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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

***Repetitive Model Refactoring for Design Space  
Exploration of Intensive Signal Processing  
Applications***

Calin Glitia — Pierre Boulet — Eric Lenormand — Michel Barreteau

N° ????

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*Rapport  
de recherche*



# Repetitive Model Refactoring for Design Space Exploration of Intensive Signal Processing Applications

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**Abstract:** The efficient design of computation intensive multidimensional signal processing application requires to deal with three kinds of constraints: those implied by the data dependencies, the non functional requirements (real-time, power consumption) and the availability of resources of the execution platform. We propose here a strategy to use a refactoring tool dedicated to this kind of applications to help explore the design space. This strategy is illustrated on an industrial radar application modeled using the Modeling and Analysis of Real-time and Embedded systems (MARTE) UML profile. It allows to find good trade-offs in the usage of storage and computation resources and in the parallelism (both task and data parallelism) exploitation.

**Key-words:** specification language, parallelism, high-level code transformations, optimizations, design space exploration, refactoring strategy

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## Refactorisation de modèles répétitifs pour l'exploration d'architecture d'applications de traitement de signal intensif

**Résumé :** La conception efficace d'applications pour le traitement intensif de signal demande de prendre en compte trois types de contraintes : celles dérivées des dépendances de données, les exigences non fonctionnelles (temps réel, consommation d'énergie) et la disponibilité des ressources de la plateforme d'exécution. Nous proposons ici une stratégie visant à utiliser un outil de refactoring dédié à ce type d'applications pour aider dans l'exploration de l'espace de conception. Cette stratégie est illustré sur une application industrielle de traitement radar, modélisée en utilisant le profile UML Marte conçu pour la modélisation et l'analyse de systèmes embarqués temps réel. Cela nous permet de trouver de bons compromis entre l'utilisation des ressources de stockage et de calcul et dans l'exploitation du parallélisme (parallélisme à la fois de tâches et de données).

**Mots-clés :** langage de spécification, parallélisme, transformations de code de haut-niveau, optimisations, exploration de l'espace de conception, stratégie de refactoring

## 1 Introduction

Computation intensive multidimensional signal processing applications are predominant in several application domains such as image and video processing or detection systems (radar, sonar). By multidimensional, we mean that they manipulate primarily multidimensional data structures such as arrays. The difficulty and the variety of these intensive signal processing applications come from the way the elementary functions access their input and output data as parts of multidimensional arrays. The complex access patterns lead to difficulties to schedule these applications efficiently on parallel and distributed execution platforms such as multiprocessor systems-on-chip. Three kinds of scheduling constraints sum up: data dependency constraints, environmental constraints (real-time, power consumption, etc) and hardware platform constraints (available computation, storage and communication resources). A design exploration tool that would guaranty the respect of some of these constraints (data dependencies) while allowing to improve on the others (usage of resources) would be very helpful.

The MARTE (Modeling and Analysis of Real-time and Embedded systems) UML profile [14, 16] is the standard for modeling real-time and embedded systems. It is well suited to model intensive signal processing applications thanks to its Repetitive Structure Modeling extension of UML. This extension is designed to provide ways of compact expression of systems having a repetitive structure or topology by the decomposition into repetitions of regularly interconnected components.

We propose here a refactoring tool (included in Gaspard2 [4, 15], an Integrated Development Environment dedicated to the visual co-design for embedded systems based on MARTE) for the design space exploration of repetitive MARTE models of intensive multidimensional signal processing applications and we experiment it on a typical radar processing application.

After briefly summarizing the principles of the modeling language in Section 2, the radar application used in the paper is described in Section 3 and modeled in Section 3.2. Accordingly to the rules of transition from the specification model to the execution model presented in Section 4, the refactoring transformations are used to adapt the application to the execution model by the reduction of array sizes and granularity changes in Section 4.2.

## 2 Multi-dimensional modeling

To model multi-dimensional signal processing applications, the language must support their core characteristics: multi-dimensional data structures, access to these data structures in a regular way, sliding windows, several sampling rates, cyclic data access (for cyclic space or frequency dimensions) and stateful computations. These requirements have been taken into account in the design of the MARTE Repetitive Structure Modeling package.

Models using the MARTE profile represent visual abstractions that, by the extensive use of the Repetitive Structure Modeling package, aim at expressing a maximum of the parallelism of computations, of targeted architectures and the repetitive distribution of computations on parallel execution units. Throughout the paper we will refer to such models as MARTE RSM models/specifications.

The repetitive specification provides ways to access multidimensional arrays via sub-arrays, the support of sliding windows, the possibility to deal with cyclic data accesses, several sampling rates in the same specification and can express stateful computations such as recursive filters through uniform inter-repetition dependences [7]. To make such models statically scheduleable, rules are imposed on the specification, namely regarding the single assignment and the complete production of array elements. These rules and a complete formalism of the Array-OL (Array Oriented Language) model of computation (MoC), the theoretical basis of the MARTE RSM models, are available in [3]. A comparison to similar models of computation dedicated to signal processing, mainly in terms of allowed data structures and expressiveness, like synchronous data-flow languages (SDF [12] and its multidimensional extensions (G)MDSDF [13], WSDF [9]), functional languages (Alpha [8]) or imperative languages (Sisal [1], SaC [17]), is available in [7].

Formally, with MARTE RSM, an application is a set of *tasks* connected through *ports*, representing multi-dimensional arrays characterized by their *shape* and their *direction*. The tasks are equivalent to mathematical functions reading data on their input ports and writing data on their output ports. The tasks are of three kinds: *elementary*, *compound* and *repetition*. An elementary task is atomic (a black box), it can come from a library for example. A compound is a dependence graph whose nodes are tasks connected via their ports, it allows to express task parallelism. A repetition is a task expressing how a single sub-task is repeated, each instance of the repeated task operates with sub-arrays of the inputs and outputs of the repetition, making the repetitions independent and therefore parallel by construction, it allows to express data parallelism. For a given input or output, all the sub-array instances have the same shape, are composed of regularly spaced elements and are regularly placed in the array. The *tiler* (one tiler per input or output) concept represents the mathematical expression of the elements of the patterns as tiles of the array, and is composed of: a *fitting* matrix  $F$ , whose column vectors represent the regular spacing between the elements of a pattern in the array;  $\mathbf{o}$ , the *origin* of the *reference pattern* (for the *reference repetition*) and a *paving* matrix  $P$ , whose column vectors represent the regular spacing between the patterns. We can summarize the pattern construction with one formula. For a given repetition index  $\mathbf{r}$ ,  $\mathbf{0} \leq \mathbf{r} < \mathbf{s}_{\text{rep}}$  (the repetition space is unique for a repetition task and all its inouts and outputs) and a given index  $\mathbf{i}$ ,  $\mathbf{0} \leq \mathbf{i} < \mathbf{s}_{\text{pattern}}$  in the pattern, the corresponding element in the array has the coordinates

$$\mathbf{o} + (P F) \cdot \begin{pmatrix} \mathbf{r} \\ \mathbf{i} \end{pmatrix} \bmod \mathbf{s}_{\text{array}}, \quad (1)$$

where  $\mathbf{s}_{\text{array}}$  is the shape of the array,  $\mathbf{s}_{\text{pattern}}$  is the shape of the pattern,  $\mathbf{s}_{\text{rep}}$  is the shape of the repetition space.

