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Preliminary results for introducing dependent random variables in stochastic feasibility analysis on CAN

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Abstract

We consider periodic and sporadic messages that are scheduled on a Controller Area Network (CAN) bus. We propose a stochastic analysis framework that provides probability distributions of message response times where the arrival of messages are given by independent random variables. This analysis can be extended easily to the case of dependent random variables. We also define the necessary steps to validate such analysis.

1 Introduction

Context of the study Requests in real-time environment are often of a recurring nature. Such systems are typically modeled as finite collections of simple, highly repetitive activities (e.g., tasks, messages). When the different instances of those activities are generated in a very predictable manner, we deal with periodic activities. The real-time performances of periodic activities on uniprocessor, distributed or network systems have been extensively studied. For these activities, the corresponding response times can be evaluated when all their parameters are known, but this is not the case of all real-time applications. Some parameters can be unknown until the time instant when the activity is released or the environment can change forcing the application to adapt. For instance, the problem is quite different for sporadic activities since, in many practical cases, it is not possible to have a precise knowledge of the activation pattern before the implementation of the system. For instance this is the case for sporadic messages, usually subject to soft real-time constraints, exchanged among Electronic Control Units (ECUs) in the body network of a vehicle. Such messages are usually assigned a low priority and they do not delay the hard real-time periodic traffic. However it is difficult to estimate their own response times.

Problem definition In this paper we discuss the problem of evaluating the real-time performances of periodic and sporadic messages that are scheduled on a CAN bus. We deal in this context with non-preemptive fixed-priority scheduling under the hypothesis that each message has its own priority. We are interested in obtaining probability distribution functions for the response times of messages, be they periodic or sporadic. We are clearly here in the realm of soft real-time constraints, but their satisfaction is important since large response times may jeopardize the execution of a function and may even raise safety concerns in some cases. In addition, low responsiveness is negatively perceived by the user.

Existing work Our approach belongs to the class of stochastic analyses for real-time systems. Probabilistic approaches are promising because they answer questions that cannot be addressed in a deterministic manner (e.g., distribution of response times) and consider models that are more realistic for instance regarding the task activation patterns or the expression of soft real-time constraints.

These approaches can be classified in two main classes. One class consists in extracting quantitative information for one or more parameters (e.g. distribution of execution time) from samples of observations collected by monitoring the system [1, 2]. Such approaches actually belong more to the realm of statistical methods. The other class of stochastic approaches concerns the temporal analysis of systems that have at least one parameter being a random variable. Among the studies in this area, one can for instance mention [3, 4].

To our best knowledge, there is no study on the response time of CAN-based systems for the case where the arrival of sporadic messages are given by independent random variables, which is the focus of this paper.

Our contribution Based on the exact response time analysis given in [5], we give a stochastic equation for calculating the distribution functions of messages response times on CAN. This equation can be solved using an algorithm provided in [4]. Moreover, it can be easily extended to dependent random variables by replacing the convolutions with the corresponding operations defined in [6]. These operations are based on the introduction of *copulas*, which to our best knowledge has never been used in this context. Copulas have been introduced to real-time systems in a seminal paper of Bernat et al. [6] and this theory allows to take into account possible dependencies between the random variables giving the inter-arrival times.

Organisation of the paper The paper is organized as follows. Section 2 contains our model and the associated notations. In Section 3 we introduce existing results that we use in this paper. In Section 4 we propose a stochastic feasibility analysis and in Section 5 necessary steps to validate a stochastic analysis. We present some ideas on future work in Section 6.

2 Model and notations

We analyse here the response time of messages under non-preemptive fixed-priority scheduling on a CAN bus. We consider $\tau = \{\tau_1, \tau_2, \cdots, \tau_n\}$ a set of messages, ordered in the decreasing order of their priority.

Each message is characterized by a transmission time C_i and a relative deadline D_i. Each new release j of a message τ_i is defined by different random variables. We denote by $\mathcal{T}_{i,j}$[1] the random variable giving the inter-arrival time between the $(j)^{th}$ and the $(j+1)^{th}$ instance of a message τ_i. The first release happens at a time instant given by $\mathcal{T}_{i,0}$. It is assumed that all random variables are independent of other jobs of this message.

