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# Parameter-free method for polygonal representation of the noisy curves ⋆

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**Abstract.** We propose a parameter-free method for the detection of dominant points and polygonal representation of possibly noisy curves. Based on [1, 2], this work aims at a parameter-free method through a multiscale approach. We propose a new evaluation criterion to automatically determine the most appropriate width parameter for each input curve. Thanks to a recent result [3] on the decomposition of a curve into a sequence of maximal blurred segments, the complexity of this algorithm is $O(n \log n)$.

## 1 Introduction

The dominant points are points of local maximum curvature on the curves. They play a critical role in shape representation, shape recognition, and object matching. Starting from Attneave's work [4], there are many existing methods for dominant points detection. In general, we can classify these methods into 2 groups. The first one contains direct methods that use the curvature or alternative significance to determine the dominant points as local maxima. The second one contains indirect methods that deduce the dominant points after a polygonal approximation phase.

In the first group, the approach based on multiscale, multi-resolution [5,6] is often used for many dominant points detection methods. Arrelbola [6] processed the multiresolution structure that contains successive lower resolutions of the same object using linked pyramid approach. He adapted the multiresolution pixel linking algorithm for the processing of curve contours that are represented by a chain-code. Zhang et al. [5] computed the curvature of a curve with Gaussian derivative filters at various scales. Their method is based on a curvature scale-space technique in which the dominant points are detected at local extrema of the curvature product [5] whose value exceeds a threshold. A multiorder framework to analyse a digital curve at different levels of thickness was proposed in [7]. This approach is based on the arithmetical definition of a discrete line.

Recently, in the framework of the discrete geometry, some methods [1,8,9] are proposed to work with the curvature of noisy data. Moreover, Kerautret et al. [10] presented an application to the corner points detection based on the curvature estimation. In [2], Nguyen and Debled-Rennesson also proposed a novel method

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to determine a polygonalisation of a digital curve. These methods also used the notion of blurred segment [11] to work with noisy curves. However, all above methods depend on an input parameter: the width of blurred segment.

The previous method [2] is based on a geometric approach. It relies on the research of region of support by using decomposition of the curve into a sequence of maximal blurred segments. It works well with regular (non-noisy) curves by using the default parameter. The obtained results are good in comparison with the other methods (see tab. 1, 2 in [2]). The width parameter allows this method to work with noisy curves. In addition, the best working width depends on the input curve and the noise caused by the acquisition process. A bad working width can deduce too many detected dominant points that come from a noisy effect or can cause a high value of error approximation. Therefore, the determination of the most appropriate working width for each entry curve is an interesting problem.

In this paper, we proposed a parameter-free method for the dominant points detection and the polygonal approximation of noisy curves. The idea is to apply the previous method [2] through a multi-width framework. We propose a new evaluation criterion to determine the most appropriate width parameter for each input curve. This schema is similar to [7, 12]. This proposed framework is well adapted to noisy images. The rest of the paper is presented as follows. The next section recalls the previous method [2] with a fixed parameter. The section 3 presents the proposed method without parameter. The sections 4, 5 introduce some experimental results and a conclusion.

## 2 Previous dominant point detection method

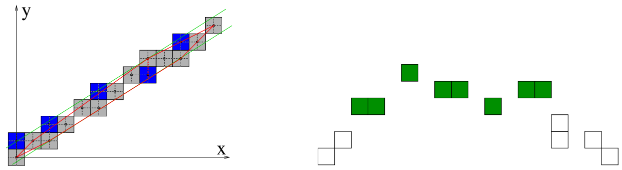
### 2.1 Blurred segment and region of support

**Blurred segment:** The notion of blurred segment [11] is introduced from the notion of discrete line [13]. A ***discrete line***, noted $D(a, b, \mu, \omega)$, is a set of points $(x, y) \in \mathbb{Z}^2$that verifies: $\mu \leq ax - by < \mu + \omega$. A ***blurred segment*** [11] (see figure 1.a) with slope $\frac{a}{b}$, lower bound $\mu$ and thickness $\omega$ is a set of integer points $(x, y)$ that is optimally bounded (see [11] for more detail) by the discrete line $D(a, b, \mu, \omega)$. The value $\nu = \frac{\omega-1}{max(|a|,|b|)}$ is called the width of this blurred segment. We proposed in [1] the notion of maximal blurred segment. A ***maximal blurred segment of width*** $\nu$ (see figure 1.b) is a width $\nu$ blurred segment that cannot be extended neither at the right side nor at the left side.

