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***Abstract*— This paper deals with the Permanent Magnet Synchronous Motor (PMSM) observability analysis for sensorless control design. The problem of loss of observability at low frequency range is always recognized in experimental settings. Nevertheless, there are no sufficient theoretical observability analyses for the PMSM. In the literature, only the sufficient observability condition has been presented. Therefore, the current work is aimed especially to the necessary observability condition analysis. Furthermore, an Estimator/Observer Swapping system is designed here for the surface Permanent Magnet Synchronous Motor (PMSM) to overcome position observability problems at zero speed which is an unobservable state point.**

I. INTRODUCTION

Industries concerned by PMSMs are continuously seeking for cost reductions in their products. These reductions often impose the minimization of number of sensors used for control purposes because they substantially contribute to increase the complexity and cost of the full installation (additional cables, maintenance, etc.), and the default probability. This imposes the use of observers in order to realize sensor-less control design. Since many observers for electrical machines with sensor-less control are available, as the extended Kalman filter [1], the full order and the reduced order observers [2], the LMI based methods [3], the high-frequency signal injection methods ([4],[5]) the sliding mode observers ([6],[7]), the adaptive observers [8], and so on, the main research stream has been focused on searching for reliable speed and position estimation methods for PMSM with the aim to replace the mechanical sensors with the observer in the control system ([9]-[12]). However, the current problems to successfully apply sensor-less control for PMSM are the existence of operating regimes for which the observer performance is remarkably deteriorated due to the difficulties in estimating correctly the motor position. The failure of sensor-less schemes in some particular operating conditions has been always recognized in experimental setting. In the case of induction motors, the observability has been studied by many authors ([13],[14]). Nevertheless, there are no sufficient theoretical observability studies for the PMSM. Only the sufficient observability condition has been presented in literature. For instance, in [15], observability is analyzed in the case of constant high speed operation. In [16] and [17], the author gives only the sufficient observability condition (not necessary) of the PMSM in the particular case of constant speed. The current work is aimed especially to the necessary observability condition analysis. In [18] and [19], we have given the sufficient condition of loss of observability for the Surface PMSM (SPMSM). In this paper, observability of both the Interior PMSM (IPMSM) and the SPMSM is studied and discussed at different operating conditions then the necessary and sufficient observability condition is presented. Furthermore, an Estimator/Observer Swapping system is proposed, for the SPMSM, to overcome position observability problems at zero speed which is an unobservable state point. The designed observer based on Higher Order Sliding Mode (HOSM) is used in order to ensure the robustness against disturbances and to avoid the chattering phenomenon [20]. This paper is organized as follows: In the second section, mathematical models of both IPMSM and SPMSM are presented. In section three, the non linear observability is recalled. The observability analysis of both IPMSM and SPMSM is given in section four. The proposed observer is designed in section five. Simulation results are illustrated in section six. Finally, some concluding remarks are drawn in the last section.

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II. SYNCHRONOUS MOTOR MODELS

A. Interior Permanent Magnet Case

The dynamic model of the IPMSM in the (α,β) fixed coordinate is given by equation (1) and (2) ([21], [22]):

$$\begin{pmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{pmatrix} = \Gamma^{-1}\left[\begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix} - \begin{pmatrix} R-2L_1\omega_e\sin(2\theta_e) & 2L_1\omega_e\cos(2\theta_e) \\ 2L_1\omega_e\cos(2\theta_e) & R+2L_1\omega_e\sin(2\theta_e) \end{pmatrix}\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} - \omega_e K_e \begin{pmatrix} -\sin(\theta_e) \\ \cos(\theta_e) \end{pmatrix}\right] \quad (1)$$

$$\begin{aligned} \dot{\omega}_e &= \frac{P}{J}[2L_1(cos(\theta_e)i_\alpha + sin(\theta_e)i_\beta) + \phi_m](-sin(\theta_e)i_\alpha \\ &+ cos(\theta_e)i_\beta) - \frac{f_v}{J}\omega_e - \frac{T_l}{J} \end{aligned} \quad (2)$$

where $\Gamma^{-1} = \frac{1}{L_0^2-L_1^2} \times \begin{pmatrix} L_0 - L_1\cos(2\theta_e) & -L_1 sin(2\theta_e) \\ -L_1 sin(2\theta_e) & L_0 + L_1\cos(2\theta_e) \end{pmatrix}$