The pattern construction allows the construction ranging from simple block tiled patterns to complex uniform accesses where accesses non-parallel with the axes are combined with strides and modulo. Nonetheless, in real intensive signal processing applications, most of the access remain parallel with the axes, by block or with sliding windows:

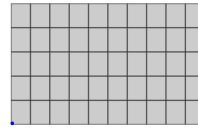
**Parallel with the axes.** With such accesses, all the vectors of the paving and the fitting matrixes contain at most one non-zero value, making the corre-

spondence between the repetition's or pattern's corresponding dimension and an array dimension.

**Access by blocks.** Uniform and perfectly tiled block-like patterns are accessed. The fitting and the paving matrix express parallel with the axes construction where the patterns are continuous tiled blocks in the array (each fitting vector contains just one value equal to 1<sup>1</sup>) and the blocks are perfectly tiled one after another (paving vectors have non-zero values equal to the dimension of the pattern on the corresponding array dimension).

#### Example 1

An array with a shape of  $(80, 50)$  can be tiled into  $(10, 5)$  blocks of  $(10, 8)$  elements using the following tiler: fitting =  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ , paving =  $\begin{pmatrix} 8 & 0 \\ 0 & 10 \end{pmatrix}$  as illustrated on the figure on the right.

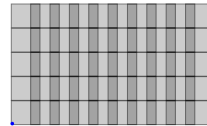


**Entire tiled dimensions.** It is a special case of block access where pattern dimensions take entire array dimensions.

**Sliding windows.** It represents a block access where the blocks do not perfectly tile on some dimensions, with overlapping blocks which cause array elements to be tiled in multiple patterns. On the dimensions of the sliding, paving vectors do not have non-zero values equal to the dimension of the pattern and the value represents the *step* of the window on this dimension.

#### Example 2

Leaving unchanged the paving and the fitting matrix of the block access example and by enlarging the pattern to a block of  $(10, 12)$ , we have a sliding window access on the first dimension of the array, with a step of 8.



### 3 STAP radar application

This paragraph describes a MTI (Moving Target Indication) application, whose goal is to detect from an aircraft the objects that move on the ground, and especially move slowly among all the other generally still reflecting surfaces under the radar beam (ground clutter). This is done by receiving the echo from the ground of a periodic sequence of radar pulses (bursts). Radar processing permits to estimate both the position of a target through the delay between transmission of a signal (pulse) and reception of its echo, and its speed through the Doppler effect that affects echoes of several identical pulses sent periodically: the speed of the target results in a (small) variation of its distance from the radar, which is only visible as a phase shift on the radar signal (e.g. around

<sup>1</sup>Pseudo-identity matrix, on each line or column we can find at most one non-zero value of 1, but the matrix may not be square.



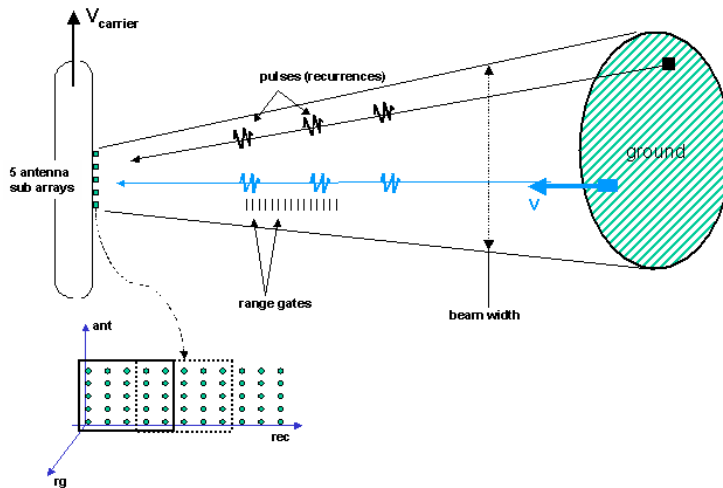


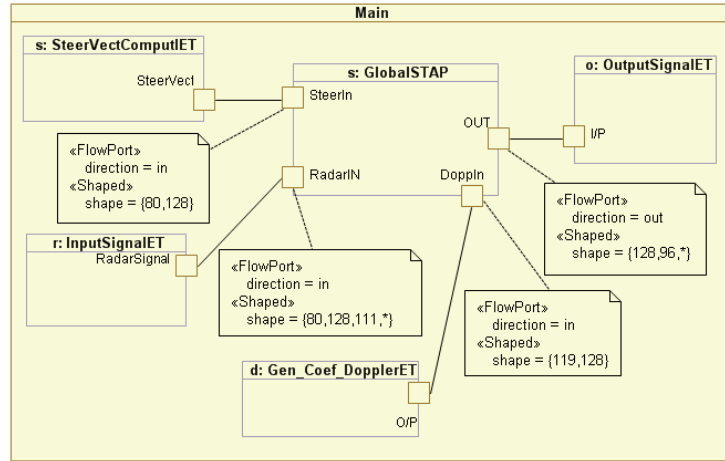
Figure 1: STAP.

10 GHz). In this basic approach, Doppler processing consists in a bench of filters (e.g. a Fast Fourier Transform) each tuned towards a particular phase shift between successive echoes. This kind of Doppler processing is in some situations sufficient to separate reflecting objects on the basis of their speeds. When the beam is directed toward the ground, the largest part of the echoed energy is supposed to come from the still objects that compose the ground (called clutter), while the moving objects of interest send weak but phase-shifted echoes. However, the radar beam is not perfectly sharp and has a width of a few degrees, which results in giving to some still objects on the ground at the borders of the beam a relative speed with respect to the aircraft (due to the aircraft's own speed) and creating undesired interferences over the moving targets echoes: this creates an ambiguity between intrinsic speeds and azimuths of targets. Adaptive filtering techniques, where fixed filters are replaced by filters that are computed at run-time from the received signal itself, help to minimize at best the effects of this unwanted clutter signal: in this MTI case, the Space Time Adaptive Processing is used. In this method, a set of filters is computed at every burst, by solving linear systems whose right hand side terms are reference vectors of theoretical phase patterns expected on antenna sub-arrays at several consecutive pulses, each of which corresponding to a particular (velocity, angle) hypothesis of the target relatively to the aircraft. This is shown in Figure 1 where 2D patterns on dimensions antenna and pulse (rec) are considered to compute filters that remove the natural ambiguity between velocity and azimuth.

