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Improving the Precision on Multi Robot Localization by Using a Series of Filters Hierarchically Distributed

Agostino Martinelli

Abstract— **This paper introduces a new approach to the problem of simultaneously localizing a team of mobile robots equipped with proprioceptive sensors able to monitor their motion and with exteroceptive sensors able of sensing one another. The method is based on a series of extended Kalman filters hierarchically distributed. In particular, the team is decomposed in several groups and, for each group, an extended Kalman filter estimates the configurations of all the members of the group in a local frame attached to one robot, the group leader. Finally, at the highest level of the hierarchy, one single filter estimates the locations of all the group leaders. The key advantage of this approach is its ability to distribute the computation necessary to perform the multi robot localization under limited computation and communication capabilities. In particular, the approach significantly outperforms an optimal approach based on a single estimator. This is shown by analytically computing the precision on the localization of each robot in the case of one single degree of freedom. In particular, the best hierarchy is analytically determined by deriving the dependency of the localization precision on the communication and computation capabilities and on the sensors accuracy.**

I. Introduction

In most cases, autonomous mobile robots are required to know precisely their configuration in order to successfully perform their mission. This is usually achieved by fusing proprioceptive data (gathered by sensors monitoring the motion of the vehicle, like encoders) with exteroceptive data (e.g. [1], [3], [4], [15]).

When a team of mobile robots cooperates to fulfill a mission, an optimal localization strategy must take advantage of relative observations (detection of other robots). This problem has been considered in the past following different approaches.

Fox and collaborators [5] introduced a probabilistic approach based on Markov localization. Their approach has been validated through real experiments showing a drastic improvement in localization speed and accuracy when compared to conventional single robot localization. Other approaches take advantage of relative observations for multi-robot localization [7], [9], [10], [17], [18], [19]. In [9] a method based on a combination of maximum likelihood estimation and numerical optimization was introduced. This method allows reducing the error on the robot localization by using the information coming from relative observations among the robots in the team.

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In [18], a distributed multi robot localization strategy (DMRL) was introduced. This strategy is based on an Extended Kalman Filter (EKF) to fuse proprioceptive and exteroceptive sensor data. In [11], DMRL was adapted in order to deal with any kind of relative observations among the robots. In [18], it was shown that the equations of DMRL can be written in a decentralized form, allowing the decomposition into a number of smaller communicating filters. However, the distributed structure of the filter only regards the integration of the proprioceptive data. As soon as an observation between two robots occurs, communication between each member of the team and a single processor (which could be embedded in a member of the team) is required. Furthermore, the computation required to integrate the information coming from this observation is entirely performed by this processor with a computational complexity which scales quadratically with the number of robots. Obviously, the centralized structure of the DMRL in dealing with exteroceptive observations becomes a serious problem when the communication and processing capabilities do not allow to integrate the information contained in the exteroceptive data in real time. In particular, this happens as soon as the number of robots is large, even if each robot observes only few other robots at once. In [14] this problem was considered. However, the structure of the filter was maintained the same as in [18] (namely centralized in dealing with exteroceptive data). Each robot was supposed to be equipped with several sensors and the optimal sensing frequencies were analytically derived by maximizing the final localization accuracy. The limit of this approach is that as the number of robots increases, the sensing frequencies reduce. In other words, by performing the estimation process in a centralized fashion, as in DMRL, it is necessary to reduce the number of observations to be processed as the number of robots increases. The question which arises is: *given a team of robots with constraints on computation and communication capabilities, is it possible to improve the precision on the localization by decentralizing the estimation process?*

The information filter is not the right choice to overcome this problem in this framework although the integration of exteroceptive data is very simple and could be easily distributed. The problems come when dealing with proprioceptive data. This point was discussed in [18].

In this paper we introduce a new approach based on a series of EKFs hierarchically distributed. In particular, the team is decomposed in several groups and, for each group, an EKF estimates the locations of all the members of the group in a local frame attached to one robot, the group leader.

Finally, at the highest level of the hierarchy, one single filter estimates the locations of all the group leaders.

The system is described in section II. The proposed approach is introduced in section III. In section IV we analytically compute the precision on the robot localization achievable by using this approach and in particular we compute how this precision depends on the hierarchy. The computation is performed for a homogeneous team of robots moving in a $1D$environment. This derivation allows analyzing the performance of the approach with respect to the parameters characterizing the system and in particular it allows finding the best hierarchy depending on these parameters. Finally, conclusions are reported in section V.

