Time-Varying Delay Passivity Analysis in 4 GHz Antennas Array Design

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Noname manuscript No.
(will be inserted by the editor)

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the date of receipt and acceptance should be inserted later

Abstract In this paper, a new approach for synchronization of dynamical networks with time-delays is proposed. It is based on stability theory on coupled time-delayed dynamical systems. Some new criteria for the stability analysis which ensure the synchronization of the networks are analytically derived. Conditions for synchronization, in the form of a matrix inequality, are established. It uses the Lyapunov and Krasovskii stability theories. The problem of stabilization is transformed into a linear matrix inequality (LMI) which is easily solved by a numerical toolbox. In this approach, parameter uncertainties are introduced in the network model. Numerical simulations show the efficiency of the proposed synchronization analysis. A network of 4-GHz smart antenna array is used and analyzed in some details. This array provides a control of the direction of the radiation pattern.

Keywords Passivity, Network, time-delay system, LMI, Robust control, milliwave electronics

1 Introduction

Complex networks have recently attracted increasing attention from various fields of science, engineering and biology [1]. The global exponential synchronization of complex time-delayed dynamical networks possessing general topology has been investigated in many contributions. In [2], time-delay dependent linear controllers are designed via Lyapunov stability theory, in [3] an experimental design on chua's circuits has been made. In [4], the authors show that synchronization and desynchronization of a complex dynamical network can be determined by the network topology and the maximum Lyapunov exponent of the individual chaotic nodes. An adaptive version can be found in [5]. Recently, in [6][7], it is proved that synchronization of time-varying dynamical network is completely determined by the inner-coupling matrix. In [8], stability analysis of the networks has been extended using the phase and the frequency of the oscillators. It has been applied for random coupling large scale system. In [9], studies has been made when the network topology is allowed to change.

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Some of these approaches use Linear matrix inequality (LMI)-based techniques to verify the stability (*e.g.* [10],[11]). The main advantage of the LMI-based approaches is that the LMI stability conditions can be solved numerically. A strategy based upon the absolute stability theory and upon Kalman Yakubovich Popov lemma (KYP) can be found in [12]. Following the dissipativity approach introduced by Willems [13], some authors also study the passivity of neural networks. The passivity properties of static multilayer neural networks are studied in [14]. Although there exists a lot of results in linear and nonlinear systems where the passivity of systems with time-delays has not been fully investigated. In [15][16], the authors analyze the passivity of linear time-delay systems. The passivity properties for time-delayed neural networks (DNNs) have been considered in [17]. The same interests have been followed for cyclic oscillators. Another approach is employed in [18], which is based on a piecewise analysis method where some new delay-dependent synchronization stability criteria are derived. The passivity description of oscillators was previously examined by Chua in [19]. The dissipation and its incremental form are determined on a specific class of passive oscillators [20]. Many earlier results in the literature have nevertheless exploited the structure of Lur'e systems in the study of nonlinear oscillations. The use of numerical tools restricted to a linear element in the forward path and to a piecewise linear static element in the feedback path is discussed in [21] and [22]. In a recent paper [23], the authors have developed a method to prove the complete synchronization in networks of coupled limit-cycle or chaotic oscillators with arbitrary connection graphs.

The main goal of this paper is to investigate synchronization dynamics of a general model of complex time-delayed dynamical cyclic or chaotic networks with the use of passivity approach. Time-delay effects are made up of two parts. One part is used to have a certain time-delay between cells in the networks. Another part is composed of multiple time-delay. This part is considered as a perturbation for synchronization. This time-delays between cells are necessary in some cases as in antenna arrays. Only the information on the variation range of the time-delay is needed to analyze the network. A numerical example will be given to show the efficiency of our method.

This paper is organized as follows. In Section I, a model of complex time-delayed dynamical networks is presented, and some definitions related to synchronization of the time-delayed dynamical networks are given. Section III deals with the stability analysis of synchronization manifold in presence of time-delays. In Section IV, some generalities on the field of antenna arrays are given and theoretical results are verified by several simulations. Finally, some concluding remarks are given in Section V.

