Arc segmentation in linear time

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Abstract. A linear algorithm based on a discrete geometry approach is proposed for the detection of digital arcs and digital circles using a new representation of them. It is introduced by inspiring from the work of Latecki [1]. By utilizing this representation, we transform the problem of digital arc detection into a problem of digital straight line recognition. We then develop a linear method for arc segmentation of digital curves.

1 Introduction

The digital arcs and circles are basic geometric objects of which the recognition is an interesting topic. In the literature, some methods have been proposed for the recognition of digital circles. Nakamura et al. [2] proposed a recursive algorithm for determining the center of a digital circle, but its complexity is exponential in the general case. Kim [3, 4] proposed several results on digital disks. The first result [3] detects if a set of grid points in a $N \times N$ image is a digital disk with complexity $O(n^3)$. The second result [4] reduces this task to $O(n^2)$. Based on the classical separating arc problem, Kovalevsky [5] (resp. Fisk [6]) proposed an algorithm for the recognition of a digital disk in $O(n^2 \log n)$ (resp. $O(n^2)$) time. Coeurjolly [7] transformed the problem of circle recognition into a problem of search a 2D point that belongs to the intersection of n^2 half-plane. Sauer [8] (resp. Damaschke [9]) presented a linear algorithm to decide if a curve is a digital circle (resp. arc) based on Megiddo's algorithm [10]. Worring [11] introduced a digital circle segmentation method by using a fixed size window process. Roussillon [12] proposed a linear algorithm of circle recognition in 3 particular cases.

We present in this paper a linear method for the detection of digital circles or digital arcs based on a discrete geometry approach. Firstly, a polygonalization is applied in linear time on the input curve [13]. Secondly, we use a transform proposed by Latecki et al. [1] to represent the obtained polygon in a novel space called ***tangent space***. We show that a sequence of chords of a circle will correspond to a sequence of collinear points in the tangent space. So the problem of arc/circle detection can be considered as a problem of digital straight line recognition.

This paper is organized as follows. Section 2 recalls some definitions concerning digital circles and blurred segments. The next section presents a technique to transform an arc into the tangent space and proposes some principal properties of the arc in this representation. Section 4 proposes a linear algorithm for the detection of digital arcs or digital circles. In Section 5, we present a linear method for the segmentation of a curve into arcs and some experimentations.

2 Discrete circle and blurred segment

Discrete circle: In the literature, there exist several definitions of discrete circle. They are proposed by considering a real circle on the grid digitization. The difference among them is the process of discretization. Nakamura et al. [2] considered a discretization of a real circle by the points of $\mathbb{Z}^2$ that are the nearest points of that circle. Kim [3] proposed a definition of discrete circle as a boundary of a digital disk superimposed by a real circle. Andres [14] used an arithmetic approach to define a digital circle as a sequence of points superimposed by a ring.

Discrete line and blurred segment: The notion of blurred segment [13] was introduced from the notion of arithmetical discrete line. An ***arithmetical discrete line***, noted $D(a, b, \mu, \omega)$, is the set of points $(x, y) \in \mathbb{Z}^2$ that verifies: $\mu \leq ax - by < \mu + \omega$ with a main vector (b, a), lower bound μ and thickness ω . A ***width*** ν ***blurred segment*** (BS) is a set of points $(x, y) \in \mathbb{R}^2$ that is optimally bounded (see [13] for more details) by a discrete line $D(a, b, \mu, \omega)$ verifying $\frac{\omega-1}{max(|a|,|b|)} \leq \nu$. Fig. 1 shows a BS of with 1.25 (the sequence of gray points) whose optimal bounding line is $\mathcal{D}(5, 8, -8, 11)$. A linear method for recognition of BS has been also proposed in [13].

