How Graph Theory can help Communications Engineering

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Abstract

We give an overview of different aspects of graph theory which can be applied in communication engineering, not trying to present immediate results to be applied neither a complete survey of results, but to give a flavor of how graph theory can help this field. We deal in this paper with network topologies, resource competition, state transition diagrams and specific models for optical networks.

1   Introduction

It is not news that the development of sciences and technology makes it impossible to be aware of all fields of knowledge, and specialization cannot be avoided in the scientific research where the rapid advances make it difficult even to follow the new results in the own specific discipline.

This unavoidable fact turns out to be a strong limitation in the development of science, since many achieved results in some disciplines that could be useful in other fields of knowledge are in most cases unknown by scientists.

It is desirable that researchers know at least the existence of other field's advances. Although not being specialist on these other areas, they should know where to look for information in benefit of their own research.

Multidiscipline teams and multidiscipline areas are words that nowadays seem to be important in the scientific research. Indeed collaboration with other groups is always fruitful not only because each discipline can help the others with known results but also because different points of view can help one each other to better understanding the insight of the problems.

This is even more important if we focus on *ancillae scientiae*, i.e. sciences which are used as tools in other sciences such as maths is with respect to engineering. Mathematicians are usually looking for new problems and engineers are usually looking for new solutions. Dialogue of both areas is interesting for both communities.

Nevertheless dialogue is not always easy and requires some effort from both, mathematicians and engineers. The translation of a problem arising from engineering to a mathematical language requires a deep knowledge of the discipline in which the problem is contextualized. Setting of assumptions and hypothesis of the model is always a compromise between realism and tractability. Finally, the results given by the mathematical model must be validated with some experimental or simulation results.

One cannot expect to have a library of "plug and play" models for new problems, but the art of creating a mathematical model is one of the examples in which cooperation of engineers and mathematicians is crucial in order to get valuable results.

In this paper we give an overview of different aspects of graph theory which can be applied in communication engineering, not trying to present immediate results to be applied neither a complete survey of results, but to give a flavor of how graph theory can help this field.

The remaining of this paper is structured in four sections. We present, in Section 2, some basic concepts of graph theory. Section 3 is devoted to graphs as models for network topologies. Different problems related with communications in which Graph Theory might be useful are treated in Section 4. Finally, Section 5 deals with specific problems that appear in Optical communications, and a detailed example is also shown in that section.

2   Graph theory concepts

A graph $G = (V, E)$ is an ordered pair consisting of a set $V = V(G)$ of elements called *vertices* and a set $E = E(G)$ of unordered pairs of vertices called *edges*.

Two vertices $u, v \in V(G)$ are said to be *adjacent* if there is an edge $(u, v) \in E(G)$ joining them. The edge $(u, v) \in E(G)$ is also said to be *incident* to vertices u and v. The degree of a vertex v, $\delta(v)$ is the number of vertices in $V(G)$ adjacent to it. The minimum and maximum degrees of the graph, δ and Δ respectively, are defined as the minimum and maximum over the degrees of all vertices of the graph.

A *path* $w(v_0, v_n)$ is a sequence of vertices $v_0, v_1, \ldots, v_n$ such that $(v_{i-1}, v_i) \in E(G)$, $i = 1..n$. Such a path is said to be of length n. A graph is said to be *connected* if there is a path between any pair of vertices. If the graph is connected, the distance between two vertices $d(a, b)$ is the length of the shortest path joining a and b and the *diameter* of the graph is the maximum among the distances between all pair of vertices of the graph.

A graph can be defined and represented in different ways. One typical representation is by means of a drawing in which vertices are drawn as points in the plane or in the space and edges are drawn as lines joining points. In spite of being an intuitive view of a graph, it seldom leads to some insight on its properties. Graphs can also be characterized by its *adjacency matrix*, The adjacency matrix of a graph with n vertices is a $n \times n$ matrix $A = (a_{ij})$ with $a_{ij} = 1$ if $(i, j) \in E(G)$ and 0 otherwise. Notice that the adjacency matrix is always a symmetric matrix. Another way to define a graph is giving the adjacency rules on the set of vertices. Graphs that can be defined that way are usually "well structured", and use to have nice properties.

