A class of perturbed Cell-Transmission models to account for traffic variability

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Submitted For Publication
89th Annual Meeting of the Transportation Research Board
August 1, 2009

Word Count:

Number of words:	6122
Number of figures:	4 (250 words each)
Number of tables:	0 (250 words each)
Total:	7122

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Abstract

We introduce a general class of traffic models derived as perturbations of cell-transmission type models. These models use different dynamics in free-flow and in congestion phases. They can be viewed as extensions to cell transmission type models by considering the velocity to be a function not only of the density but also of a second state variable describing perturbations. We present the models in their discretized form under a new formulation similar to the classical supply demand formulation used by the seminal *Cell-Transmission Model*. We then show their equivalence to hydrodynamic models. We detail the properties of these so-called perturbed cell-transmission models and illustrate their modeling capabilities on a simple benchmark case. It is shown that they encompass several well-known phenomena not captured by classical models, such as forward moving disturbances occurring inside congestion phases. An implementation method is outlined which enables to extend the implementation of a cell transmission model to a perturbed cell transmission model.

1 Introduction

Classical macroscopic models of traffic. The modeling of highway traffic at a macroscopic level is a well established field in the transportation engineering community, which goes back to the seminal work of Lighthill, Whitham [17] and Richards [23]. Their work introduced to the traffic community the kinematic wave theory which enables one to reconstruct fundamental macroscopic features of traffic flow on highways such as queues propagation. The so-called *LWR model*, based on conservation of vehicles, encompasses most of the non linear phenomena observed on highways in a computationally tractable framework.

In order to close the model, one needs to assume a relation between the velocity and density of vehicles. Greenshields [13] empirically measured a relation between the density and the flow of vehicles, now known as the *fundamental diagram*, which led to the formulation of the LWR problem as a single unknown state variable problem, which could be solved by discretization techniques.

A way to approach the resolution of the discretization of the mass conservation equation in a tractable manner was later proposed by Lebacque [15]. It was shown that a discrete solution of the LWR equation could be constructed by considering the local *supply demand* framework. In the case of concave fluxes, this solution is equivalent to the one obtained using a classical numerical method in conservation laws, the Godunov scheme [12].

The triangular model. Newell [18, 19, 20] introduced the *triangular fundamental diagram*, which is to this date one of the most standard models for queuing phenomena observed at bottlenecks, and for highway traffic modeling in general. Daganzo [7, 8] derived a discrete equivalent of the LWR equation in the case of the triangular fundamental diagram. This model known as the *Cell-Transmission Model* provided the transportation community with a meaningful modeling tool for highway traffic. One of the main assumptions of all the classical models is that the speed of vehicles is a single-valued function of the density.

Second order and perturbed models. Following hydrodynamic theory, attempts at modeling highway traffic with a second conservation equation and a second state variable to augment the mass conservation equation led to the development of so-called *second order models*, such as the Payne [22] and Whitham [25] model. Unfortunately, this model exhibited flaws pointed out by Daganzo [9], Del Castillo [10] and Papageorgiou [21], including the possibility for vehicles to drive backwards along the highway. These flaws were corrected in a new generation of second order models proposed for instance by Aw and Rascle [3], Lebacque [16] and Zhang [28, 29]. By considering a second state variable, these models offer additional capabilities with respect to classical models and for example enable the possibility to include velocity measurements such as the ones obtained from GPS cell phones [26].

The phase transition model Colombo [6] developed a phase transition traffic model with different dynamics for congestion and free-flow, to model fundamental diagrams observed in practice [1]. Like the work of Newell, this approach was motivated by the fundamentally different features of traffic in free-flow and in congestion [24]. In particular, this model includes a set-valued congested part of the fundamental diagram and a single-valued free-flow part of the fundamental diagram. The set-valued congestion phase enables one to account for much more measurements in the congestion phase than the classical fundamental diagram does. Indeed, in the classical setting, a measurement falling outside of the fundamental diagram has to be discarded or approximated. Thus for any tasks involving real data, information is lost at the data processing step. In the setting proposed by the phase transition model and the subsequent perturbed cell transmission model, a whole cloud of measurements can be considered valid.

In part due to the complexity of practical implementation, Colombo's model was extended in [5], leading to a new class of models taking in account the perturbation around the classical fundamental diagram known to exist in practice. Similar to the work of Zhang [28], an assumption is made that a classical fundamental diagram can be viewed as an equilibrium (or average) of the highway traffic state in the perturbed model. In this article, we describe the physical approach developed in [5] and present simple and meaningful local rules to implement a class of discrete perturbed models. We also provide a set of simple steps which can be followed to extend the well-known implementation of the cell-transmission model to an implementation of a perturbed cell transmission model.

