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1. Abstract

Predicting the evolution of geophysical fluids (ocean, atmosphere, continental water) is a major scientific and societal challenge. Achieving this goal requires to consider all the available information: numerical models, observations, error statistics... In order to combine these heterogeneous source of information one uses the data assimilation techniques.

During the last two decades, many visible and infrared band sensors have been launched on different satellites. This provide a large amount sequence of images of the earth system. This kind of information is underused in current data assimilation systems. In this paper we will describe how to use optimal control methods for data assimilation and in particular we will emphasise on techniques allowing to assimilate sequences of images.

2. Keywords: Images, Optimal Control, Geophysical Fluids, Data Assimilation.

3. Introduction

Understanding and forecasting the evolution of geophysical fluids (ocean, atmosphere, continental water) is a major scientific and societal challenge with important applications (extreme meteorological events, climate change, droughts and floods, etc.) and is currently the subject of an intensive research effort by the international scientific community. In order to achieve this goal one needs to take into account all the available information. This information can take several forms:

- Numerical models: they attempt to capture the system dynamics on a wide range of time and space scales. They are based on the Navier-Stokes equations and are complicated by the fact that they must deal with the irregular shape of the domain, with the parameterization of unresolved processes, and with highly nonlinear processes at the mesoscale and below. They are mathematical information under the form of a set of non linear PDE's.
- Observations: Despite their regular improvement, models will always be characterised by imperfect physics and some poorly known parameters (e.g., initial and boundary conditions). This is why it is important to also have observations of the system. Observations are now increasingly numerous as result of both satellite and in situ observing systems. However, their accuracy is not always satisfactory, and the processing of such a large quantity of data can be difficult. Moreover, observations provide only a partial view of reality, localised in time and space, and sometimes only indirectly related to model variables.
- statistical information: there is an *a priori* knowledge about the error of the two above sources of information, this needs to be taken into account.
- qualitative information: forecasters use their knowledge of the system and additional material such as satellite images to interpret the output of numerical prediction systems and provide corrected forecasts.

Since models and observations taken separately do not allow for a deterministic reconstruction of real geophysical flows, it is necessary to use these heterogeneous but complementary sources of information simultaneously through so called Data Assimilation methods. These are inverse modelling techniques based on the mathematical theory of statistical estimation. Their aim is to combine observations and prior model estimates, taking into account their respective statistical accuracies, in such a way that the combined estimate is more accurate than either source of information taken individually.

In the early 80's [2] proposed to use optimal control techniques techniques in order to perform this task. This method (commonly referred as Variational Data Assimilation (VDA)) has now been adopted by most of the main meteorological operational centres. In variational assimilation, the analysis problem is defined by the minimization of a cost function that measures the statistically weighted squared differences between the observational information, which includes a prior estimate of the system control variables, and their model counterpart. The cost function is minimized with respect to the control variables and this is done iteratively using a gradient descent method.

During the last two decades many satellites have been launched for the observation of the Earth for a better knowledge of the atmosphere and of the ocean. They provide, among other data, images of the earth system. It is clear that the dynamics of the images observed has a strong predictive potential, unfortunately this information is not currently optimally used in conjunction with numerical models. The purpose of this paper is to present variational data assimilation and to describe an extension of optimal control to the assimilation of sequences of images. To that purpose, two basic techniques can be considered:

- Pseudo-Observation: a velocity field can be estimated from the images, using image processing techniques and then this estimated velocity field can be used as pseudo-observations in a classical variational assimilation scheme.
- Direct assimilation: in the cost function of the variational formulation a quadratic term measuring the discrepancy between computed and observed images is included. Preliminary results on an academic test case will be presented.