### 3.1 Implementation constraints

Those characteristics are mainly:

- A large part of the application is data-flow, manipulating multi-dimensional arrays of data.
- The processing chain uses different operators, with in particular different needs in terms of precision and dynamic range.



The main STAP application, *GlobalSTAP* takes as input an multidimensional infinite array of data from the sensors (*InputSignalET*) and, using the steer vectors provided by *SteerVectCompIET* and the Doppler coefficients provided by *Gen\_Coef\_DopplerET*, processes the data and provides the radar detection results as an infinite array, which are consumed by *OutputSignalET* for display, storage or post-treatment.

Figure 2: STAP: top level.

- the processing chain is dynamic, and varies during run-time by frequent variation of algorithm parameters (sizes of arrays, loop bounds, ...) while keeping roughly the same processing graph. This is generally called multi-modding in the radar community.
- the computational load is high enough to clearly involve parallel computing hardware.
- real-time performance is one of the key requirements, both in terms of computation throughput and latency. This may result from some operational requirements and/or embedded architecture constraints such as memory limitations.

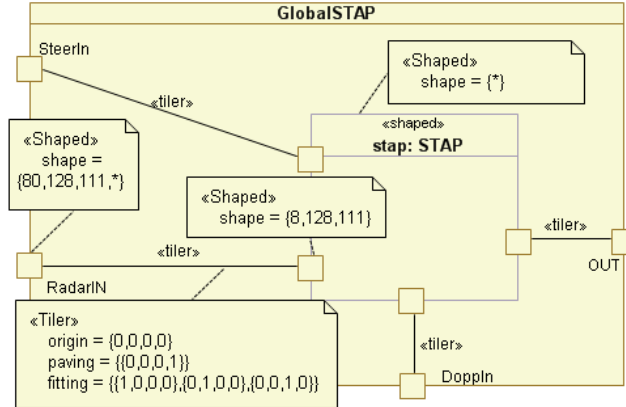
### 3.2 STAP model

The STAP application was modeled in Papyrus UML using the MARTE Profile<sup>2</sup>.

Starting from the top level, the application is successively depicted using a compound or repetitive decomposition, until the wanted detail level is reached, represented by elementary tasks that can be deployed on library functions. The top level of the specification, illustrated on Figure 2, describes the global functionality and the interaction with the environment.

The infinite dimension of the arrays processed by *GlobalSTAP* represents the abstraction of time and Figure 3 describes the data-flow level of the application, the infinity repetition (time) of a single STAP data treatment.

<sup>2</sup>Throughout the paper, figures represent real manipulable UML models.



Just one sample tiler is shown, expressing how the infinite radar signals, represented by an array of shape  $\{8, 128, 111, *\}$  is decomposed into an infinity,  $\{*\}$ , patterns of shape  $\{8, 128, 111\}$ . The paving matrix of  $\{\{0, 0, 0, 1\}\}$  expresses the correspondence between the infinite repetition and the last dimension of the array, while the fitting matrix of  $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}\}$  expresses the correspondence between the pattern dimensions and the first three dimensions of the array.

Figure 3: STAP: data-flow level.

Figure 4 illustrates the compound decomposition of the repeated task of Figure 3 into successive repetitive filters, with array sizes and repetition spaces shown on the figure. The tilers that express the pattern/tiles construction for each repetition were not made visible.<sup>3</sup>

Each repetitive filter has a different functionality and different data access patterns:

**PulseCompression** takes sliding windows with a step of  $\{1\}$  of  $\{16\}$  values on the third dimension of the input array and computes an average value for each of the pattern.

**CovMatrixEstim** takes  $\{119, 96\}$  blocks of  $\{10, 8\}$ , with a sliding window with a step of  $\{1\}$  on the second dimension of the input array and expands them into blocs of  $\{80, 80\}$  which will be arranged into an array of size  $\{80, 80, 119, 96\}$ .

**AverageCovar** eliminates the third dimension of the arrays by computing an average of  $\{119\}$  values.

**MatInv** takes  $\{96\}$  square matrices of  $\{80, 80\}$  and inverses them.

**Stap\_Filter** applies the  $\{128\}$  steering vectors of dimension  $\{80\}$  to each line of the inverted matrices, producing an array of size  $\{128, 96, 80\}$ .

**Stap\_Application** compares the filtered values with the initial compressed pulses, by blocks of  $\{8, 10\}$  to identify the movements.

<sup>3</sup>A complete model is available at [http://gforge.inria.fr/frs/download.php/5755/examples\\_papyrus.zip](http://gforge.inria.fr/frs/download.php/5755/examples_papyrus.zip).

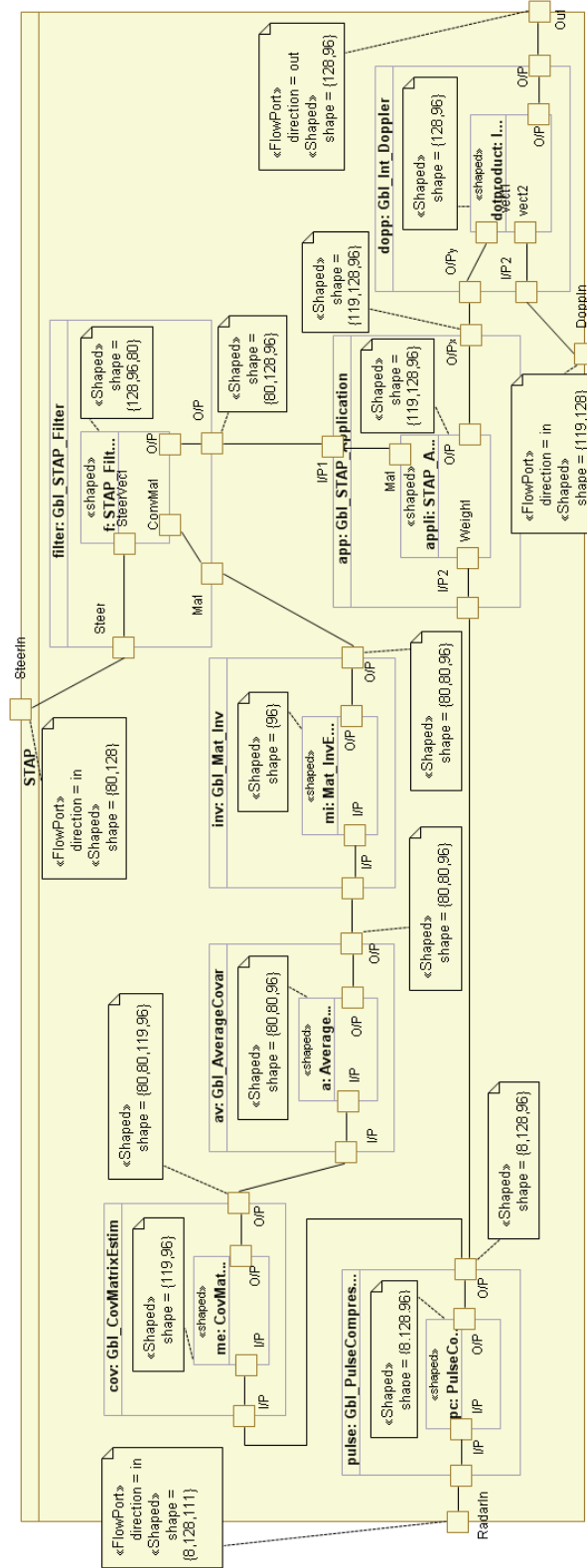
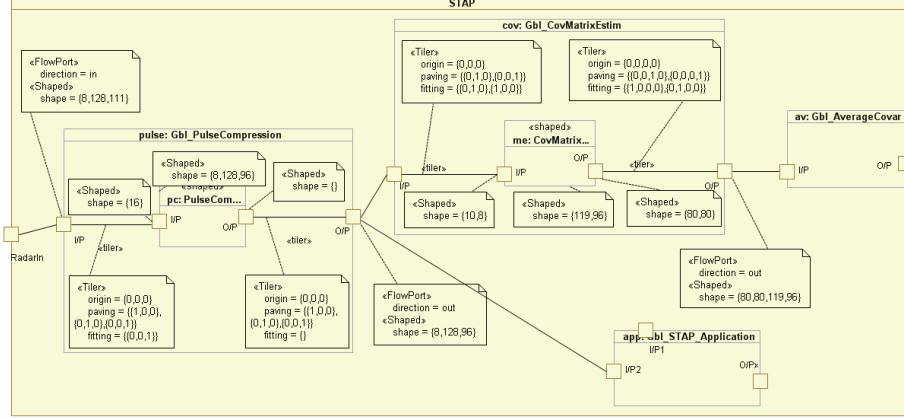