Outline. The outline of this work is as follows. In Section 2, we recall the classical framework for discrete macroscopic models, and introduce the discrete formulation of a class of phase transition models relying on physical consideration about traffic flow properties. In particular we show that these models reduce to a set-valued version of the cell-transmission model in the case of a triangular flux. Section 3 provides some examples of the modeling abilities of the class of perturbed models derived, and illustrates the better performances of the class of perturbed models. Section 4 gives a guidebook for perturbed model deployment. Conclusions and future research tracks are outlined in Section 5.

2 Discrete formulation of macroscopic traffic flow models

We consider the representation of a stretch of highway by N space cells C_s, $0 \leq s \leq N$ of size Δx and assume that representing time evolution by a discrete sequence of times with a Δt step size yields a correct approximation for traffic flow modeling.

We make the usual assumption that there is no ramp on the link of interest, and assume by considering a one-dimensional representation of the traffic conditions that even on a multi-lanes highway, traffic phenomena can be accurately modeled as one lane highway. The results presented here can easily be generalized to networks, for example using the framework developed by Piccoli [11].

The following section presents the fundamental macroscopic traffic modeling equation, i.e. the mass conservation.

2.1 Classical models

2.1.1 Mass conservation equation

We call k_s^t the density of vehicles in the space cell C_s at time t, and $Q_{\text{s-up}}^t$ (respectively $Q_{\text{s-down}}^t$) the flux upstream (respectively downstream) of cell s between time t and time $t+1$. The absence of ramp in cell s allows us to write the following conservation equation for the density of vehicles in cell s:

$$k_s^{t+1}\,\Delta x - k_s^t\,\Delta x = Q_{\text{s-up}}^t\,\Delta t - Q_{\text{s-down}}^t\,\Delta t \tag{1}$$

which states that between two consecutive times the variation of the number of vehicles cell C_s is exactly equal to the difference between the number of vehicles having entered the cell from upstream and the number of vehicles having exited the cell from downstream.

Equation (1) which is the mass conservation from fluid dynamics (in a discrete setting) is widely used among the transportation engineering community and considered as one of the most meaningful ways to model traffic flow on highways. Defining the fluxes $Q_{\text{s-down}}$, $Q_{\text{s-up}}$ between two cells is a more complex problem, which can be approached by considering a supply demand formulation.

2.1.2 The supply demand approach

The supply demand approach [15] states that the flow of cars that can travel from an upstream cell to the next downstream cell depends on both the upstream density and the downstream density. If we define the demand function $\Delta(\cdot)$ as a continuous increasing function of the density and the supply function $\Sigma(\cdot)$ as a continuous decreasing function of the density, then the flux between two cells is given by the minimum of the *upstream demand* and the *downstream supply*. The supply and demand function are bounded above on each cell by the flow capacity of the cell. Using the notations introduced above, the supply demand formulation reads:

$$\begin{aligned} Q_{\text{s-up}}^t &= \min(\Delta(k_{s-1}^t), \Sigma(k_s^t)) \\ Q_{\text{s-down}}^t &= \min(\Delta(k_s^t), \Sigma(k_{s+1}^t)). \end{aligned} \tag{2}$$

The demand and supply functions are related to the fundamental diagram as follows. In free-flow the supply $\Sigma(\cdot)$ is simply limited by the capacity of the cell whereas in congestion, the supply is limited by current traffic conditions. In free-flow, the demand $\Delta(\cdot)$ is limited by current traffic conditions whereas in congestion the demand is constrained by the capacity of the cell. Given a fundamental diagram $Q(\cdot)$, with a unique maximum at the critical density k_c, the supply $\Sigma(\cdot)$ and demand $\Delta(\cdot)$ functions can thus be defined as:

$$\Delta(k) = \begin{cases} Q(k) & \text{if} \quad k \le k_c \\ Q(k_c) & \text{otherwise} \end{cases} \quad \text{and} \quad \Sigma(k) = \begin{cases} Q(k_c) & \text{if} \quad k \le k_c \\ Q(k) & \text{otherwise} \end{cases}$$

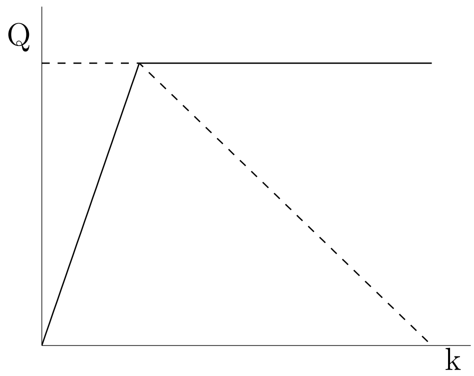

Figure 1: **Supply demand.** The supply curve (bold line) is an increasing function of density and the demand curve (dashed line) is a decreasing function of density.