4. Variational Data Assimilation

4.1. Principle of VDA

Let us briefly describe the basic principle of Variational Data Assimilation (VDA). The state of the flow is described by a state variable $\mathbf{x}$ depending on time and space, it represents the variables of the model (velocity, temperature, elevation of the free surface, salinity, concentrations in biological or chemical species...). The evolution of the flow is governed by the differential system:

$$\left\{ \begin{array}{rcl} \dfrac{d\mathbf{x}}{dt} & = & M(\mathbf{x}, U) \\ \mathbf{x}(0) & = & V \end{array} \right. \tag{1}$$

U is some unknown parameters of the model: boundary condition, model error, parametrization of subgrid effect... U may depend on space and time, V the initial condition is unknown and depends on space. Let's assume that U and V being given, the model has a unique solution between 0 and T. $\mathbf{y}_{obs}(t)$ are given observations of the fields available between 0 and T, For the sake of simplicity we suppose their continuity in time. The discrepancy between the observation and the state variable is defined by a so-called cost-function of the form:

$$J(U, V) = \frac{1}{2} \int_0^T \parallel H(\mathbf{x}) - \mathbf{y}_{obs} \parallel^2 dt + \frac{1}{2} \parallel U - U_0 \parallel^2 + \frac{1}{2} \parallel V - V_0 \parallel^2 \tag{2}$$

H is a mapping from the space of the state variable toward the space of observations where the comparison is carried out. The second and the third terms are regularization terms in Tikhonovs sense, they also allow to introduce some a priori information. Its important to point out that the norms are on three different spaces, they can take into account the statistical information by introducing the error covariance matrice. Here we will consider only identities in the three spaces. The problem of VDA can be considered at the determination of U^* and V^* minimizing $J(U, V)$. As a first approximation (humidity, salinity, concentration are non-negative) one has to solve a problem of unconstrained optimization. From the numerical point of view U^* and V^* will be estimated by a descent type algorithm $i.e$ by finding the limit of a sequence:

$$\begin{pmatrix} U_{k+1} \\ V_{k+1} \end{pmatrix} = \begin{pmatrix} U_k \\ V_k \end{pmatrix} + \lambda_k D_k \tag{3}$$

where D_k is the direction of descent deduced from the gradient of J and λ_k is the stepsize realizing the minimum of J along the direction of descent. For computing the gradient one introduces a so-called adjoint variable $\mathbf{p}$ as the solution of the adjoint model:

$$\begin{cases} \dfrac{d\mathbf{p}}{dt} + \left[\dfrac{\partial M}{\partial \mathbf{x}}\right]^T = \left[\dfrac{\partial H}{\partial \mathbf{x}}\right]^T .(H(\mathbf{x}) - \mathbf{x}_{obs}) \\ \mathbf{p}(T) = 0 \end{cases} \tag{4}$$

After a backward integration of the adjoint model, the gradient of J is given by:

$$\nabla J = \begin{pmatrix} \nabla_U J \\ \nabla_V J \end{pmatrix} = \begin{bmatrix} -\left[\dfrac{\partial M}{\partial U}\right]^T .\mathbf{p} \\ -\mathbf{p}(0) \end{bmatrix} \tag{5}$$

The derivation of the system can be found in [2]. The model (1) plus the adjoint model (4) with the optimality condition ∇J (5) is the Optimality System (O.S.), let us point out that the optimality system contains all the available information and therefore sensitivity studies with respect to the observations must be carried out from the O.S. rather than from the model.

4.2. Deriving the adjoint model
The adjoint variable has the same dimension as the direct model leading to a large increase in the size of the code, another inconvenient is that, in the case of nonlinear model the solution of the direct must be stored. From the direct model toward the adjoint model there are two basic operations: 1) computing the jacobian of the model with respect to the state variable. This operation is not very difficult, the code can be derived statement by statement, 2) transposing the jacobian. This is the most difficult operation because of multiple dependancies. For preexisting codes the derivation of an adjoint model is an expensive operation in term of (wo)manpower. Fortunately these last years have known the development of software on automatic diferentiation such as TAPENADE [3]

4.3. Application in meteorology
Nowadays most of the important meteorological centers have adopted VDA (European Center for Medium Range Weather Forecast, Japan Meteorological Agency, Météo France...). Models are based on the equation of fluid dynamics plus thermodynamics, after discretization in space (finite difference and spectral methods) the model is represented by a system of ODEs with about 500 millions of variables ,and this size will increase further in the future.