ω_e is the electric rotor speed; R is the stator resistance; P is the pair pole number; J is the moment of inertia; ϕ_m is the rotor flux; f_v is the viscous friction; T_l is the load torque; $[i_\alpha \quad i_\beta]^T$ and $[u_\alpha \quad u_\beta]^T$ are the $(\alpha-\beta)$ stator current and

voltage vector respectively.
$L_1 = \frac{L_d - L_q}{2}$ and $L_0 = \frac{L_d + L_q}{2}$, where L_d, L_q are the (d-q) stator inductance components;

B. Surface Mounted Permanent Magnet Case

In the SPMSM we have $L_d = L_q$ then $L_1 = 0$.
Thus, from equation (1) and (2), we can deduce the dynamic model of the SPMSM:

$$\begin{pmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{pmatrix} = \frac{1}{L_0}\left[\begin{pmatrix} u_\alpha \\ u_\beta \end{pmatrix} - R\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} - \omega_e K_e \begin{pmatrix} -\sin(\theta_e) \\ \cos(\theta_e) \end{pmatrix}\right] \quad (3)$$

$$\begin{aligned} \dot{\omega}_e &= \frac{P}{J}\phi_m(-sin(\theta_e)i_\alpha + cos(\theta_e)i_\beta) - \frac{f_v}{J}\omega_e \\ &- \frac{T_l}{J} \end{aligned} \quad (4)$$

Where K_e is the BEMF constant.

III. NONLINEAR OBSERVABILITY

In this section, the nonlinear observability is recalled [23]. We consider systems of the form:

$$\sum : \begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \quad (5)$$

Where $x \in X \subset R^n$ is the state vector, $u \in U \subset R^m$ is the control vector, $y \in R^p$ is the output vector, f and h are C^∞ functions.

Definition 3-1(Locally weak observability)
Consider the system $\sum$ and let x_0 be a point of the state space X.

- $\sum$ is locally weakly observable at x_0 if there exist an open neighborhood V of x_0 such that for every open neighborhood v of x_0 contained in V, $I_v(x_0) = x_0$ and is locally weakly observable if it is so at every $x \in X$.
- $\sum$ is locally regularly weakly observable at x_0 if it is locally weakly observable at x_0 and the $n-1$ derivatives outputs are sufficient to locally observe the system.

Rank Criterion
A sufficient locally regularly weakly observable condition at x_0 of (5) is that there exists $(u, \dot{u}, ...,)$ such that:

$$rank(J) \mid_{x_0} = rank \begin{pmatrix} dh \\ dL_f h \\ dL_f^2 h \\ . \\ . \\ . \\ dL_f^{n-1} h \end{pmatrix} \mid_{x_0} = n \quad (6)$$

Remark 3-1

1. The notion of locally regularly weakly observability is introduced in order to design an observer of dimension equal to n.

2. The condition (6) depends on $(u, \dot{u}, ...)$ and this is an implicit justification of the universal inputs introduced in [24].

IV. OBSERVABILITY ANALYSIS OF THE PMSM

Let's consider the state vector $x = [i_\alpha, \quad i_\beta, \quad \theta_e, \quad \omega_e]^T$ and the output vector $y = [i_\alpha, \quad i_\beta]^T$. Voltages and currents are assumed to be measurable. The order of the state vector of the PMSM is $n = 4$. Thus, according to the observability rank criterion mentioned earlier, the PMSM is locally regularly weakly observable at x_0 for $(u, \dot{u}, ...)$ if the following condition is fulfilled:

$$rank(J)|_{x_0,(u,\dot{u},...)} = 4 \quad (7)$$

A. Observability analysis of the IPMSM

Consider the system (1) and (2) as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} (\Lambda_{11}\gamma_1 + \Lambda_{12}\gamma_2)/(L_0^2 - L_1^2) \\ (\Lambda_{21}\gamma_1 + \Lambda_{22}\gamma_2)/(L_0^2 - L_1^2) \\ x_4 \\ T_e - mx_4 - \tau \end{pmatrix} \quad (8)$$