Since a graph is a binary relationship within elements on a generic set, it is not surprising that graphs appear, at least implicitly, in many different contexts of the scientific knowledge such as sociology, economy, or engineering.

Graphs can be generalized to other related structures that have been also widely studied:

- Multigraphs and weighted graphs. In some cases it is useful to consider edges having a weight, usually a numerical value, or either that more than one edge can join the same two vertices.

- Digraphs. A digraph or directed graph is a graph in which we consider edges (called arcs in the context of digraphs) having a direction assigned. In other words, a digraph $G = (V, A)$ is an ordered pair consisting of a set $V = V(G)$ of vertices and a set $A = A(G)$ of *ordered* pairs of vertices called *arcs*.

- Hypergraphs. Instead of considering edges as binary relations between vertices, *hyperedges* are defined as subsets of vertices (with not a determined size). A hypergraph $G = (V, H)$ is then defined as an ordered pair consisting of a set $V = V(G)$ of vertices and a set $H = H(G)$ of subsets of vertices called *hyperedges*.

- Directed hypergraphs. A directed hypergraph $G = (V, \vec{H})$ is then defined as an ordered pair consisting of a set $V = V(G)$ of vertices and a set $\vec{H} = \vec{H}(G)$ of ordered pairs of subsets of vertices called *hyperarcs*.

2

All the definitions given for graphs can be translated to all these structures defined above. The reader can find other related concepts and definitions in [5, 12].

3 Graphs and network topologies

Concerning to interconnection networks it is straightforward that a network topology can be modeled by a graph in which the vertices of the graph play the role of nodes in the network and the edges model the point-to-point links between different nodes. The graph can be undirected in the case of full-duplex communication links, or directed in case the links are unidirectional or in case of highly asymmetric communication protocols such as ADSL. Alternatively, if some nodes share a common physical or logical bus, the bus can be modeled by a hyperedge, and hence the whole network by a hypergraph. This might be the case of networks that use CSMA/CD-like protocols such as Ethernet.

Yet in the early 50's graph theory was used to model telephony networks, and from the very beginning interesting results came from these studies like, for instance, the studies for non-blocking multistage switching networks (the Clos Network, [17]). Since then lots of studies have been done in graph theory when considering graphs as topological models for different kinds of networks.

Lots of aspects have to be considered in the design of an interconnection network and many of them are highly dependent on the topology and can be studied in terms of Graph Theory. In most cases an important restriction is the maximum number of links that can be connected to each node (maximum degree of the graph). In case that it is not possible (or convenient) to have a link between any pair of nodes, a multihop solution must be adopted, i.e. a message must hop through intermediate nodes before reaching its destination. It is desirable, however, to have the number of hops between nodes as small as possible (small diameter of the graph). The problem of finding graphs with fixed degree and number of vertices having the smallest possible diameter is known as the (Δ, n)–problem, and has been widely studied in the literature ([12, 7]).

The designer should in some cases foresee that the network might grow in the future by adding new nodes. It is convenient that the network size can be increased without loosing its structural properties. There are many graphs that have this property (scalability).

Another important item of interconnection network is routing and communications algorithms. In case that the graph is well structured, efficient algorithms can be specifically design for that network topology. The efficiency of a routing protocol depends on the simplicity of the processing in the intermediate nodes and on the memory needed in these processors to handle the delivery (such as small routing tables). Routing has also been widely studied for specific network topologies under different communication models (virtual paths, store–and–forward, wormhole routing, deflection (hot potato) routing,...). Efficiency of a routing can also be measured in terms of throughput which is related with the number of routes using the same link or node (forwarding index [16, 30]).

Among the communication algorithms, broadcasting and dissemination (gossiping) of information are of special interest since they appear often in many communication processes. They have also been widely studied under different models and assumptions [31].

It can be also interesting to know the behavior of topology and routing when some of the components of the network fail. Fault tolerance can measured in different ways: by the increasing of the diameter when removing links or nodes (Δ, D', s)–problem [36]); by the number of links or nodes that may fail keeping the network still connected (connectivity [26]) or in terms of routing, by the accessibility of nodes without reconfiguring the routing tables under network failures (surviving graph [23]).

In many cases it is not possible to define a well structured topology or even the topology may

be unknown or unstable like in the internet. Theoretical results and heuristic algorithms can also be obtained for generic or even unknown graphs [22].

