Filter Transformations for Shift-Insensitive Feature Detection

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Introduction

- Visual filters can be modelled by derivatives G_k of the Gaussian function.
- The combined responses characterize the local structure of the image.
- This *Gaussian jet* representation is convenient because it is:
  - Steerable: Get any $G_k(x,\sigma,\theta)$ from $G_k(x,\sigma,\theta_1)\cdots G_k(x,\sigma,\theta_{k+1})$.
  - Dimensionally separable, hence easily defined in 2D and 3D.
  - The natural code for typical image features (ridges, blobs, etc).
- But what about *complex cells*, cf. the Gabor 'energy model'?
- Can the jet be made insensitive to small shifts of the image?

Replica Filters

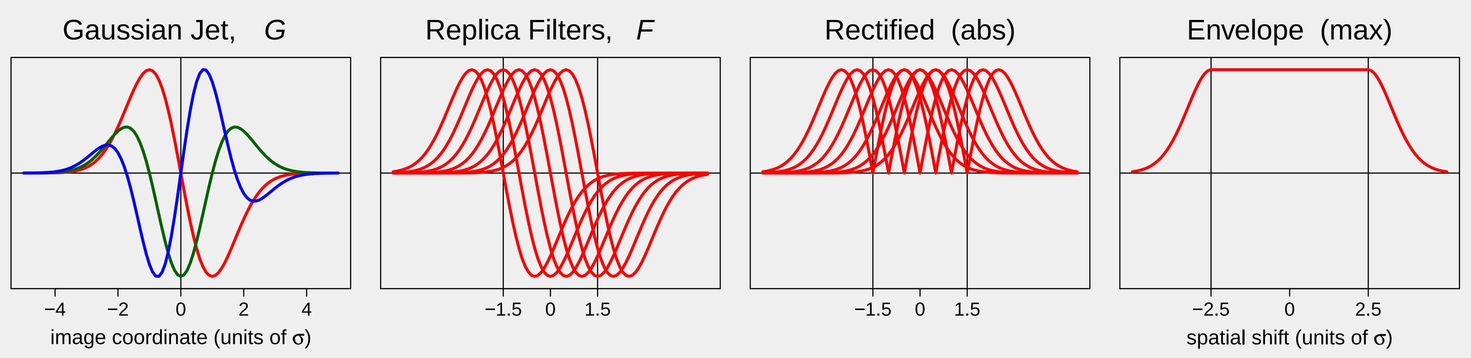
- Let $F_\star(x,u)$ be a family of ideal filters, shifted by u.
- These can be Taylor-approximated from $F_\star(x,0)$ and its derivatives.
- In particular, choose the edge-templates $F_\star(x,u) = G_1(x-u,\sigma)$.
- Now define the replica filters, $F \approx F_\star$, from the Dth-order jet:

$$F(x,u) = \sum_{k=0}^{D-1} \frac{-u^k}{k!} G_{k+1}(x,\sigma)$$

- Problems: Unstable, and nature of the approximation is unclear.
- Solution: Allow polynomial weights $P_k(u)$, and solve by least-squares.

Impulse Response

- Schematic representation:



- More derivatives are needed in practice (see 'Filter Construction' box).
- The interval of approximation is $\pm\rho\sigma$, with $\rho = 1.5$ here.
- Note that the family of replica filters is *continuous* (only 7 shown).

Neural Implementation

- The linear-response vector is $q_j = F_j \cdot s$, one value for each shift.
- A neurally plausible 'soft-max' is used to compute the envelope:

$$q_\star = \textstyle\sum_j w_j\,|q_j| \approx \max|q_j|$$

- The weights are defined by a nonlinearity and normalization:

$$w_j = \exp\big(\mu|q_j|\big) \Big/ \textstyle\sum_j \exp\big(\mu|q_j|\big)$$

Matrix Formulation

- F : Replica filters (rows) $\quad P$: Polynomials (columns)
  G : Gaussian derivatives (rows) $\quad M$: Monomials (columns)
  s : Input signal (column) $\quad C$: Estimated coefficients
- The defining equations are:

$$F = PG \quad \text{where} \quad P = MC$$

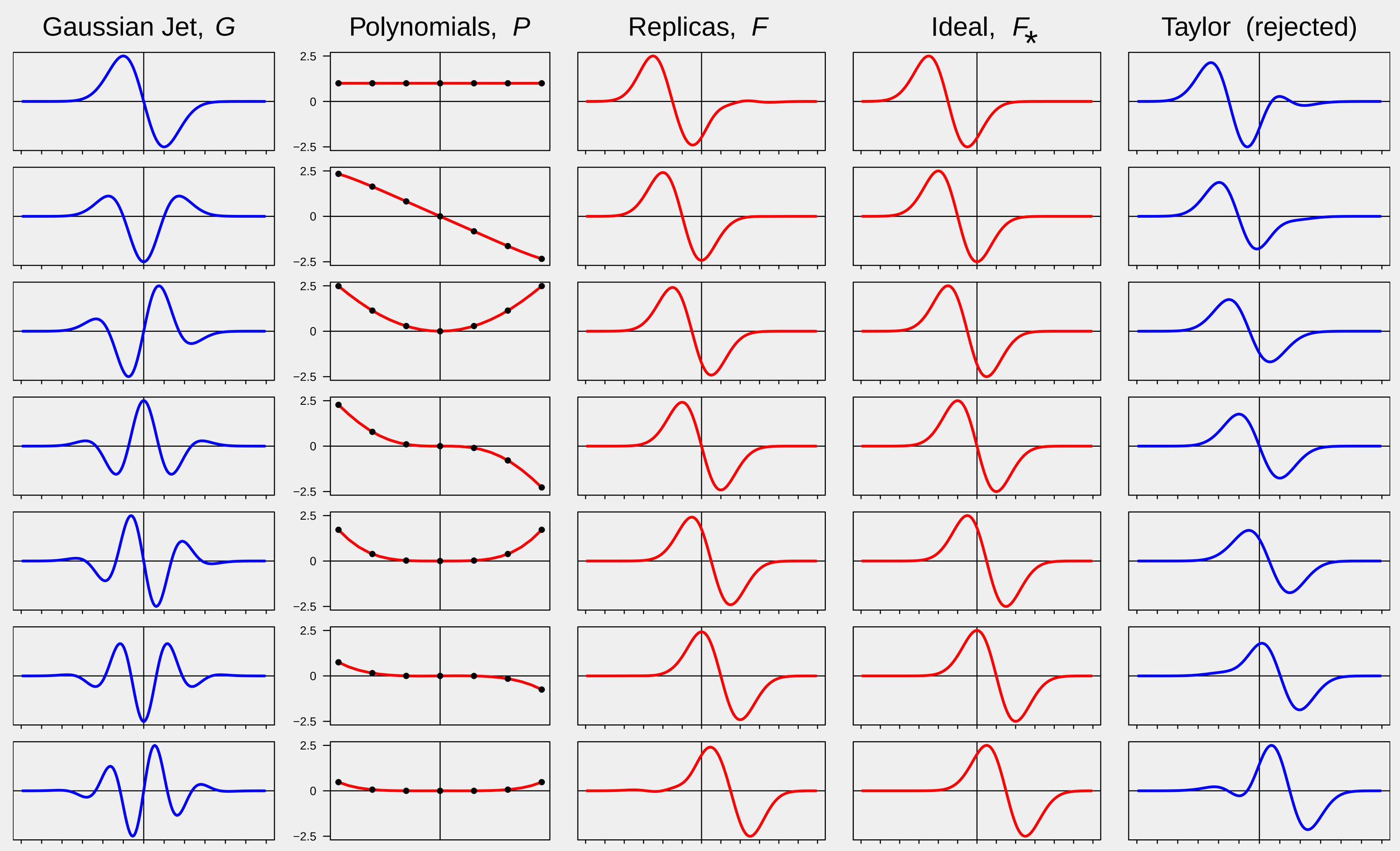
- The 'filter design' problem is to estimate C, given ideal filters $F_\star$.
- Least-squares solution is the pseudo-inverse of a Kronecker product:

$$\operatorname{vec}(C) = \big(G^\top \otimes M\big)^+ \operatorname{vec}(F_\star)$$