Where

$$\Lambda = \begin{pmatrix} L_0 - L_1\cos(2\theta) & -L_1 sin(2\theta) \\ -L_1 sin(2\theta) & L_0 + L_1\cos(2\theta) \end{pmatrix}$$

Λ_{ij} is the i^{th} row of the j^{th} column of the matrix Λ
$\gamma_1 = u_\alpha - (R - 2L_1x_4\sin(2x_3))x_1 + 2L_1x_4\cos(2x_3)x_2 + x_4K_e sin(x_3)$
$\gamma_2 = u_\beta - (R - 2L_1x_4\sin(2x_3))x_2 - 2L_1x_4\cos(2x_3)x_1 - x_4K_e cos(x_3)$
$T_e = \frac{P}{J}[2L_1(cos(x_3)x_1 + sin(x_3)x_2) + \phi_m](-sin(x_3)x_1 + cos(x_3)x_2)$ is the electromagnetic torque.
Let $f(x) = \begin{pmatrix} (\Lambda_{11}\gamma_1 + \Lambda_{12}\gamma_2)/(L_0^2 - L_1^2) \\ (\Lambda_{21}\gamma_1 + \Lambda_{22}\gamma_2)/(L_0^2 - L_1^2) \\ x_4 \\ T_e - mx_4 - \tau \end{pmatrix}$ and $h(x) = y$.

Then, look at the vector of information generated from the output and its only first derivatives:

$$O_1 = \begin{pmatrix} h_1 \\ h_2 \\ L_f h_1 \\ L_f h_2 \end{pmatrix} \quad (9)$$

The associated observability matrix is:

$$J_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\partial L_f^1 h_1}{\partial x_1} & \frac{\partial L_f^1 h_1}{\partial x_2} & \frac{\partial L_f^1 h_1}{\partial x_3} & \frac{\partial L_f^1 h_1}{\partial x_4} \\ \frac{\partial L_f^1 h_2}{\partial x_1} & \frac{\partial L_f^1 h_2}{\partial x_2} & \frac{\partial L_f^1 h_2}{\partial x_3} & \frac{\partial L_f^1 h_2}{\partial x_4} \end{pmatrix} \quad (10)$$

The computation of the corresponding determinant gives:

$$
\begin{aligned}
\Delta_1 &= [[2L_1\sin(2x_3)\gamma_1+(L_0-L_1\cos(2x_3))\frac{\partial\gamma_1}{\partial x_3}\\
&- 2L_1\cos(2x_3)\gamma_2\\
&- L_1\sin(2x_3)\frac{\partial\gamma_2}{\partial x_3}].[-L_1\sin(2x_3)\frac{\partial\gamma_1}{\partial x_4} \\
&+ (L_0+L_1\cos(2x_3))\frac{\partial\gamma_2}{\partial x_4}]-[-2L_1\cos(2x_3)\gamma_1\\
&- L_1 sin(2x_3)\frac{\partial\gamma_1}{\partial x_3}-2L_1\sin(2x_3)\gamma_2\\
&+ (L_0-L_1\cos(2x_3))\frac{\partial\gamma_2}{\partial x_3}][(L_0-L_1\cos(2x_3))\frac{\partial\gamma_1}{\partial x_4}\\
&- L_1\sin(2x_3)\frac{\partial\gamma_2}{\partial x_4}]]/(L_0^2-L_1^2)
\end{aligned}\tag{11}
$$