When the fundamental diagram is triangular, the demand and supply functions are piecewise affine as illustrated on Figure 1, and the supply demand approach is exactly the cell-transmission model [7].

The supply demand approach enables one to define two types of traffic conditions; free-flow and congestion, which have fundamentally different features.

2.2 Two traffic phases

The behavior of traffic depends on the relative values of supply and demand. When the supply is higher than the demand, traffic flow is said to be in *free-flow*, the flux is defined by the number of cars that can be sent from upstream (upstream demand). On the opposite, when the demand is higher than the supply, the traffic is said to be in *congestion* because the flux is defined by the number of cars that the road can accept downstream (downstream supply).

These two dynamics exhibit at least one capital difference:

- In free-flow the flux is defined from upstream and information is moving forward, whereas in congestion the flux is defined from downstream and information is moving backwards.

One may note that the seminal Cell-Transmission Model considers this property as a required model feature, and thus can be viewed as a phase transition model. Figure 2 illustrates two typical sets of experimental measurements. Two distinct phases appear characterized by:

- In free-flow, the speed is constant and the flux is uniquely determined by the density of cars (straight line through the origin for low densities in Figure 2). The knowledge of density or count seems to provide enough information to represent the traffic state.

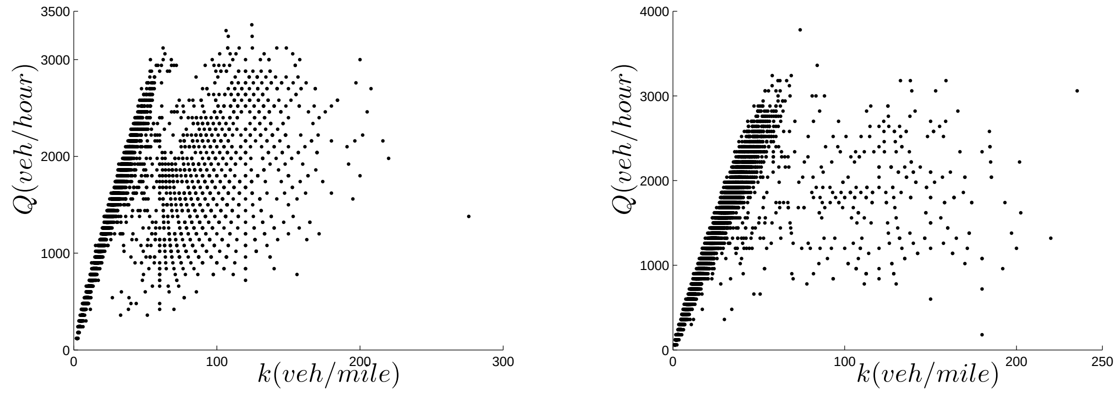

Figure 2: Experimental flow-density relations over a one week-period at two locations on a highway in Roma. Flow was directly measured and density was computed from the measured flow and the measured speed. In free-flow the speed is constant. The shape of the congestion phase changes for different locations.

- In congestion, a given density does not correspond to a unique speed, i.e. the fundamental diagram is set-valued. A second variable must be introduced to model the traffic state.

The first observation is taken in account by the triangular model whereas the second observation motivates the use of a phase transition model [5, 6] using different dynamics for free-flow and congestion, and justify the introduction of a perturbed model in congestion to define the dynamics of two variables necessary to model the congested traffic state [24, 27]. We introduce in the following section a class of perturbed cell transmission type models directly derived from classical models.

2.3 Perturbation of cell-transmission type models

2.3.1 A perturbed fundamental diagram

We propose to describe traffic state on a link of highway by using a perturbed phase transition model. Assuming that the highway link is composed of the cells C_s for $s = 1, \cdots, N$, we define the speed of traffic in each cell as follows:

$$v_s = \begin{cases} V_{\text{ff}} & \text{if} \quad C_s \quad \text{is in free-flow} \\ V(k_s)\,(1+q_s) & \text{if} \quad C_s \quad \text{is in congestion} \end{cases} \tag{3}$$

where V_{ff} is the free-flow speed and $V(\cdot)$ is the velocity function of a classical model.

