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Résumé — This work is motivated by the modeling of blood flows through the beating myocardium, namely cardiac perfusion. Perfusion is modeled here as a flow through a poroelastic medium. The main contribution of this study is the derivation of a general poroelastic model valid for a nearly incompressible medium which experiences finite deformations, illustrated by several numerical examples [2].

Mots clés — fluid-solid interaction, porous media, large strain, myocardium perfusion.

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1 Background and motivation

Despite recent advances on the anatomical description and measurements of the coronary tree and on the corresponding physiological, physical and numerical modeling aspects, the complete modeling and simulation of blood flows inside the large and the many small vessels feeding the heart is still out of reach. Therefore, in order to model blood perfusion in the cardiac tissue, we must limit the description of the detailed flows at a given space scale, and simplify the modeling of the smaller scale flows by aggregating these phenomena into macroscopic quantities, by some kind of "homogenization" procedure. To that purpose, the modeling of the fluid-solid coupling within the framework of porous media appears appropriate.

Poromechanics is a simplified mixture theory where a complex fluid-structure interaction problem is replaced by a superposition of both components, each of them representing a fraction of the complete material at every point. It originally emerged in soils mechanics with the work of Terzaghi [4], and Biot [1] later gave a description of the mechanical behavior of a porous medium using an elastic formulation for the solid matrix, and Darcy's law for the fluid flow through the matrix. Finite strain poroelastic models have already been proposed (see references in [2]), albeit with *ad hoc* formulations for which compatibility with thermodynamics laws and incompressibility conditions is not established.

2 Methods

We introduce a general poroelastic formulation valid for finite strains and compatible with incompressibility, as these two features are deemed to be important in the modeling of living tissues. We follow the strategy – presented in [3] in a linear framework – of deriving the formulation from an appropriate free energy functional, which is crucial to guarantee that fundamental thermodynamics principles are satisfied.
We then propose a numerical procedure to solve the resulting system of equations : a fixed point algorithm iteratively couples the "solid" and the " fluid" parts of this system.

3 Main results

We illustrate the behavior of this poroelastic model by several numerical examples. The first test cases consist of typical poroelastic configurations : swelling (see Figure 1) and complete drainage.

Finally, a simulation of cardiac perfusion is presented in an idealized left ventricle embedded with active fibers. Results show the complex temporal and spatial interactions of the muscle and blood, repro-

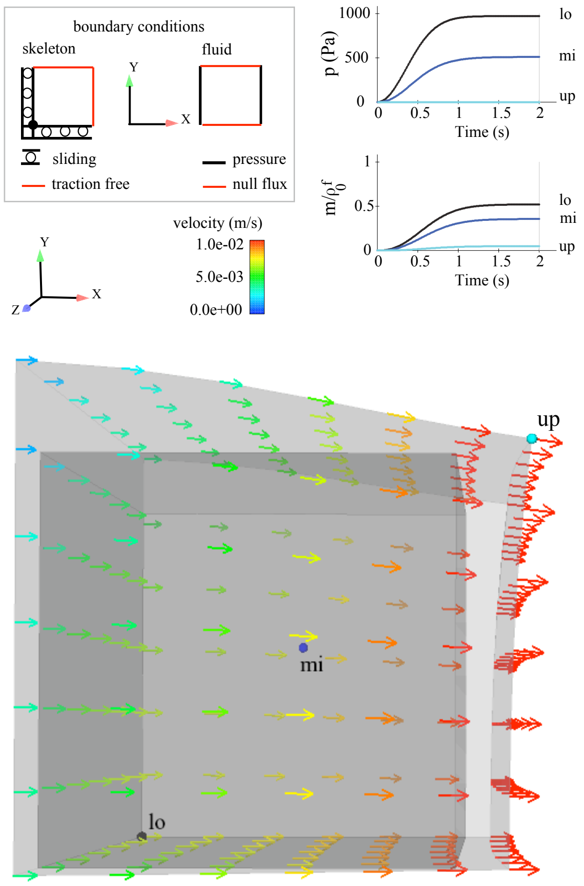

FIGURE 1 – Swelling test of a cube. No external force is applied on the skeleton but a fluid pressure gradient is imposed between two opposite faces whereas a null flux condition is applied on the four other faces. Dark grey represents the initial cube, and light grey the deformed cube. The arrows are the velocity vectors, colored by their magnitude. Pressure and mass are plotted against time for three points (the lower point attached to the inlet face in black, the middle point in blue and the upper point attached to the outlet face in cyan).

ducing several key phenomena observed in cardiac perfusion.

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