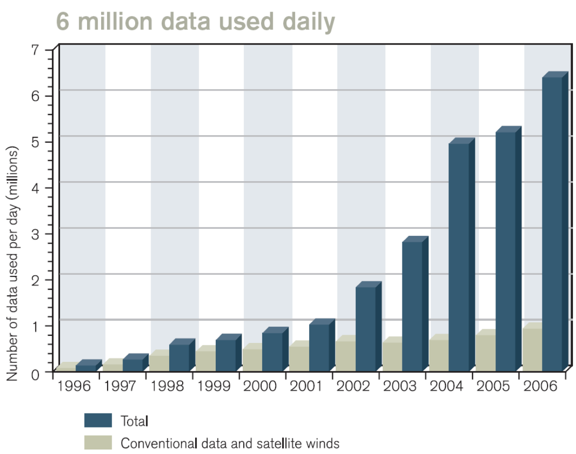

Figure 1: Total number of observations of the atmosphere used per days. (Source ECMWF)

Data are from numerous sources (in situ, radiosonds, aircraft, geostationary and polar orbiting satellites, etc.) and they are heterogeneous in density, quality and nature and therefore the covariance matrice associated to the error of observation has to be carefully estimated. The observation of Earth by Satellites is now the main source of data. These instruments do not observe directly the state variables of the model but radiances, therefore to be useful in the data assimilation, according to the general scheme defined above, an appropriate H operator transforming the observation into a state variable (*e.g.* radiances into a vertical profile of temperature) has to be provided. In this case this operator is nonlinear and may not be trivial to derive. Moreover, the adjoint of this operator will be needed. Fig 1 displays the increase in the total number of observations during the last decade including conventional and satellite observations. The total number has known an exponential increase, nevertheless it remains smaller than the number of unknown of the VDA problem, consequently we have to deal with an ill posed inverse problem and the role of the regularization term is of importance.

5. Assimilation of Images

5.1. Dynamics of images

The observation of the Earth by satellite is a source of information on the ocean and the atmosphere thanks to the measures of radiances. As it was said above this data can be plugged in a data assimilation scheme in a classical. However, there is another important source of information captured by satellites: the dynamics of some meteorogical or oceanic "objects": fronts, clouds, vortexes is visible on the sequences of images. Up to now this information has been used in a qualitative way rather than in a quantitative one. Two preliminary questions arise: 1) What is an image? Basically an image is

Figure 2: Black and white satellite images. Left: cloud coverage over Europe (visible channel, source Météo-France/Meteosat). Right: Black Sea - sea surface temperature (infra-red channel, source NOAA/AVHRR)

composed of pixels which is associated a scalar (grey level) field for black and white images (Fig 2) and three scalars for coloured images. From the mathematical point of view an image can be considered as an eulerian field. 2) What is seen? Fig 2left shows the nebulosity: this is a complex functions of the state variable of the model: clouds are not directly in the solution of the model because they depends both on thermodynamics (temperature and humidity) and also on the microphyics in the cloud. (water, ice, snow and the size of the particles) On Fig 3 we can see the evolution of a system on the Atlantic Ocean. These images provide an information on the dynamics of the system.

Is it possible to quantify this information in order to combine it with numerical models?

5.2. Retrieving "pseudo-observations"

The first approach to assimilate sequences of images is to evaluate from the sequence a velocity field then use this field as (pseudo-) observation in a regular VDA scheme [4]. There are several ways to extract a velocity field from a sequence of image. The most common is usually called optical-flow, it is a classical approach in computer vision and it is based on the conservation of grey level values for individual pixels. Given a pixel of coordinates (x, y), I is the luminance of the pixel, this quantity is conservative and this