Application to the cell transmission model The velocity function for the classical cell transmission model reads $V(k_s) = w(1-k_j/k_s)$ where w is the backwards speed propagation and k_j is the jam density. Thus the perturbed speed reads:

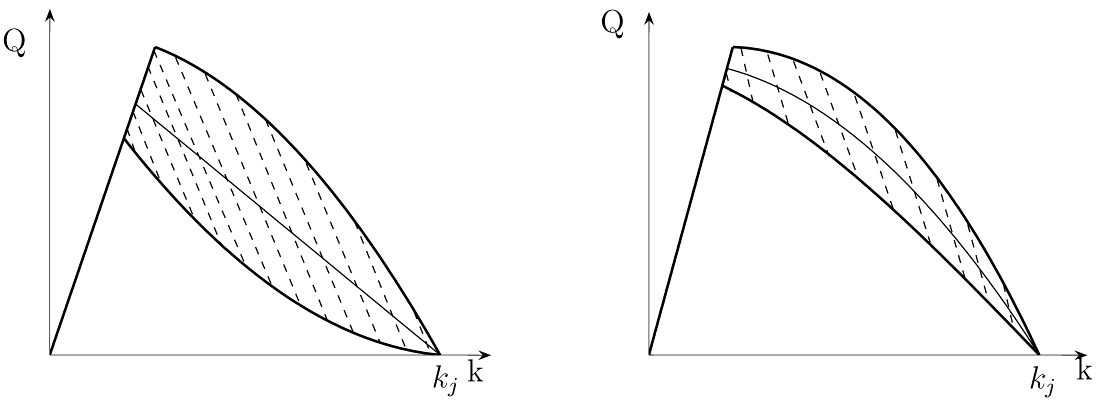

Figure 3: **Left:** Perturbed triangular fundamental diagram (the equilibrium flux function is linear decreasing in congestion). **Right:** Perturbed Greenshields fundamental diagram (the equilibrium flux function is parabolic decreasing in congestion). One can note that the free-flow speed is constant in both models and the flux is set-valued in congestion, i.e. to one density corresponds several values of the flux.

$$v_s = V(k_s)\,(1+q_s) = w(1-\frac{k_j}{k_s})\,(1+q_s)$$

and yields the fundamental diagram from Figure 3 left.

In free-flow, we describe the speed to be constant as per the triangular model, whereas in congestion we introduce a second variable q_s, modeling the fact that for a given density k_s the speed of cars is not uniquely determined by the density. The multiplicative factor $1+q_s$ means that q_s can be viewed as a perturbation around the reference state of traffic which is given by the classical fundamental diagram. In the following we call *equilibrium speed* the value of the speed for $q_s = 0$ (which is the speed of the classical model according to equation (3)). The state of traffic is described by:

$$\begin{cases} k_s & \text{if} \quad C_s \quad \text{is in free-flow} \\ (k_s, q_s) & \text{if} \quad C_s \quad \text{is in congestion.} \end{cases}$$

In free-flow the density k_s completely describes the traffic state and the speed of vehicles is constant equal to V_{ff}. The flux of vehicles in the cell is the product of the density of vehicles and their speed $k_s\,V_{\text{ff}}$. In congestion, the state of traffic is described by the two variables density k_s and perturbation q_s. According to the expression outlined in (3), the speed of vehicles is $V(k_s)\,(1+q_s)$. The flux of vehicles is the product of the density and the speed and is given by $k_s\,V(k_s)\,(1+q_s)$.

Remark 1. *In the following, we assume that the equilibrium speed function in congestion is continuous, decreasing, vanishes at the maximal density, equals the free-flow speed at the critical density, and that the equilibrium flux is concave.*

Remark 2. *For the sake of mathematical and physical consistency, the size of the perturbation q_s cannot be chosen arbitrarily and must satisfy the following constraints:*

- *The perturbed speed must be positive, i.e. $q_s \geq -1$.*
- *The curves on which q_s/k_s is constant (see section 2.3.3 for a physical interpretation of these curves) have a concavity with constant sign. This yields a bound on the perturbation which can be analytically computed by writing that the second derivative of the flux $k_s\, V(k_s)\,(1+q_s)$ with respect to the density k_s has a constant sign for a given value of q_s/k_s.*