Case 1: IPMSM at zero speed: It is important to note that for interior permanent magnet synchronous motor L_1 is always different from 0, consequently the J_1 determinant at zero speed ($x_4=0$) is:

$$
\begin{aligned}
\Delta_1 &= [[2L_1\sin(2x_3)(u_\alpha-Rx_1)-2L_1\cos(2x_3)(u_\beta\\
&- Rx_2)].[-L_1\sin(2x_3)(2L_1\sin(2x_3)x_1\\
&+ 2L_1\cos(2x_3)x_2+K_e\sin(x_3))+(L_0\\
&+ L_1\cos(2x_3))(-2L_1\cos(2x_3)x_1\\
&- 2L_1\sin(2x_3)x_2-K_e\cos(x_3))]]/(L_0^2-L_1^2)
\end{aligned}\tag{12}
$$

Remark 4-1: Looking at the previous expression (12), we remark that at zero speed operation Δ_1 depends on current and voltage. Therefore, we have always the opportunity to find again the observability property by injection of a continue current. Thus, we can conclude that the IPMSM is always observable.

B. Observability analysis of the SPMSM

In this section, we present the observability analysis of the SPMSM and we give a sufficient condition of loss of the observability property.
In this case, we have $L_d=L_q$ then $L_1=0$. Therefore, the expression of the determinant of J_1 given in (11) becomes:

$$\Delta_1 = -K_e^2 x_4 \tag{13}$$

Remark 4-2: The determinant Δ_1 is dependant only on x_4. Thus, for the considered output and only its first derivative, the SPMSM is locally weakly observable at x_0 if $x_4\neq 0$. This condition is independent on the considered input $u_{\alpha,\beta}$. Now the question is to look if higher derivatives of output overcome the observability singularity at zero speed ($x_4=0$). For that, let's consider the model of the SPMSM given by equations (3) and (4) in the form of (5) where :

$$
f(x,u)=\begin{pmatrix} ax_1+bx_4sin(x_3)+cu_\alpha\\ ax_2-bx_4cos(x_3)+cu_\beta\\ x_4\\ k_t(-sin(x_3)x_1+cos(x_3)x_2)-mx_4-\tau\end{pmatrix}
$$

and $h(x)=[x_1,x_2]^T$, with $a=\frac{-R}{L_0}$, $b=\frac{K_e}{L_0}$, $c=\frac{1}{L_0}$, $k_t=\frac{p\phi_m}{J}$, $m=\frac{f_v}{J}$ and $\tau=\frac{T_l}{J}$.

Let's look to the following vector of information generated from:

$$
O_2 = \begin{pmatrix} h_1\\ h_2\\ L_fh_1\\ L_fh_2\\ L_f^2h_1\\ L_f^2h_2\end{pmatrix} \tag{14}
$$

The associated observability matrix is:

$$J_2 = \frac{\partial}{\partial x}O_2 \tag{15}$$

Condition (7) can be tested by searching for a regular matrix constructed from any four rows of matrix J_2. Let's consider only the 1^{st}, 2^{nd}, 5^{th} and the 6^{th} rows of J_2 as.

$$
J_2 = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ \frac{\partial L_f^2h_1}{\partial x_1} & \frac{\partial L_f^2h_1}{\partial x_2} & \frac{\partial L_f^2h_1}{\partial x_3} & \frac{\partial L_f^2h_1}{\partial x_4}\\ \frac{\partial L_f^2h_2}{\partial x_1} & \frac{\partial L_f^2h_2}{\partial x_2} & \frac{\partial L_f^2h_2}{\partial x_3} & \frac{\partial L_f^2h_2}{\partial x_4}\end{pmatrix} \tag{16}
$$

The computation of the corresponding determinant gives:

$$
\begin{aligned}
\Delta_2 &= \frac{\partial L_f^2h_1}{\partial x_3}.\frac{\partial L_f^2h_2}{\partial x_4}-\frac{\partial L_f^2h_2}{\partial x_3}.\frac{\partial L_f^2h_1}{\partial x_4}\\
&= b^2[-a^2+am+2k_t(-cos(x_3)x_1\\
&- sin(x_3)x_2)]x_4-2b^2x_4^3-b^2(a-m)\dot{x}_4
\end{aligned}\tag{17}
$$

From equation (17) the observability loss for the considered output and only its first and second derivatives is $\Delta_2=0$. The associated manifold of unobservability is given by $\bar{\Omega}=\{x:\ \Delta_2(x)=0 \text{ and } \Delta_1=0\}$.