3 Arc representation in tangent space

Modified tangent space representation: We recall in this section some notions concerning a representation of a polygon in the tangent space. Latecki et al. [1] proposed ***the tangent space representation*** as a tool of similarity measure for shape matching. Inspired from this representation, we propose a modified tangent space to represent a polygonal curve. The difference is that we do not normalize the axis $0x$ in the tangent space. Let $C = \{C_i\}_{i=0}^{n}$ be a polygonal curve, $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$ and l_i - the length of the line segment $C_iC_{i+1}, i \in \{0, \ldots, n-1\}$. If C_{i+1} is on the right of $\overrightarrow{C_{i-1}C_i}$ then $\alpha_i > 0$, otherwise $\alpha_i < 0$. From now, we denote $P.x$ (resp. $P.y$) to indicate the x (resp. y)-coordinate of point P. We consider a transformation that associates the polygon C of $\mathbb{Z}^2$ to a polygon of $\mathbb{R}^2$ that is constituted by line segments $T_{i2}T_{(i+1)1}, T_{(i+1)1}T_{(i+1)2}$ for i from 0 to $n-1$ with

$T_{02} = (0, 0)$,

$T_{i1} = (T_{(i-1)2}.x + l_{i-1}, T_{(i-1)2}.y)$, i from 1 to n,

$T_{i2} = (T_{i1}.x, T_{i1}.y + \alpha_i)$, i from 1 to $n-1$.

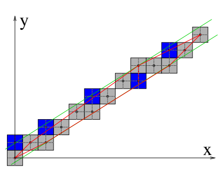


Fig. 1. A BS [13].

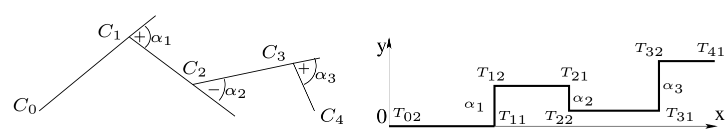


Fig. 2. Transformation to the tangent space: on the left, the input polygonal curve and on the right, its tangent space representation.

Properties of arcs in the modified tangent space representation The theorem below allows us to study the properties of a representation in the tangent space of a polygon that corresponds to an arc or a circle.

Theorem 1. *Let* $C = \{C_i\}_{i=0}^{n}$ *be a polygon,* $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$. *The length of* C_iC_{i+1} *is* l_i*, for* $i \in \{0, \ldots, n-1\}$. *The vertices of* C *are on a real arc of radius* R *and of center* O *such that* $\angle C_iOC_{i+1} \leq \frac{\pi}{4}$ *for* $i \in \{1, \ldots, n-1\}$. *This results below is obtained.*

$$\frac{1}{R} < \frac{\alpha_i}{\frac{l_i + l_{i+1}}{2}} < \frac{1}{0.9742979R}$$

Proof. Let us consider figure 3. We have $\alpha_i = \angle C_iOH_{i-1} + \angle C_iOH_i$. We denote that $\alpha_{i1} = \angle C_iOH_{i-1}$ and $\alpha_{i2} = \angle C_iOH_i$. Moreover, $\angle C_1OH_0 = \frac{\angle C_0OC_1}{2} \leq \frac{\pi}{8}$, $\angle C_1OH_1 = \frac{\angle C_1OC_2}{2} \leq \frac{\pi}{8}$. In addition, we have $\sin \angle C_1OH_0 = \frac{l_0}{2R}$, $\sin \angle C_1OH_1 = \frac{l_1}{2R}$. Therefore, $\frac{l_0+l_1}{2R} = \sin\alpha_{11} + \sin\alpha_{12}$. Similarly, we have $\frac{l_{i-1}+l_i}{2R} = \sin\alpha_{i1} + \sin\alpha_{i2}$, for $i \in \{1, \ldots, n-1\}$. Because of $x \geq \sin x \geq x - \frac{x^3}{6}$ with $x > 0$, we have $\alpha_{i1} > \sin\alpha_{i1} > \alpha_{i1}(1 - \frac{\alpha_{i1}^2}{6}) > \alpha_{i1}(1 - \frac{\frac{\pi}{8}^2}{6}) > 0.9742979\alpha_{i1}$. Similarly $\alpha_{i2} > \sin\alpha_{i2} > 0.9742979\alpha_{i2}$. Therefore, we have $\alpha_i > \frac{l_{i-1}+l_i}{2R} > 0.9742979\alpha_i$. This theorem is proved.

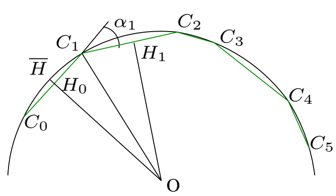


Fig. 3. Property of a set of sequential chords of a partial circle.

