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Abstract. Ant algorithms are usually derived from a stochastic modeling based on some specific probability laws. We consider in this paper a full deterministic model of "logistic ants" which uses chaotic maps to govern the behavior of the artificial ants. We illustrate and test this approach on a TSP instance, and compare the results with the original Ant System algorithm. This change of paradigm —deterministic versus stochastic— implies a novel view of the internal mechanisms involved during the searching and optimizing process of ants.

Key words: Metaheuristics, Chaotic Map, Optimization, Swarm Intelligence, Ant Algorithm

1 Introduction

Ant algorithms constitute a family of stochastic models mainly based on the following probability function used by artificial ants as a decision function:

$$p_{ij} = \frac{(\tau_{ij})^{\alpha}(\eta_{ij})^{\beta}}{\sum_{l \in \mathcal{N}_i}(\tau_{il})^{\alpha}(\eta_{il})^{\beta}} \tag{1}$$

where τ_{ij} denotes the amount of pheromone on the edge (i, j) linking node i to node j, and η_{ij} denotes an extra heuristics adapted to the problem[1], and $\mathcal{N}_i$ is the set of other existing edges from node i. The exponents α and β are greater or equal to 1, but commonly equals 2 to get the best performances. This probability law derives from the law found by Deneubourg to fit statistically with the experimental data of the famous "double bridge experiment" [1] involving only two edges. This stochastic decision enables a colony of agents (artificial ants) to find or approximate good solutions for many hard optimisation problems [2].

This paper focuses on the stochastic foundations of the ant algorithm modeling: more precisely our concern is with the existence of an alternative deterministic model which would exhibit the dynamical aspects of the involved processes. Some works have been published in this way of modeling, by requiring the hypothesis of chaotic dynamics in ant behaviors [3, 4].

[1] the inverse of the edge distance in the TSP case.

In the same way, we propose the deterministic model of "logistic ants" in this paper. This model is inspired by some theoretical studies on iterated non-linear maps, namely logistic maps or quadratic maps, which are well known to produce chaotic time series [5]. This feature is needed for simulating stochastic behaviors. The main advantage of this approach lies in the possibility of controlling the chaotic properties of the iterated map through a single parameter in our case. We deal therefore with the deterministic chaos theory, to "replace" the probability theory. This field provides tools like bifurcation diagrams to monitor the processes. The presentation of the logistic ant model constitutes the first section of this paper.

The logistic ant model has been already applied on the binary bridge experiment and has proved to simulate the symmetry breaking of the problem. It has moreover produced the same shape of data series as the experimental ones [6]. But it has never been applied to optimization problems, contrary to ant algorithms. We intend therefore to validate our approach by comparing the logistic ant model to the "Ant System" algorithm [2] —one of the first ant algorithm instance in the family— on a Travelling Salesman Problem (TSP) benchmark with 48 nodes. The objective is at this stage to prove that the concept is relevant for optimization, not to deal with a hard TSP. This constitutes the second section of this paper. The results we get are encouraging and our last section is dedicated to discuss these results.

2 The logistic ant model for TSP

This section is devoted to the design of the logistic ant system. In fact this system is a reactive multi-agent system (MAS) composed of an environment plus many (logistic) agents. The agents interact through the environment by a pheromone field, that's why we call them "ants". However, the difference of logistic agents compared to stochastic ants lies in the decision process of logistic ants which is governed by a deterministic logistic map, in contrary to the stochastic law (1).

2.1 Metaheuristic principles of ant algorithms

Let us specify this in the case of a TSP problem where the objective is to find the shortest Hamiltonian cycle in a weighted graph[2]. The generic considered graph is denoted $G = (V, E)$ where V is the set of $|V| = n$ vertices and E the set of edges. In our case the graph is symmetric and has $\frac{n(n-1)}{2}$ edges.

Before describing the different parts of the logistic ant system, let us recall some rooting principles of ant algorithms. We use the same global metaheuristic method using a pheromone field to perform optimization on a symmetric TSP, that is:

- N ants forming a colony are involved at the same time on a given TSP,
- the algorithm proceeds in a global loop composed of global steps,

[2] we will only consider symmetric TSP in this paper

– during a global step, each ant achieves individually an hamiltonian cycle from a random initial position and marks it by an amount of pheromone,
– each global step ends with a pheromone reinforcement of the best cycle, when all ants have finished their cycle.

