

# Building Roadmaps of Local Minima of Visual Models

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## Contributions

- Optimization-based algorithms for systematically finding nearby local minima of high-dimensional cost functions, and building 'roadmaps' of the low-peak-cost paths linking them.
  - No prior knowledge of minimum distribution or cost topography needed.
  - Can also handle constraints (inequality, non-linear).

## Approach

- Adapt computational chemistry methods for finding *Transition States*
  - codimension 1 saddle points (0 gradient,  $l$  negative curvature direction)
- Two deterministic approaches based on local optimization:
  - Eigenvector tracking*: modified form of damped Newton minimization.
  - Hypersurface sweeping*: propagates a hypersurface through the space, tracking minima on it.
- From each transition we slide downhill to next minimum
  - minimum peak cost pathway linking the minima

## Conclusions

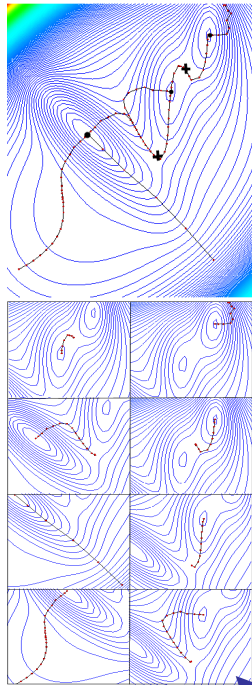
- Efficient deterministic algorithms for exploring high-dimensional non-convex likelihood surfaces.
- Useful for many other vision problems (model-based vision, SFM).

## Eigenvector Tracking

- Choose an ascent eigenvector ' $k$ ' and track it as you progress
- Move uphill along ' $k$ ' and downhill along all other eigenvectors
- Modify damping signs to minimize an equivalent local virtual cost model

$$\delta\tilde{x}(\lambda) = \begin{pmatrix} \frac{g_1}{\lambda_1 + \sigma_1 \lambda} \\ \vdots \\ \frac{g_n}{\lambda_n + \sigma_n \lambda} \end{pmatrix}, \quad \lambda \geq -\min(0, \sigma_1 \lambda_1, \dots, \sigma_n \lambda_n, +\infty), \quad \sigma_k = -1, \sigma_{\text{rest}} = +1$$

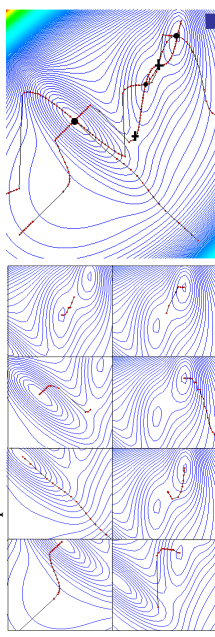
- Tracking 'a given eigenvalue' is ill-defined: when eigenvalues cross, their eigenvectors slew around rapidly.
- Search may diverge to infinity (wall-climbing).



## Hypersurface Sweeping

- A codimension 1 saddle is
  - a local maximum in one direction
  - a local minimum in the other  $n-1$  directions
- Sweep space with a moving hypersurface
  - hyperplane, expanding hyper-ellipsoid...
  - parametrized implicitly  $c(x)=t$  or explicitly  $x=x(t,u,l)$
- Track local-minima within hypersurface and find temporal maxima along tracks

- Only finds saddles that surface cuts in positive curvature directions
- Some ('sawtooth') maxima are disappearances not saddle points
- Plane / ellipse orientation fixes initial track directions.

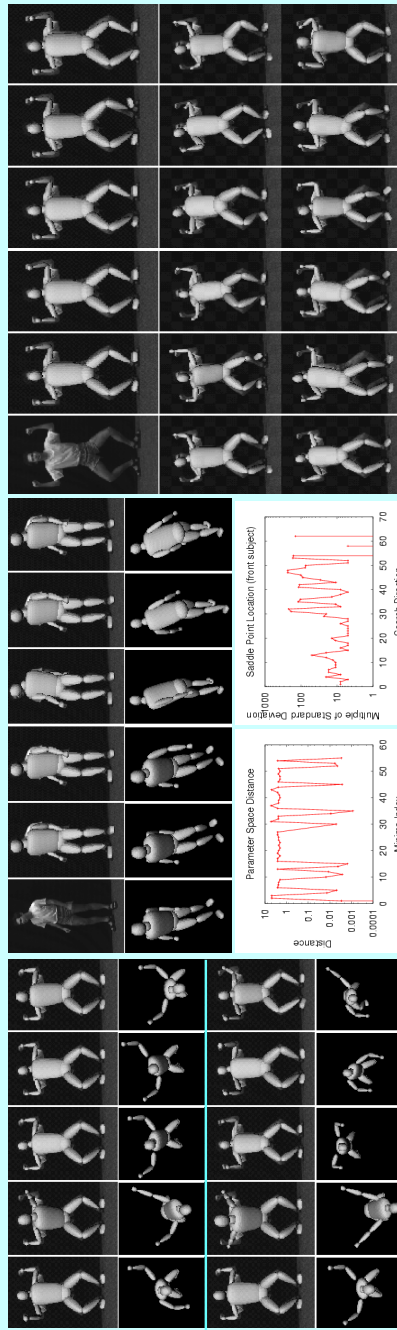


## Local Minimization versus Saddle Point Search

- Minimization**
  - many local descent based algorithms
  - measure progress by decrease in function value: local 'sufficient decrease' ensures 'global convergence' to some minimum
- Saddle Point Search**
  - ascend towards saddle using modified descent algorithms
  - no universal progress criterion
  - needs initial ascent direction or direction weighting
- Newton Minimization**
  - Pure Newton minimization ( $f = \text{cost function}, g = \text{gradient}, H = \text{Hessian}$ )
    - $0 = g(x + \delta x) \approx g(x) + H\delta x \Rightarrow \delta x = -H^{-1}g$
    - efficient near minimum
    - further away, may diverge or converge to any type of stationary point.
  - Damped Newton minimization
- globalize convergence by adding 'damping' matrix  $D$ 
  - in an eigenbasis where  $D = I$ :
    - $$\delta\tilde{x}(\lambda) = \begin{pmatrix} \frac{g_1}{\lambda_1 + \lambda} \\ \vdots \\ \frac{g_n}{\lambda_n + \lambda} \end{pmatrix}, \quad \lambda \geq -\min(0, \lambda_1, \dots, \lambda_n)$$

## Application: Monocular Human Pose, Reconstruction and Tracking Ambiguities

- 32 dof articular human model with superquadric parts, joint angle limits and part non-interpenetration constraints
- Initialize using model-image joint correspondences, track using edge+silhouette and edge+flow cost.
- Method finds many (~300) minima caused by ambiguous (forwards/backwards) depths or incorrect assignments.
- Many more minima are eliminated by physical (joint, non-interpenetration) constraints



Pose ambiguities for given model-image joint correspondences (frontal and half-profile views), and some statistics on minima found.

Tracking Ambiguities: flow+edge (top) and silhouette+edge (middle & bottom) cost function