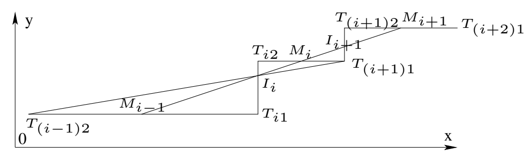


Fig. 4. Property of a polygon in our modified tangent space representation.

This theorem allows to deduce that the corresponding curve of midpoints of $T_{(i-1)2}T_{i1}$, $1 \leq i \leq n$ in the tangent space of the curve C is quasi collinear. From now on, the midpoint curve is called MpC. In addition, the more $\sin\alpha_i$ closes to α_i, $1 \leq i < n$, the more MpC is collinear. Therefore, we can decide if a digital curve approximates an arc of circle by verifying the collinearity of its MpC in the tangent space. A qualitatif study on this approximation will be also considered in Section 5.

4 Arc segmentation

Detection of digital arcs Thanks to theorem 1, we introduce now an heuristic algorithm for (see algo. 1) deciding if a digital curve is an arc. Our main idea is to work on the representation of a digital curve in the modified tangent space. In this representation, the set of midpoints $MpC = \{M_i\}_{i=0}^{n-1}$ (see the above section) will be constructed. And we will use a linear procedure [13] to test the collinearity of these points. If the response is positive, we consider that the input digital curve is a digital arc (a partial circle).

A higher value of the range of vertical ordinate in the tangent space leads to false positive detection as an example; an helix can be detected as an arc. So, to avoid this problem, the maximal difference of vertical ordinate is fixed to 2π for detecting an arc. The input parameter α_{max} of Algorithm 1 allows to control the obtained error.Parameter ν_1 is used for polygonalization by using the recognition of blurred segments. The input parameter ν_2 is used as the width in the algorithm for recognition of blurred segments [13] to test the collinearity of the midpoint set in the representation of the tangent space. In practice, α_{max} (resp. ν_1) is chosen as $\frac{\pi}{4}$ (resp. 1). In addition, ν_2 can be chosen as a fixed value from 0.15 to 0.25 without problem.

This heuristic algorithm detect well circular arc. In Section 5 (see corollary 1), we consider an adaptive estimation of ν_2 to guarantee the quality of detected arcs. It is

Algorithm 1: Detection of a digital arc/circle

Data: $P = \{P_i\}_{i=0}^{n}$ digital curve, α_{max} - maximal admissible angle, ν_1- width of BS for polygonalization, ν_2- width of BS for collinear test [1]
Result: ARC (resp. CIRCLE): C is a digital arc (resp. circle), FALSE if not.
begin
  Use algorithm [13] to polygonalize P into BS of width ν_1: $C = \{C\}_{i=0}^{m}$;
  Represent C in the modified tangent space by $T(C)$; $BS = \emptyset$;
  if *there exists* i *such that* $|T_{i2}.y - T_{i1}.y| > \alpha_{max}$ **then return** FALSE;
  Determine midpoint set $MpC = \{M_i\}_{i=0}^{m-1}$ of $\{T_{i2}T_{(i+1)1}\}_{i=0}^{m-1}$; $i = 1$;
  while $|M_i.y - M_0.y| \leq 2\pi$ **do** $BS = BS \cup M_i$; i++;
  Use algorithm [13] to verify if BS is a blurred segment of width ν_2;
  if BS *is a blurred segment of with* ν_2 **then**
    if $|M_{m-1}.y - M_0.y| == 2 * \pi$ **then return** CIRCLE;
    else return ARC;
  else return FALSE;
end

evident that we can lightly change Algorithm 1 to check the condition of Corollary 1 and Proposition 2 in linear time also.

The algorithms of polygonalization and of blurred segment recognition are linear. The complexity of the tangent space transform, and the construction of midpoint curve $MpC = \{M_i\}_{i=1}^{m-1}$ is in $O(m)$. Because $m << n$, the total complexity of our detection method is then in $O(n)$.

Segmentation of curves into digital arcs Based on the above idea for detection of an arc, we then develop a linear method for the segmentation of digital arcs by using a width ν blurred segment [13] polygonalization on the curve of midpoints. Its main idea, illustrated in figure 5, is based on the polygonalization of the midpoint curve (Fig. 5.e). Contrariwise to Algorithm 1, we polygonalize the midpoint curve MpC in spite of recognizing if MpC is a BS and then each line segment corresponds to a circular arc.