To facilitate the presentation, we consider $\mathcal{T}_{i,j}$ to be discrete, thus its probability mass function (pmf) is:

$$\mathcal{T}_{i,j} = \begin{pmatrix} t_k \\ P(T = t_k) \end{pmatrix}_{k \in \{1, \cdots, k_{i,j}\}} \quad (1)$$

where $t_k \in [t_i^{min}, t_i^{max}]$ and $k_{i,j} \in \mathbb{N}^*$ is the number of values that random variable $\mathcal{T}_{i,j}$ has. We consider that t_i^{min}, t_i^{max} are given, thus known. If $t_i^{min} =$

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[1]In this paper we utilise calligraphic letters to denote random variables

$t_i^{max} = T_i$ then the message τ_i is a **periodic message** and we have $\mathcal{T}_{i,j} = \begin{pmatrix} T_i \\ 1 \end{pmatrix}$. If $t_i^{min} \neq t_i^{max}$, then the message τ_i is a **random message**.

We consider in this paper that $\sum_{i=1}^{n} \frac{C_i}{t_i^{min}} \leq 1$.

The set $hp(i)$ contains the indices of messages with higher priority than τ_i and $lp(i)$ the indices of messages with lower priority than message τ_i. The set $hp(i)$ contains $\mathbf{R}(i)$ which is the set of indices of random higher priority messages and $\mathbf{P}(i)$ the set of indices of periodic higher priority messages.

All considered values are multiples of τ_{bit} which is the time duration of a single bit on the CAN bus.

3 Existing results that we use

3.1 Deterministic exact analysis for CAN

In this section, since we present existing results for periodic messages, we consider that each message τ_i is characterized by its minimal inter-arrival time T_i.

We present here existing results [5] giving the worst-case response time of a message τ_i when all messages are periodic. First we need to calculate the length of the busy period for a message of priority i. We can obtain the length by solving iteratively the following equation

$$t_i = B_i + \sum_{k \in hp(i) \cup \{i\}} \left\lceil \frac{t_i}{T_k} \right\rceil C_k \quad (2)$$

where $B_i = \max_{k \in lp(i)} C_k$.

Then the worst-case response time of τ_i is obtained as

$$R_i = \max_{q=0 \ldots Q_i - 1} \{W_i(q) - qT_i + C_i\} \quad (3)$$

where $Q_i = \left\lceil \frac{t_i}{T_i} \right\rceil$ and $W_i(q)$ is the solution of the following equation:

$$W_i(q) = B_i + qC_i + \sum_{k \in hp(i)} \left\lceil \frac{W_i(q) + \tau_{bit}}{T_k} \right\rceil C_k \quad (4)$$

Equation (4) can be solved iteratively by considering at the first step of the iteration $W_i(q)^0 = 0$.

3.2 Stochastic analysis for preemptive uniprocessor fixed-priority scheduling

We present here existing stochastic equation giving the response times distribution of activities un-

der preemptive uniprocessor fixed-priority scheduling [4].

When the set of activities is strictly composed of periodic ones, then the following well-known formulation allows computing the worst-case response time of activity τ_i (we consider the activities ordered in the decreasing order of their priorities):

$$R_i = C_i + \sum_{k \in hp(i)} \left\lceil \frac{R_i}{T_k} \right\rceil C_k \qquad (5)$$

Equation (5) can be solved iteratively, by forming a recurrence relationship. This equation was extended to the case of activities with inter-arrival times given by independent random variables [4]:

$$\mathcal{R}_i = \mathcal{C}_i \otimes (\otimes_{k \in \mathbf{P}(i)} \left\lceil \frac{\mathcal{R}_i}{T_k} \right\rceil C_k) \otimes (\otimes_{k \in \mathbf{R}(i)} \mathcal{N}_{\tau_k}(\mathcal{R}_i) C_k) \qquad (6)$$

where $\mathcal{C}_k = \begin{pmatrix} C_k \\ 1 \end{pmatrix}, \forall k \in \mathbb{N}, \mathcal{C}_i = \begin{pmatrix} C_i \\ 1 \end{pmatrix}, \forall i \in \mathbb{N}$ and $\mathcal{N}_{\tau_k}(\mathcal{R}_i)$ is the random variable giving the number of arrivals of τ_k with their probabilities within $\mathcal{R}_i$. By $\otimes$ we understand the convolution of two independent random variables.

4 Stochastic feasibility analysis

Without loss of generality and in order to facilitate the presentation, we consider that $\mathcal{T}_{i,j} = \mathcal{T}_{i,k}, \forall j, k$.