2 Preliminaries on network of dynamical systems

Consider N systems or oscillators of the following form

$$\begin{aligned} \dot{x}_i &= A(\Delta)x_i + Bu_i^* \\ y_i &= Cx_i, \end{aligned} \tag{1}$$

with the feedback interconnection,

$$u_i^* = ky_i - B_1(\Delta)\sigma(y_i) + u_i, \tag{2}$$

where $i = 1, \cdots, N$. Each system or cell is a Lur'e system with state vector $x_i \in \mathbb{R}^n$, inputs of subsystems $u_i^* \in \mathbb{R}^m$ and outputs of subsystems $y_i \in \mathbb{R}^m$. $\sigma(y_i)$ is a static nonlinearity where $\sigma(.) \in \mathbb{R}$ is a stiffening nonlinearity defined below. Here, we suppose that $B_1(\Delta) > 0$.

Assumption 1 $\sigma(.)$ *is a smooth sector nonlinearity in the sector* $(0;\infty)$*, which satisfies* $\sigma^{'}(0) = \sigma^{''}(0) = 0$*,* $\sigma^{'''}(0) > 0$ *and*

$$\lim_{y_i \to \infty} \frac{\sigma(y_i)}{y_i} = \infty.$$

The model (1) is a continuous-time model. However, matrices $A(\Delta)$ and $B_1(\Delta)$ are not precise but uncertain and time-invariant and belong respectively to $\mathcal{A}$, $\mathcal{B}_1$, polytope of matrices defined by

$$\mathcal{A} = \{A = A(\alpha) \,|\, A(\alpha) = \sum_{j=1}^{n_p} (\alpha_j A_j); \alpha \in \Delta\} \tag{3}$$

$$\Delta = \{\alpha \in \mathbb{R}^{n_p} \,|\alpha_j \geq 0, \forall j \in \{1, ..., n_p\}\, ; \sum_{j=1}^{n_p} (\alpha_j) = 1\}.$$

Matrix $\mathcal{A}$ (resp. $\mathcal{B}_1$) is a convex combination of the matrices A_j (resp. (B_{1j})), $j = 1, ..., n_p$ corresponding to the vertices of the polytope.

Remark 1 All the cells are identical. The considered variations represent the thermal variations of the components and the variations due to the process of manufacturing.

Definition 1 The system (1) is called dissipative if there exists a supply function $V_i(x) \geq 0$ such that

$$V_i(x_i(t)) - V_i(x_i(0)) \leq \int_0^t y_i(\mu)^T u_i^*(\mu) d\mu. \tag{4}$$

In order to perform a robust criteria for the synchronization schemes, the following assumption must be introduced.

Assumption 2 *Each system (1) with the feedback (2) is supposed to be a dissipative oscillator, see [21] for another definition. A dissipative oscillator satisfies two conditions :*

First, the feedback system satisfies the dissipation inequality

$$\dot{V}_i(x_i) \leq (k - k^*_{passive}) y_i^T y_i - y_i^T B_1(\Delta)\sigma(y_i) + y_i^T u_i, \tag{5}$$

where $V_i(x_i) = \frac{1}{2} x_i^T P x_i$ *with* $P = P^T > 0$, $PA_j + A_j^T P \leq 0$ *for* $j \in \{1, \cdots, n_p\}$ *with* $PB = C^T$. $V_i(x_i)$ *represents the storage function associated to the feedback system.* $k^*_{passive}$ *is the critical value of* k *above which the closed loop system looses passivity.*

Secondly, when unforced ($u_i = 0$)*, the feedback system has got a global limit cycle.*

Remark 2 For matrices A and B, the notation $A \otimes B$ (the Kronecker product) stands for the following matrix

$$A \otimes B = \begin{bmatrix} A_{11}B & \cdots & A_{1n}B \\ \vdots & \ddots & \vdots \\ A_{n1}B & \cdots & A_{nn}B \end{bmatrix}, \tag{6}$$

where A_{ij} $i, j \in \{1, \cdots, n\}$ stands for the $ij - th$ entry of the $n \times n$ matrix A.