II. The System

A. Process and observation model

Let us consider a team of N robots equipped with both proprioceptive and exteroceptive sensors. We will refer to them by using $r_1, r_2, ..., r_N$. The dynamics of the state $X = [x_1, x_2, ..., x_N]^T$, which contains the configurations of $r_1, r_2, ..., r_N$ in a common reference frame, can be described through the following stochastic differential equation:

$$dX \;=\; f(X(t), U(t))dt \;+\; n(X(t), U(t))dW(t) \qquad (1)$$

where $U(t)$ are the robot velocities as estimated by proprioceptive sensors (e.g. inertial measurement unit and/or Doppler or encoder systems); $W(t)$ is a standard Wiener process with the same dimension of U (d_U), which accounts all the non-systematic uncertainties affecting the proprioceptive measurements. The function f is a vector function with the same dimension of X (d_X). The function n is a $d_X \times d_U$ matrix function characterizing the covariance Q of the proprioceptive sensors (in particular, the error accumulated per unity of time is $\dot{Q} = nn^T$) [16].

An observation provided by exteroceptive sensors can be characterized by the following equation:

$$z(t) \;=\; h(X(t)) \;+\; v(t) \qquad (2)$$

where $v(t)$ is a zero mean white process. In the following we will consider relative observations between two robots. When r_i observes r_j we have:

$$h(X(t)) \;=\; h_r(x_i(t), x_j(t)) \qquad (3)$$

This is the observation provided by any sensor which is able to sense another robot in the team and evaluate a relative quantity (e.g. its bearing, its distance etc).

B. Estimating the relative coordinates

The problem we are considering is the simultaneous localization of all the robots $r_1, r_2, ..., r_N$, i.e. the estimation of the state X. This estimation has to be performed by integrating the information coming from the proprioceptive readings (contained in the vector U in (1)) and from the exteroceptive readings (the vector z in (2)). This integration can be performed by using a Kalman Filter (as in DMRL).

By carrying out an observability analysis which accounts the system non-linearities (e.g. by using the observability rank criterion [8]), it is possible to show that the state X is actually not observable [12]. This is basically due to the fact that no robot has absolute localization capabilities. For this reason, instead of considering the problem of estimating X, we focus our attention on the problem of estimating the state $D = [d_1, d_2, ..., d_{N-1}]^T$, containing the configuration of each r_i ($i = 1, ..., N-1$) in the frame of r_N. In particular, depending on the dimension of the environment where the robots move, the dimension of each d_i can be 1 ($1D$ environment), 3 ($2D$ environment), 6 ($3D$ environment).

The problem arising when estimating the relative coordinates is the necessity of communication among the robots to integrate proprioceptive data (i.e. this communication is required at the frequency of the proprioceptive data). This makes a real time implementation not feasible. In other words, the distributed property of DMRL when integrates the proprioceptive data is lost. However, it is possible to overcome this problem by updating the state D only at the frequency of the exteroceptive observations.

Let us suppose that r_N performs the estimation of D. Furthermore, let us assume that at the time t an exteroceptive observation is integrated to update D and that the successive observation occurs at the time $t + \Delta t$. During the interval $(t, t + \Delta t)$ each robot r_i estimates the transformation of its frame occurred starting from the time t up to the current time by using its proprioceptive sensors. Furthermore, it also estimates the covariance of this transformation. Let us indicate with $\Delta x_i(\tau)$ the estimated transformation occurred up to time $\tau \in (t, t + \Delta t)$. The predicted value of d_i at the time $t + \Delta t$ is:

$$d_i(t + \Delta t) = \ominus \Delta x_N(\Delta t) \oplus d_i(t) \oplus \Delta x_i(\Delta t) \qquad (4)$$

where the symbol $\oplus$ is adopted to indicate the composition of two transformations and $\ominus$ to indicate the inverse transformation. We remark that equation (4) requires communication between r_i and r_N. We also remark that the formula in (4) does not introduce approximations, i.e. the same result would have been obtained by predicting d_i at the frequency of the proprioceptive data.