For each message $\tau_k \in \mathbf{R}(i)$ we take into account the smallest values t_k^{min} in Equation (2) and we obtain the following formulation:

$$t_i = B_i + \sum_{k \in hp(i) \cup \{i\}} \left\lceil \frac{t_i}{t_k^{min}} \right\rceil C_k \qquad (7)$$

We extend now the response time formulation to the case with random variables by replacing max with the *mean* in Equation (3)[2]. We obtain that the average response time of τ_i is obtained as the distribution function:

$$\mathcal{R}_i = \frac{1}{q} \sum_{q=0...Q_i-1} \{ \mathcal{W}_i(q) \otimes \begin{pmatrix} -qt_i^{min} \\ 1 \end{pmatrix} \otimes \mathcal{C}_i \} \qquad (8)$$

where $Q_i = \left\lceil \frac{t_i}{t_i^{min}} \right\rceil$ and $\mathcal{W}_i(q)$ is the solution of the following equation:

$$\mathcal{W}_i(q) = (\otimes_{k \in \mathbf{P}(i)} \left\lceil \frac{\mathcal{W}_i(q) + \tau_{bit}}{\mathcal{T}_k} \right\rceil \mathcal{C}_k) \otimes$$

$$(\otimes_{k \in \mathbf{R}(i)} \mathcal{N}_K(\mathcal{W}_i(q)) \mathcal{C}_K) \otimes \begin{pmatrix} B_i + qC_i \\ 1 \end{pmatrix}$$

The last equation can be solved iteratively following the algorithm given in [4]. We consider at the first step of the iteration $\mathcal{W}_i(q)^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

5 Validation of stochastic approaches

We consider that the validation of any stochastic feasibility analysis should always verify the following three conditions:

- **Initial condition**: if all random variables have an unique value, then the analysis should be able to find the same result as existing deterministic analyses. For instance, let us consider our case where the inter-arrival times are given as random variables. Verifying the initial condition implies that for a system with all random variables having an unique value, i.e.$\mathcal{T}_i = \begin{pmatrix} T_i \\ 1 \end{pmatrix}, \forall i$, the analysis gives us the same answer as the corresponding deterministic analysis.

- **Robustness condition**: if at least one random variable contains at least two different values, then the analysis should be tested by using rare events theory. Usually the complexity of this type of approach is high (e.g., utilisation of convolutions) and the probability of having very high computations must stay under a certain threshold, i.e., having very high computations must be a rare event.

- **Worst-case condition**: if at least one random variable contains at least two different values, then the analysis must be able to:

  1. deal with systems of periodic and sporadic tasks. The stochastic analysis should be able to answer even when periodic activities exist.

2. insure the worst-case behaviour of the system. The analysis should be able to detect the instance or the instances having the worst-case response time.

Concerning the analysis we present here, the first condition is not fulfilled, but it allows us to provide the following discussion. When we consider systems with all random variables having an unique value, we obtain the mean of q response times instead the $\max$ because of the choice of calculating the average response time in Equation (8). As we said, another way of merging all response times can be used and by applying the initial condition, we can easily detect those that are appropriate.

Concerning the second condition, experimentations are needed in order to conclude on this issue. Once a sufficient number of experimentation exist, one can use theory like Large Deviation [1] to conclude on the robustness of his/her approach.

Concerning the third condition, our approach allows to fulfill only the first part, since our analysis is able to provide answer even when periodic activities exist. The second part of this condition is still an open question in our case except for the case where $\mathcal{T}_{i,j} = \mathcal{T}_{i,k}, \forall j, k$.

6 Conclusion and future work

In this paper we extend an existing stochastic analysis from the preemptive uniprocessor case to non-preemptive CAN case of systems under fixed-priority scheduling. Discussions on validation of this approach were proposed. The approach presented here is a first step to the introduction of dependent random variables for messages scheduled on CAN bus.

Complete validation of our feasibility analysis – as mentioned before our approach needs a complete validation by fulfilling all three conditions proposed in Section 5. We need a correct way to merge all response times and to choose the instances that allow us to insure worst-case behaviour.

Case of dependent random variables: introduction of copulas – the introduction of copulas in our analysis is the next logical step. This implies simply replacing the convolutions in Equation (8) by corresponding operations introduced in [6].

Complexity issues – any approach using convolutions is concerned by its complexity if we want to stay under a certain threshold of pessimism. In our case, a first step to decrease our complexity should be the utilisation of simpler CAN analysis like the one given in [7].

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