**Region of support of width $\nu$:** Deducing from [14], we proposed [2] the notion of ROS that is compatible with the blurred segment notion.

**Definition 1.** *For each point M of the curve, the blurred segment of width $\nu$ between M and left (resp. right) extremity is called left arm chair (resp. right arm chair) of this point. Left and right arm chairs of a point constitute its region of support (ROS) (see figure 2). The angle between them is called the ROS angle of this point.*

**Remark 1** *The smaller ROS angle of a point is, the higher the dominant character of this points is.*




(a) (b)

**Fig. 1.** a. $\mathcal{D}(5, 8, -8, 11)$, optimal bounding line (vertical distance $= \frac{10}{8} = 1.25$) of the sequence of gray points. - b. A maximal blurred segment of width 2 (in dark gray points).

This remark is deduced from this work [1], where curvature at a point is estimated as inverse of circumcircle radius. Therefore, we have a corollary of this remark: if ROS angle of a point is nearly $180^0$, this point can't be a dominant point candidate.

### 2.2 Previous method for dominant point detection

Dominant points [4] are local maximum curvature points on a curve that have a rich information content and are sufficient to characterize this curve.

We developed a novel efficient method for dominant point detection [2]. In this section, we recall some propositions that were utilized in this method [2]. Assume a given width $\nu$, we have:

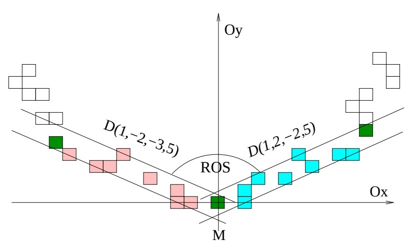



**Fig. 2.** Region of support based on left, right extremities of the point M

**Proposition 1.** *A dominant point of a curve must be in a common zone of successive maximal blurred segments (see figure 3.a).*

The consequence of this proposition is that the dominant point must be in a common zone of successive maximal blurred segment. Let us consider the common zone of more than 2 successive maximal blurred segments.

**Proposition 2.** *The smallest common zone of successive maximal blurred segments whose slopes are monotone contains a candidate as dominant point (see figure 3.b).*

To eliminate the weak dominant point candidates, we proposed this proposition below.

**Proposition 3.** *A maximal blurred segment contains a maximum of 2 candidates as dominant point (see figure 3.c).*

To locate the dominant points in the common zones, we used this heuristic:
**Heuristic strategy:** *In each smallest zone of successive maximal blurred segments whose slopes are increasing or decreasing, the candidate as dominant point is detected as the middle point of this zone.*

Based on the above theoretical framework and using this heuristic strategy above, we then proposed in [2] a method for detection of dominant points that can be described in the algorithm 1. This algorithm detects smallest common zones of successive blurred segments whose slopes are monotone and determine dominant points as central points of these common zones.

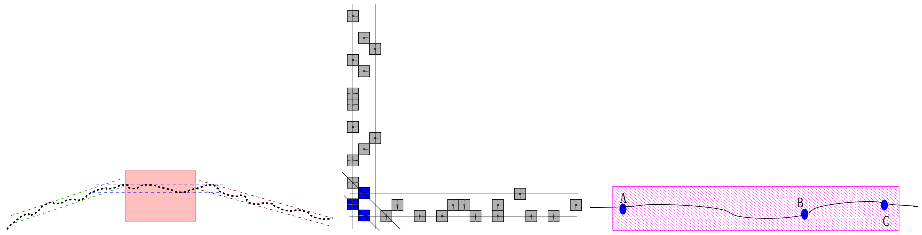
**Algorithm 1**: Dominant point detection [2]

```
Data: C discrete curve of n points, ν width of the segmentation
Result: D set of extracted dominant points
begin
        Build MBS_ν = {MBS(B_i, E_i, ν)}_{i=1}^m, {slope}_{i=1}^m;        (*)
        m = |MBS_ν|; E_0 = −1; B_{m+1} = n; p = 1; q = 1;
Scan      while p ≤ m do
            while E_q > B_p do p++;
            Add (q, p) to stack;
            q=p;
Detect    while stack ≠ ∅ do
            Take (q, p) from stack;
            Decomposition of {slope_q, slope_{q+1}, ..., slope_p} into monotone sequences;
            Determine the last monotone sequence {slope_r, ..., slope_p};
            Determine C_{⌊(r+p)/2⌋} as dominant point;
end
```

