}\left(\frac{\sum|y(t_i)|^2}{\sum|n(t_i)|^2}\right)$, corresponds to $SNR = 25dB$. The size of the signals is $L = 10000$ samples. The first is a sinusoid signal which frequency is a constant function of time. The value of the angular frequency is $\varphi_1 = 12\Pi rad.s^{-1}$. The second signal is distinguished from the first one from the introduction of an abrupt change in its frequency. The signal's angular frequency is defined in $[0, T_f]$[8] as follows:

$$\varphi_1(t) = \begin{cases} 6\Pi \text{ si } \frac{Tf}{3} < t < \frac{2Tf}{3} \\ 24\Pi \text{ otherwise} \end{cases}$$

These signals are shown in figure 2.

For convenient implementation of our estimator, we introduce a sliding window in which we suppose that the model (10) describes the signal well. For each estimation window, we get the numerical estimation of φ_1^2. Their variations on the whole length of the signal are depicted in figure 3 with $T = 500$ samples. In all subsequent numerical simulations the integrals are computed via the classical trapezoidal rule. Besides, the numeric estimations of the angular frequencies are

[8] T_f is the length of the signal

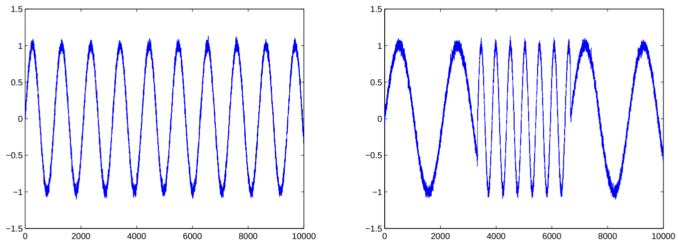
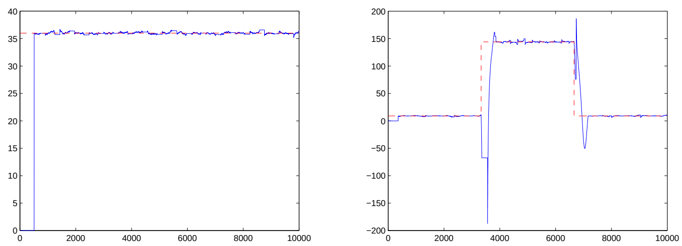


Figure2: Academic signals

initialized to zero at $t = 0$. To avoid zero-crossing of the denominator in the equation (16), a threshold is fixed at 10^{-4}. The performance of our estimator is based on a compromise during the choice of estimation window. Indeed, the more the window is important the better the free noise is but more signal model is questionable.
Figure 3 depicts angular frequency evolution of the two academic signals according to the time. We note that the estimator (16) has excellent capacities to estimate and pursue the frequency evolution even in the presence of noise. It also appears that the noise characteristics do not have a significant effect on the estimation.



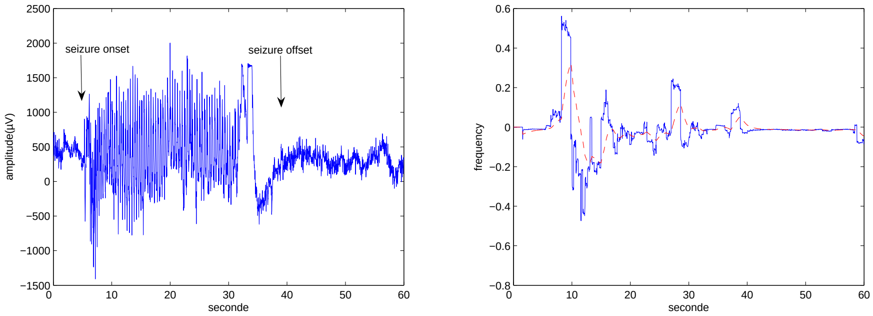
(a) estimated (–) et real (- -) φ_1^2 *(b)* estimated (–) et real (- -) φ_1^2

Figure3: estimated frequency of the two academic signals

3.2 EEG signals

In this section, we depict the result of using our method on EEG recording for which the clinical signs of an epileptic activity have been observed. The sampling rate is of 64 Hz. Figure 4-(a) shows one of the recordings (the measured voltage is function of the number of samples). The data contains a total of 1 minute with pre-seizure, the seizure and some of post-seizure activity. Based on the visual inspection of experimented neurologist, the seizure which translates an anomaly in the EEG signal and substitutes from the normal activity, occurs from the 5^{th}

seconde and lasts 35 seconds.