Figure 4: STAP decomposition.



For the *PulseCompression* repetition, starting from each element of the input array (identity paving matrix), a pattern of {16} elements on the third dimension is used to compute a single value, {}, values rearranged into a tree dimensional array with a shape of {8, 128, 96}. The difference of size between the input and the output array on the third dimensions ( $111 - 96 = 15$ ) is caused by the sliding window accesses, for the last 15 elements the access window will exit the array dimensions. *CovMatrixEstim* repetition has a similar sliding window functionality, this time 2D blocks with a shape of {10, 8} sliding on the second dimension with a step of 1. These blocks are expanded into {119, 96} square blocks of {80, 80}.

Figure 5: First two filters: tiler construction.

**Int\_Doppler** applies the {128} Doppler vectors of size {119} to identify the speeds of the moving targets.

Each repetition is characterized by different repetitive and uniform pattern accesses, data consumption or production, expressed by the tiler informations of each connection inside the repetition, accordingly to relation 1. Figure 5 illustrates the tiler construction for two of the STAP filters, *PulseCompression* and *CovMatrixEstim*.

At filter level, each repeated task has an elementary functionality which can be deployed on library functions: matrix inversion, average computations, etc.

MARTE RSM models are static and therefore we have chosen to implement a single mode of the multi-moding functionality mentioned in Section 3.1. Multiple modes can be modeled with the use of control structures based on mode automata have been proposed by Labbani et al. [10, 11] or in the upcoming beta3 or 1.0 version of MARTE (no public reference yet available at the time of writing) but they are outside the scope of this study.

## 4 Towards an execution model

A MARTE RSM *specification* expresses the maximum of parallelism through the use of the repetitive decomposition. It describes the data dependencies between the elements of the arrays and, as a direct consequence, a strict partial order between the calls of the tasks. A valid specification must be statically

defined and therefore defines a strict partial order. Indeed, any schedule that respects this order will compute the same output values from the same inputs. The construction rules that assure that a specification is valid are described and proved in [3].

The *execution model* represents the abstraction of the execution and it must abide with the strict partial order defined by the specification. It is a design intention that, by expressing the minimal order of execution, lots of decisions can and have to be taken when mapping a MARTE RSM specification onto an execution platform: how to map the various repetition dimensions to time and space, how to place the arrays in memory, how to schedule parallel tasks on the same processing element, how to schedule the communications between the processing elements? It is a strength of this specification that the space-time mapping decision is separated from the functional specification. This allows to build functional component libraries for reuse and to carry out some architecture exploration with the least restrictions possible.

We have chosen to make the transition from a MARTE RSM specification to an execution model the most straight-forward possible. Indeed the execution model should reflect the specification. This choice has as advantage that the execution model remains similar to the structure of the specification (parallelism, granularity), at least until the code generation. A representative example is the generation of VHDL code that will be used for FPGA synthesis<sup>4</sup> and where the component structure of the specification can be found in the VHDL modules and even on the FPGA layout. It comes down to the constraint that a component may start its execution when all its input arrays are available. This constraint introduces a major problem, represented by the so called “synchronization barriers” between the components. Such a barrier is created by the data dependencies; a representative illustration of the problem is the presence of any intermediate array that contains an infinite dimension, which would cause the execution to be stalled in that point. As a solution, some repetitions can be transformed to flows, the execution of the repetitions is pipelined and the patterns are read and written as a flow of tokens (each token carrying a pattern).

## 4.1 High-level data-parallel transformations

The data-parallel code transformations can be used to adapt the application to the execution, allowing to choose the granularity of the flows and a simple expression of the mapping by tagging each repetition by its execution mode: data-parallel or pipeline. A great care has been taken with these transformations to ensure that they do not modify the precise element to element dependencies [5,18]. A comparative study between these transformations and the loop transformations in the context of program optimizations can be found in [6]. Loop transformations [2] are used intensively for optimizations at compilation time but the complexity of optimization algorithms is one reason why many compilers still use heuristics.

Our high-level transformations are designed as tools at the specification level that can be used for adapting the application to the execution on parallel embedded platforms and for design space exploration, leading to some interesting characteristics:

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<sup>4</sup>available within Gaspard2 framework.

- The transformations work as a refactoring tool, their result is translated into model changes at the same level of abstraction.
- The application keeps its exact functionality, the same output values are computed for the same inputs.
- As optimization targets, the reduction of array sizes and the parallelism management are the priority. A specification expresses the maximum of parallelism and the transformations can be used to change its granularity for a better mapping on a parallel execution platform.
- The specification, the refactoring and the transition to an execution model are separate stages. The same specification can be adapted differently to the execution, accordingly to platform and execution constraints.

Regardless of their role, all these transformations have a similar impact on an application, by redistributing repetitions through the hierarchy levels, with the creation or suppression of hierarchy levels.