In this paper, the elementary discrete time step t corresponds to the process time needed to achieve a "local loop", that is a loop where all ants in the colony has moved into a new vertex. A global step lasts therefore $T = n-1$ time steps for all ants to cover an Hamiltonian cycle in parallel. We keep the naming of local loop relative to the vertex and global loop relative to the graph, to distinguish the different levels of schedulling in the algorithm. The global loop lasts until a fixed limit of time steps is reached: this is the criterion to stop the algorithm in this study. The best optimization performance among the colony is then recorded.

2.2 The environment design

We use the concept of environment of the MAS paradigm to include all the data related to the graph problem. In this way, our MAS environment is composed of a basic geometric space denoted E and many fields on it. E corresponds here to the $2D$ discrete space $\mathbb{N}_n^* \times \mathbb{N}_n^*$ where $\mathbb{N}_n^* = \{1, \cdots, n\}$ and n is the number of vertices in the considered graph.

The notion of field The field concept is the means we use to structure the data in the environment and to describe dynamically all the processes within the logistic ant system. A field is defined as a mapping between the geometric space E and the real set. It may also be seen as a data layer of the environment, but remains mathematically a function. We distinguish the fields notably by the origine of their data: an endogenous field comprises data produced by agents, whereas an exogenous field comprises external data, set by the problem. On the other hand a field can be dependent on time or not. In all the following, (k, l) denotes a pair of coordinates in the space E.

Main fields The environment is composed of the following main fields:

– The adjacency field $\mathcal{A}$ corresponds simply to the connectivity matrix of the graph and is an exogenous field.
– The weight field stores the exogenous data relative to the given distances in the graph. Here we use a formula to transform all distances into the interval $[0, 1]$. In this way, we intend to have a generic design, scale-free and independent of the units used:

$$\mathcal{W}(k, l) = \frac{\min_{(i,j)\in E} d(i, j)}{d(k, l)} \tag{2}$$

where $d(i, j)$ gives the distance of the edge (i, j).

– The pheromone field $\mathcal{T}^t$ is dynamically build by the ant colony. It is therefore a cumulative endogenous field: The pheromone field is initialized to 0. It is characterized by a cumulative updating process and an evaporation coefficient ρ.

Other fields Other fields are needed in our environment modeling:

– A field of visited vertices and edges denoted by $\mathcal{H}_i^t$ stores, for an ant i at each time t during a global step of the algorithm, the taboo list of edges and nodes already visited by the ant.
– A field of ordered edges $\mathcal{O}^t$ maintains a list of edges related to each node of the graph, ordered by the amount of pheromone. This field is needed because of the determinitic nature of the choosing decision process of logistic ants (cf section 2.4).
– A field of influence: the pheromone field is a dynamical field, and it has to be updated after each ant has achieved a hamiltonian cycle. That is why we involve also an "influence" pheromone field denoted $\widetilde{\mathcal{T}}$, which is a temporary field for the global update of the pheromone field.

2.3 Design of the logistic ant

Rooting principles

Internal state definition of the logistic ant. Let us describe now the internal behavior of the logistic ant. The logistic ant i is a reactive agent with the internal state $s_i^t = \langle x_i^t, a_i^t \rangle$ at time step t:

– $x_i \in [0, 1]$ is the decision variable of the agent,
– $a_i \in [0, 1]$ is the internal control variable of the agent which governs its type of dynamics.

The interpretation of the internal variables becomes clear within the decision function of the agent, which is a conjugate form of the logistic map:

$$f(x, a) = 1 - a(2\ x - 1)^2 \tag{3}$$

Here, both x and a belong to the interval $[0, 1]$: x is the main variable and a the control parameter of the map. When the a value is set, one can iterate this function according to the recurrence $x^{t+1} = f(x^t, a)$. Numerical studies on this recurrence for many iterations (about hundreds) lead to three type of results: a fixed point or a periodic cycle or chaotic (aperiodic) series. The asymptotic dynamical properties of this map are summed up in the bifurcation diagram (1).