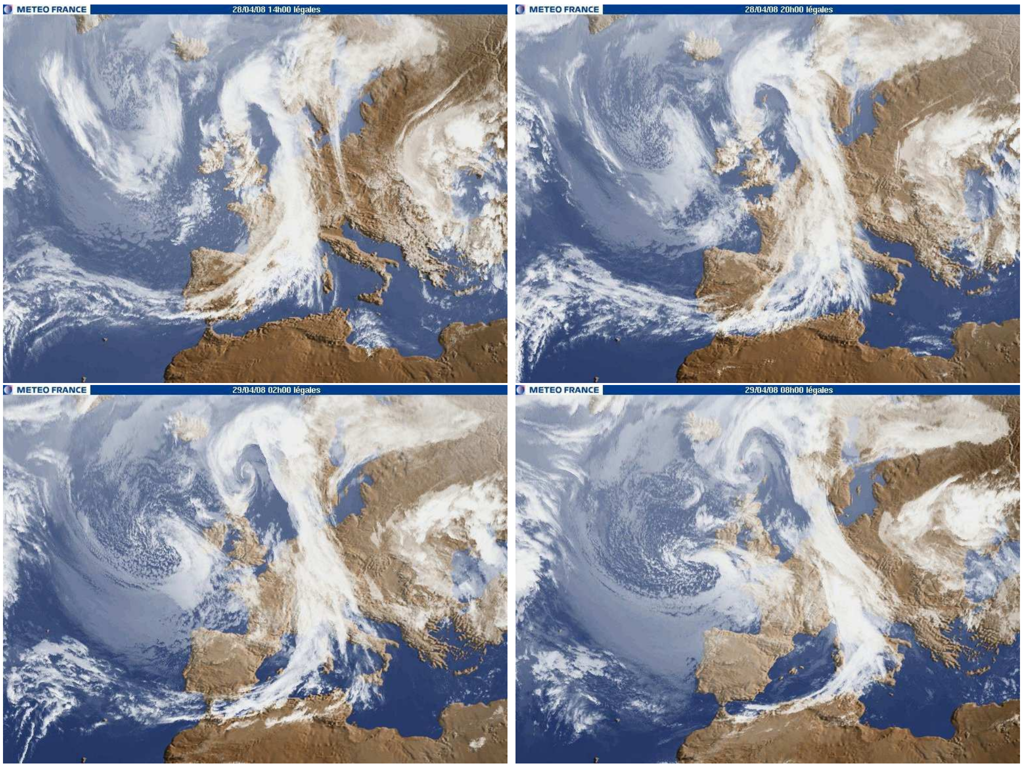

Figure 3: sequence of cloud coverage images above Europe from Meteosat (source Météo-France) from 28/04/2008 to 29/04/2008

property is mathematically written as the total derivative of I is equal to 0:

$$\frac{dI}{dt}\left(x(t), y(t), t\right) = 0. \tag{6}$$

Leading to :

$$\frac{\partial I}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial I}{\partial y}.\frac{\partial y}{\partial t} + \frac{\partial I}{\partial t} = \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0 \tag{7}$$

u, v are the components of the velocity of the flow and are unknown. This equation is not sufficient to retrieve the velocity field, nevertheless it can be included in a variational formulation where one will look for the velocity field minimizing J with :

$$J(u,v) = \int_I \left(\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t}\right)^2 d\Omega + \int_I \parallel \nabla u \parallel^2 + \parallel \nabla v \parallel^2 \, d\Omega \tag{8}$$

The second term can be considered as a regularisation term to smooth the computed fields. In the first term we recognize the scalar product of the gradient of luminance with the velocity field, a consequence is that these vectors are orthogonal then this equation does not bring any quantitative information about the velocity field. This is not the only law of conservation which could be considered according to the nature of the image: with an image of the color of the ocean an equation of conservation of chlorophyll (with source and sink) terms could be considered, with an image of Sea Surface Temperature (SST), the Boussinesq approximation could be used... Once the velocity field is determined, it can be fed in a classical VDA scheme such as presented in 4.1.

Most of the time optical flow methods only take into account 2 successive frames for the estimation of a velocity field and cannot deal with missing data. Recently [5] proposed an extension called "image model" that allow to use several successive frames at once by adding a governing equation for (u, v) to the optimality system that generate the pseudo odservations.