Using the properties of the Kronecker product, see [22], the dynamics of the network can be represented by the following equations

$$\begin{array}{ll} \dot{X} &= (\mathbb{I}_N \otimes A(\Delta))X + (\mathbb{I}_N \otimes B)U^* \\ Y &= (\mathbb{I}_N \otimes C)X, \\ U^* &= kY - (\mathbb{I}_N \otimes B_1(\Delta))\Sigma(Y) + U \end{array} \tag{7}$$

where $X = [x_1(t), x_2(t)..., x_N(t)]^T$ and $\Sigma(Y) = [\sigma(y_1(t), ..., \sigma(y_N(t))]^T$ the stiffening nonlinearity. $U^* = [u_1^*, ..., u_N^*]^T$ is the input vector and $Y = [y_1(t), ..., y_N(t)]^T$ the output vector with $u_i, y_i \in \mathbb{R}^m$ $\forall i \in \{1 \cdots N\}$. Suppose that the system (1) is interconnected by the $N \times N$ coupling matrix Γ

$$U = -k_c(\Gamma \otimes \mathbf{I}_m)Y. \tag{8}$$

Note that there is a coupling between the $i-th$ and $j-th$ system if $\Gamma_{ij} \geq 1$ and k_c is a scalar which corresponds to the coupling strength.

The vector $\mathbf{1}_N$ $((1, \cdots, 1)^T \in \mathbb{R}^N)$ is supposed to belongs to the kernel of Γ. Moreover, the rank of Γ is assumed to be equal to N-1. In this case, the network is connected. Notice that these assumptions do not require the symmetry of Γ.

When studying the time-delayed network synchronization, the change of coordinates $e = (\overline{R} \otimes I_n)X$ can be used [24][22], where $\overline{R}$ is the nonsingular matrix $N \times N$,

$$\overline{R} = \begin{bmatrix} 0 & \mathbf{0_{N-1}^T} \\ -\mathbf{1_{N-1}} & \mathbf{I}_{N-1} \end{bmatrix} \tag{9}$$

Notice that the assumptions on Γ imply

$$\overline{R}\Gamma = \begin{bmatrix} 0 & \mathbf{0^T} \\ \mathbf{0} & \tilde{\Gamma} \end{bmatrix} \overline{R}. \tag{10}$$

The class of interconnection matrices Γ is further assumed to be such that $\tilde{\Gamma}$ is positive definite. $\tilde{\Gamma}_s$ is the symmetric part of $\tilde{\Gamma}$, *i.e.* $x^T\tilde{\Gamma}_s x = \frac{1}{2}x^T(\tilde{\Gamma} + \tilde{\Gamma}^T)x$.

3 Robust synchronization Analysis of network with needed time-delays and multiple time-delays distrubances

In the case of time-delay synchronization,

$$\begin{array}{ll} e(t) = (\overline{R} \otimes I_n)X = & [0, x_2(t-\tau_2) - x_1(t) \\ & , \cdots, x_N(t-\tau_N) - x_1(t)]^T. \end{array} \tag{11}$$

When $e = 0$, there exists a desired time-delay τ_i between each cell of the network. The dynamics of the error system is reduced to

$$\mathcal{E} : \begin{cases} \dot{e} &= (\mathbb{I}_N \otimes A(\Delta))e + (\mathbb{I}_N \otimes B)U_e^* \\ Y_e &= (\mathbb{I}_N \otimes C)e \\ U_e^* &= kY_e - (\mathbb{I}_N \otimes B_1(\Delta))\sigma_e(e, y_1) + U_e \end{cases} \tag{12}$$

where $\sigma_e(e, y_1) = [0, \sigma(y_2) - \sigma(y_1), ..., \sigma(y_N) - \sigma(y_1)]^T$
Let introduce h_j $(j = 1, 2, \cdots, p)$, the undesired time-delays (like propagation time-delays),

Assumption 3 *the time-varying delays h_j are supposed to satisfy the following conditions*

$$\begin{array}{l} 0 \leq h_j(t) \leq \bar{h}_j < \infty \\ 0 < \underline{l}_j < \dot{h}_j \leq \bar{l}_j < 1. \end{array} \tag{13}$$

Suppose that the system (12) is interconnected by

$$U_e = k_c(\Gamma \otimes \mathbb{I}_m)Y_e(t) + \sum_{j=1}^{p}(\Gamma_2 \otimes \mathbb{I}_m)Y_e(t-h_j) \tag{14}$$

It is considered that Γ_2 has got the same properties as Γ. A particular case is $\Gamma_2 = k_c\Gamma$.