We have focused on some important parameters to be considered in order to choose a network topology. There are, however, many other aspects we have not considered here and may also be important in some cases, like, for instance, the efficient layout of the network in the case of VLSI systems (planarity, crossing minimization, [5]) or the efficient embedding of logical topologies onto a specific physical one in the case of parallel implementation of algorithms that must run over a specific network [34] (the typical instance for this last example is the case of parallelizing the FFT algorithm on the hypercube network).

4 Other applications of graph theory

Besides the importance and usefulness of using graphs to model the topology of the network, there are some other ways in which graph theory can help communication engineers. In this section we show two different examples:

4.1 State transition diagrams

Another application of graph theory in communications engineering is the one in which vertices model states of some process and arcs (directed edges) model the possible transitions between these states. The most important example for this kind of modeling are the well known Markov chains, but they are not the only ones.

An interesting area in which Graph Theory can help to obtain results in these kind of processes is the case of convolutional codes or the more recent time-space codes ([40]) that are decoded by means of maximum likelihood, such as the way the Viterbi algorithm works.

The symbol to be transmitted in these kind of codes depends not only on the symbol wanted to be decoded at the receiver, but also on the state the transmission is. Moreover, both the transmitted symbol and the present state determine a new state in the transmission.

The usual transition of states is given by a shift register on the state, i.e. states are sequences consisting on the last transmitted symbols. Shift register leads to a very well known family of directed graphs called De Bruijn digraphs [28].

Transition diagrams given by Shift Register have not been proved to be optimum in terms of code gain (gain in terms of error probability in decoding), and other transition diagrams could be considered in order to construct better codes.

4.2 Resource competition

Indeed graphs can easily model any situation in which binary relationships outcome like for instance the competition of agents towards the same resources: vertices of the graph represent the different agents, and two vertices are adjacent if they want to use the same resource. In order to compute the minimum number of resources needed one can use the resulting "incompatibility" graph. In fact, this problem turns to be a standard problem in graph theory called vertex coloring: i.e. assign colors to the vertices of the graph such that not two adjacent vertices must have the same color [5]

In the remaining of this section two examples will be shown for better understanding of this kind of graphs.

4

4.2.1 Interfering graph in cell telephony systems

The frequency assignment problem for a cellular network has been thoroughly discussed in the literature owing, in part, to the important economic repercussions that it has for network operators. The largest outlay to be confronted by an operator is the license fee for the allocation of the radio frequency spectrum. As this spectrum is limited, the profitability of the network will depend on the number of stations required to satisfy the users demand.

In the design of a cellular mobile communication system, a region is divided into series of cells, each one covered by a base station. Each station works at a certain frequency, and the same frequency can be used in different cells. However, the reuse of the same frequency is limited by the maximum tolerable interference between cells. The frequency assignment problem consists on finding an assignment of frequencies to the base stations using the smallest possible number of frequencies considering the interference constrains.

A cellular network can be described by a graph where the nodes represent the cells and the edges joint nodes corresponding to cells that cannot use the same frequency because of interferences.

This model can also be refined considering different levels of interferences, different transmitters per station, or some fixed restrictions in the assignment. See [1] for some recent results and algorithms to solve that problem.

4.2.2 Conflict graph in wavelength assignment for virtual paths

Let us consider in this section WDM optical networks consisting of routing nodes interconnected by point-to-point fiber-optic links which can support a certain number of wavelengths. Switching will be supposed to be also optical, and packets are supposed not to change of wavelength at the intermediate nodes. When a communication must be established between to nodes of the network a virtual path is reserved and a wavelength is assigned to that transmission.

An important restriction in this model is that two different virtual paths sharing at least one common link must have different wavelengths, as if they did not, it would not be possible to distinguish both signals at the next intermediate node.

To compute the minimum number of wavelengths required to establish a set of virtual paths we define the *Conflict Graph* the following way: Vertices of the graph stand for the different virtual paths and two vertices will be adjacent if the corresponding virtual path share at least one link.

As in the previous section, the wavelength assignment problem can be solved in the Conflict Graph by assigning colors (wavelengths) to its vertices in such a way that adjacent vertices have different colors [4].