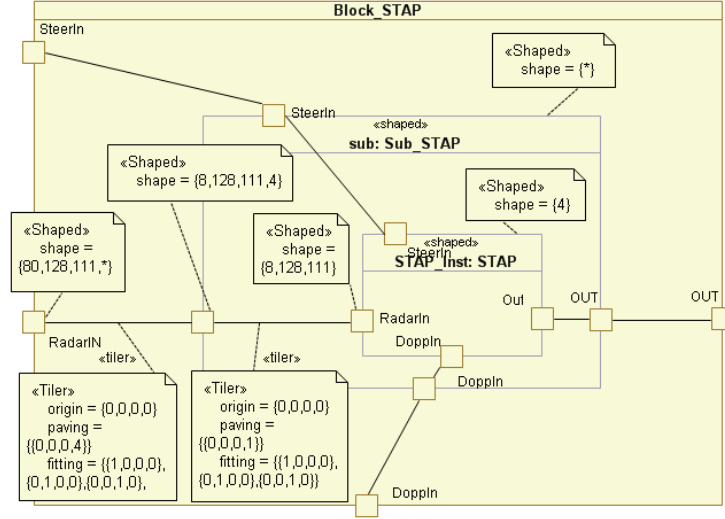
Taking each transformation one by one, we have:

- *Fusion* takes two successive repetitive tasks, creates a superior hierarchy level for the computed common repetition, while what is left of the initial repetitions is placed on the inferior hierarchy level, minimizing the size of the intermediate array.
- *Tiling* splits a repetition into blocks, by creating a hierarchy level.
- *Change paving* (either by dimension creation or by linear growth) acts on redistributing repetitions through successive levels of hierarchy and modifying the granularity of the application.
- *Collapse*, by being the opposite of fusion and tiling, suppresses the superior hierarchy level, its repetitions being added to each of the inferior level repetitions.

## 4.2 Refactoring using high-level transformations

Next, we will see how the STAP specification can be adapted to the execution using the refactoring and how a heuristic of transformation chaining can be deduced.

**Isolation of the infinite dimensions.** The presence of infinite dimensions (of arrays or repetitions) throughout the application is the first thing to take care of when passing to the execution. An infinity value of a repetition will block the execution in that point and an array with an infinite dimension cannot be placed in a limited size memory. As the infinity is used to express the time (or the data flow), a solution is to isolate the infinity values at a high level of the hierarchy and consider this level as the data-flow level at the execution: the arrays will not be allocated entirely in memory (just the patterns) and an sequential execution will be chosen for the repetition. By the use of the *fusion*

Figure 6: Tiling into blocks of shape  $\{4\}$ .

transformation on successive infinite repetitions, a common repetition can be computed and will represent the data-flow level at execution<sup>5</sup>.

The STAP application was modeled already having all the infinity values at one hierarchy level, Figure 3, and therefore this step can be skipped. A specification where the infinity values would have been at the filter decomposition level, Figure 4, would express the same exact functionality but would necessitate the isolation of the infinity at a data-flow level before passing to the execution.

**Granularity change.** Execution constrains may impose changes in the *granularity* of an application. Taken for instance, if we have 4 processors and want to execute the repetition of Figure 3 in parallel, a *tiling* transformation can be used to split the repetition into blocks of 4 repetitions that will be placed on the 4 processors and executed in parallel, while the sequence of blocks will represent the data-flow level. It is what we call changing the granularity of a repetition and the result of such transformation is shown on Figure 6.

**Reduction of the size of the intermediate arrays.** The reduction of the size of the intermediate arrays can be achieved by the use of the fusion transformation. An elementary fusion computes the minimum intermediate array between two successive repetitions.

**Definition 1 (Minimum intermediate array)** *Between two successive repetition tasks<sup>6</sup>, a minimal intermediate array is represented by a minimum group of patterns produces by the first task that allows the second task to execute at least once, therefore producing elements and in consequence allowing a non-blocking execution.*

<sup>5</sup>The fusion of two infinite repetitions that fails to isolate the infinity at top level indicates an invalid specification.

<sup>6</sup>The first task produces an array consumed by the second one.



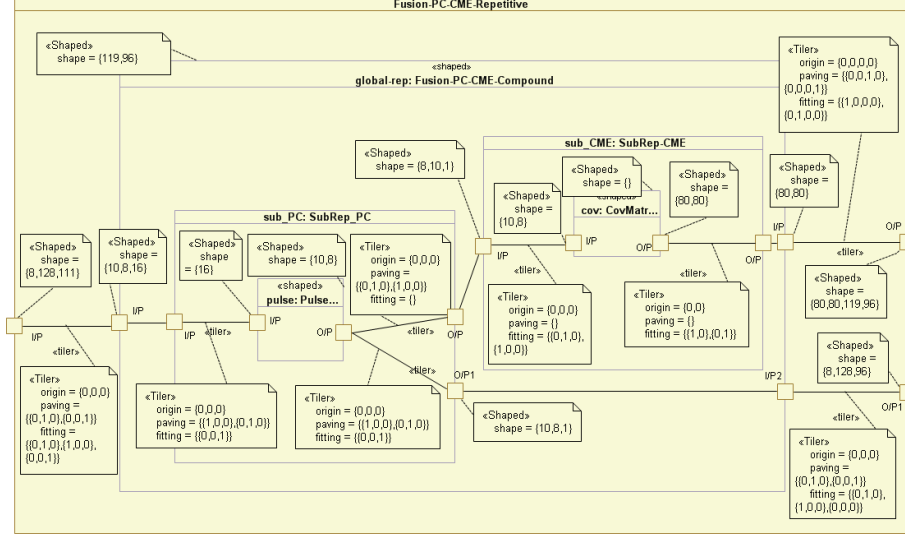


Figure 7: Repetitions of first two tasks after the fusion.

**Remark 1** *The fusion computes this minimum group of intermediate patterns and transforms the application by creating a common repetition with two sub-repetitions and an intermediate array reduced to the minimum group of patterns.*

**Example 3** *Figure 7 shows the result of fusion for the repetitions of Figure 5. A common repetition with a shape of  $\{119, 96\}$  is computed and two sub-repetition of  $\{10, 8\}$  and respectively  $\{\}$  on the second level of hierarchy. The minimum group of patterns produces by the first sub-repetition that allow the execution at least once<sup>7</sup> of the second sub-repetition is of  $\{10, 8, 1\}$ , and therefore the intermediate array is reduced from the shape of  $\{8, 128, 96\}$  ( $8 \times 128 \times 96 = 98304$  memory elements reduced to  $10 \times 8 \times 1 = 80$ , with a factor of  $98304/80 = 1228.8$  times).*

**Definition 2 (Multiple fusion)** *A fusion between several connected repetitive tasks can be achieved by the chaining of elementary fusion and collapse transformations. Following the dependence order between tasks, each task is fused with the result of the previous fusion<sup>8</sup>, while a collapse transformation limits the explosion of hierarchy levels.*

**Remark 2** *By a multiple fusion, just the last<sup>9</sup> intermediate array is minimized. The other intermediate arrays represent the minimum group of patterns needed for the execution at least once of the last repeated task, and not of the repetitive task that consumes this array.*

**Example 4** *Figure 8 shows the fusion of the repetitions PulseCompression  $\oplus$  CovMatrixEstim  $\oplus$  AverageCovar  $\oplus$  MatInv. The first three sub-repetitions*

<sup>7</sup>In this case exactly once,  $\{\}$ .