Remark 3 From the particularity of (11), *i.e.* the first row is equal to 0, the stability of the system (12) can be studied with a $(N-1)$ order system. Let note the new state variables with an underline, *e.g.* $\underline{e} = [e_1 \cdots e_N]^T$.

The formulation of the system can be transformed into

$$\begin{aligned} \underline{\mathcal{E}}\ :\ \underline{\dot{e}} &= \big(\mathbb{I}_{N-1} \otimes A(\Delta))\underline{e} \\ &+ (\mathbb{I}_{N-1} \otimes B)k_c((\tilde{\Gamma} + k\mathbb{I}) \otimes \mathbb{I}_m)(\mathbb{I}_{N-1} \otimes C)\big)\underline{e} \\ &+ \sum_{j=1}^{p}(\mathbb{I}_{N-1} \otimes B)(\tilde{\Gamma_2} \otimes \mathbb{I}_m)(\mathbb{I}_{N-1} \otimes C)\underline{e}(t-h_j) \\ &- (\mathbb{I}_{N-1} \otimes B)(\mathbb{I}_{N-1} \otimes B_1(\Delta))\underline{\sigma_e}(e, y_1), \end{aligned} \tag{15}$$

This system can be changed into an interconnection system

$$\begin{aligned} \underline{\mathcal{E}} : \underline{\dot{e}} &= A_0(\Delta)\underline{e} + A_1\underline{e_h} + (\mathbb{I}_{N-1} \otimes B)\underline{U_l} \\ \underline{Y_e} &= C_e\underline{e} \\ \underline{U_l} &= (\mathbb{I}_{N-1} \otimes k\mathbb{I})\underline{Y_e} - (\mathbb{I}_{N-1} \otimes B_1(\Delta))\underline{\sigma_e}(e, y_1), \end{aligned} \tag{16}$$

where $\underline{e_h} = \big[\,\underline{e}(t-h_1), \cdots, \underline{e}(t-h_p)\,\big]$.

The interconnected system is dissipative (definition 1) when

$$V(\underline{e}(t)) - V(\underline{e}(0)) \leq \int_0^t \underline{Y_e}(\mu)^T \underline{U_l}(\mu) d\mu. \tag{17}$$

Remark 4 The supply function must be semi-definite positive but it can be different than $e^T Pe$

Some works tackle the problem in the linear case without dissipativity, see [25], [26] and [27]. The solution of Xia *et al* seems to be less conservative than the others.

Remark 5 When h is time-invariant and its exact value is known, the robust stabilization control problem was solved based on a reduction method [28].

A theorem inspired from [29] is used. In this paper, the attention is focused on the investigation of network analysis for the time-varying delay case with passivity. In this case, the following supply function (Lyapunov-Krasovskii functional) is taken

$$V(t) = V_1 + V_2 + V_3 + V_4, \tag{18}$$

where

$$V_1 = \underline{e}^T(\mathbb{I}_{N-1} \otimes P)\underline{e} \tag{19}$$

$$V_2 = \sum_{j=1}^{p} \int_{t-h_j}^{t} \int_{s}^{t} \dot{\underline{e}}(z)^T W_j \dot{\underline{e}}(z) dz ds \tag{20}$$

$$V_3 = \sum_{j=1}^{p} \int_{t-h_j}^{t} \underline{e}(s)^T Q_j \underline{e}(s) ds \tag{21}$$

$$V_4 = \sum_{j=1}^{p} \int_{0}^{t} (1 - \bar{l}_j) \tag{22}$$

$$\int_{z-h_j}^{z} [\, \underline{e}^T(z) \; \dot{\underline{e}}(s)^T \,] \begin{bmatrix} Z_j & Y_j \\ Y_j^T & X_j \end{bmatrix} \begin{bmatrix} \underline{e}(z) \\ \dot{\underline{e}}(s) \end{bmatrix} ds dz.$$