On the other hand, computing the covariance of $D(t+\Delta t)$ from the expression in (4) (i.e. by computing the Jacobian from this expression) introduces an approximation. In particular, the result can be different from the one obtained by carrying out the computation of the covariance of D at the frequency of the proprioceptive data. However, the difference becomes negligible if Δx_i is evaluated with good accuracy from the proprioceptive measurements (which is often the case since Δt is a short time).

III. The Hierarchical Structure of the Proposed Estimation Approach

When a single EKF is adopted to estimate D, one single processor performs the update of D as soon as a new exteroceptive observation is available. In particular, to integrate a new observation, all the robots must communicate with this processor and the computational burden to update D scales quadratically with the number of robots.

We divide the N robots into K groups of M robots. We have:

$$N = K \times M \tag{5}$$

Each group contains one group leader and $M-1$ other robots. Let us indicate with r^j the leader of the j^{th} group ($j = 1, ..., K$) and with r_i^j the i^{th} robot belonging to the j^{th} group ($i = 1, ..., M-1$). We are assuming, without loss of generality, $r^j \equiv r_M^j$.

Our strategy is based on $K+1$ EKFs ($F_0, F_1, F_2, ..., F_K$). F_j ($j \neq 0$) estimates the $M-1$ configurations of r_i^j ($i = 1, ..., M-1$) expressed in a reference frame attached to r^j. Let us indicate with d_i^j the configuration of r_i^j in the frame attached to r^j. F_j estimates the state:

$$D_j = \left[d_1^j, d_2^j, ..., d_{M-1}^j\right]^T \quad j = 1, ..., K \tag{6}$$

The last filter (F_0) estimates the configuration of all the group leaders in the reference of one of them, for instance in the reference attached to r^K. By indicating with d^j the configuration of r^j in the reference of r^K, the filter F_0 estimates the state:

$$D_0 = \left[d^1, d^2, ..., d^{K-1}\right]^T \tag{7}$$

A. Exteroceptive data integration

The $(K+1)$ EKFs integrate independent relative observations occurring among all the robots. In particular, each filter F_j ($j \neq 0$) integrates the observations between robots belonging to the same group (j^{th}). Therefore, an observation between $r_{i_1}^j$ and $r_{i_2}^j$ can be expressed in the following way:

$$z \ = \ h(d_{i_1}^j, d_{i_2}^j) \ + \ v(t) \tag{8}$$

F_0 integrates all the observations performed between robots not belonging to the same group. Let us consider an observation between $r_{i_1}^{j_1}$ and $r_{i_2}^{j_2}$. This observation has the following expression:

$$z \ = \ h(d^{j_1} \oplus d_{i_1}^{j_1}, d^{j_2} \oplus d_{i_2}^{j_2}) \ + \ v(t) \tag{9}$$

Since we want to use this observation to only update D_0 we use the following approximation:

$$z \ = \ h(d^{j_1}, d^{j_2}) \ + \frac{\partial h}{\partial d_{i_1}^{j_1}} \delta d_{i_1}^{j_1} + \frac{\partial h}{\partial d_{i_2}^{j_2}} \delta d_{i_2}^{j_2} + \ v(t) \tag{10}$$

where $\delta d_{i_1}^{j_1}$ and $\delta d_{i_2}^{j_2}$ are the (unknown) error respectively on $d_{i_1}^{j_1}$ and $d_{i_2}^{j_2}$ estimated from F_{j_1} and F_{j_2}. Since F_{j_1} and F_{j_2} estimate independent quantities by using independent measurements, $\delta d_{i_1}^{j_1}$ and $\delta d_{i_2}^{j_2}$ are independent one each other. On the other hand, F_0 adopts in part the same proprioceptive measurements to estimate D_0. Therefore, there is a correlation between δd_i^j and d^k. In the next section, where we compute the precision on the localization, we neglect this correlation. Note that this correlation becomes smaller as the precision of the exteroceptive data compared with the precision of the proprioceptive data increases.

We remark that for $K = 1$ or $K = N$ the proposed approach coincides with DMRL (i.e. it is based on a single estimator to integrate the exteroceptive data).