(*)$MBS_\nu$ is the sequence (determined by an algorithm presented in [1]) of width $\nu$ blurred segments $MBS_i(B_i, E_i, \nu)$ of the curve $C$ with $B_i$ and $E_i$ the indices of respectively the first and the last points of $MBS_i$ in $C$.

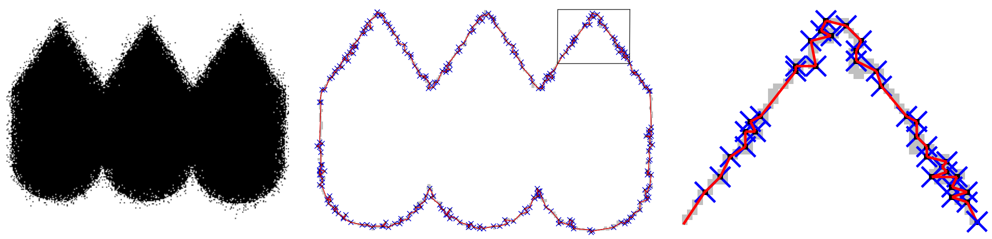
### 2.3 Problems and restriction

Almost all existing methods for dominant point detection do not work well with noisy curves that are extracted from objects with bad scan conditions. These curves have many weak dominant points. The previous method [2] can work well with the noisy curves by using an appropriate width parameter. However, there are many detected dominant points with the default parameter of 0.8 value (see figure 4.c) when we work with noisy curves. Therefore, the detection quality does not achieve our expectations. In this work, our purpose is to work well with noisy objects through a multi-width mechanism.




(a) (b) (c)

**Fig. 3.** a. Gray zone isn't common zone of successive maximal blurred segments; b. Common zone in black points contains candidate of dominant point; c. If a maximal blurred segment contains more than 2 dominant point candidates, the middle candidates are weaker than 2 extremities



(a) (b) (c)

**Fig. 4.** a-Input image, b-Extracted curve with detected dominant points (using the previous method [2] with default parameter), c-A part of fig. 4.b

## 3 Parameter-free method approach

In this paper, we present a solution for the starting point problem. We also introduce an automatic mechanism to determine the working width through a multi-width process. Therefore, the detected dominant points are well located on the curve shape regardless of the local corners caused by the noisy effect.

### 3.1 Improvement of the previous method

When we work with a same closed curve, the change of the starting point location can affect the final result. To avoid the starting point problem, our solution is to detect the most significant dominant point firstly. And then, this point is considered as the starting point of the new curve that can be easily deduced from the current curve by moving the part between the starting point and this point to the end of the curve. This point is determined as the point whose absolute curvature value is the greatest on the curvature profile that is constructed by the method presented in [1]. This simple strategy is good for almost curves but it doesn't improve if the curve has shape as a circle.

### 3.2 Our multi-width approach

We solve here the following problem: for an input curve, can we determine the most appropriate width parameter to obtain the best representation of its shape?

We propose a method to determine the best working width through a multi-width framework. In brief, for a given curve, we will examine the successive polygonal representations obtained by increasing the width parameter and computed with the method proposed in [2]. The most appropriate width parameter is determined by using **a new evaluation criterion** on the result of the polygonal representation at each width. In the following paragraph, we summarize the used methods in the literature to compute error criteria for polygonalization and we deduce from this study a new evaluation criterion that is well adapted to our multi-width segmentation method.