Assumption 4 *The pair $(A_0(\Delta), B)$ is stabilizable.*

Theorem 1 *Consider N coupled time-delay dissipative systems (12). Under the previous assumptions, if there exists a set of matrices F, $P > 0$, $Q_j > 0$, $W_j > 0$, $X_j > 0$, Y_j and Z_j for $j \in \{1, \cdots, p\}$ and $i \in \{1, \cdots, n_p\}$ such that the following LMIs are verified*

$$\begin{bmatrix} Z_j & Y_j \\ Y_j^T & X_j \end{bmatrix} \geq 0$$

$$(\bar{l}_j - \underline{l}_j)X_j - (1 - \bar{l}_j)W_j < 0$$

$$M_i = \begin{bmatrix} \psi_1 & -\psi_3 & 0 & 0 & P \\ -\psi_3^T & -\psi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathbb{W} & \mathbb{W} \\ P & 0 & 0 & \mathbb{W} & 0 \end{bmatrix} + \mathsf{Sym}\left\{F[\, A_{0i} \; A_1 \; (\mathbb{I}_{N-1} \otimes B) \; 0 \; -I \,]\right\} < 0,$$

then the system (12) is asymptotically stable and synchronizes such that $\{x_i \in \mathbb{R}^n$ for $i = 1, \cdots, N : x_1(t) = x_2(t - \tau_2) = \cdots = x_N(t - \tau_N)\}$. Where $\mathsf{Sym}\{R\} = R + R^T$ and

$$\mathbb{W} = \sum_{j=1}^{p} \bar{h}_j W_j \tag{23}$$

$$\psi_1 = \sum_{j=1}^{p} Q_j + (\bar{l}_j - \underline{l}_j)[\bar{h}_j Z_j + Y_j + Y_j^T] \tag{24}$$

$$+ (\mathbb{I}_{N-1} \otimes (k - k^*_{passive})\mathbb{I})C_e^T C_e \tag{25}$$

$$\psi_2 = Diag[\,(1 - \bar{l}_1)Q_1 \; \cdots \; (1 - \bar{l}_p)Q_p\,] \tag{26}$$

$$\psi_3 = [\,(\bar{l}_1 - \underline{l}_1)Y_1 \; \cdots \; (\bar{l}_p - \underline{l}_p)Y_p\,] \tag{27}$$

$$F = \left[F_1^T \; F_2^T \; F_3^T \; F_4^T \; F_5^T \right]^T \tag{28}$$

Proof: For convenience, $A_0(\Delta)$ is noted as $\mathbf{A}_0$ and $e_s = e(s)$ and $e_z = e(z)$ and $(\mathbb{I}_{N-1} \otimes B) = \mathbf{B}$ Taking into account the functional $V(t)$ of (18), the time-derivatives of the Lyapunov functions are given by

$$\begin{aligned}\dot{V}_1(\underline{e}(t)) = &\ \underline{e}(t)^T\big(\mathbf{A}_0^T P + P\mathbf{A}_0\big)\underline{e}(t)\\ &+ \underline{e}(t)^T PA_1\underline{e}_h(t) + \underline{e}_h(t)^T A_1^T P\underline{e}(t),\\ &+ 2\underline{e}(t)^T P\mathbf{B}\underline{U_l}\end{aligned} \tag{29}$$

$$\begin{aligned}\dot{V}_2(\underline{e}(t)) = &\ \textstyle\sum_{j=1}^{p}\big(h_j(t)\dot{\underline{e}}^T W_j\dot{\underline{e}}\\ &- (1-\dot{h_j})\int_{t-h_j}^{t}\dot{\underline{e}}_s^T W_j\dot{\underline{e}}_s ds\big),\end{aligned} \tag{30}$$

$$\begin{aligned}\dot{V}_2(\underline{e}(t)) \leq &\sum_{j=1}^{p}\overline{h}_j\big(\underline{e}^T\mathbf{A}_0^T W_j\mathbf{A}_0\underline{e}\\ &+ \sum_{n=1}^{p}\underline{e}^T\mathbf{A}_0^T W_j A_{1n}\underline{e}(t-h_n)\\ &+ \sum_{n=1}^{p}\underline{e}(t-h_n(t))A_{1n}^T W_j\mathbf{A}_0\underline{e}^T\\ &+ \sum_{j=1}^{p}\sum_{n=1}^{p}\underline{e}(t-h_n(t))A_{1n}^T W_j A_{1n}\underline{e}(t-h_n(t))\\ &+ 2\underline{e}^T\mathbf{A}_0^T W_j\mathbf{B}\underline{U_l} + \underline{U_l}^T\mathbf{B}W_j\mathbf{B}\underline{U_l}\\ &+ 2\sum_{n=1}^{p}\underline{e}(t-h_n(t))A_{1n}^T W_j\mathbf{B}\underline{U_l}\big)\end{aligned} \tag{31}$$