Remark 4-3 In (17), at zero speed $x_4=0$, it is obvious that the SPMSM is locally weakly observable for $\dot{x}_4\neq 0$. This is less restrictive than condition $\Delta_1=0$ given by (13).

Case 2: SPMSM at zero speed and acceleration
The problem now it is to consider the particular case where $\dot{x}_4=x_4=0$ (zero speed and acceleration), and to look if possible to recover the observability of SPMSM by using the higher order derivatives (greater than 2) of the output.
First: consider zero acceleration ($\dot{x}_4=0$)
The model of the SPMSM used in this case is given by (3)-(4) in the form of (5) where the function $f(x,u)$ is replaced by

$$
f_0(x,u)=\begin{pmatrix} ax_1+bx_4sin(x_3)+cu_\alpha\\ ax_2-bx_4cos(x_3)+cu_\beta\\ x_4\\ 0\end{pmatrix}
$$

and $h(x)=[x_1,x_2]^T$,

Consider now the vector of information generated by the output and its first, second and third derivatives:

$$O_3 = \begin{pmatrix} h_1 \\ h_2 \\ L_{f_0}h_1 \\ L_{f_0}h_2 \\ L_{f_0}^2 h_1 \\ L_{f_0}^2 h_2 \\ L_{f_0}^3 h_1 \\ L_{f_0}^3 h_2 \end{pmatrix} \quad (18)$$

The associated observability matrix is:

$$J_3 = \frac{\partial}{\partial x} O_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & bx_4 cos(x_3) & bsin(x_3) \\ 0 & a & bx_4 sin(x_3) & -bcos(x_3) \\ a^2 & 0 & c_5 & d_5 \\ 0 & a^2 & c_6 & d_6 \\ a^3 & 0 & c_7 & d_7 \\ 0 & a^3 & c_8 & d_8 \end{pmatrix} \quad (19)$$

with

$$\begin{aligned} c_5 &= abx_4 cos(x_3) - bx_4^2 sin(x_3) \\ d_5 &= absin(x_3) + 2bx_4 cos(x_3) \\ c_6 &= abx_4 sin(x_3) + bx_4^2 cos(x_3) \\ d_6 &= -abcos(x_3) + 2bx_4 sin(x_3) \\ c_7 &= a^2 bcos(x_3)x_4 + (-absin(x_3)x_4 - bx_4^2 cos(x_3))x_4 \\ d_7 &= a^2 bsin(x_3) + abx_4 cos(x_3) - bx_4^2 sin(x_3) \\ &+ (abcos(x_3) - 2bx_4 sin(x_3))x_4 \\ c_8 &= a^2 bsin(x_3)x_4 + (abcos(x_3)x_4 - bx_4^2 sin(x_3))x_4 \\ d_8 &= -a^2 bcos(x_3) + abx_4 sin(x_3) + bx_4^2 cos(x_3) \\ &+ (absin(x_3) + 2bx_4 cos(x_3))x_4 \end{aligned}$$

Remark 4-4 In (19), at zero acceleration $\dot{x}_4 = 0$, it is obvious that the SPMSM is locally weakly observable for $x_4 \neq 0$.