One can object that this *a priori* planing is not realistic, and it is true. But this *static* and simple setting of the problem can be improved by using more realistic dynamic approximations based on on-line algorithms [1].

Notice that the examples shown in this section have identical translation into a coloring problem of graphs.

Related to the the wavelength assignment problem, it is interesting to point a question with a (maybe) surprising answer.

The recent Dense WDM (DWDM) allows to use a considerable number of wavelengths in the same fiber. Is it equivalent to use a single fiber with DWDM that using multiple fibers with conventional WDM to join two nodes in the network (supposing that both solutions allow the same total number of wavelengths per link)?

Many factors should be considered to decide which solution is better. But some Graph theory background can help to realize that both solutions are not topologically equivalent [11]. In fact, it

is easy to see that the second alternative turns out to be more efficient from the point of view of the usage of the resources (wavelengths), as shown in the following example:

Suppose a simple network consisting on four nodes connected in a star topology as shown in Figure 1 and imagine we want to establish three simultaneous communications between nodes 1–2, 2–3 and 1–3. The goal will be assigning wavelengths to these transmission requirements using the minimum number of wavelengths.
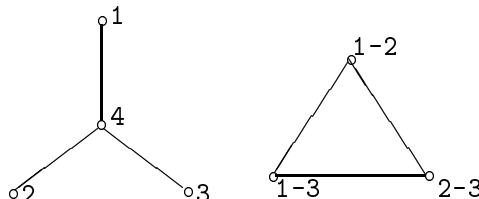

Figure 1: A network and the conflict graph for 3 requirements.

If links consist on two fibers each, the requirement can be satisfied by using one wavelength per fiber, while two wavelengths per fiber are not enough to solve the assignment with a single fiber per link. This can be easily seen in terms of graph colorings, since the conflict graph in this former case is a triangle, and vertices in a triangle cannot be colored with two colors.

5   Graph theory and all-optical networks

In the preceding sections we have reviewed different aspects in which Graph Theory can help Communications engineering. We want to focus in this section on optical networks. At a first sight one might think that, from the graph theoretic point of view, there is not any difference between optical networks and generic interconnection networks, since graph theory do not care about what are the wires made of.

But besides the general results and models for interconnection networks that can be also useful in our context, new specific problems appeared in the study of all optical networks. These problems are not exclusive of this field but have now a particular interest.

In particular, we are not concerned in this section with classical networks in which some electrical wires are replaced by optical fibers and at each endpoint there is a electric-to-optical shift, but on networks in which switching is also done using lightwave technology.

Some of the properties of all-optical networks, that lead to new problems in Graph theory are the following:

- While electrical buffering is technologically easy to implement, this is not the case of optical systems. Routing must be done without storing the information in the routers and packets cannot wait to be delivered through a link without optical to electrical conversion [25].

  Fast routing algorithms must be realized (such as MPLS [2, 3]) and, in case of network congestion, deflection routing [38, 14] could be a good solution.

- New routing constraints appear when using WDM, different from those in TDM. An example of problems arising from WDM was given in the previous section.

- New models for networks are being developed due to the new devices in optical technology. In this context there have appeared many studies on bus networks [6, 18] in which the network topology is no more modeled by a graph but by a hypergraph.

Also, in this direction we will explain a more detailed example in the next section in which we model a network by a directed hypergraph using existing optical devices.

5.1 Using directed hypergraphs with OTIS and OPS couplers

Topologies allowing one-to-many communications in a single communication step have been extensively studied. The building block is the Optical Passive Star (OPS) coupler [27] which can be used as a *single-OPS broadcast-and-select* network in which an incoming optical signal is broadcast to all output ports [10]. Although a great deal of research efforts have been concentrated on single-OPS topologies [24, 15, 37], the OPS coupler presents a severe drawback since its size is technologically limited by its splitting capabilities. That is the reason why *multi-OPS* topologies seems more viable and scalable [6, 18].

In this section, we present some multi-OPS topologies with the single-hop Partitioned OPS networks (POPS) [13] and the multi-hop stack-Kautz network [18]. We also present the stack-graphs, a family of directed hypergraphs which is very useful to model and manipulated multi-OPS networks. Finally, we present the OTIS architecture [35], an optical device which was used to implement the optical interconnections of the POPS and the stack-Kautz.