IV. Analytical Derivation of the Precision Achievable on the 1D Case

The characterization of the estimation error is provided by a covariance matrix P satisfying the Riccati differential equation [2]. This characterization can be adopted in a real implementation (where the system is actually time-discrete) since, starting from a discrete-time system, it is possible to derive an equivalent continuous-time system model as shown in [13]. The structure of the Riccati equation related to the general case defined by the equations (1) and (2) is:

$$\frac{dP}{dt} = FP + PF^T + \dot{Q} - \nu PH^T R^{-1} HP \tag{11}$$

where F is the Jacobian of the dynamics with respect to the state to be estimated ($F = \nabla_X f(X(t), U(t))$), $\dot{Q} = nn^T$ (n is defined in (1)) characterizes the noise in the dynamics, $R = < vv^T >$ (v is defined in (2)) characterizes the observation error, H is the Jacobian of the observation with respect to the state to be estimated ($H = \nabla_X h(X(t))$), ν is the frequency of the exteroceptive observations integrated by the estimator.

In order to perform analytical computation we consider the case of one degree of freedom. Investigating $1D$ case where analytical solutions can be provided is an efficient way to derive important properties for a given problem. In particular, this kind of investigation is not new in the field of mobile robotics [6]. Our goal here, is to find the best hierarchy and in particular its dependency on the parameters characterizing the system. Future works will focus on the possibility to extend these results to higher dimensional cases.

We will consider a homogeneous team of robots moving in $1D$ environment. In this case $X \in \Re^N$. The dynamics of each x_i ($i = 1, ..., N$) is described by the following stochastic differential equation

$$dx_i \ = \ u_i(t)dt \ + \ \sigma dw_i(t) \tag{12}$$

where $w_i(t)$ is a standard Wiener process of dimension 1 and we assumed a constant noise term (σ). Because of the team homogeneity, σ is the same for all the robots.

Let us consider the relative observations. When r_i observes r_j we consider the following relative observation

$$z_r(t) \ = \ x_j(t) - x_i(t) + v_r(t) \tag{13}$$

where $v_r(t)$ is a white zero mean Gaussian sequence, with constant variance, $< v_r^2(t) >= \sigma_r^2$. We assume that each robot performs the same number of relative observations per unity of time. Furthermore, we assume that each robot observes uniformly the other robots in the team. We will indicate with ν_r the frequency of the relative observations performed by a given robot on another given robot. Because of the team homogeneity, both σ_r^2 and ν_r are the same for all the robots. From the previous assumptions, the total number of relative observations per unity of time is:

$$\#_{Obs} = \nu_r N(N-1) \tag{14}$$

A. Precision on D_j

Let us consider the j^{th} group of robots where r^j performs the estimation of D_j by running the filter F_j. In this $1D$ case, $d_i^j = x_i^j - x_M^j$ ($i = 1, ..., M-1$). Therefore, for the dynamics of d_i^j we have:

$$dd_i^j = (u_i^j - u_M^j)dt + \sigma(dw_i^j - dw_M^j) \tag{15}$$

and for the observation performed by r_{i_1} on r_{i_2}:

$$z_r = d_{i_2}^j - d_{i_1}^j + v_r \tag{16}$$

Let us indicate with P_j the covariance matrix of D_j. From (15) and (16) equation (11) becomes:

$$\frac{dP_j}{dt} = \sigma^2 Q_m - 2\nu_r^G \sigma_r^{-2} P_j C_m P_j \tag{17}$$

where:

- $m = M - 1$ is the size of P_j;
- $Q_m \equiv I_m + \Im_m$ with I_m the identity $m \times m$ matrix and $\Im_m$ the $m \times m$ matrix whose entries are all equal to 1;
- $C_m \equiv (m+1)I_m - \Im_m$
- ν_r^G characterizes the frequency of the relative observations actually used by the filter F_j; in particular, ν_r^G defines the relative observations performed by a given robot belonging to the considered group on another robot belonging to the same group; in general, this parameter is fixed by the limited computational and communication capabilities as we will discuss in section IV-C.

We remark that C_m and Q_m are symmetric commuting matrices, therefore they are simultaneously diagonalizable. This means that it exists an invertible matrix T such that $Q_m = TLT^{-1}$ and $C_m = T\Lambda T^{-1}$ with L and Λ diagonal matrices. By setting $P_j = T\hat{P}_j T^{-1}$ equation (17) becomes:

$$\frac{d\hat{P}_j}{dt} = \sigma^2 L - 2\nu_r^G \sigma_r^{-2} \hat{P}_j \Lambda \hat{P}_j \tag{18}$$

Note that $tr(P_j) = tr(\hat{P}_j)$.