Second: consider also $x_4 = 0$ (this corresponds to zero acceleration and speed)
In this case the observability matrix J_3 (19) becomes:

$$J_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & 0 & bsin(x_3) \\ 0 & a & 0 & -bcos(x_3) \\ a^2 & 0 & 0 & absin(x_3) \\ 0 & a^2 & 0 & -abcos(x_3) \\ a^3 & 0 & 0 & a^2 bsin(x_3) \\ 0 & a^3 & 0 & -a^2 bcos(x_3) \end{pmatrix} \quad (20)$$

Remark 4-5: From the equation (20) a recurrence relation can be obtained

$$\frac{\partial}{\partial x} L_{f_0}^k h_i = a \frac{\partial}{\partial x} L_{f_0}^{k-1} h_i \mid_{x_4=0} \quad (21)$$

$k = 2,3, \quad i = 1,2.$ and can be generalized for higher derivatives. Thus, the higher derivatives of the output greater than three do not recover additional information.

Remark 4-6: Equations (20) and (21) show that for the considered output (currents) and its derivatives at any order, the condition $\dot{x}_4 = x_4 = 0$ (zero speed and acceleration) generates a structural subset of indistinguishability $\{x : \quad x_4 = 0 \quad and \quad \dot{x}_4 = 0\}$. A physical interpretation is related to have the Back Electromotive Forces (BEMFs) equal to zero at any time for $x_4 = \dot{x}_4 = 0$, then any information with respect to the SPMSM rotor position is in the dynamics of stator currents.

V. THE OBSERVER DESIGN

In order, to overcome the observability problems of the SPMSM at zero speed, we propose here an Estimator/Observer swapping system based on the "Super Twisting Algorithm" (STA). Thus, in this section, the used STA is recalled and then applied to the SPMSM. The convergence of the developed observer is studied.

A. The Super Twisting Algorithm

The general form of the STA is defined as follows [25]:

$$\begin{aligned} u(e_1) &= u_1 + \lambda_1 |e_1|^{\frac{1}{2}} sgn(e_1) \\ \dot{u}_1 &= \alpha_1 sgn(e_1) \end{aligned} \quad (22)$$

with $e_1 = x_1 - \hat{x}_1$,
$\lambda_1, \alpha_1 > 0$ are the observer parameters, u_1 is the output of the observer, x_1 is the estimated variable and:

$$sgn(e_1) = \begin{cases} 1 & if \quad e_1 > 0 \\ -1 & if \quad e_1 < 0 \\ \in [-1 \quad 1] & if \quad e_1 = 0 \end{cases}$$

B. Application to Surface PMSM

Let e_α and e_β be the BEMFs. Consider only current dynamic equations of the SPMSM, we can write:

$$\begin{cases} \dot{x}_1 = ax_1 + x_a + cu_\alpha \\ \dot{x}_2 = ax_2 + x_b + cu_\beta \end{cases} \quad (23)$$

where

$$\begin{cases} e_\alpha = -\omega_e \sin(\theta_e) \\ e_\beta = \omega_e \cos(\theta_e) \end{cases} \quad (24)$$

and

$$[x_a \quad x_b] = -b[e_\alpha \quad e_\beta] \quad (25)$$

$[x_a \quad x_b]$ is the vector of unknown variables. Currents and voltages are assumed to be measurable. Applying the STA (22) to system (23), we obtain systems (26) and (27):

$$\begin{cases} \dot{\hat{x}}_1 = \tilde{x}_a + ax_1 + cu_\alpha + \lambda_1 |e_1|^{\frac{1}{2}} sgn(e_1) \\ \dot{\hat{x}}_a = \alpha_1 sgn(e_1) \end{cases} \quad (26)$$

$$\begin{cases} \dot{\hat{x}}_2 = \tilde{x}_b + ax_2 + cu_\beta + \lambda_2 |e_2|^{\frac{1}{2}} sgn(e_2) \\ \dot{\hat{x}}_b = \alpha_2 sgn(e_2) \end{cases} \quad (27)$$

Where $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ and $\lambda_1, \lambda_2, \alpha_1, \alpha_2$ are positive constants that will be given later. $\tilde{x}_a$ and $\tilde{x}_b$ are the estimated values of the unknown variables x_a and x_b.
According to equations (23), (26) and (27), error dynamics of the observer are given by:

$$\begin{cases} \dot{e}_1 = e_a - \lambda_1 |e_1|^{\frac{1}{2}} sgn(e_1) \\ \dot{e}_a = f_1(x_b) - \alpha_1 sgn(e_1) \end{cases} \tag{28}$$

$$\begin{cases} \dot{e}_2 = e_b - \lambda_2 |e_2|^{\frac{1}{2}} sgn(e_2) \\ \dot{e}_b = f_2(x_a) - \alpha_2 sgn(e_2) \end{cases} \tag{29}$$