**Evaluation criteria:** There are two criteria [15, 16] that are used popularly for dominant point detectors and polygonal approximations. They're based on measuring the distorsion between the input curve and polygon that is generated from detected dominant points. Two most common measuring approachs for quantifying the distorsion are the error approximation and the number of detected dominant points.

The error approximation between the input curve and the approximating polygon can be the integral square error (ISE) that measures the quadratic error or the maximal distance error ($L_\infty$) that measures the maximal distance between the points of input curves and the corresponding segments of the polygon. The compression ratio ($CR = \frac{N_{DP}}{N}$) measures the ratio of the number of points between the approximating polygon and the input curve. A high compression ratio leads to a high approximation error, however the keeping of a low approximation error causes a low compression ratio. Therefore, Sarkar [15] introduced the criterion FOM (figure of merit) to aim at balancing a high compression ratio and a low approximation error that were obtained with the dominant point detector algorithm. $FOM = \frac{CR}{ISE}$.

Rosin [16] splited the assessment into 2 components: efficiency and fidelity. The principle idea is to compare the suboptimal polygon that corresponds to dominant point detector with correspoding optimal polygon in the same conditions. The efficiency measures the compression capacity of the suboptimal polygon in the same error approximation (ISE) and the fidelity measures error of the suboptimal polygon in the same number of detected dominant points: $Efficiency = \frac{N_{opt}}{N_{approx}} * 100$, $Fidelity = \frac{E_{opt}}{E_{approx}} * 100$ where $N_{opt}$ and $N_{approx}$ are the numbers of detected dominant points by using optimal and suboptimal polygonalization algorithms with the same error approximation; $E_{opt}$ and $E_{approx}$ are the ISE of error approximation by using optimal and suboptimal polygonalization algorithms with the same number of detected dominant points. A combined measure is defined as geometric mean of the 2 measures: $Merit = \sqrt{Fidelity \times Efficiency}$

**Our proposed method:** We propose a multi-width framework by applying our previous method [2] through increasing width parameters. The principal idea is to determine the most appropriate width parameter by using an evaluation criterion that is applied on results of dominant point detector at each width.

Concerning the evaluation criteria, the merit measure allows comparison of results obtained using dominant point detectors with different number of dominant points. However, since the complexity of an optimal algorithm is high ($O(n^2)$ [17] for min-$\varepsilon$ problem , $O(Mn^2)$ [18] for min-# problem), the use of Rosin's criterion leads to an inefficient method to determine the most appropriate width parameter through the multi-width approach. So, we propose to use an approach based on the Sarkar's criterion. Some authors [6, 19] argued that FOM [15] criterion is not sufficient for balancing the tradeoff between the ISE and the CR. The reason is that the ISE changes more rapidly than the CR for most of the tested shapes. Therefore, Marji et al. [19] proposed a modified version of Sarkar: $FOM_n = \frac{CR^n}{ISE}$ with n=2 in practice.

Besides the ISE that defines error approximation at global view, the $L_\infty$ measures the error approximation at a local level. We recognize that the decision of a human observer as dominant point also depends on the local approximation. Due to the addition property of ISE, in many cases, the $L_\infty$ for the region of support (ROS) of a dominant point does not change even though the ISE increases considerably. Let us see an example in figure 5 a, b. Both curves have the same number of dominant points and the same $L_\infty$. However the curve in figure 5b has a greater ISE than the one of the curve in figure 5a. On the contrary, the dominant property of the second dominant point in 5b is higher than the corresponding dominant point in 5a because of its longer left blurred segment in its ROS. Since $L_\infty$ controls the maximal fluctuation distance between each point of the curve and the approximating polygon, it is more appropriate than ISE to consider left and right blurred segments of ROS of a dominant point.