5.1.1 Optical passive star

An optical passive star coupler (OPS) is a singlehop one-to-many optical transmission device [24, 37] which can be view as an optical bus. An $OPS(s, z)$ has s inputs and z outputs. When $s = z$, the OPS is said to be of degree s (see Figure 2). When one of the input nodes sends a message through an OPS coupler, the s output nodes have access to it. An OPS coupler is a passive optical system, i.e. it requires no external power source. It is composed of an optical multiplexer followed by an optical fiber or a free optical space and a beam-splitter [29] that divides the incoming light signal into s equal signals of a s-th of the incoming optical power. Finally, in a single-wavelength OPS couplers of degree s, only one optical beam can be guided through each device. Consequently, no two nodes can have concurrent access to any OPS.

An OPS coupler of degree s can be modeled by a hyperarc linking two hypergraph nodes composed of s vertices each [10], meaning that the processors of one set (the OPS sources) can send messages only through the hyperarc, whereas the other set (the OPS destinations) can receive messages only through the same hyperarc. Figure 2 shows an OPS coupler modeled by a hyperarc.
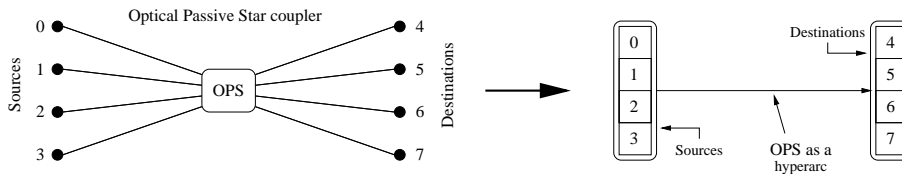

Figure 2: Modeling an optical passive star coupler of degree 4 by a hyperarc.

5.1.2 Stack-graphs

The stack-graphs were defined in [10] as a particular class of directed hypergraphs. Informally, they can be obtained by means of piling up s copies of a digraph G and subsequently viewing each stack of s nodes as a hypergraph node and each stack of s arcs as a hyperarc. The value s is called the

stacking factor of the corresponding stack-graph, and $\varsigma(s, G)$ denotes the stack-graph of stacking factor s, obtained from the digraph G.

Stack-graphs are not optimum in terms of the (Δ, D)–problem, and other families of directed hypergraphs could be also considered [6]. But stack-graphs are easy to deal with and we will show in the following, through examples, that are very efficient to model multi-OPS networks.

5.1.3 Partitioned Optical Passive Stars network

The Partitioned Optical Passive Stars (POPS) topology, introduced in [13], is an interconnection architecture that uses multiple non-hierarchical OPS to achieve single-hop networks. The POPS network $POPS(t, g)$, is composed of $N = tg$ nodes and g^2 OPS couplers of degree t. The nodes are divided into g groups of size t (see Figure 3). Each OPS coupler is labeled by a pair of integers (i, j), $0 \leq i, j < g$. The input of the OPS (i, j) is connected to the i-th group of nodes, and the output to the j-th group of nodes. This network benefits of the routing facilities of singlehop networks and of the broadcast capabilities of OPS-based networks.
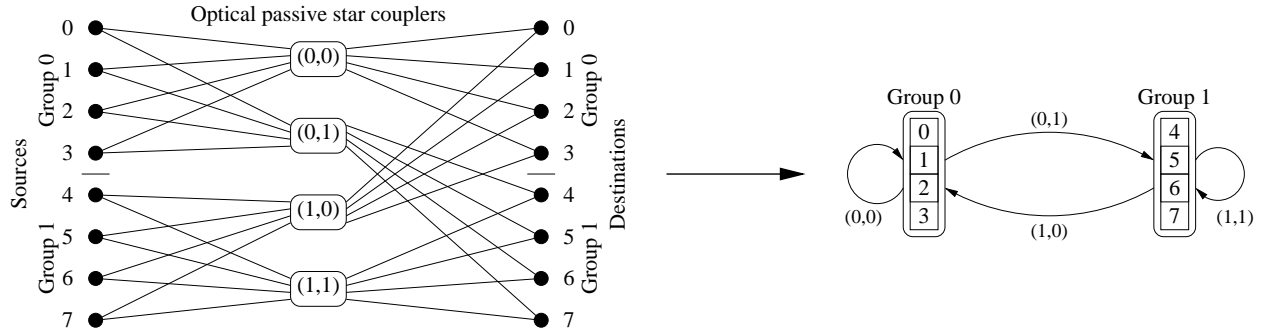

Figure 3: $POPS(4, 2)$ with eight nodes, modeled as $\varsigma(4, K_2^*)$.