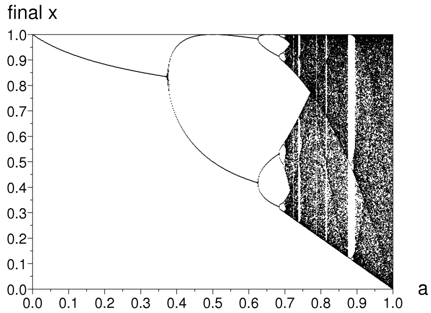

Fig. 1. Bifurcation diagram of the iterated map $x^{t+1} = 1 - a(2\,x^t - 1)^2$ with 500 loops.

Parametric control for exploration and exploitation. The basic idea of the algorithm consists in using the dynamical properties of the logistic map by modifying dynamically the a value as the algorithm runs and in respect to the optimisation objective. Chaos occurs within the right part of the diagram (1) which may correspond to the exploration phase, whereas fixed points occur in the left part which may correspond to the exploitation phase. This modulation of a is achieved by the perception and action functions/operators. Our algorithm at the local level of ants will lead from an exploration phase to an exploitation phase, that is from a high a-value ($a \simeq 1$) to a low a-value ($a \simeq 0$).

2.4 Inside the logistic ant

The internal processing of a logistic ant follows a sensorimotor scheduling — typical of a cybernetics approach—, that is a perception-decision-action process. This scheduling is achieved during an elementary time step of the algorithm, in parallel by each ant of the colony.

The perception process Let us consider an ant i on a given vertex k of the graph, let V_k denote the set of all vertices connected with vertex k and not yet visited (not belonging thus to the taboo list). The perception operator acts on the pheromone field according to the formula:

$$P_i(k) = max_{l \in V_k}\{\mathcal{T}(k,l)\} \quad (4)$$

This perception returns simply the maximum amount of pheromone from a given vertex.

The decision process The decision process performs the transition of the internal state of the logistic ant. For an ant i, it is formalized as a dynamical system between two time steps t and $t+1$:

$$\begin{cases} a_i^{t+1} = \dfrac{1}{1 + e^{\alpha\ (P_i^t(k) - \tau_0)}} \\ \\ x_i^{t+1} = f(x_i^t, a_i^{t+1}) \end{cases} \tag{5}$$

The updating of a_i regulates the adaptation behavior of ants by means of a sigmoid function which fixes the envelop of the decreasing variation of the control variable in function of perceptions.

The action process After updating the decision variable, the logistic ant effects local actions:

- Choose an edge among the ordered edges list (through the field $\mathcal{O}$) from the current vertex in proportion to the value of the decision variable x and move on the choosen vertex, denoted l^*.
- Update the set of visited edges and vertices, that is the field $\mathcal{H}$.
- Update the influence pheromone field by the following formula:

$$\widetilde{\mathcal{T}}_i^{t+1}(k, l^*) = x_i^{t+1}\ \mathcal{W}(k, l^*) \tag{6}$$

2.5 Reaction of the environment

The environment reactions occur once at the end of each time step when the N ant local loop are made up, once at the end of each global step to reinforce the best cycle.

The reaction process at each time step consists in the updating of the field $\mathcal{O}$, the field $\mathcal{H}_i$ for each ant, and the global pheromone field according to:

$$\mathcal{T}^{t+1} = \mathcal{T}^t + \sum_{i=1}^{N} \widetilde{\mathcal{T}}_i \tag{7}$$

At the level of a global step, the reaction consists in the reinforcement of the pheromone field for the best cycle among the colony. Let L_i denote the distance of the cycle for an ant i and G_{min} denote an inferior bound of the minimal distance of a cycle defined by the sum of minimal distances from all edges. The amount of pheromone $\triangle\tau \in [0, 1]$ for reinforcing the best cycle is given by:

$$\triangle\tau = \frac{G_{min}}{\min_i\{L_i\}} \tag{8}$$

This formula is independent of the distance units used and does not depend on any parameter. The updating of the pheromone field according to the evaporation coefficient ρ is then very similar as the Ant system algorithm. Finally the reaction process resets the ordered edges field, the visited edges field, and the influence pheromone field to their initial values.