So, the keeping of a low ISE is not sufficient to lead to a good polygonal representation. We must also consider the $L_\infty$ although normally they also refer to error approximation. Therefore, we propose a modified version of Sarkar's criterion by using the Marji's remark. Our proposition is based on an observation that ISE changes more rapidly than CR and $L_\infty$. We suppose that ISE has a quadratic relation to $L_\infty$. This hypothesis comes from a quadratic property in ISE formula. We show in figure 6 the change of ratio $CR/L_\infty$ depend to width parameter. This diagramme confirms an hypothesis concerning a linear relation between $CR$ and $L_\infty$. Therefore, we propose our ***evaluation criterion*** as follows:

$$modifiedFOM = \frac{CR^2}{ISE} \times \frac{CR}{L_\infty} = \frac{CR^3}{ISE \times L_\infty}$$

We then propose an algorithm without parameter to segment a possibly noisy curve (see algorithm 2). The principal idea is to determine the first local maximum on the profile of our evaluation criterion that is constructed through multi-width process.

**Complexity:** The loop in the algorithm 1 will be stopped at the most appropriate width that is well adapted to noisy effect and principal shape of the curve. For the next width, as the stability of the shape is obtained, the number of dominant points does not change anymore so the CR does not change anymore, but the error approximation considerably increases due to a slight variation in the position of the dominant points that are not well located. So, the peak on

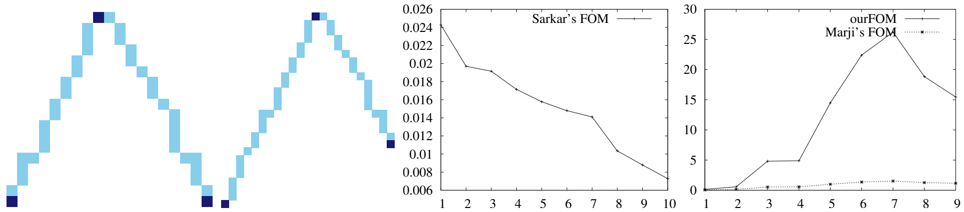



**Fig. 5.** From left to right: a,b -2 curves have a same $L_\infty$ but concerning the ISE, the second one has a greater ISE; c- The FOM criterion decreases when the width is increased, d- Comparison between FOM2 and our proposed FOM profiles on the curve in the figure 4.b with proposed method [2].

---

**Algorithm 2**: Parameter-free method for polygonal representation

---

**Data**: $C$ discrete curve of $n$ points
**Result**: $optWidth$ - obtained width, $P$ - polygonal representation
$width = 1$; $isLoop = TRUE$; $maxFOM = 0$;
**while** $isLoop$ **do**
    Use algorithm 1 to obtain $P'$ as polygonal representation of the curve with $width$ as parameter;
    Calculate corresponding $modifiedFOM$;
    **if** $modifiedFOM > maxFOM$ **then**
        $optWidth = width$; $maxFOM = modifiedFOM$; $P = P'$;
    **else** $modifiedFOM \leq maxFOM$ $isLoop = FALSE$;
    $width{+}{+}$;

---

the profile curve of our evaluation criterion corresponds locally to the most appropriate width at its scale. It means that the number of iterations in this loop corresponds to the most appropriate width value. On the other hand, this parameter value depends to the shape of the input curve at this scale. It doesn't depend to the length of the curve. Therefore, the number of iterations in this loop can be seen as a constant value in relation with the length of the input curve. As a result, the algorithms 1 and 2 have the same complexity. Thank to [3], it can be done in $O(n \log n)$ time.

## 4 Experimentation

We present in figure 9, 7 and table 1 the results that were obtained through our multi-width framework. The input images are presented in figure 8 and the curves of figure 9 are extracted from these noisy images (Kanungo's noise [20]). The figure 10 shows also a sequence of results on the curve d in figure 9 by increasing width parameters. We determined that the most appropriate width is 7 based on our criterion.

**Discussion:** We have tested our criterion and Marji's criterion on many noisy curves. They give the same results in nearly 90% cases. We recognize that, our

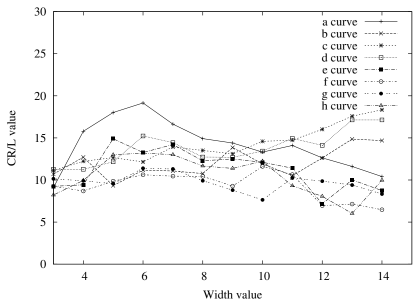



**Fig. 6.** Profile of CR/L on 6 curves presented in figure 9

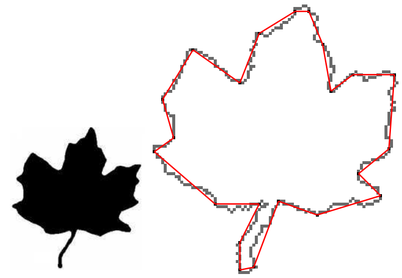


**Fig. 7.** From left to right: a. A real leaf image ; b. Shape description obtained at the most appropriate width parameter (width=3), noise is generated by using Kanungo model [20].