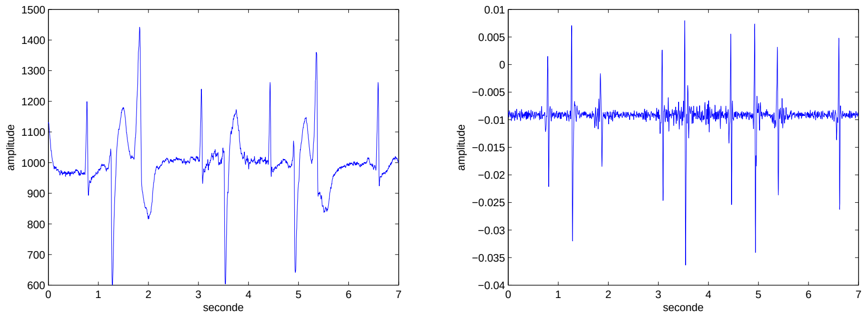
(a) EEG signals with an epileptic seizure pre-processed *(b)* Variation of EEG frequency content pos-processed

Figure4: EEG data

Here, the frequency estimation is used to analyze EEG signal with epileptic seizure. The aim is to find the relationship between frequency content variation of EEG signals and the pre-seizure, seizure and post seizure to detect a brain disorder such as an epilepsy. Figure 4-(b) shows the frequency variation of the EEG portion by using the estimator (16). To remove peaks which result from the zero-crossing of the estimator's denominator, the threshold is fixed at 10^{-4}. Based on the hypothesis that signal's angular frequency is weak out of the critical phase, any abrupt deviation translates a seizure. In order to limit the noise effect again, we make an average of the estimated frequency value in each point in short durations segments. In figure 4-(b), the curve in solid line shows the average variations of the estimated frequency. It is also possible to filter the previous estimation with a filter of cut-off frequency $w_c = 0.01Hz$ (dotted line figure 4-(b)). However, this latter smooths the frank variations.
In the set of the obtained curves, the estimator ensures the frequency changes detection translating the seizure appearance in EEG signal. Indeed, the three different stages : pre-seizure, seizure and post-seizure, could be observed by using the curves. While supervising this estimation, it is possible to localize the seizure onset and offset. Besides, this detection coincides with the expert interpretations.

3.3 ECG signals

ECG signals that we use were excerpted from the MIT-BIH Arrhythmia Database [32]. In order to depict the contribution of our approach, we choose two examples of ECG portions which cause a problem in the QRS complex detection by their forms and by the noise dominance. The first example is illustrated by the record 200. The choice of this record is based on the fact that it presents a very particular morphology which is difficult to extract the QRS complex by the existing methods. We can notice that in figure 5-(a) this recording presents on the one hand, a QRS complex reversed and an S wave completely flooded in the T wave, on the other hand, the amplitude of the T wave is superior to that of the QRS complex which makes the QRS complex detection difficult. But, the results obtained by the proposed estimator as shown in figure 5-(b) show that we can detect the occurrence of QRS complex in the ECG signal. The study of this signal through the draw of the frequency variation offers us the possibility of indicating the locations of a particular frequency in time. For example, the peaks in the curve of frequency coincide with those corresponding with the occurrence of a QRS complex in the original cardiac signal. The second exam-



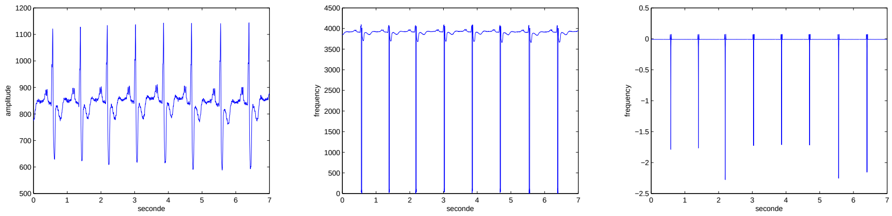
(a) Original signal, "record 200" from the MIT/BIH database

(b) QRS detected

Figure5: ECG data with particular morphology

ple is illustrated by the noise-contaminated recording 118 (figure 6-(b)). The signal-to-noise ratio of this recording is $SNR = 24dB$. This clinical additive noise was extracted from the MIT-BIH Noise Stress Database [33]. The noise recordings from the latter base were made using physically active volunteers and standard ECG recorders, leads, and electrodes in positions where the subjects' ECG were not visible. Three noise records were generated by selecting intervals that contained predominantly baseline wander, muscle (EMG) artifact, and electrode motion artifact. Electrode motion artifact is generally considered the most troublesome which cannot be removed easily by simple filters, as noise of other types can do. We choose to display the result obtained for the noisy signal (figure 6-(c)). The estimator (16) is able to detect correctly the location of the assumed QRS candidates even in the presence of noise. It seems that the noise has no effect on the estimation result. We based all judgments of correctness upon the annotations in the database. Each annotation in the locations and the morphology of a beat was determined by arbitration between two cardiologists.

These latters had to be in agreement to obtain the computer-readable reference annotations for each beatt included in the database.



(a) Noise free signal, "record 118" from the MIT/BIH Database

(b) Noisy signal, "recording 118" from the MIT/BIH Database, SNR=24dB

(c) QRS detected

Figure6: Noisy ECG data

4 Conclusion

Our work tackles the problem of the physiological signals analysis, markers of abrupt changes such as an epileptic activity in EEG and QRS complex occurrence in ECG, with new mathematical tool of estimation. This tool is based on a combination of differential algebra and operational calculus. We suggest an estimator of the "instantaneous" [9] angular frequency with the help of a simple local model ensuring a physiological signal representation during a short lapse of time. By applying this method to the EEG recordings of patients recovering from an epileptic activity, we are able to detect the seizure. Our approach shows also satisfactory results in ECG signal by detecting QRS complex occurrence in a severe context. This opens a promising research field in biomedical for this new estimation theory. Among several possible applications, we will consider the problem of the ECG modeling.

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