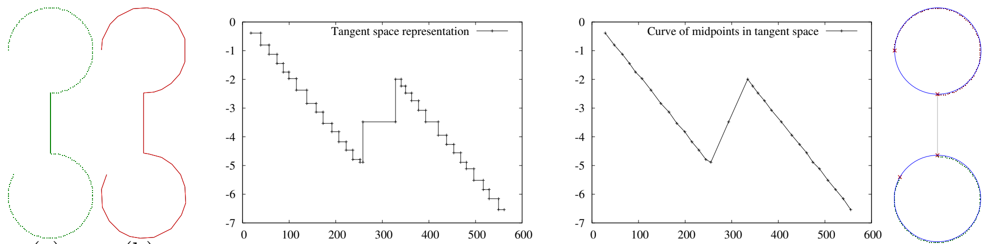
Experimental results and application to real images We have implemented this linear method. An example of a curve segmented into arcs is presented in Fig. 5. Firstly, the approximating polygon (see Fig. 5.b) is constructed from the input curve in Fig. 5.a. After that, we transform it into the modified tangent space representation (see Fig. 5.c). Then, by polygonalizing the curve of midpoints in this tangent space (see Fig. 5.d), the corresponding arcs can be detected (see Fig. 5.e).

Fig. 6 shows an experimentation on technical drawing images. Figs. 6.a, 6.d are input images. Figs. 6.c, 6.f present the extracted arcs from the borders presented in 6.b, 6.e. Our method gives good results on this type of images which frequently contain arc and circle primitives. Fig. 7 presents our obtained result with a real image.

5 Study of quasi collinearity property of midpoint curve

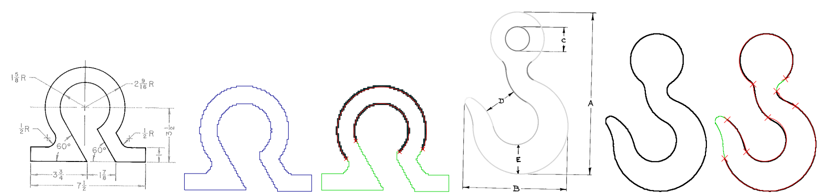
Algorithm 1 works well in practice. However, we have a problem to estimate error approximation in general case when the value of ν_2 is fixed. In this section, we present a first study concerning the utilization of this algorithm where ν_2 is chosen adaptively.

[1] By default, $\alpha_{max} = \frac{\pi}{4}$, $\nu_1 = 1$ for normal curves and $\nu_2 = 0.2$ (see algo. 1).



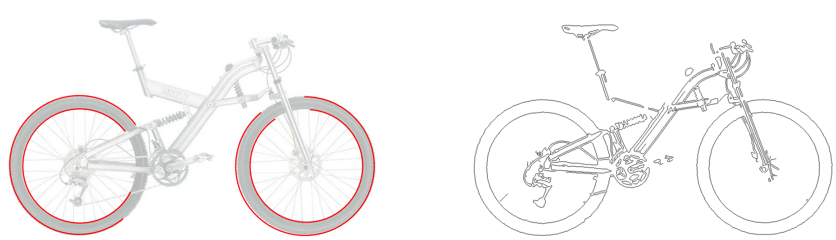
(a) (b) (c) Tangent space representation (d) Curve of midpoints (e)

Fig. 5. Arc segmentation results on a digital curve: (a) Input curve, (b) Approximated polygon, (c) Results of arc segmentation.



(a) Input image (b) Outline (c) Result (d) Input image (e) Outline (f) Result

Fig. 6. Experimentation on technical drawing images.



(a) Detected arcs on the input image (b) Extracted edge using Canny filter

Fig. 7. Experimentation on a real image at width 2.

Let us suppose that $\alpha_{max} = max\{\alpha_i\}_{i=1}^n$. Let us suppose that R_i is the radius of the approximating circle that passes through 3 points C_{i-1}, C_i, C_{i+1}; $\alpha_{i1} = \angle H_{i-1}OC_i$, $\alpha_{i2} = \angle H_iOC_i$ (see Fig. 3). We suppose that $\alpha_{i1}, \alpha_{i2} \leq \frac{\pi}{8}$ for $i = 1, \ldots, n-1$ to guarantee the condition $\sin x \simeq x$ in Theorem 1. It means that we consider the condition $\sin x \simeq x$ with $x \in [0, \frac{\pi}{8}]$. Therefore, we have $\alpha_i \leq \frac{\pi}{4}$.