The steady state solution can be easily derived by requiring $\frac{d\hat{P}_j}{dt} = 0$. We obtain:

$$\hat{P}_j = \frac{\sigma\sigma_r}{\sqrt{2\nu_r^G}} L^{\frac{1}{2}} \Lambda^{-\frac{1}{2}} \tag{19}$$

In the appendix we compute the eigenvalues of Q_m and C_m (i.e. the main diagonal entries of L and Λ). By using these values we get for the trace at the steady state:

$$\lim_{t\to\infty} trP_j(t) = \sqrt{\frac{2}{M\nu_r^G}}(M-1)\sigma\sigma_r \tag{20}$$

and for the precision at the steady state on the estimation of the relative coordinates of each group member:

$$\sigma_{Member}^2 = \sqrt{\frac{2}{M\nu_r^G}}\sigma\sigma_r \tag{21}$$

B. Precision on D_0

The filter F_0 integrates the relative observations between robots belonging to different groups, as explained in section III-A. Let us consider two groups: the j_1^{th} and the j_2^{th} ($j_1, j_2 = 1, ..., K,\ j_1 \neq j_2$). We have to distinguish among three cases of observation between the robots belonging to these two groups:

- the observation occurs between the two group leaders (i.e. between r^{j_1} and r^{j_2});
- the observation occurs between a group leader and a member of the other group (i.e. either between r^{j_1} and $r_{i_2}^{j_2}$ or between $r_{i_1}^{j_1}$ and r^{j_2}, ($i_1, i_2 = 1, ..., M-1$));
- the observation occurs between two members belonging to the first and the second group (i.e. between $r_{i_1}^{j_1}$ and $r_{i_2}^{j_2}$).

By using the expressions in (10) and (16) we have respectively for the three previous cases:

- $z = d^{j_1} - d^{j_2} + v$;
- $z = d^{j_1} - d^{j_2} + \delta d_{i_1}^{j_1} + v$
- $z = d^{j_1} - d^{j_2} + \delta d_{i_1}^{j_1} - \delta d_{i_2}^{j_2} + v$

where $\delta d_{i_1}^{j_1}$ and $\delta d_{i_2}^{j_2}$ are the (unknown) error respectively on $d_{i_1}^{j_1}$ and $d_{i_2}^{j_2}$ estimated from F_{j_1} and F_{j_2} which are independent one each other. Their variance is given in (21).

By introducing the following quantity:

$$\mu = \frac{\sigma_{Member}^2}{\sigma_r^2} \tag{22}$$

and by neglecting the correlation between δd_i^j and d^k we obtain for the covariance of D_0:

$$\frac{dP_0}{dt} = \sigma^2 Q_k - 2\nu_r^L \sigma_r^{-2} b_m P_0 C_k P_0 \tag{23}$$

where:

- $k = K - 1$ is the size of P_0;
- ν_r^L characterizes the frequency of the relative observations actually used by the filter F_0; in particular, ν_r^L defines the relative observations performed by a given robot belonging to a given group on another robot beloging to a different group; in general, this parameter is fixed by the limited computational and communication capabilities as we will discuss in section IV-C.
- b_m is defined by the following expression:

$$b_m = 1 + \frac{2m}{1+\mu} + \frac{m^2}{1+2\mu} \tag{24}$$

The structure of (23) is the same as in (17). Therefore, as in (21), the precision at the steady state on the estimation of the relative coordinates of each group leader in the frame of r^K is:

$$\sigma^2_{Leader} = \sqrt{\frac{2b_m}{K\nu_r^L}}\sigma\sigma_r \qquad (25)$$

C. The best Hierarchy

We want to find the best hierarchy in order to minimize the error on the localization of each robot. In other words, we want to find the value of K which minimizes the error on the estimated relative coordinates of a given robot in the reference of r^K. If the considered robot is a group leader, the error to be minimized is the one in (25). On the other hand, we want to consider the worst case, namely a robot which is not a group leader. In this case, by neglecting the correlation between the error on D_j and D_0, the error is the sum of the two components in (25) and (21).

$$\sigma^2_{ss} = \qquad (26)$$

$$= \sigma^2_{Leader} + \sigma^2_{Member} = \sigma\sigma_r \left(\sqrt{\frac{2b_m}{K\nu_r^L}} + \sqrt{\frac{2}{M\nu_r^G}} \right)$$

The minimization of (26) has to be performed by taking into account the limited computational and communication resources. In particular, the frequencies ν_r^L and ν_r^G must satisfy several constraints. Let us derive them.