**Table 1.** Comparison between modifiedFOM and Marji's FOM profiles through increasing widths on a, c and h curves of figure 9 and a real leaf curve in figure 7.

| | Width: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a curve | modifiedFOM | 0.147 | 0.603 | 5.472 | 5.705 | 18.241 | 29.435 | **34.438** | 25.826 | 21.582 |
| | Marji's FOM | 0.055 | 0.155 | 0.586 | 0.627 | 1.155 | 1.633 | **1.798** | 1.552 | 1.447 |
| c curve | modifiedFOM | 0.203 | 1.078 | 3.233 | 10.996 | **12.722** | 10.159 | 10.124 | 10.984 | 9.205 |
| | Marji's FOM | 0.084 | 0.219 | 0.430 | **0.987** | 0.957 | 0.803 | 0.833 | 0.861 | 0.681 |
| h curve | modifiedFOM | 0.158 | 0.381 | 2.739 | 2.904 | 4.133 | 6.575 | **7.421** | 7.191 | 6.624 |
| | Marji's FOM | 0.047 | 0.106 | 0.314 | 0.354 | 0.415 | 0.506 | **0.562** | 0.553 | 0.567 |
| real leaf | modifiedFOM | 1.336 | 2.232 | **2.405** | 0.630 | 0.076 | 0.175 | 0.156 | 0.163 | 0.183 |
| | Marji's FOM | 0.260 | 0.395 | **0.485** | 0.470 | 0.478 | 0.429 | 0.391 | 0.373 | 0.374 |

criterion gives more clearly the peaks than Marji's criterion in their profiles of evaluation criterion. Let us see the case of real leaf curve in table 1. These 2 profiles have the same peak at width 3 in their profiles. However, it is unclear to say the most appropriate width parameter for corner detector among 3, 4 and 5 values of width parameter. On the contrary, our criterion gives a true peak at value 3. It is similar to the h curve when there are 2 peaks on Marji's criterion profile at values 7 and 9 of width parameters. For the c curve, there is a different peak between both criteria. The Marji's criterion gives the first peak at width 4, instead of 5 for our criterion. Therefore, our criterion works better than Marji's criterion in the context of noisy curves. Our profile of evaluation criterion gives nearly the same peak (10.996 in comparison with 11.702 at with 5) at width 4 (see table 1). So, there is a convergence between these 2 criteria. The figure 11, table 2 give the results on the c curve in figure 9 at 2 width parameters: 4 and 5. The number of dominant points considerably reduces from width 4 to width 5.

## 5 Conclusion

We propose in this paper a framework based on discrete geometry results to work with possibly noisy curves through a multi-width approach. We also proposed a modified version of the Sarkar's criterion for evaluating the obtained results of the polygonalization method at each scale. This proposition is introduced after studying the evaluated criteria of some authors [15, 16, 19]. A theoretical and comparative study on the evaluation of these criteria is in progress. In the future, we hope to apply this parameter-free polygonalization method directly to real images. An application on object recognition will be considered after this milestone.