With $e_a = x_a - \tilde{x}_a$, $e_b = x_b - \tilde{x}_b$, $f_1(x_b) = \omega_e x_b$ and $f_2(x_a) = -\omega_e x_a$

Following the results proposed in [26] and [27] with respect to the STA (22) dedicated to the observer design given by equations (26) and (27), we set:

Corollary: For any initial conditions $x(0)$, $\hat{x}(0)$, there exists a choice of λ_i and α_i such that the error dynamics e_1, e_2, $\dot{e}_1$ and $\dot{e}_2$ converge to zero and by consequence $\tilde{x}_a \longmapsto x_a$ and $\tilde{x}_b \longmapsto x_b$. (See proof in [22])
Figure 1 shows the finite time convergence of the proposed observer. Parameters of the observer are given by :

$$\alpha_1 > f_1^+ \quad and \quad \lambda_1 > (f_1^+ + \alpha_1)\sqrt{\frac{2}{\alpha_1 - f_1^+}} \tag{30}$$

$$\alpha_2 > f_2^+ \quad and \quad \lambda_2 > (f_2^+ + \alpha_2)\sqrt{\frac{2}{\alpha_2 - f_2^+}} \tag{31}$$

Where $f_2^+ = max(f_2(x_a))$ and $f_1^+ = max(f_1(x_b))$,

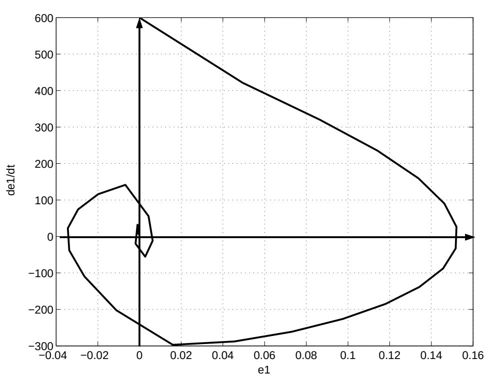

Fig. 1. The trajectory $\dot{e}_1 = f(e_1)$: Finite time convergence.

C. Position and speed estimation

Having the estimated value of x_a and x_b we can easily deduce the rotor position and speed using equations (24), (25), (26) and (27). When the motor operates out of the unobservable region, the rotor position can be calculated as:

$$\hat{\theta}_e = \arctan_2(\frac{-\tilde{x}_a}{\tilde{x}_b}) \tag{32}$$

However, as it is shown in section four, the rotor position is not observable at zero speed and acceleration because $\tilde{x}_a$ and $\tilde{x}_b$ are non existent in this condition and then we can not use the observer equation (32). For this reason, we propose here an Estimator/Observer swapping system which allows the use of the observer at high speed and swap automatically to the estimator when the speed becomes under a defined very low value. The estimated position is calculated as:

$$\hat{\theta}_e = \int_0^t |\hat{\omega}_e| dt + cte \tag{33}$$

Where

$$|\hat{\omega}_e| = \frac{\sqrt{\tilde{x}_a^2 + \tilde{x}_b^2}}{b} \tag{34}$$

The initial value of the estimated position ($\hat{\theta}_e(0) = cte$) is equal to the last value computed by the observer (32) before swapping to the estimator. The estimated speed is calculated as:

$$\hat{\omega}_e = \frac{\sqrt{\tilde{x}_a^2 + \tilde{x}_b^2}}{b} sgn(\tilde{x}_a \sin(\hat{\theta}_e) - \tilde{x}_b \cos(\hat{\theta}_e)) \tag{35}$$