2.3.2 Conservation equations for traffic states

Having defined the state of traffic in congestion and in free-flow, we define the dynamics of these quantities as follows. The density k_s is assumed to satisfy the mass conservation given by equation (1). We assume that the macroscopic perturbation $q_s\,\Delta_x$ is also conserved, and thus that q_s satisfies the perturbation conservation equation:

$$q_s^{t+1}\,\Delta x - q_s^t\,\Delta x = R^t_{\text{s-up}}\,\Delta t - R^t_{\text{s-down}}\,\Delta t \tag{4}$$

where $R^t_{\text{s-up}}$ (respectively $R^t_{\text{s-down}}$) is the flow of macroscopic perturbation entering the cell C_s from upstream (respectively exiting from downstream). The dynamics satisfied by the traffic states is:

$$\begin{cases} k_s^{t+1}\,\Delta x - k_s^t\,\Delta x = Q^t_{\text{s-up}}\,\Delta t - Q^t_{\text{s-down}}\,\Delta t & \text{in free-flow} \\ \begin{cases} k_s^{t+1}\,\Delta x - k_s^t\,\Delta x = Q^t_{\text{s-up}}\,\Delta t - Q^t_{\text{s-down}}\,\Delta t \\ q_s^{t+1}\,\Delta x - q_s^t\,\Delta x = R^t_{\text{s-up}}\,\Delta t - R^t_{\text{s-down}}\,\Delta t \end{cases} & \text{in congestion} \end{cases} \tag{5}$$

One must be careful that at any location, the flux of mass $Q_{\text{s-up}}$ and the flux of perturbation $R_{\text{s-up}}$ are coupled by the relation (3) defining the speed and thus can not be defined independently by two uncoupled supply demand relations similar to (2). A coherent approach to the definition of the cell boundary fluxes is to consider the microscopic meaning of the state variable q_s.

2.3.3 From a macroscopic perturbed model to a behavioral driver model

Equation (4) expresses the conservation of the macroscopic perturbation $q_s\,\Delta x$. The usual classical fundamental diagram corresponds to the equilibrium velocity function (i.e. at $q_s = 0$), and for a given density this velocity function can take values above or below the equilibrium velocity function depending on the sign of q_s.

This variation of the velocity function around its equilibrium value leads us to consider the state variable q_s as characterizing the propension of an element of traffic to move forward, in a very similar way to the *driver's ride impulse* from [2]. Indeed, in a cell C_s with a density of vehicles k_s, high values of q_s model aggressive drivers who are eager to move forward and adopt high speed. Low values of q_s model passive drivers who adopt low values of speed.

The speed v_s of drivers and their average aggressiveness defined by the quantity q_s/k_s will play a decisive role in the definition of the boundary fluxes.

Remark 3. *One may note that it is not possible to measure the aggressiveness level of drivers. According to the definition of our class of model, this quantity is completely determined by the knowledge of the speed and density. Thus measures of counts or speeds can be combined with measures of density in order to compute values of the aggressiveness level.*

2.3.4 Traffic rules defining flow between cells

The supply demand formulation does not yield a simple formalism for perturbed models. We choose to define the fluxes from equation (5) by other equivalent physical considerations. We propose two different sets of rules depending on whether the traffic state in the upstream cell is in free-flow or in congestion.

Congested upstream cell

We consider two neighboring cells C_{s-1} and C_s with traffic states $(k_{s-1}^t,\ q_{s-1}^t)$ and (k_s^t, q_s^t) such that the upstream cell is in a congested state. We define the following two rules who will define the flux between these two cells between times t and $t+1$:

- To enter the downstream cell, the vehicles from the upstream cell must modify their speed from v_{s-1}^t to the speed of the vehicles from the downstream cell v_s^t.
- The vehicle from the upstream cell modify their speed according to their average driving aggressiveness q_s/k_s.