Accordignly with the architecture proposed in section III-A and with equation (14), the number of observations per unity of time processed by each F_j ($j = 1, ..., K$) is:

$$\#^G_{Obs} = \nu_r^G M(M-1) \qquad (27)$$

and the number of observations per unity of time processed by F_0 is:

$$\#^L_{Obs} = \nu_r^L M^2 K(K-1) \qquad (28)$$

We are considering an homogeneous team of robots and we suppose that all the robots are equipped with the same processor and that they have the same communication capabilities. We remind that the computational cost to integrate one exteroceptive observation scales quadratically with the size of the estimated state and that the requested communication scales linearly [18]. Regarding the filter F_j ($j = 1, ..., K$) the computational constraint is:

$$\alpha_{cp}(M-1)^2 \#^G_{Obs} = \alpha_{cp}\nu_r^G M(M-1)^3 \leq \nu_{cp}^{MAX} \qquad (29)$$

where α_{cp} is a dimensionless parameter depending on the processor embedded on each robot and ν_{cp}^{MAX} is the maximal allowed frequency due to the limited computational capability characterizing the processor. Regarding the communication constraint we obtain:

$$\alpha_{cm}(M-1)\#^G_{Obs} = \alpha_{cm}\nu_r^G M(M-1)^2 \leq \nu_{cm}^{MAX} \qquad (30)$$

where α_{cm} is another dimensionless parameter which depends on the communication capability of each robot and ν_{cm}^{MAX} is the maximal allowed frequency due to the limited communication capability.

Therefore, the maximal allowed value of ν_r^G is:

$$\nu_r^{G\ MAX} = \qquad (31)$$

$$= min \left(\frac{\nu_{cm}^{MAX}}{\alpha_{cm}M(M-1)^2}, \frac{\nu_{cp}^{MAX}}{\alpha_{cp}M(M-1)^3} \right)$$

similarly, we obtain for the maximal allowed value of ν_r^L:

$$\nu_r^{L\ MAX} = \qquad (32)$$

$$= min \left(\frac{\nu_{cm}^{MAX}}{\alpha_{cm}M^2K(K-1)^2}, \frac{\nu_{cp}^{MAX}}{\alpha_{cp}M^2K(K-1)^3} \right)$$

By using these two values in (26) and by knowing that $N = MK$ we finally obtain how the precision on the localization of a given robot (σ^2_{ss}) depends on the hierarchy (i.e. on the parameter K). In order to find the best hierarchy we can analytically solve the equation $\frac{d\sigma^2_{ss}}{dK} = 0$. Obviously, the value we find in this way is not necessarily an integer. Furthermore, even if we choose the closest integer, the total number of robots (N) is not necessarily a multiple of this integer. However, simple solutions can be provided as illustrated in the following example.

We consider a system consisting of $N = 101$ robots. Furthermore, the communication and computational resources are characterized by the following values for the parameters defined in section IV-C: $\frac{\nu_{cm}^{MAX}}{\alpha_{cm}} = 0.11Hz$ and $\frac{\nu_{cp}^{MAX}}{\alpha_{cp}} = 0.27Hz$. Finally, for the proprioceptive and exteroceptive sensors we assume $\sigma^2 = (0.01)^2 m^2 s^{-1}$ and $\sigma_r^2 = (0.01)^2 m^2$.