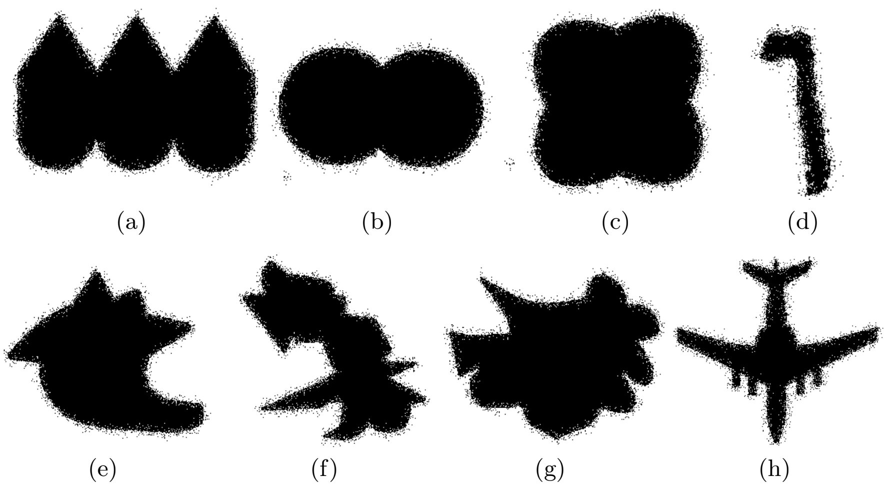



**Fig. 8.** Noisy input images by using Kanungo model [20].

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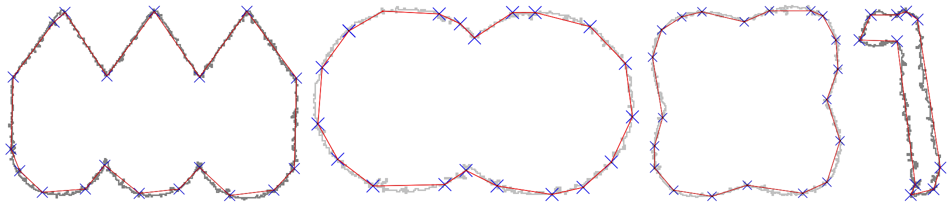
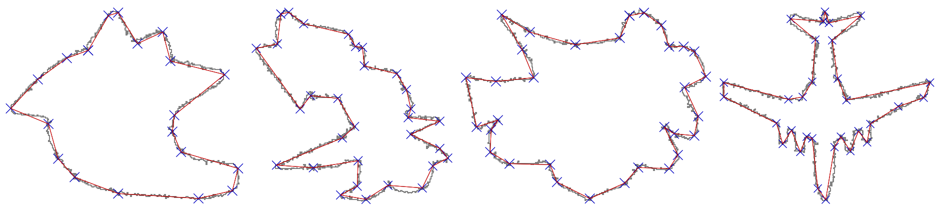
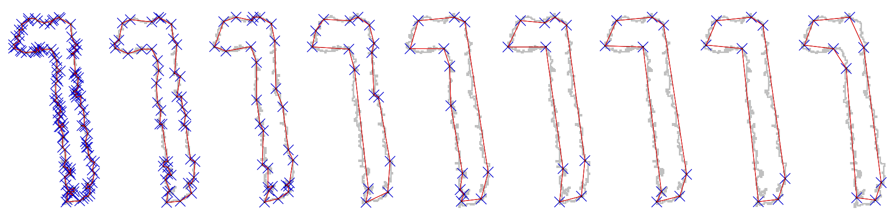


(a) width=7 (b) width=5 (c) width=5 (d) width=7



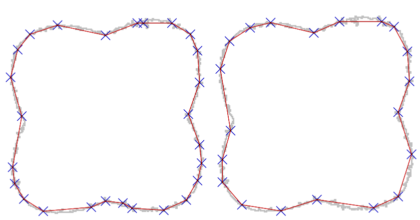
(e) width=6 (f) width=6 (g) width=5 (h) width=7

**Fig. 9.** Automatic dominant point detection. The appropriate width parameters are determined through a multi-width framework.



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5 (f) 6 (g) 7 (h) 8 (i) 9

**Fig. 10.** A sequence of results obtained by increasing width parameters from 1 to 9. The most appropriate with is determined as 7.



(a) Width=4 (b) Width=5

**Fig. 11.** Dominant detection on the c curve in figure 9 at widths 4 and 5.

**Table 2.** The index of dominant point detector on c curve in figure 9 at widths 4 and 5.

| $Width$ | $ISE$ | $L_\infty$ | $CR$ | nDPs |
|---|---|---|---|---|
| 4 | 3554.38 | 5.316 | 59.231 | 26 |
| 5 | 5698.53 | 6.297 | 77 | 20 |

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