These two rules imply that the vehicles which will exit the upstream cell C_{s-1} to enter the downstream cell C_s will have speed v_s and will have an average aggressiveness q_s/k_s. Thus the flux between cell C_{s-1} and cell C_s correspond to a new traffic state $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$ which can be defined by the system of equations:

$$\frac{q_{s-1/2}^{t+1/2}}{k_{s-1/2}^{t+1/2}} = \frac{q_{s-1}}{k_{s-1}} \quad \text{and} \quad v_{s-1/2}^{t+1/2} = v_s \tag{6}$$

where the second equation can be rewritten as an equation in $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$ using the expression from (3). This yields a system of two independent equations in $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$. The corresponding speed $v_{s-1/2}^{t+1/2}$ can be computed from the expression of $k_{s-1/2}^{t+1/2}$ and $q_{s-1/2}^{t+1/2}$ using equation (3). The mass flux and perturbation flux can be then defined as:

$$Q_{\text{s-up}}^t = k_{s-1/2}^{t+1/2}\, v_{s-1/2}^{t+1/2} \quad \text{and} \quad R_{\text{s-up}}^t = q_{s-1/2}^{t+1/2}\, v_{s-1/2}^{t+1/2}$$

Free-flowing upstream cell

We consider two neighboring cells C_{s-1} and C_s with traffic states k_{s-1}^t (free-flow) and (k_s^t, q_s^t) (congestion). The boundary flux of vehicles between the upstream cell C_{s-1} and the downstream cell C_s falls into one of these two cases:

- If the upstream flow is lower than the downstream flow then traffic conditions are imposed from upstream and the boundary flow is the upstream flow. This leads to the boundary flow:

$$Q_{\text{s-up}}^t = k_{s-1}^t \, V \quad \text{and} \quad R_{\text{s-up}}^t = q_{s-1/2}^{t+1/2} \, V$$

  where $q_{s-1/2}^{t+1/2}$ is the perturbation defined by $V(k_{s-1}^t)\,(1 + q_{s-1/2}^{t+1/2}) = V$.

- If the upstream flow is higher than the downstream flow then traffic conditions are imposed from downstream and we obtain similar conditions to the case of two congested cells. Incoming vehicles will adapt their speed to the downstream speed and adopt the lowest corresponding average level of aggressiveness allowable by the fundamental diagram. These two conditions yield the equations:

$$\frac{q_{s-1/2}^{t+1/2}}{k_{s-1/2}^{t+1/2}} = \frac{q_{\min}}{k_j} \quad \text{and} \quad v_{s-1/2}^{t+1/2} = v_s \tag{7}$$

  where $q_{\min}, k_j$ are the minimal density of perturbation and jam density (maximal density). If we note $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$ the solution of (7), the boundary fluxes are given by:

$$Q_{\text{s-up}}^t = k_{s-1/2}^{t+1/2} \, v_{s-1/2}^{t+1/2} \quad \text{and} \quad R_{\text{s-up}}^t = q_{s-1/2}^{t+1/2} \, v_{s-1/2}^{t+1/2}$$

3 Benchmark cases

3.1 Encounter of two flows with different properties

3.1.1 Perturbed model features

We consider the situation of two cells with congested flows. In the upstream cell the traffic state is (k_A, q_A) with high density and low speed and in the downstream cell the state is (k_B, q_B) with low density and high speed. These two traffic states are represented by the points A and B on Figure 4 (right).

According to the rules described in section 2.3.4, the cars from the upstream cell will increase their speed while keeping the same average aggressiveness level q_A/k_A. Physically this means that the drivers from the traffic state A which is slower and denser increase their speed when they reach the front end of the flow A, but do not change their behavior.

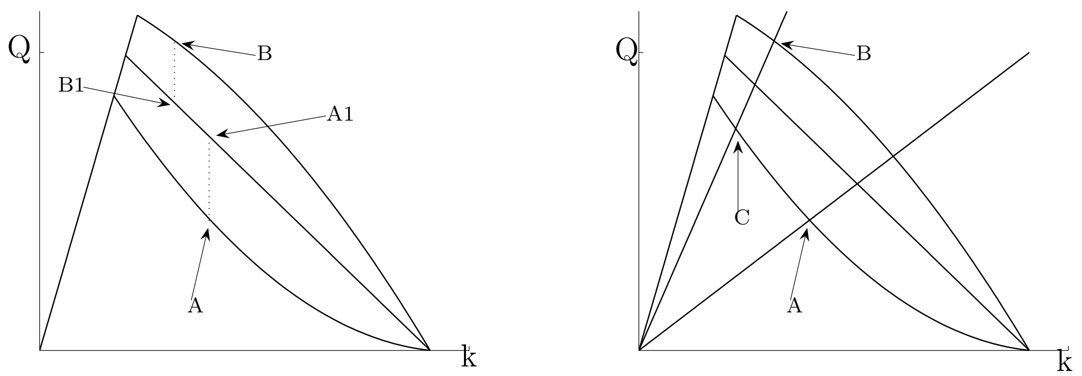

Figure 4: **Left: Classical model.** A and B fall outside of the classical fundamental diagram and are viewed as $A1$ and $B1$; the resulting steady state is $B1$. **Right: Perturbed model.** A and B fall in the perturbed fundamental diagram; the resulting steady state is C.