<sup>8</sup>We start by fusing two repetition.

<sup>9</sup>The order of tasks in a multiple fusion is given by the strict partial order between these tasks.

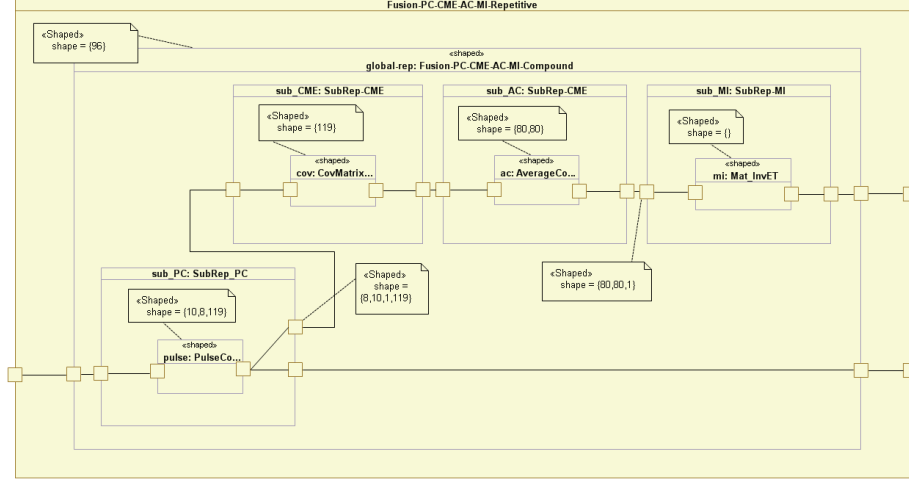


Figure 8: Repetitions of first four tasks after the fusion.

represent minimum execution that allow the last sub-repetition to execute at least once and therefore minimizing the last intermediate array, but not the first two. With such fusion, the first intermediate array is reduces to a size of  $\{10, 8, 1, 119\}$ , 119 times bigger than the minimal reduction reached by the fusion of just the first two repetitions of Figure 7.

Another factor that comes into play is the amount of *re-computations* introduced into the application by the fusion.

**Definition 3 (Re-computation)** *Re-computations are represented by an increase of the repetitions for the first task involved in a fusion, caused by the pattern production/consumption between the two tasks.*

**Remark 3** *The complete repetition of a task is given by concatenating all the shapes of the hierarchical repetitions starting at the level of the task and climbing the hierarchy levels to the top level<sup>10</sup>. Re-computations are represented by an increase in the complete (taken into account all the levels of hierarchy) repetition of a task after the fusion.*

Sliding windows accesses with some initial elements found in several pattern groups determines the first task to compute them several times, after the fusion. In the case of re-computation, the decrease of intermediate array size has as side effect an increase of computations.

**Example 5** *On Figure 7, the complete repetition of the first repeated task is, by concatenating the repetition spaces of the two hierarchy level, of  $\{119, 96, 10, 8\}$ , while the initial repetition was of  $\{8, 128, 96\}$  and therefore the number of repetitions increase with a factor of  $119 \times 96 \times 10 \times 8 / (8 \times 128 \times 96) = 9.29$ .*

Thus, in the case of intermediate values that are used several times, the designer has to make a trade-off between the storage in memory or multiple computations of these intermediate values.

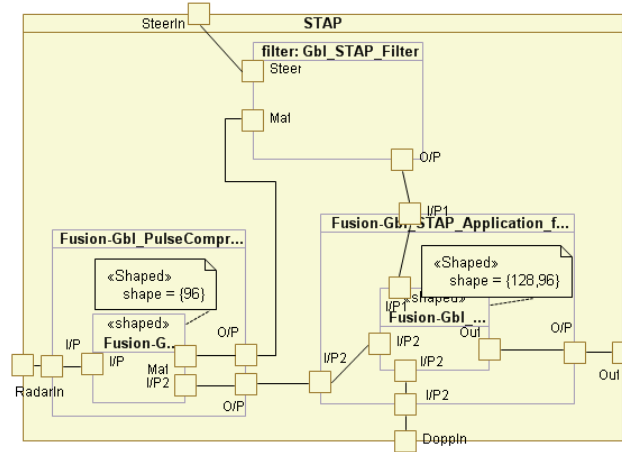
<sup>10</sup>The infinite repetition at data-flow level is neglected in our paper.

### 4.3 Reduction of intermediate arrays on STAP

Task	Rep. before	Rep. after	Re-comp.( $\times$ )
<b>PulseCompression (PC)</b>	$8 \times 128 \times 96$	$128 \times 96 \times \begin{pmatrix} 8 \times 128 \times 119 \\ 119 \times 119 \\ 80 \times 80 \times 119 \\ 119 \\ 80 \times 119 \\ 119 \\ 1 \end{pmatrix}$	$119 \times 128$
<b>CovMatrixEstim (CME)</b>	$119 \times 96$		$119 \times 128$
<b>AverageCovar (AC)</b>	$80 \times 80 \times 96$		$119 \times 128$
<b>MatInv (MI)</b>	96		$119 \times 128$
<b>StapFilter (SF)</b>	$128 \times 96 \times 80$		119
<b>StapApplication (SA)</b>	$119 \times 128 \times 96$		1
<b>IntDoppler (ID)</b>	$128 \times 96$	1	1

The hierarchy of repetitions is represented by brackets in the table. After a complete fusion of the repetitions, a common repetition of  $\{128, 96\}$  is computed and the total repetition for each sub-repetitions is given by multiplication with this common repetition. The re-computation is computed as the division of the repetition after and before the refactoring.

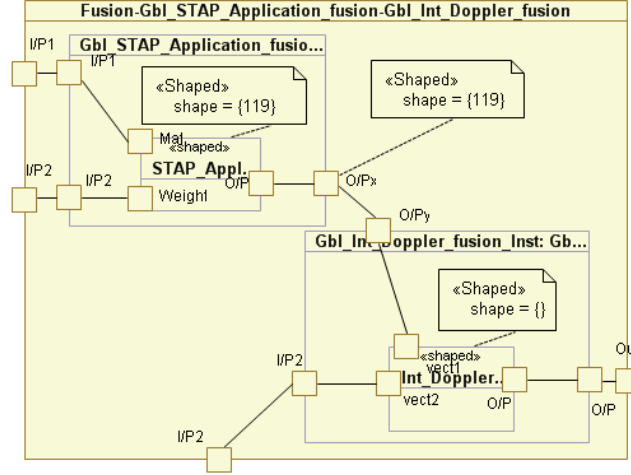
Table 1: Repetitions for the complete fusion.