Comparison of radius of local circumcircles to the global radius

Proposition 1. *Let $C = \{C_i\}_{i=0}^n$ be a polygon, $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$. The length of C_iC_{i+1} is l_i, for $i \in \{0, \ldots, n-1\}$. We denote O_i (resp. R_i) respectively the center (resp. the radius) of circumcirle that passes through 3 points C_{i-1}, C_i, C_{i+1}, H_i the projection of O_i on C_iC_{i+1}. Suppose that $R_i - OH_i \leq h$ for $i \in \{1, \ldots, n-1\}$. This results below is obtained. $R_i\alpha_i \geq \frac{l_{i-1}+l_i}{2} \geq R_i\alpha_i - 0.3377h\alpha_i$*

Proof. We denote $\alpha_{i1} = \angle H_{i-1}O_iC_i$, $\alpha_{i2} = \angle H_iO_iC_i$ (see Fig. 3). Firstly, $\cos\alpha_{i1} = 1 - 2\sin^2\frac{\alpha_{i1}}{2} = \frac{OH_i}{R_i} \geq \frac{R_i - h}{R_i} = 1 - \frac{h}{R_i}$. In addition, thanks to $\alpha_{i1} \leq \frac{\pi}{8}$ and $\frac{\sin(x)}{x}$ is decreasing in $[0, \frac{\pi}{16}]$, we have $\sin x \geq x\frac{\sin\frac{\pi}{16}}{\frac{\pi}{16}}$. Therefore $\frac{h}{R_i} \geq 2\sin^2\frac{\alpha_i}{2} \geq 2\cdot\left(\frac{\sin\frac{\pi}{16}}{\frac{\pi}{16}}\right)^2\left(\frac{\alpha_{i1}}{2}\right)^2 > \frac{0.9872}{2}\alpha_{i1}^2$. Similarly, we have $\frac{h}{R_i} > \frac{0.9872}{2}\alpha_{i2}^2$, for $i \in \{1, \ldots, n-1\}$ (1).

In addition, we have this remark $x \geq \sin x \geq x - \frac{x^3}{6}$ with $\frac{\pi}{4} \geq x \geq 0$. So, $\alpha_i \geq \sin\alpha_{i1} + \sin\alpha_{i2} > \alpha_{i1} + \alpha_{i2} - \frac{1}{6}(\alpha_{i1}^3 + \alpha_{i2}^3) = (\alpha_{i1} + \alpha_{i2})(1 - \frac{1}{6}(\alpha_{i1}^2 - \alpha_{i1}\alpha_{i2} + \alpha_{i2}^2)) = \alpha_i(1 - \frac{1}{6}(\alpha_{i1}^2 - \alpha_{i1}\alpha_{i2} + \alpha_{i2}^2)) \geq \alpha_i(1 - \frac{1}{6}(\alpha_{i1}^2 + \alpha_{i2}^2)))$, for $i \in \{1, \ldots, n-1\}$ (2).

Thanks to (1) and (2), we obtain $\sin\alpha_{i1} + \sin\alpha_{i2} > \alpha_1(1 - \frac{1}{3\cdot 0.9872}\frac{h}{R})$, for $i \in \{1, \ldots, n-1\}$ (3).

Moreover, we have $\alpha_i = \alpha_{i1} + \alpha_{i2}$ and $\alpha_{i1}, \alpha_{i2} \leq \frac{\pi}{8}$. In addition, we have $\sin\alpha_{i1} = \frac{l_{i-1}}{2R}$, $\sin\alpha_{i2} = \frac{l_i}{2R}$. Therefore, we have $\frac{l_{i-1}+l_i}{2R} = \sin\alpha_{i1} + \sin\alpha_{i2}$, for $1 \leq i < n$ (4).

Thanks to (3) and (4), we obtain $R_i\alpha_i \geq \frac{l_{i-1}+l_i}{2} \geq R_i\alpha_i(1 - \frac{1}{3\cdot 0.9872}\frac{h}{R_i}) = \alpha_i(R_i - \frac{h}{3\cdot 0.9872}) \Leftrightarrow R_i\alpha_i \geq \frac{l_{i-1}+l_i}{2} \geq R_i\alpha_i - 0.3377h\alpha_i$.