3 Simulation and results of the logistic ant algorithm

Simulations have been performed on 20000 elementary time steps on the “att48” TSP instance from the TSPLIB library for wich the optimal cycle equals 10628. Different ant number N and evaporation coefficient $\rho \in \{0, 0.01, 0.02, \cdots, 0.1\}$ have been tested. We have compared the best results between the classical Ant System algorithm and our logistic ant algorithm on 5 runs for each initial configuration. The comparison of the best results is given in table 1. The sigmoid function used by logistic ants has been fixed to the following parameter: $\alpha = 0,04$ and $\tau_0 = 30,0$. In terms of computation time, the running time of the logistic

Number of agents	10	20	30	40
AS	$(0.0, 11028)$	$(0.06, 10845)$	$(0.06, 10847)$	$(0.08, 10777)$
LA	$(0.01, 11074)$	$(0.0, 11049)$	$(0.02, 10894)$	$(0.01, 11026)$

Table 1. Comparison of best results between the “Ant System” (AS) algorithm and the Logistic Ant algorithm (LA). Each cell gives a pair (ρ_{best}, L_{best}).

ant algorithm is in average twice the runnig time of the Ant System algorithm with the same implementation conditions. Both algorithms failed in finding the optimal solution in the limited laps of time. Some points have to be mentioned:

- best results are very close between both algorithms,
- the AS algorithm converges very fast towards good solutions, whereas it takes more time for the LA algorithm to find good solutions,
- the increase of the ant number has low impact on the results in the AS case, whereas it speeds up the convergence in the LA case,
- the evaporation coefficient has to be near 0.1 to give the best results in the AS case, whereas much lower values are needed in the LA case.

4 Discussion and interpretation of the logistic ant algorithm

We consider that the performances of the logistic ant system are convincing to carry on this model for optimization. We foresee some advantages of this approach:

- the deterministic nature of the logistic ant model enables to make clear the underlying mechanisms involved in the ant colony. We have shown indeed that the pheromone field modifies the internal control variable of agents and impacts its future decisions through the nonlinear logistic map. The pheromone field reveals to be both a control field in dynamical terms and a decision field.

– This model constitutes therefore a metaheuristic approach, because of the genericity of the convergence principle: the convergence profile is given for each ant by the bifurcation diagram of the logistic map. This convergence principle is an abstract one and can be adapted to many types of optimization functions.
– The logistic ant model may be linked with approaches involving biological chaos as it is mentionned in the introduction of this paper.

The main drawback we may see in this model lies in the theoretical impossibility to prove at the present time the asymptotic convergence to the optimal solution, instead of the ant colony algorithm family. By contrast the convergence process may be known within each ant through the generic bifurcation diagram of the logistic map and according to the variation of the ant control variable. But it does not lead to know the convergence process at the global level. We intend to compensate for this drawback by keeping several chaotic agents in the environment to maintain an exploring level.

5 conclusion

We have shown in this paper a way to build a full deterministic model of ant algorithm by means of logistic maps as decision functions. The results obtained by our "logistic ants" reveal to be comparable to the Ant System algorithm on the same TSP instance. The main advantage of this deterministic model lies in the mechanism principles it involves: the optimization process results from an internal parametric control of ants on the logistic map through the field of pheromone. An other interesting aspects of the model lies in the systemic formalization using a field-based environment and a sensori-motor loop in ants, which is generic and independent of the implementation on computers. It enables therefore to be applied to many types of problems. Our assumption is finally that the logistic ant model may be linked with realistic biological chaotic phenomena.

References

1. Deneubourg, J., Aron, S., Goss, S., Pasteels, J.: The self-organizing exploratory pattern of the argentine ant. Insect Behavior **3** (1990) 159–168
2. Dorigo, M., Stützle, T.: Ant Colony Optimization. The MIT Press (2004)
3. Cole, B.J.: Is animal behaviour chaotic? evidence from the activity of ants. In: Proceedings: Biological Sciences. Volume 244. (1991) 253–259
4. Miramontes, O., Solé, R.V., Goodwin, B.C.: Neural networks as sources of chaotic motor activity in ants and how complexity develops at the social scale. International Journal of Bifurcation and Chaos **11**(6) (2001) 1655–1664
5. Collet, P., Eckmann, J.P.: Iterated Maps on the Interval as Dynamical System. Birkhäuser Boston (1980)
6. Charrier, R., Bourjot, C., Charpillet, F.: A nonlinear multi-agent system designed for swarm intelligence : the logistic MAS. In: SASO'07, International Conference on Self-Adaptive and Self-Organizing Systems, Boston. (2007)