The first four repetitions are grouped into a common repetition of  $\{96\}$  and the last two into a repetition of  $\{128, 96\}$  of a compound containing two sub-repetitions of the initial tasks, shown on Figure 10.

Figure 9: Top level transformed application.

The reduction of intermediate arrays is the principal use of fusion transformations. Having several successive repetitive tasks, we need a way of reducing at maximum the set of intermediate arrays, while avoiding blocking points in the execution and limiting as much as possible the re-computations introduced by the fusions. Different mappings might impose slightly different heuristics, like forbidding re-computations or the priority of minimizing the array sizes in the detriment of re-computations. On STAP application of Figure 4, we have chosen as objective to minimize the intermediate arrays while limiting the re-computations.



One of the reduction of the intermediate array  $StapApp \rightarrow IntDopp$  to a size of  $\{119\}$  can be observed.

Figure 10: Sub-repetitions of *StapApplication* and *IntDoppler*.

**Complete fusion.** A *complete* fusion (the fusion of all the repetitions at a hierarchy level) is not appropriate in this case, as shown in Table 1 (containing informations on the repetitions before, after and the introduced re-computations), where we find ourselves with a great amount of re-computations. Accordingly to Definition 2, the fusion of multiple repetitions minimizes only the last intermediate array. Table 2 shows how just two intermediate arrays have a reduced size, as for the others we have even an increase in array size, caused by the multidimensional re-arrangement of patterns as result of the succession of fusion/collapse transformations<sup>11</sup>.

Array	Size before	Size after	Reduction
PC→CME	$8 \times 128 \times 96$	$8 \times 128 \times 119$	0.807
CME→AC	$80 \times 80 \times 119 \times 96$	$80 \times 80 \times 119 \times 119$	0.807
AC→MI	$80 \times 80 \times 96$	$80 \times 80 \times 119$	0.807
MI→SF	$80 \times 80 \times 96$	$80 \times 80 \times 119$	0.807
SF→SA	$80 \times 128 \times 96$	$80 \times 119$	103.26
PC→SA	$8 \times 128 \times 96$	$8 \times 128 \times 119$	0.807
SA→ID	$119 \times 128 \times 96$	119	12288

Array size in memory is given in memory units of the data type of array elements, by the multiplication of the multidimensional dimensions of its shape. In the table, arrays are identified by the tasks that produce/consume the array (Producer→Consumer) and the reduction in size is given by the division of array sizes before and after the refactoring.

Table 2: Array sizes for the complete fusion.

<sup>11</sup>Initial overlapping accesses are expanded through multiple dimensions with elements duplication.

Nonetheless, such a transformation provides useful informations that can be used for finding the best sequence of fusions for our objective. First of all, it provides the fusion order representing the strict partial order between the repetition tasks, in this case:  $PulseCompression \oplus CovMatrixEstim \oplus AverageCovar \oplus MatInv \oplus StapFilter \oplus StapApplication \oplus IntDoppler$ . Secondly, on Table 1 we can identify where changes in the re-computations appear and therefore the fusions that introduce re-computations, in our case:  $MatInv \oplus StapFilter$  and  $StapFilter \oplus StapApplication$ . In our goal to reduce at minimum the re-computation, it would suggest that the grouping of the first four repetitions and of the last two for fusion, while isolation  $StapFilter$ , would not introduce any re-computations.

**Two-by-two fusions.** Other fusions that provide useful informations are the fusions of each two successive repetitions. The results contain informations on the minimal intermediate array achievable between any two repetitions and on the re-computations introduced by the fusions, as shown on Table 3.

Fusion	Reduction (/)	Re-comp.
<b>PC</b> $\oplus$ <b>CME</b>	$128 \times 96/10 = 1228.8$	9.29
<b>CME</b> $\oplus$ <b>AC</b>	96	1
<b>AC</b> $\oplus$ <b>MI</b>	96	1
<b>MI</b> $\oplus$ <b>SF</b>	96	128
<b>SF</b> $\oplus$ <b>SA</b>	$128 \times 96 = 112288$	119
<b>PC</b> $\oplus$ <b>SA</b>	$128 \times 96/10 = 1228.8$	$119 \times 10 = 1190$
<b>SA</b> $\oplus$ <b>ID</b>	$128 \times 96 = 12288$	1

The maximum reduction of the intermediate array together with the re-computations introduces on the first sub-repetition, achieved by the fusion of each two successive repetitions.

Table 3: Fusion two-by-two.

**Best choice fusion.** The informations provided by the complete and the two-by-two fusions can guide the choice of a sequence of fusions that achieves the best results for our chosen objective of reducing at maximum the intermediate arrays while limiting the re-computations. The complete fusion suggests the grouping of the first four repetitions and of the last two but the two-by-two fusions indicate that the fusion of the first two repetitions introduces re-computations with a factor of nearly 10 for a reduction of array size with a factor of 1228.8. We have two alternatives for the first group of repetitions:

- If the re-computation factor of 10 is acceptable, we can group the first four repetitions for the fusion.
- Otherwise, just repetitions two to four will be fusioned, with no re-computations.

For illustration, we have chosen the first alternative, grouping the first four tasks and the last two. The result shows a reduction to maximum (accordingly to Table 3) of arrays:  $CovMatEst \rightarrow AvCov$ ,  $AvCov \rightarrow MatInv$  and  $StapApp \rightarrow IntDopp$  but a reduction non-maximal for  $PulCompr \rightarrow CovMatEst$ . On the second level of the hierarchy, by the fusion of the two sub-repetitions,  $PulseCompression$  and  $CovMatrixEstim$ , we can further reduce (to maximum) this array, with as result

the presence of three levels of repetitions for these two repetitions. The array-size reductions are shown in Table 4, while the repetitions and re-computations in Table 5.

Array	Size before	Size after	Reduction
<b>PC</b> → <b>CME</b>	$8 \times 128 \times 96$	$8 \times 10$	1228.8
<b>CME</b> → <b>AC</b>	$80 \times 80 \times 119 \times 96$	$80 \times 80 \times 119$	96
<b>AC</b> → <b>MI</b>	$80 \times 80 \times 96$	$80 \times 80$	96
<b>MI</b> → <b>SF</b>	$80 \times 80 \times 96$	$80 \times 80 \times 96$	1
<b>SF</b> → <b>SA</b>	$80 \times 128 \times 96$	$80 \times 128 \times 96$	1
<b>PC</b> → <b>SA</b>	$8 \times 128 \times 96$	$8 \times 128 \times 96$	1
<b>SA</b> → <b>ID</b>	$119 \times 128 \times 96$	119	12288

Table 4: Array sizes for the best choice fusion.