Thus the flow of cars moving from the upstream cell to the downstream cell will be in state C, defined by the intersection of two curves. The first curve is the straight line defined by the speed being the speed of B, namely $v_C = V(k_B, q_B)$ according to expression (3). The second curve is defined by the average aggressiveness of drivers being the average aggressiveness of drivers from state A, namely $q_C/k_C = q_A/k_A$. One can note that this set of two equations is the one introduced at (6).

3.1.2 Comparison of perturbed and classical model

We compare the evolution predicted by a classical model and by its associate perturbed model, for the two flows described in previous section. The evolution given by the perturbed model was described in previous section.

The classical model can not take in account the states A and B as such because they fall outside of the classical fundamental diagram. Joint measurements of speed and density returning traffic states A and B would have to be approximated. They could be understood as states $A1$ and $B1$ if the density measurement were more reliable.

The interaction of states $A1$ and $B1$ is described by the cell-transmission model as producing the steady state $B1$. One can note that this state is significatively different from the steady state C predicted by the perturbed model.

3.2 Homogeneous in speed states

Traffic flows composed of various densities in which all the vehicles drive at the same speed are commonly observed but cannot be accounted for by classical models which assume that for one given density, only one speed can occur.

Perturbed models allow traffic states with different densities to have the same speed, and can model the homogeneous in speed states observed by Kerner [14]. For instance, if we consider the encounter of two traffic flows with the same speed and different densities such as the state B and C from Figure 4, the model predicts that the difference in flows and densities between the two traffic states is such that the discontinuity propagates downstream at exactly the same speed. It is the similar situation that is observed in free-flow for the triangular model. Indeed one could imagine that the straight line of constant speed defined by $v = v_C$ is the free-flow part of a classical triangular fundamental diagram, in which case the same type of propagation of the two states B and C would be predicted by the cell-transmission model.

4 Implementing a perturbed cell-transmission model

In this section we propose to give a brief outline of the way to implement a perturbed cell-transmission model.

1 Define a classical fundamental diagram which fits the dataset best. Depending on the implementation constraints, this can be done in a variety of methods, from a visual agreement to an optimization routine [4]. In particular, identify the free-flow speed V_{ff}, the jam density k_j and the critical density k_c. This corresponds to the classical implementation method for the CTM.

2 Compute bounds on the perturbation according to the limitations expressed in remark 2. This requires to compute the maximum and minimum of the second derivative of the flux function along a curve of constant aggressiveness level.

3 Given a traffic condition, i.e. a point (ρ, q), check that all the discrete congested states fall into the fundamental diagram, otherwise use an approximation method to map it back to the fundamental diagram, similarly to the case of the classical fundamental diagram.

4 Evolve the model in time using the rules proposed in section 2.3.4.

This shows that implementing a perturbed cell-transmission model is almost as simple as implementing the classical cell-transmission model. We illustrated in section 3 the added value of these models.

5 Conclusion

In this article we propose a class of perturbed models which match empirical features of highway traffic more closely than classical models by incorporating a set-valued fundamental diagram in congestion. We show that by considering a second state variable in congestion, this class of models has greater modeling capabilities.

We follow the principles of the cell-transmission model which assumes that the two phases of traffic, free-flow and congestion, have fundamentally different behaviors. We consider that the speed of traffic is constant in free-flow whereas in congestion it has a perturbed value around the equilibrium speed. The class of models introduced is customizable in the sense that traffic engineers can select the most appropriate classical fundamental diagram and perturb it according to experimental measurements.

We make the assumption that the state variable introduced satisfies a conservation equation, which is motivated by its physical interpretation. At the macroscopic level, it can be considered as a perturbation of the traffic state around the classical fundamental diagram. At a microscopic level, this variable models the behavior of drivers, who make different speed choices for the same observed density. We provide simple meaningful rules to march the model forward in time.

Finally, we provide a simple way to implement this perturbed class of traffic models in the framework currently used by traffic engineers. We show that these models which result from an extension of usual cell-transmission type models can be derived in a straightforward manner.

References

[1] Freeway Performance Measurement System. http://pems.eecs.berkeley.edu/.