VI. SIMULATION RESULTS

The used motor in the simulation testing is a three-phase SPMSM. The specifications and parameters are : Rated Power $P_n = 1.7kW$; Rated speed $\omega_n = 157 rad.s^{-1}$; Rated voltage $U_n = 380V$; Rated current $I_n = 3.8A$; Number of pole pairs $P = 3$; Stator inductance $L_0 = 0.027H$; Stator resistance $R = 3.3\Omega$; Rotor flux $\phi_m = 0.341$; Rotor inertia $J = 0.0026 kg.m^2$; Viscous friction $f_v = 0.0034 kg.m^2.s^{-1}$. The proposed observer is tested in open loop to the benchmark trajectories [28] presented in Fig. 2. In this benchmark, two reference trajectories are defined: The reference rotor speed (Fig. 2(a)) and the load torque (Fig. 2(b)). In this work, we are interesting only to the observability of the SPMSM. Two tests are carried out. In the first test, we use only the observer (Fig. 3). In this case, we remark that the estimated position and speed reach the real ones with good accuracy and robustness when the motor operates out of the unobservable condition. However, at zero speed and acceleration, the rotor position is not observable. In the second test, we use the proposed Estimator/Observer swapping system (Fig. 4). Thus, in this case, we show that the rotor position can be obtained at all range of speed.

VII. CONCLUSION

In this paper, the observability analysis of both the SPMSM and the IPMSM has been presented and discussed at different operating conditions. A necessary and sufficient observability condition has been presented. Furthermore, in order to improve the rotor position estimation at zero speed, an Estimator/ Observer swapping system has been designed for the SPMSM. The convergence of the observer is proved. Some simulation results has been presented to illustrate the performance of the proposed Estimator/Observer swapping system compared to the results obtained when using only the observer.

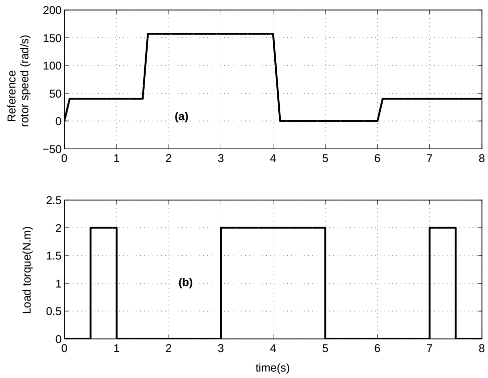

Fig. 2. Benchmark trajectories: (a) Reference speed (rad/s), (b) Load torque(N.m)

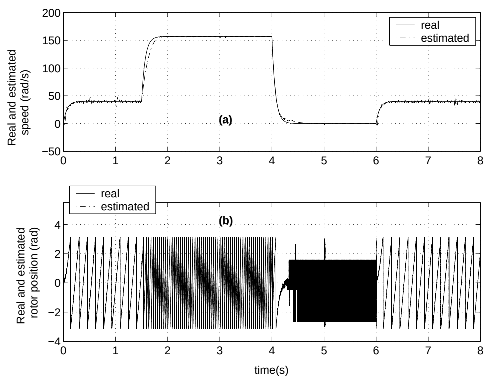

Fig. 3. The observer only: (a) Real and estimated rotor speed (rad/s), (b) Real and estimated rotor position (rad)

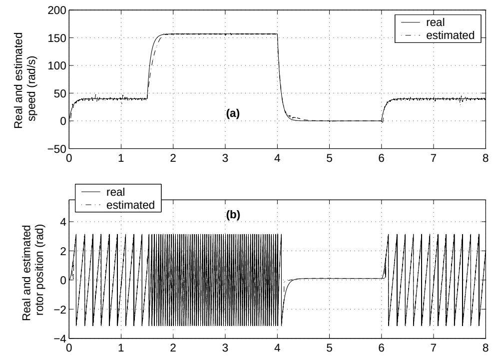

Fig. 4. Estimator/Observer Swapping: (a) Real and estimated rotor speed (rad/s), (b) Real and estimated rotor position (rad)

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