Since an OPS coupler is modeled by a hyperarc, the POPS network $POPS(t, g)$ can be modeled as a directed hypergraph, and in fact it is not difficult to see that the underlying topology consists on a stack-K_g^+ (or $\varsigma(t, K_g^+)$, for short) of stacking-factor t, where K_g^+ is the complete digraph with loops having g nodes and g^2 arcs (see Figure 3), as proposed in [9, 27].

Using the stack-graph model, embedding of rings, tori, de Bruijn and other well knows topology on POPS were easily obtained [9, 8].

5.1.4 Stack-Kautz

We saw with the POPS networks that stack-graphs represent a powerful tool for building multi-OPS networks based on graphs presenting good topological characteristics. The Kautz digraph [33] is one such graph.

The Kautz digraph $K(d, k)$ of degree d and diameter k has $d^{k-1}(d + 1)$ vertices. A vertex is labeled with a word of length k, $(x_1, \cdots, x_k)$, on the alphabet $\Sigma = \{0, \cdots, d\}$, $|\Sigma| = d + 1$, in which $x_i \neq x_{i+1}$, for $1 \leq i \leq k - 1$. There is an arc from a vertex $x = (x_1, \cdots, x_k)$ to all vertices y such that $y = (x_2, \cdots, x_k, z)$, $z \in \Sigma$, $z \neq x_k$. The Kautz digraph can also be defined in terms of line digraph iteration [28] and we have $K(d, k) = L^{k-1}(K_{d+1}^*)$, where K_{d+1}^* denotes the complete digraph without loops. It is both Eulerian and Hamiltonian and optimal with respect to the number of nodes if $d > 2$ [33]. A shortest path routing algorithm (every path is of length at most k) is
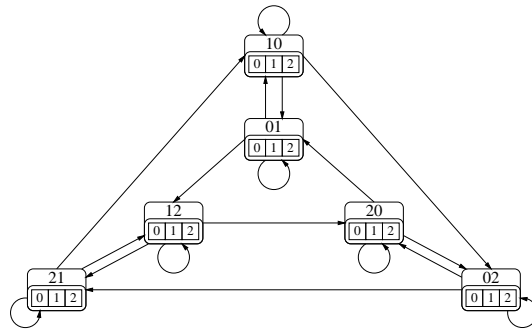
Figure 4: Stack-Kautz network $SK(3, 2, 2)$.

induced by the labels of the nodes. It can be extended to generate a path of length at most $k + 2$ which survives $d - 1$ link or node faults [32]. Also, node disjoint paths can be easily generated.

The stack-Kautz were first proposed in [18] as a multi-hop alternative of the POPS networks. They are modeled by a stack-graph build from the Kautz digraph, as follows. Let $K_l(d, k)$ denotes the Kautz digraph in which each vertex receive a loop and hence has degree $d + 1$.

Definition 1 *The* **stack-Kautz** *$SK(s, d, k)$ is the stack-graph $\varsigma(s, K_l(d, k))$ of stacking-factor s, degree $d + 1$ and diameter k (see Figure 4).*

The stack-Kautz inherits most of the properties of the Kautz digraph, like simple and fault tolerant routing algorithms, as described in [18]. Also broadcasting is very efficient Furthermore, the POPS network is a stack-Kautz of diameter one, $POPS(t, g) = SK(t, g - 1, 1)$.