$$-\sum_{j=1}^{p}(1-\dot{h_j})\int_{t-h_j}^{t}\dot{\underline{e}}_s^T W_j\dot{\underline{e}}_s ds \tag{32}$$

where A_{1n} represents the part of A_1 with respect to the $n-th$ time-delay. When using the same method, $\dot{V}_3(\underline{e}(t))$ can be expressed as

$$\begin{aligned}\dot{V}_3(\underline{e}(t)) \leq &\ \textstyle\sum_{j=1}^{p}\big[\underline{e}^T Q_j\underline{e}\\ &- (1-\bar{l}_j)\underline{e}(t-h_j(t))^T Q_j\underline{e}(t-h_j(t))\big]\end{aligned} \tag{33}$$

and for the last functional

$$\begin{aligned}\dot{V}_4(\underline{e}(t)) \leq &\sum_{j=1}^{p}(1-\bar{l}_j)\big[\overline{h}_j\underline{e}^T(t)Z_j\underline{e}(t) + 2\underline{e}(t)^T Y_j(\underline{e}(t)\\ &-\underline{e}(t-h_j))\big] + \sum_{j=1}^{p}(1-\bar{l}_j)\int_{t-h_j}^{t}\dot{\underline{e}}_s^T X_j\dot{\underline{e}}_s ds.\end{aligned} \tag{34}$$

using Eq.(29-34),

$$\dot{V} \leq \xi^T(t) M_r \xi(t)$$
$$+ \textstyle\sum_{j=1}^{p} \int_{t-h_j}^{t} \dot{e_s}^T \big((\bar{l}_j - \underline{l}_j) X_j - (1 - \bar{l}_j) W_j \big) \dot{e_s} ds$$
$$+ 2\underline{e}(t)^T P (\mathbb{I}_{N-1} \otimes B) \underline{U_l}$$

with $\xi^T = \left[\underline{e}(t)^T \ \underline{e}(t-h_1)^T \ \cdots \ \underline{e}(t-h_p)^T \ \underline{U_l} \right]$ and M_r is

$$\begin{bmatrix} \mathbf{A}_0^T P + P\mathbf{A}_0 + \mathbf{A}_0^T \mathbb{W} \mathbf{A}_0 + \psi_{1r} & (.) & (.) \\ (PA_1 - \psi_3 + \mathbf{A}_0^T \mathbb{W} A_1)^T & A_1^T \mathbb{W} A_1 - \psi_2 & (.) \\ \mathbf{B}^T \mathbb{W} \mathbf{A}_0 & \mathbf{B}^T \mathbb{W} \mathbf{A}_1 & \mathbf{B}^T \mathbb{W} \mathbf{B} \end{bmatrix}$$

M_r is different than $M = \sum_{j=1}^{n_p} (\alpha_j M_j)$ of Eq.(23). This difference is $\psi_{1r} = \psi_1 - (\mathbb{I}_{N-1} \otimes (k - k^*_{passive}) \mathbb{I}) C_e^T C_e$ Note that $M_r(\Delta)$ can be rewritten as

$$M_r = \begin{bmatrix} \mathbf{A}_0^T P + P\mathbf{A}_0 + \psi_{1r} & PA_1 - \psi_3 & 0 \\ (PA_1 - \psi_3)^T & -\psi_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} \mathbf{A}_0^T \mathbb{W} \\ A_1^T \mathbb{W} \\ \mathbf{B}^T \mathbb{W} \end{bmatrix} \left(\mathbb{W} \right)^{-1} \left[\mathbb{W}\mathbf{A}_0 \ \mathbb{W} A_1 \ \mathbb{W}\mathbf{B} \right]. \tag{35}$$

Using the Schur complement , the dynamics are given by

$$\begin{bmatrix} \mathbf{A}_0^T P + P\mathbf{A}_0 + \psi_{1r} & PA_1 - \psi_3 & 0 & \mathbf{A}_0^T \mathbb{W} \\ (PA_1 - \psi_3)^T & -\psi_2 & 0 & A_1^T \mathbb{W} \\ 0 & 0 & 0 & \mathbf{B}^T \mathbb{W} \\ \mathbb{W}\mathbf{A}_0 & \mathbb{W} A_1 & \mathbb{W}\mathbf{B} & -\mathbb{W} \end{bmatrix}$$