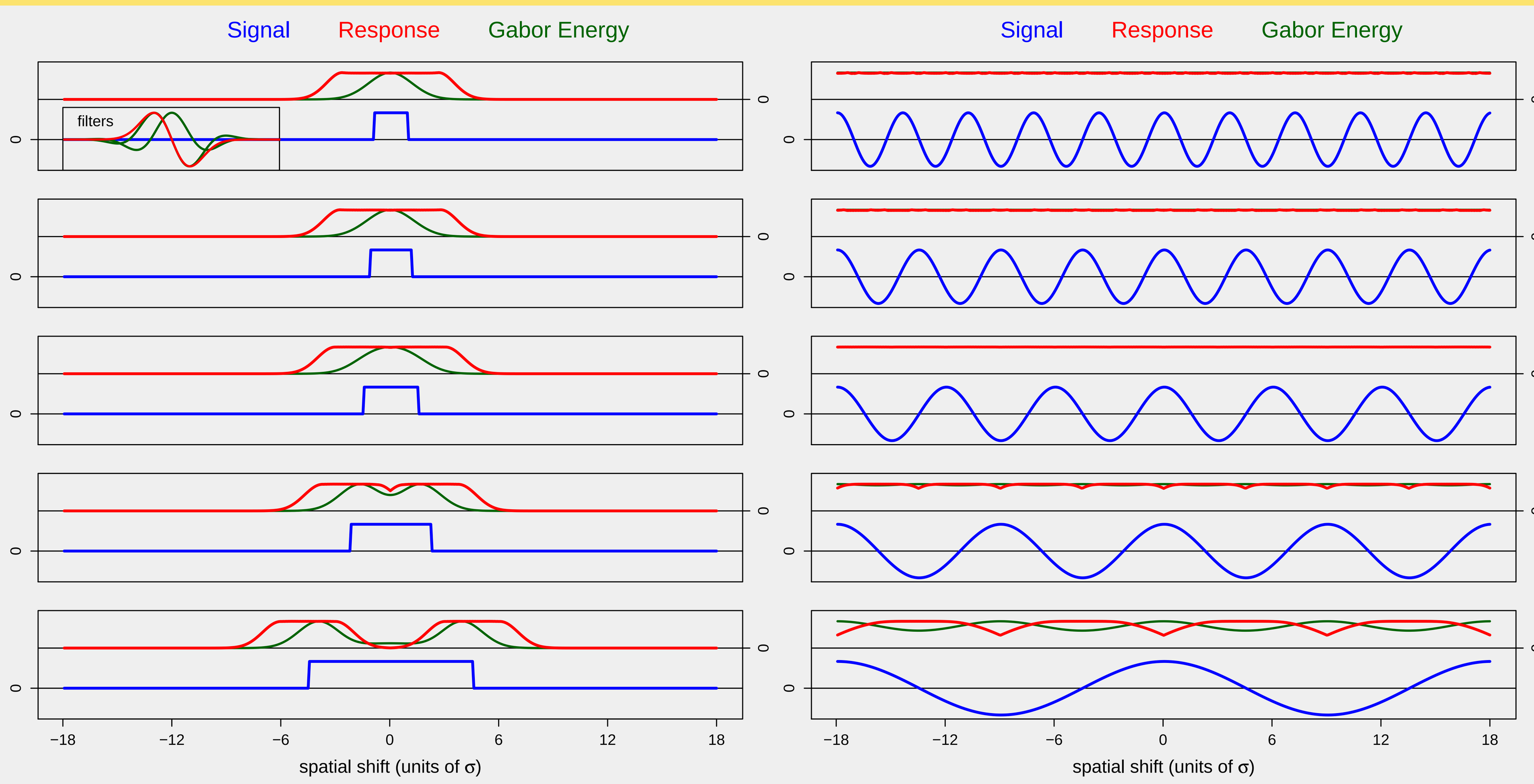
- The replica response q is a linear transformation of the jet response Gs:

$$q = Fs = P(Gs)$$

Complete Example



Bar and Grating Responses



Two-Dimensional Filters

- The image gradient at 2D position x, and scale σ, is estimated as:

$$\nabla S(x) = \big[G_1(x,\sigma,0)\cdot S,\ G_1(x,\sigma,\pi/2)\cdot S\big]$$

- The replica filters can find the local maximum of the gradient:

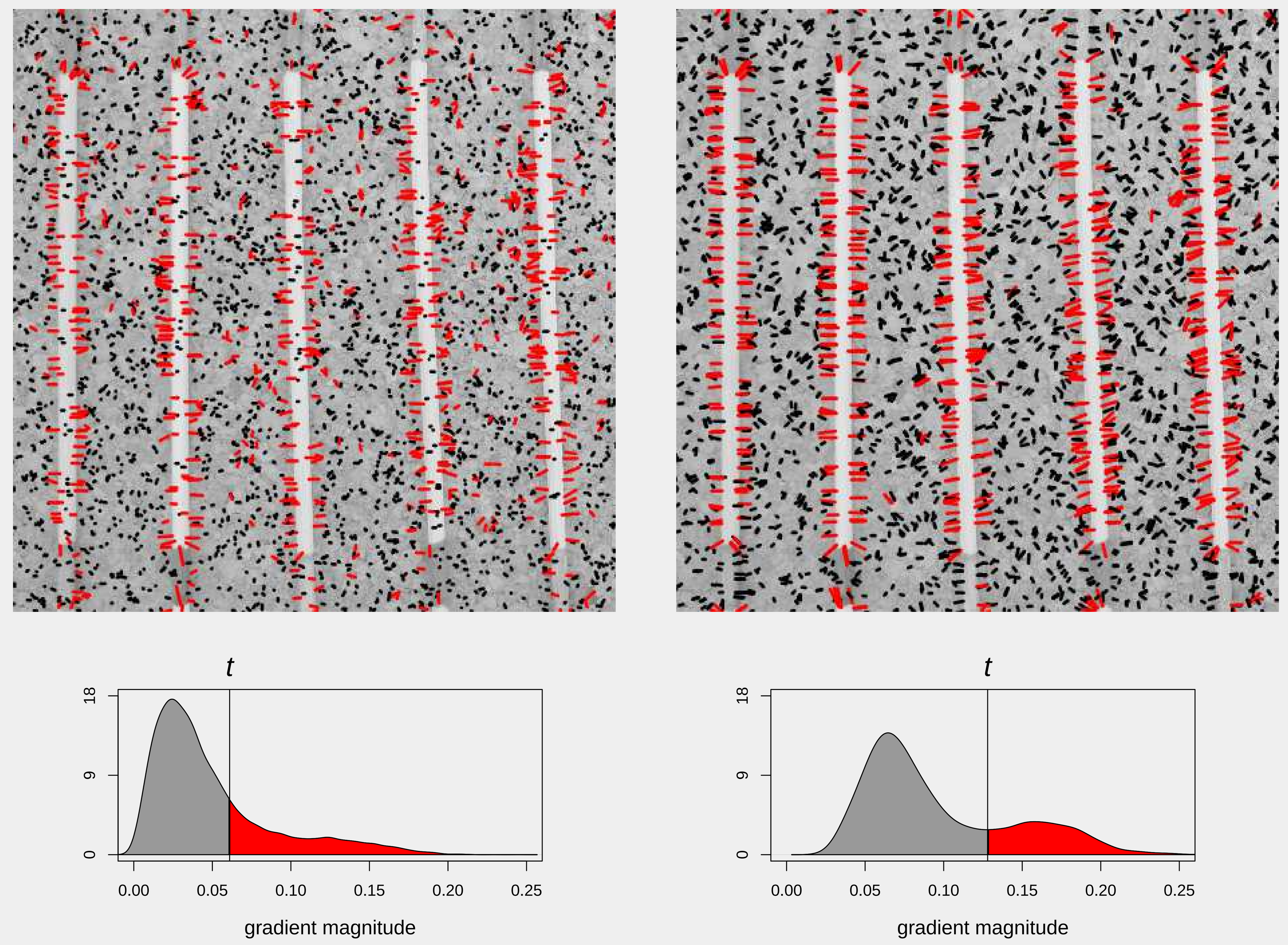
$$u_\star = \underset{u\in\mathcal{F}}{\arg\max}\,\big|\nabla S(x+u)\big| \quad \text{where} \quad \mathcal{F} = \{\,u : |u| < \rho\sigma\,\}$$

- The disk $\mathcal{F}$ is the receptive field of the mechanism.

Natural Image Experiments

- Make a coarse ($\sim$1% pixels) random sampling of the gradient.
- Compare $\nabla S(x)$ to $\nabla S(x+u_\star)$, using $\sigma = 3$ pixels and $\rho = 1.5$.
- Split each distribution at $\mathrm{P}\big(|\nabla S| < t\big) = 0.75$; plot strong vectors in red.

Raw Gradient $\nabla S(x)$ | Adjusted Gradient $\nabla S(x+u_\star)$



- The undersampled structure is better represented by $\nabla S(x+u_\star)$.
- The true gradient distribution is bi-modal (boundaries plus texture).
- But the randomly-placed filters are unlikely to fall on the boundaries.
- If x is within $\rho\sigma$ of an edge, then $|\nabla S(x+u_\star)| \gg |\nabla S(x)|$.

Conclusions

- A shift-insensitive response can be obtained from the Gaussian jet.
- The signal structure can be represented geometrically.
- The new model is steerable, and works in any number of dimensions.
- High-order filters, as seen in neural data, are needed in the jet basis.

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