Task	Rep. before	Rep. after	Re-comp.
<b>PC</b>	$8 \times 128 \times 96$	$96 \times \begin{pmatrix} 119 \times \begin{pmatrix} 10 \times 8 \\ 1 \end{pmatrix} \\ 80 \times 80 \\ 1 \\ 128 \times 96 \times 80 \\ 119 \\ 1 \end{pmatrix}$	9.29
<b>CME</b>	$119 \times 96$		1
<b>AC</b>	$80 \times 80 \times 96$		1
<b>MI</b>	96		1
<b>SF</b>	$128 \times 96 \times 80$		1
<b>SA</b>	$119 \times 128 \times 96$		1
<b>ID</b>	$128 \times 96$		1

Table 5: Repetitions for the best choice fusion.

A part of the transformed application accordingly to the previous algorithm are illustrated on Figure 9, the transformed filters level, and on Figure 10 the sub-repetitions for the last two tasks.

#### 4.4 Result analysis

In the previous section we have seen how the high-level transformations can be used to adapt a specification to the execution. Optimizations can be of two kinds, platform dependent or general-purpose. The reduction of intermediate arrays can be considered as a general-purpose optimization but, as shown, execution constrains can guide the choice of fusions that lead to the reduction of intermediate arrays. The granularity changes, through the use of transformations like change paving, tiling or collapse, are more platform-dependent optimizations, aiming at the adaptation of the specification to the execution platform and the optimization of the placement of repetitions on parallel architectures.

Signal processing applications are often represented as successive repetitive filters, as the case of the STAP application of Figure 9. The complex pattern accesses makes it impossible to reduce at maximum every intermediate array and sometimes only with the introduction of additional computations. We have shown how, by exploring some fusion configurations, we can extract informations that lead us to the best choice of chain of transformations. The complete fusion provides informations on the strict partial order and on where changes in re-computations appear, while the two-by-two fusions provide the values of the maximum achievable reductions and on the re-computations introduced by such a reduction. These informations were used to separate the repetitions into groups by forbidding fusions that introduce considerable re-computations. Different objectives and different applications might determine the alternative use

of fusions, taken for instance the presence of an infinite dimension intermediate array, where the elimination of the infinite dimension has priority on the introduced re-computations.

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## 5 Conclusions

The present article shows how high-level data-parallel transformations can be used to explore the design space of multi-dimensional intensive signal processing applications with the objective of adapting a specification to the execution. In the context of Gaspard2 environment [4], using a visual UML interface and the MARTE profile, multi-dimensional intensive processing applications can be modeled, together with repetitive architectures and repetitive placement of the applications on such architectures. Before the code generation, a MARTE RSM specification must be adapted to the execution platform by reducing array sizes and adapting the application's granularity to the architectural and execution constraints. Starting from an application model, different strategies can be chosen to adapt the specification to the execution. These strategies together with different mappings can be easily explored with possible feedback from the code generation, simulation or execution. As future work, these different strategies could be implemented and integrated in the environment together with a semi-automatic refactoring: proposition to the user of different strategies with computed gains in array size reduction and re-computations.

## References

- [1] I. Attali, D. Caromel, Y. syau Chen, J. luc Gaudiot, and A. L. Wendelborn. A formal semantics for sisal arrays. In *Proc. Joint Conf. Infor. Sci.*, Sept. 1995.
- [2] D. F. Bacon, S. L. Graham, and O. J. Sharp. Compiler transformations for high-performance computing. Technical report, University of California at Berkeley, Berkeley, CA, USA, 1993.
- [3] P. Boulet. Formal semantics of Array-OL, a domain specific language for intensive multidimensional signal processing. Research Report RR-6467, INRIA, Mar. 2008.
- [4] DaRT Team. Graphical Array Specification for Parallel and Distributed Computing (GASPARD2). <http://www.gaspard2.org/>, 2009.
- [5] P. Dumont. *Spécification Multidimensionnelle pour le traitement du signal systématique*. Thèse de doctorat (PhD Thesis), Laboratoire d'informatique fondamentale de Lille, Université des sciences et technologies de Lille, Dec. 2005.
- [6] C. Glitia and P. Boulet. High level loop transformations for multidimensional signal processing embedded applications. In *SAMOS 2008 Workshop*, Samos, Greece, July 2008.
- [7] C. Glitia, P. Dumont, and P. Boulet. Array-OL with delays, a domain specific specification language for multidimensional intensive signal processing. *Multidimensional Systems and Signal Processing*, 2009.

- 
- [8] R. M. Karp, R. E. Miller, and S. Winograd. The organization of computations for uniform recurrence equations. *J. ACM*, 14(3):563–590, July 1967.
  - [9] J. Keinert, C. Haubelt, and J. Teich. Modeling and analysis of windowed synchronous algorithms. In *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages III–892– III–895, 2006.
  - [10] O. Labbani, J.-L. Dekeyser, P. Boulet, and E. Rutten. Introducing control in the gaspard2 data-parallel metamodel: Synchronous approach. *International Workshop MARTES: Modeling and Analysis of Real-Time and Embedded Systems (in conjunction with 8th International Conference on Model Driven Engineering Languages and Systems, MoDELS/UML 2005)*, Oct. 2005.
  - [11] O. Labbani, J.-L. Dekeyser, P. Boulet, and E. Rutten. UML2 profile for modeling controlled data parallel applications. In *FDL’06: Forum on Specification and Design Languages*, Darmstadt, Germany, Sept. 2006.
  - [12] E. A. Lee and D. G. Messerschmitt. Static scheduling of synchronous data flow programs for digital signal processing. *IEEE Trans. on Computers*, 36:24–35, Jan. 1987.
  - [13] P. K. Murthy and E. A. Lee. Multidimensional synchronous dataflow. *IEEE Transactions on Signal Processing*, 50(8):2064–2079, Aug. 2002.
  - [14] Object Management Group. A UML profile for MARTE, 2007. <http://www.omgmar.te.org>.
  - [15] E. Piel, R. Ben Atitallah, P. Marquet, S. Meftali, S. Niar, A. Etien, J.-L. Dekeyser, and P. Boulet. Gaspard2: from MARTE to SystemC simulation. In *Modeling and Analysis of Real-Time and Embedded Systems with the MARTE UML profile DATE’08 Workshop*, Mar. 2008.
  - [16] L. Rioux, T. Saunier, S. Gerard, A. Radermacher, R. de Simone, T. Gautier, Y. Sorel, J. Forget, J.-L. Dekeyser, A. Cuccuru, C. Dumoulin, and C. Andre. MARTE: A new profile RFP for the modeling and analysis of real-time embedded systems. In *UML-SoC’05, DAC 2005 Workshop UML for SoC Design*, Anaheim, CA, June 2005.
  - [17] S.-B. Scholz. Single assignment c: efficient support for high-level array operations in a functional setting. *J. Funct. Program.*, 13(6):1005–1059, 2003.
  - [18] J. Soula. *Principe de Compilation d’un Langage de Traitement de Signal*. Thèse de doctorat (PhD Thesis), Laboratoire d’informatique fondamentale de Lille, Université des sciences et technologies de Lille, Dec. 2001. (In French).
  - [19] Ter@ops, 2009. <http://teraops-emb.ief.u-psud.fr/>.





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