Now we consider a set of midpoints MpC is a blurred segment whose horizontal width is ϵ. Let us suppose that the slope of this blurred segment is $\frac{1}{R}, R \in \mathbb{R}$.

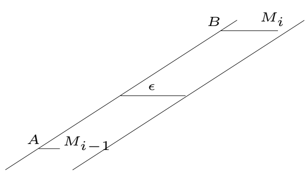


Fig. 8.

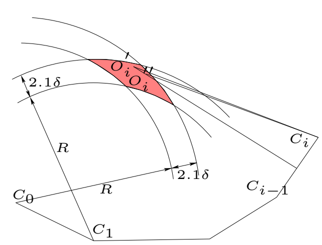


Fig. 9. $\{O_i^{'}, O_i^{''}\}_{i=1}^{n-1}$ is in a compact zone.

Let us consider Fig. 8, A and B respectively are horizontal projection of M_{i-1} and M_i on the left leaning line. We have: $M_i.x - M_{i-1}.x = (B.x - A.x) + M_iB - AM_{i-1} = R\alpha_i + M_iB - AM_{i-1}$. Because of M_i and M_{i-1} are limited by 2 leaning lines, we have $M_iB \leq \epsilon$ and $AM_{i-1} \leq \epsilon$, so $-\epsilon \leq M_iB - AM_{i-1} \leq \epsilon$. Therefore, we have $R\alpha_i + \epsilon \geq \frac{l_{i-1}+l_i}{2} \geq R\alpha_i - \epsilon$. This double inequations can be rewrited as $\frac{l_{i-1}+l_i}{2} = R\alpha_i + \epsilon_i$, where $\epsilon_i \in [-\epsilon, \epsilon]$.

Thank to proposition 1, we have: $R_i\alpha_i > R\alpha_i + \epsilon_i > R_i\alpha_i - 0.3377h\alpha_i \Leftrightarrow R_i > R + \frac{\epsilon_i}{\alpha_i} > R_i - 0.3377h$. So, $\frac{\epsilon}{\alpha_i} < R_i - R < \frac{\epsilon}{\alpha_i} + 0.3377h$. So, we have corollary 1.

Corollary 1 *Let $C = \{C_i\}_{i=0}^n$ be a polygon, $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$. The length of C_iC_{i+1} is l_i, for $i \in \{0, \ldots, n-1\}$. The set of midpoints $\{M_i\}_{i=0}^{n-1}$ is a blurred segment whose horizontal width is ϵ. We denote O_i (resp. R_i) respectively the center (resp. the radius) of circumcirle that passes through 3 points C_{i-1}, C_i, C_{i+1}, H_i the projection of O_i on C_iC_{i+1}. Suppose that $R_i - OH_i \leq h$ for $i \in \{1, \ldots, n-1\}$. This results below is obtained.* $0 < R_i - R < \frac{\epsilon}{\alpha_i} + 0.3377h \leq \frac{\epsilon}{\min\{\alpha_i\}_{i=1}^n} + 0.3377h$.

Localization of centers of local circumcircles In this section, we consider the convergence of local circumcircle centers if this condition below is satisfied: $|R_i - R| \leq \delta$, $R \in \mathbb{R}$, $1 \leq i, j \leq n-1$. Let us consider Fig. 10. To prove the convergence of centers of local circumcircle, we show this property for an approximation of a half of circle.

Proposition 2. *Let us consider a sequence of points $\{C\}_{i=0}^n$. There exist R and δ such that $R, \delta \in \mathcal{R}, 0 \leq R_i - R \leq \delta, i = 1, \ldots, n-1$. Suppose that $\angle C_kC_jC_{j+1} > \frac{\pi}{2}$ for $k \in \{0, 1\}, k < j < n$. Therefore, we have this property $0 \leq R_i^{'} - R \leq \delta$, $0 \leq R_i^{''} - R \leq \delta$, for $1 \leq i \leq n-1$.*