5.1.5 OTIS

The Optical Transpose Interconnection System (OTIS) architecture, was first proposed in [35]. It provides a large number of interconnections in a free optical space (no fiber). $OTIS(p, q)$ is an optical system which allows point-to-point (1-to-1) communications from p groups of q transmitters
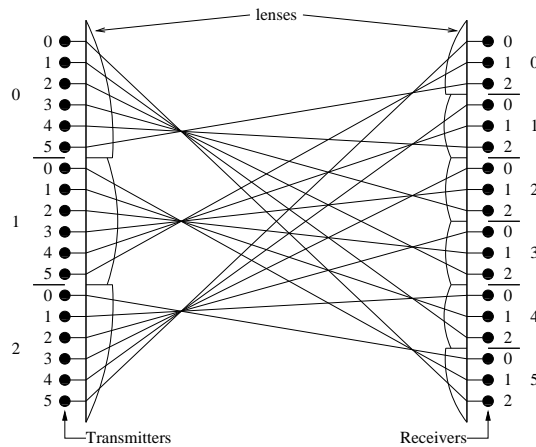

Figure 5: $OTIS(3, 12)$.

onto q groups of p receivers, using $p + q$ lenses. This architecture connects the transmitter (i, j), $0 \leq i \leq p-1$, $0 \leq j \leq q-1$, to the receiver $(q-j-1, p-i-1)$. Optical interconnections in the OTIS architecture are realized with a pair of lenslet arrays in a free space of optical interconnections as shown in Figure 5.

A family of digraphs obtained from $OTIS(p, q)$ were defined in [20] as given in definition 2. It has been shown in [20] that this family of digraphs contains the Kautz digraphs. This result was used in [19] to propose an implementation of the optical interconnections of Kautz, POPS and stack-Kautz networks using OTIS. Furthermore, an optimal implementation of the de Bruijn network with OTIS, in terms of number of lenses, was shown in [20].

Definition 2 *The $H(p, q, d)$ digraph is defined from the $OTIS(p, q)$ architecture. It has degree d, such that d divides pq, and $n = \frac{pq}{d}$ vertices and $m = pq$ arcs. Its vertices are integers modulo n and its neighborhood is defined as follows:*

$$\Gamma_H^+(u) = \left\{ v = \left\lfloor \frac{1}{d} \left((pq - 1) \left(\left\lfloor \frac{du + \alpha}{q} \right\rfloor + 1 \right) - p(du + \alpha) \right) \right\rfloor, \quad 0 \leq \alpha \leq d - 1 \right\}$$
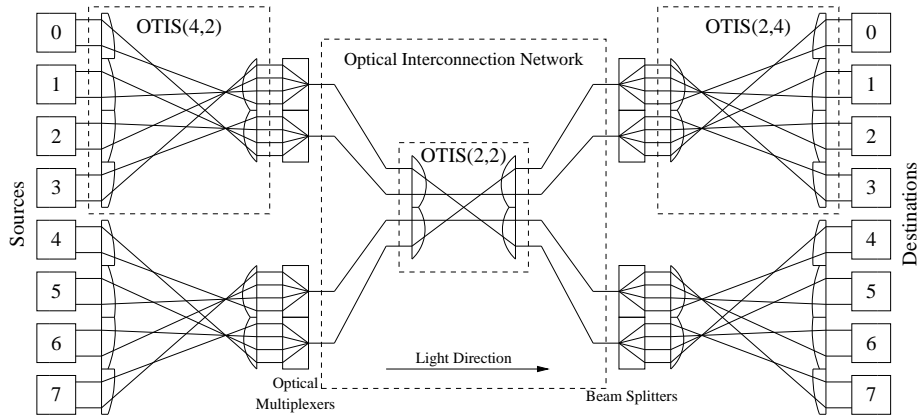

Figure 6: Optical interconnections of $POPS(4, 2)$ using the OTIS architecture.

Another family of networks, call OTIS-networks, in which both optics and electronics communications lines are used, was defined and studied in [41, 39, 21].

6 Conclusions

We presented some ideas from Graph Theory that can be useful in order to solve different problems in Communication Engineering.

Even in the case that a problem arising from engineering has not easy solution in the Graph Theory context, making the effort of translating it may be useful because:

- Translating the problem to Graphs can give some insight on it.

- A problem translated to maths is understandable by other scientific communities that can help solving it.

- Problems coming from other disciplines may have the same translation and might have been solved before.

- The recent development of Complexity Theory and Approximation Theory is contributing to solve many problems on Graphs by providing heuristic algorithms that give solutions with some bounded error.

Keeping both communities, engineers and mathematicians, working together will help developing solutions in engineering and providing new challenging and more close to reality problems to mathematicians.

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