5.3. Direct Assimilation of Images

An other way to proceed is a direct assimilation of images using a variational formalism, by doing so we will avoid the stage of generating pseudo physical observations that could introduce some additional errors in the system. The principle is to add in the discrepancy between the solution of the model and the observation a term directly linked to the images and their dynamics. It will take the form:

$$J(x_0) = \underbrace{\int_0^T \|\mathbf{y} - \mathcal{H}[\mathcal{M}_t(x_0)]\|_{\mathcal{O}}^2 dt}_{\text{Classical term } J_o} + \int_0^T \|I - \underbrace{\mathcal{H}_{\mathcal{X}\to\mathcal{F}}[\mathcal{M}_t(x_0)]}_{\substack{\text{Model to}\\ \text{Image}\\ \text{Operator}}}\|_{\mathcal{F}}^2 dt \tag{9}$$

In this expression $\mathcal{F}$ is the space of images, $\mathcal{H}_{\mathcal{X}\to\mathcal{F}}$ is a mapping from the space of the state variable toward the space of images. Therefore the question are: 1)how to define $\mathcal{F}$? The constraints which are imposed is that it should have a structure of normed space in order to use simple rules for the differentiation of J, 2)the dimensionality of $\mathcal{F}$ must remain reasonable to store the evolution of the images. $\mathcal{H}_{\mathcal{X}\to\mathcal{F}}$ is a mapping from the space of the state variable toward the space of images, it will be non linear and with a complex structure. With the usual variational approach a term in the form:

$$\left[\frac{\partial \mathcal{H}_{\mathcal{X}\to\mathcal{F}}}{\partial \mathbf{x}}\right]^T . \left(\mathcal{H}_{\mathcal{X}\to\mathcal{F}}(\mathbf{x}) - I\right) \tag{10}$$

will be added in the right hand side of the adjoint model, where $\mathcal{H}_{\mathcal{X}\to\mathcal{F}}$ is the process permitting to retrieve an image from the computed fields. The main question is: how to mathematically define images? The first idea would be to consider that an image is characterized by structures (object) and that these structures can be approximated in a functional space. The comparison observation - model will then be done in this functional space and the cost function become:

$$J(x_0) = \int_0^T \|\mathbf{y} - \mathcal{H}[\mathcal{M}_t(x_0)]\|_{\mathcal{O}}^2 dt + \int_0^T \| \underbrace{\mathcal{H}_{\mathcal{V}\to\mathcal{S}}[I]}_{\substack{\text{Image to}\\ \text{Structure}\\ \text{Operator}}} - \underbrace{\mathcal{H}_{\mathcal{X}\to\mathcal{S}}[\mathcal{M}_t(x_0)]}_{\substack{\text{Model to}\\ \text{Structure}\\ \text{Operator}}}\|_{\mathcal{S}}^2 dt \tag{11}$$

with $\mathcal{S}$ is the space of structures, $\mathcal{H}_{\mathcal{V}\to\mathcal{S}}$ extracts structures from images (for instance by multiscale transformations), $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$ extracts structures from model state variables (model outputs). typically there are two option for describing the later: either $\mathcal{H}_{\mathcal{X}\to\mathcal{S}} = \mathcal{H}_{\mathcal{V}\to\mathcal{S}} \circ \mathcal{H}_{\mathcal{X}\to\mathcal{V}}$ is a composition of the Synthetic Images Operator $\mathcal{H}_{\mathcal{X}\to\mathcal{V}}$ and the Image to Structures Operator or one define directly $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$ without using synthetic images operator. In the following we use only the first option.