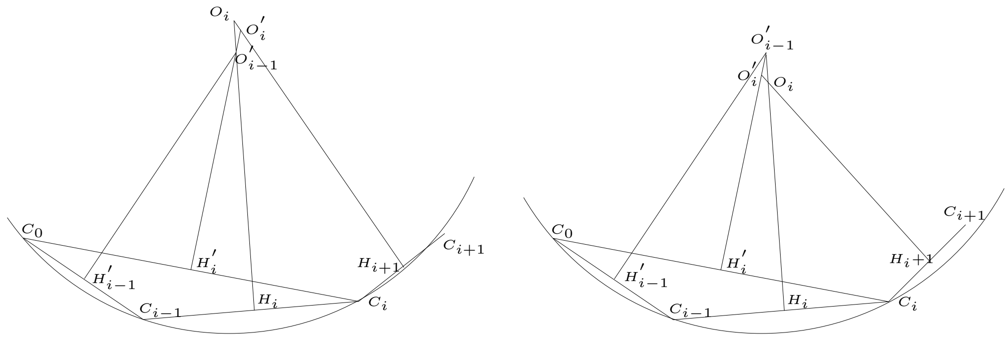
Proof. We denote $O_i^{'}$ (resp. $O_i^{''}$) and $R_i^{'}$ (resp. $R_i^{''}$) the centers and radius of circumcircles that passes through 3 points C_0 (resp. C_1), C_i, C_{i+1}. Firstly, we have a trivial remark: the perpendicular bisector of C_iC_k is between that of C_iC_j and C_jC_k. Now, we prove this proposition by induction.

Because of $R_1^{'} = R_1$, the proposition is true with $i = 1$. Suppose that $|R_{i-1}^{'} - R| \leq \delta$. Let us consider Fig. 10. We denote $H_i^{'}$, H_i are respectively the midpoint of C_0C_i, $C_{i-1}C_i$. Thank to the above remark, we have $O_{i-1}^{'}H_i^{'}$ is between $O_{i-1}^{'}H_{i-1}^{'}$ and $O_{i-1}^{'}H_i$. We consider now the position of C_{i+1} with the circle of center $O_{i-1}^{'}$, of radius $R_{i-1}^{'}$. The proposition if trivial if C_{i+1} is on this circle because of $R_i^{'} = R_{i-1}^{'}$. If C_{i+1} is outside of this circle (see Fig. 10.a), we then deduce $O_{i-1}^{'} \in [O_i^{'}H_i^{'}]$, $O_i^{'} \in [O_iH_{i+1}]$. Therefore, we have $O_{i+1}C_i > O_i^{'}C_i$, $O_i^{'}C_i > O_{i-1}^{'}C_i$. It means that $R_{i+1} > R_i^{'} > R_{i-1}^{'}$. Thank to $0 \leq R_{i+1} - R \leq \delta$ and $0 \leq R_{i-1}^{'} - R \leq \delta$, we have $0 \leq R_i^{'} - R \leq \delta$. In other case (see Fig. 10.b), by applying the same arguments, we obtain $R_{i+1} < R_i^{'} < R_{i-1}^{'}$. Therefore, we have $0 \leq R_i^{'} - R \leq \delta$, for $1 \leq i \leq n-1$. By replacing C_0 by C_1 and using the same argument, we have $0 \leq R_i^{''} - R \leq \delta$, for $1 \leq i \leq n-1$.

We have a simple remark: a triangle ABC that satisfies $\angle ABC \geq \frac{7\pi}{8}$ have : $AC > BC + \cos\frac{\pi}{8} \cdot AB$. It is trivial because of $AC^2 = BC^2 + AB^2 - 2BC \cdot AB \cos\angle ABC > BC^2 + \cos^2\frac{\pi}{8}AB^2 + 2BC \cdot AB\cos\frac{\pi}{8}$. Therefore, we have: $O_i^{'}O_i^{''} \leq \frac{|R_i^{'} - R_i^{''}|}{\cos\frac{\pi}{8}} \leq \frac{\delta}{\cos\frac{\pi}{8}} \leq 1.1\delta$, for $i = 1, \ldots, n-1$. Thanks to this result and proposition 2, it is trivial now to show that set of center $O_i^{'}$, $O_i^{''}$ is in a compact zone (see Fig. 9).

6 Conclusions

We have presented a linear method for the detection of digital circles or digital arcs. A linear method for the segmentation of a curve into digital arcs is also proposed.



(a) Outside (b) Inside

Fig. 10. Position between C_{i+1} and circumcircle of $C_0C_{i-1}C_i$.

This method is based on a discrete geometry approach. It is simple, easy and robust to implement. A more complete demonstration of the algorithm is under process.

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