In figure 1 we plot σ^2_{ss} (actually, up to a scale factor independent of K) vs K. In the plot, we consider all the real values of K in the range $(1, N)$. The minimum of σ^2_{ss} is attained for $K = 4.218$. The error obtained by choosing $K = 4$ groups of robots with respectively $M = 25, 25, 25, 26$ robots is slightly larger than the minimum in figure 1 (the difference between the two values is less than 0.2%). For $K < 28.8$ the value of $\nu_r^{G\ MAX}$ is fixed by the computation constraint (i.e. the min in (31) is the second argument for $K < 28.8$). For $K < 3.5$ the value of $\nu_r^{L\ MAX}$ is fixed by the communication constraint (i.e. the min in (32) is the first argument for $K < 3.5$). The value of σ^2_{ss} obtained for $K = 1$ is exactly the same obtained for $K = 101 = N$ as expected, since both cases correspond to the estimation carried out by a single estimator. Finally, the value of σ^2_{ss} obtained for $K = 4$ is 0.31 times the σ^2_{ss} obtained without hierarchy (i.e. for $K = 1$ or $K = N$).

Similar results are obtained by changing the parameters characterizing the system.

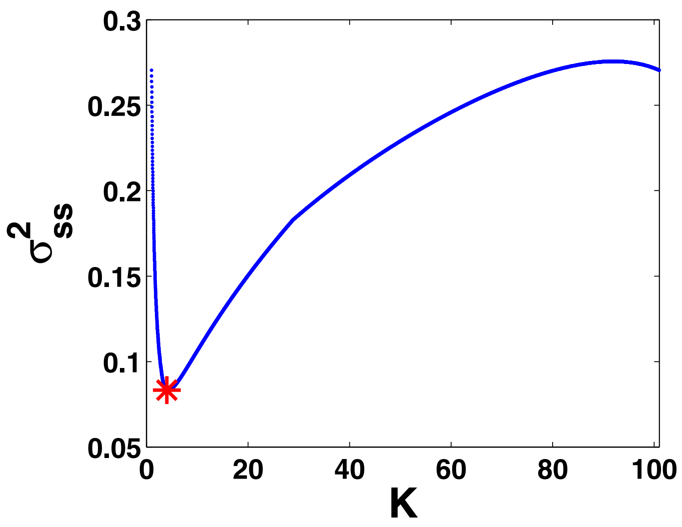

Fig. 1. σ_{ss}^2 vs K. σ_{ss}^2 is computed up to a scale factor independent of K. The red star represents the value of σ_{ss}^2 obtained by considering four groups with respectively $M = 25, 25, 25, 26$ robots.

V. Conclusion

In this paper we introduced a new approach to the problem of simultaneously localizing a team of mobile robots equipped with proprioceptive and exteroceptive sensors. The method is based on a series of extended Kalman filters hierarchically distributed. In particular, the team is decomposed in several groups and, for each group, an extended Kalman filter estimates the locations of all the members of the group in a local frame attached to one robot, the group leader. Finally, at the highest level of the hierarchy, one single filter estimates the locations of all the group leaders.

The main advantage is that the computation can be distributed over the team robots. Consequently, in the case of limited communication and computation resources, this strategy significantly outperforms an optimal approach based on a single estimator.

In order to evaluate its performance, we considered the case of one single degree of freedom. We analytically derived the error on the localization of each robot and in particular its dependency on the system parameters and on the chosen hierarchy.

Starting from this complete analysis for the $1D$ case, we are now considering the case of robots moving in $2D$ and $3D$ environments where analytical solutions cannot be provided. However, upper bound on the localization error can be analytically derived in several interesting cases.

Appendix

Eigenvalues of Q_s and C_s

Let us compute first the eigenvalues of the matrix $\Im_s$. This is a real symmetric $s \times s$ matrix. Therefore, its eigenvalues are real and their geometric multiplicity is equal to their algebraic multiplicity. Furthermore, the rank of this matrix is 1. This means that 0 is an eigenvalue whose geometric multiplicity is equal to $s - 1$. In order to find the last eigenvalue we observe that the trace of this matrix is s. Hence, the sum of the eigenvalues is s, meaning that s is also an eigenvalue with multiplicity equal to 1.

Regarding the matrix $Q_s = I_s + \Im_s$ we therefore obtain:

- 1 is an eigenvalue with multiplicity $s - 1$;
- $1 + s$ is an eigenvalue with multiplicity 1

Regarding the matrix $C_s = (s+1)I_s - \Im_s$ we obtain:

- $s + 1$ is an eigenvalue with multiplicity $s - 1$;
- 1 is an eigenvalue with multiplicity 1

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