The system synchronizes with passivity since $\dot{V} - \underline{Y_e}^T \underline{U_l} \leq 0$ is satisfied. Using (35) and (17) and assumption 3,

$$\dot{V} - \underline{Y_e}^T \underline{U_l} \leq \xi^T(t) M \xi(t) \tag{36}$$
$$+ \sum_{j=1}^{p} \int_{t-h_j}^{t} \underline{\dot{e_s}}^T \big((\bar{l}_j - \underline{l}_j) X_j - (1 - \bar{l}_j) W_j \big) \underline{\dot{e_s}} ds$$
$$- \underline{Y_e}(t)^T (\mathbb{I}_{N-1} \otimes B_1(\Delta)) \underline{\sigma_e}(\underline{e}, y_1) \tag{37}$$

where the dissipativity of each oscillator is introduced in M by ψ_1. then using the lemma [30][31], it is obvious that M is a convex combination of $M_i, \forall i \in [0, ..., n_p]$, with respect to Eq.(3). Since the different LMIs of (23) are satisfied and the nonlinearity σ_e is a stiffening nonlinearity, it implies that Eq.(37) ≤ 0 and that the equilibrium point of system is globally exponentially stable. The proof is completed.

4 numerical results: 4 GHz-Antenna Array

An antenna array is made up of an array of individual radiative elements which are placed in a particular configuration (linear, circular or matrix). In this paper, a uniform linear array of N elementary isotropic antennas separated by the same distance d is considered. In order to study the behavior of this configuration, it is assumed that the elementary antennas are fed with harmonic signals at the same frequency and each of them, multiplied with complex coefficients $\mathbf{w_m} = A_m e^{j\varphi_m}$; $m \in \{1 \ldots N\}$. In this case, the mathematical expression of the radiation pattern is,

$$f(\theta) = \sum_{m=1}^{N} \mathbf{w_m} e^{j(m-1)K_0 d \cos\theta}. \tag{38}$$

where K_0 is the propagation constant, d is the distance between elementary antennas and θ is the direction of the radiation measured from the axis who contains the antennas. The maximum of the radiation pattern in the direction of θ is obtained for $\varphi_m = -(m-1)k_0 d \cos(\theta)$. A_m are chosen to reduce the side lobes amplitudes of the radiation pattern.

It can be seen using this brief overview, that it is necessary to modify the amplitude and the instantaneous phases of the signals which are injected or extracted in/from the radiative elements. One of the solutions to generate these signals is to use a network of oscillators which are coupled to each others by time-delay lines, see [32].

In this case, the representation given in Eq.(1)-(2) can be used, where

$$x_i = \begin{bmatrix} i_L \\ C_0 v_0 \end{bmatrix} \quad A(\Delta) = \begin{bmatrix} 0 & \frac{1}{L_0 C_0} \\ -1 & -\frac{1}{R_0 C_0} \end{bmatrix} \quad u_i = i_{inj}$$
$$\sigma(y_i) = {y_i}^3 \quad B_1(\Delta) = \frac{\beta}{{C_0}^3}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [0 \; 1].$$

In this application, the values of the components of the resonant circuit ($L_0 = 1,58nH/Ln$, $C_0 = 1pF/Cn$ and $R_0 = 10k\Omega$) are chosen to assure $f_0 = 4GHz$ output frequency and the values of the nonlinear terms are $\alpha = 0.01$ and $\beta = 0.281$. $C_n = 1e-11$ and $L_n = 1e-7$ have been chosen in order to make up easier numerical convergence. In that case $k^*_{passive} = -\frac{R_0}{C_0}$ and $k_c = k = \frac{\alpha}{C_0}$. The uncertain parameters are: $\Delta = [L_0, C_0]$. In the proposed design, time-delays are introduced by microstrip time-delay lines. In order to have the structure of the network previously defined, the equations of the system correspond to that of (12)-(14) with an open chain networks interconnections (see [22] for others symmetries). In our case, it is chosen an open chain symmetry

$$\Gamma = \Gamma_2 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}. \tag{39}$$