5.4. Preliminary numerical results

The experimental framework is that of the study of a drift of a vortex on a turntable that simulate the effect of the coriolis force. The vortex is made visible thanks to the addition of a passive tracer (fluorecine) . The evolution of the state vector $\mathbf{x} = (u, v, h)$ can be approximated by the Shallow-Water equation:

$$\left\{\begin{array}{lcl} \partial_t u - u\partial_x u + v\partial_y u - fv + g\partial_x h + \mathbf{D}(u) & = & F_u \\ \partial_t v + u\partial_x v + v\partial_y v + fu + g\partial_y h + \mathbf{D}(v) & = & F_v \\ \partial_t h + \partial_x(hu) + \partial_y(hv) & = & 0 \end{array}\right. \tag{12}$$

In order to extract the structures from the image, the curvelet transform [1] is particularly adapted to this context: they are multi-scale, multi-orientation transformation with atoms indexed with a position parameter. One can define $\mathcal{H}_{\mathcal{V}\to\mathcal{S}}(I)$ as a Threshold of the Curvelet Transforms of the image I:

$$\mathcal{H}_{\mathcal{V}\to\mathcal{S}}[I] = \text{Threshold of FDCT}[I] = \mathcal{T}(\text{FDCT}[\mathbf{v}]) \tag{13}$$

(FDCT stands for Fast Discrete Curvelet Transform)

On the other hand, one can define the Model to Image Operator (synthetic image) as

$$\mathcal{H}_{\mathcal{X}\to\mathcal{V}}[\mathbf{u},\mathbf{v},\mathbf{h}] = \mathbf{q} \tag{14}$$

where $\mathbf{q}(t) = \mathbf{q}(x, y, t)$ is the passive tracer concentration verifying

$$\partial_t \mathbf{q} + u\partial_x \mathbf{q} + v\partial_y \mathbf{q} - \nu_T \Delta \mathbf{q} = 0 \tag{15}$$

And finally the Model to Structure Operator is defined by:

$$\mathcal{H}_{\mathcal{X}\to\mathcal{S}} = \mathcal{H}_{\mathcal{V}\to\mathcal{S}} \circ \mathcal{H}_{\mathcal{X}\to\mathcal{V}} = \text{Threshold of FDCT}[\mathbf{q}] = \mathcal{T}(\text{FDCT}[\mathbf{q}]) \tag{16}$$

As a preliminary setup one uses the so-called twin experiment framework, where the data (observation) are generated from the model output instead of reality. The cost function to be minimised is:

$$J(\mathbf{x}_0) = \int_0^T \|\mathcal{T}(FDCT[I]) - \mathcal{T}(FDCT[\mathbf{q}])\|^2_{\mathcal{S}} dt + \|\mathbf{x}_0 - \mathbf{x}_b\|^2_{\mathcal{X}} \tag{17}$$

Fig 4 shows the true velocity field (left) that was used to generate the observation (sequence of images)

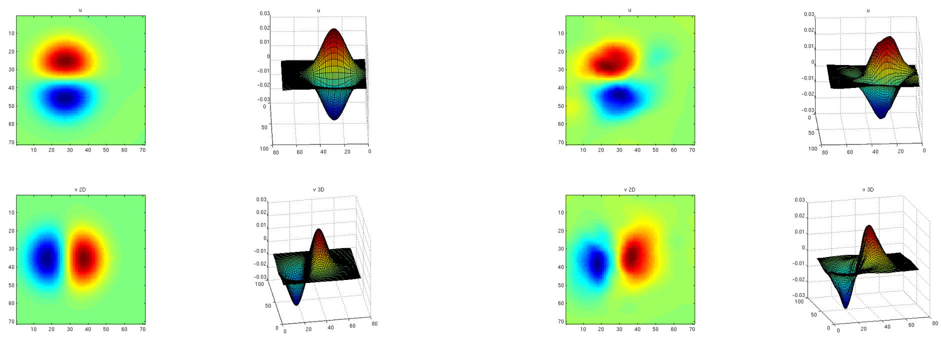

Figure 4: true velocity fied (left) vs estimated velocity field (right)

and the estimated velocity field (right) that is retrieved thanks to the minimisation of J. In this example the background $\mathbf{x}_b$ (*i.e.* the starting point of the minimisation) is the system at rest. This is not a very favourable case and therefore it is encouraging to be able to recreate a vortex with about the same characteristics.

6. Conclusions

Predicting the evolution of the environment requires to link together heterogeneous sources of information. There is a stong social demand for the improvement of prediction and VDA is a powerful tool to achieve this goal. The insertion of images into numerical models is a major challenge with applications in many scientific a technological fields.

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