[2] R. Ansorge. What does the entropy condition mean in traffic flow theory? *Transportation Research Part B*, 24(B):133–143, 1990.

[3] A. Aw and M. Rascle. Resurrection of "second order" models of traffic flow. *SIAM Journal on Applied Mathematics*, 60(3):916–938, 2000.

[4] S. Blandin, G. Bretti, A. Cutolo, and B. Piccoli. Numerical simulations of traffic data via fluid dynamic approach. *Applied mathematics and computation*, 2009 (to appear).

[5] S. Blandin, D. Work, P. Goatin, B. Piccoli, and A. Bayen. A general phase transition model for vehicular traffic. *Submitted to SIAM Journal on Applied Mathematics*, 2009.

[6] R. Colombo. Hyperbolic phase transitions in traffic flow. *SIAM Journal on Applied Mathematics*, 63(2):708–721, 2003.

[7] C. Daganzo. The cell transmission model: a dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B*, 28(4):269–287, 1994.

[8] C. Daganzo. The cell transmission model, part II: Network traffic. *Transportation Research Part B*, 29(2):79–93, 1995.

[9] C. Daganzo. Requiem for second-order fluid approximations of traffic flow. *Transportation Research Part B*, 29(4):277–286, 1995.

[10] J. Del Castillo, P. Pintado, and F. Benitez. The reaction time of drivers and the stability of traffic flow. *Transportation research Part B*, 28(1):35–60, 1994.

[11] M. Garavello and B. Piccoli. *Traffic flow on networks*. American Institute of Mathematical Sciences, Springfield, USA, 2006.

[12] S. Godunov. A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics. *Matematicheskii Sbornik*, 89(3):271–306, 1959.

[13] B. Greenshields. A study of traffic capacity. *Proceedings of the Highway Research Board*, 14(1):448–477, 1935.

[14] B. Kerner. Phase transitions in traffic flow. *Traffic and granular flow*, pages 253–283, 2000.

[15] J.P. Lebacque. The godunov scheme and what it means for first order macroscopic traffic flow models. *Proceedings of the 13th ISTTT, Ed. J.B. Lesort,*, pages 647–677, Lyon, 1996.

[16] J.P. Lebacque, X. Louis, S. Mammar, B. Schnetzler, and H. Haj-Salem. Modelling of motorway traffic to second order. *Comptes rendus-Mathématique*, 346(21-22):1203–1206, 2008.

[17] M. Lighthill and G. Whitham. On kinematic waves II a theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London*, 229(1178):317–345, 1956.

[18] G. Newell. A simplified theory of kinematic waves in highway traffic, I: General theory. *Transportation research Part B*, 27(4):281–287, 1993.

[19] G. Newell. A simplified theory of kinematic waves in highway traffic, II: Queueing at freeway bottlenecks. *Transportation research Part B*, 27(4):289–303, 1993.

[20] G. Newell. A simplified theory of kinematic waves in highway traffic, III: Multi-destination flows. *Transportation research Part B*, 27(4):305–313, 1993.

[21] M. Papageorgiou. Some remarks on macroscopic traffic flow modelling. *Transportation Research Part A*, 32(5):323–329, 1998.

[22] H. Payne. Models of freeway traffic and control. *Mathematical models of public systems*, 1(1):51–61, 1971.

[23] P. Richards. Shock waves on the highway. *Operations Research*, 4(1):42–51, 1956.

[24] P. Varaiya. Reducing highway congestion: an empirical approach. *European journal of control*, 11(4-5):301–309, 2005.

[25] G. Whitham. *Linear and Nonlinear Waves*. Pure & Applied Mathematics Series, New York: Wiley-Interscience, 1974.

[26] D. Work and A. Bayen. Impacts of the mobile internet on transportation cyberphysical systems: traffic monitoring using smartphones. In *National Workshop for Research on High-Confidence Transportation Cyber-Physical Systems: Automotive, Aviation and Rail*, Washington, D.C., 2008.

[27] J. Yi, H. Lin, L. Alvarez, and R. Horowitz. Stability of macroscopic traffic flow modeling through wavefront expansion. *Transportation Research Part B*, 37(7):661–679, 2003.

[28] H. Zhang. A theory of nonequilibrium traffic flow. *Transportation Research Part B*, 32(7):485–498, 1998.

[29] H. Zhang. A non-equilibrium traffic model devoid of gas-like behavior. *Transportation Research Part B*, 36(3):275–290, 2002.