The proposed theory has been used to prove the synchronization in a network of four oscillators. After optimization, the stability of the system was performed in the case of a $\pm 2\%$ variations of the parameters. In the results, the conditions in (13) are equals to $\bar{h}_j = 1ms$, $\underline{l}_j \approx 0$ and $\bar{l}_j = 1e-4$. In figure 1, two outputs of four oscillators have been shown. The desired time-delay τ_i between two oscillators can be seen. In figure 2, output errors between time-delayed oscillators are depicted in the case of time-varying delays perturbations. In this picture, the extreme values, *i.e.* $\pm 2\%$ of the parameter variation and $\bar{h}_j = 1ms$) of the time-delay have been taken.

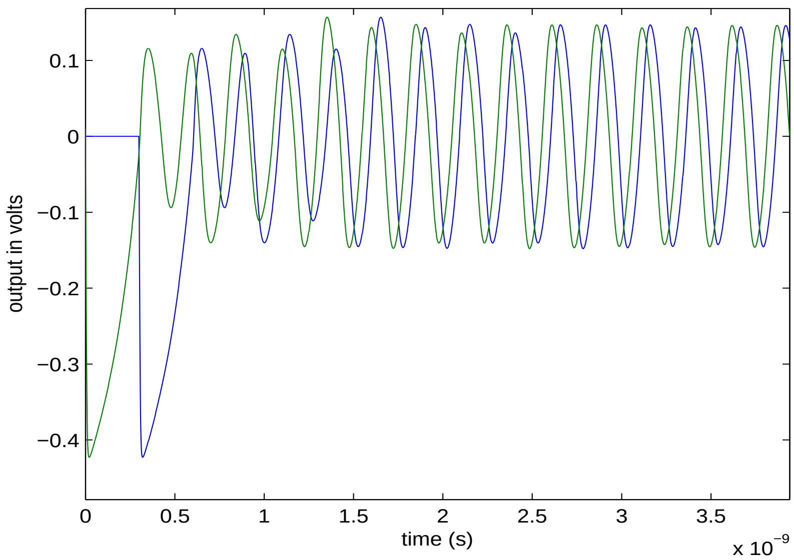

Fig. 1 Output responses of 2 time-delayed oscillators

5 Conclusion

In this paper, a new scheme has been proposed for synchronization in dynamical networks. It is based on the approach passivity and Lyapunov-Krasovskii theories. The criteria are further transformed to the LMI form. Dissipativity coupled with parameter variations can be used to extend the global stability analysis of time-delayed network of oscillators. Extension of the synchronization of the networks can be used to fed antenna arrays. In this context, the radiation pattern can be oriented.

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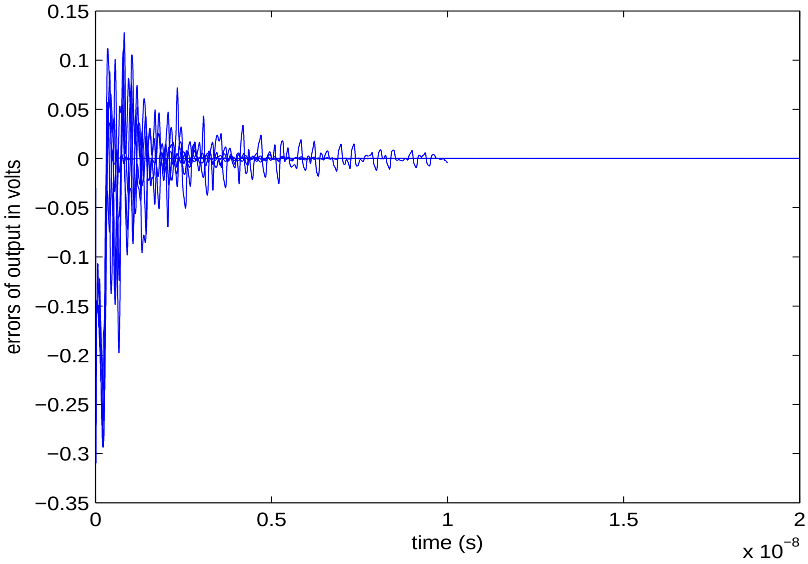

Fig. 2 Output errors between oscillators with different time-delays

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