Incremental gradient table for multiple Q-shells diffusion MRI

Emmanuel Caruyer, Christophe Lenglet, Guillermo Sapiro, Rachid Deriche

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Introduction

Most studies on sampling optimality for diffusion MRI deal with single Q-shell acquisition [1, 2, 3]. For single Q-shell acquisition, incremental gradient table has proved useful in clinical setup [3, 5, 6], where the subject is likely to move, or for online reconstruction [5, 6, 7]. In this article, we propose a generalization of the electrostatic repulsion [1] and the approach proposed in [6] to generate gradient tables for multiple Q-shells acquisitions, designed for incremental reconstruction or processing of data prematurely aborted.

Methods

We propose to arrange points in the Q-space, constrained on a finite set of shells, so that the angular coverage is optimal. As an input to our problem, the number of shells, the radius of each shell, and the relative importance of each are provided, based on considerations of the diffusion problem (see [4] for instance). The problem is to arrange a set of N points $q_n \in \mathcal{R}^3$, constrained on a set of K shells of radii r_k, with a proportion α_k of points on the k-th shell ($\sum_{k=1}^{K} \alpha_k = 1$). The distribution of points on each shell should be approximately isotropic, as the global angular coverage. As a generalisation of [1] to multiple Q-shells, we propose to minimize the energy $E_1 = \sum_k r_k \alpha_k \sum_{(i,j),|q_i|=|q_j|=r_k} E_{i,j}$, where $E_{i,j} = |q_i - q_j|^{-1} + |q_i + q_j|^{-1}$.

Any arrangement of points optimal with respect to E_1 will remain optimal if the shells are rotated with respect to each other. We propose to add an extra term to introduce interactions between points on different shells. As we stated above, the role of the radii is beyond the scope of this study, thus the coupling term between shells should play a strictly angular role. We propose to add the term $E_2 = \sum_{(i,j),|q_i|\neq|q_j|} (q_i/|q_i| - q_j/|q_j|)^{-1} + (q_i/|q_i| + q_j/|q_j|)^{-1}$. This is the electrostatic repulsion, after reprojecting all points on the unit sphere. Finally, the energy we minimize is $E = E_1 + \lambda E_2$.

Following the approach in [6], the energy minimization is incremental: for each iteration n, we look for the sample point q_n minimizing E, while the $n-1$ previous sample points remain fixed. We implemented the minimization using an exhaustive search, over a set of 10000 points randomly taken on each Q-shell of interest.

Results

We generated a set of 300 sampling points, constrained on $K = 3$ Q-shells, of radii proportional to 1.0, 2.0, and 3.0 respectively, and the importance for each shell was equal, $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$. We plot in Fig. 1 the points generated by our incremental method. It first appears that the proportion of points on each shell is exactly 1/3, and the angular coverage on each shell (Fig.1) and globally (Fig.2) is good.

As a comparison, we plot on Fig. 3 the minimum angle between any two points on each shell. We can see that it compares to the *optimal* configuration for one shell, which is the exact minimizer of electrostatic repulsion [1]. We compared in Fig.4 the value of the electrostatic energy within each shell, normalized by the minimum electrostatic energy for the same number of points. This is very close to 1 throughout the acquisition process, which means the angular coverage is also approximately optimal.

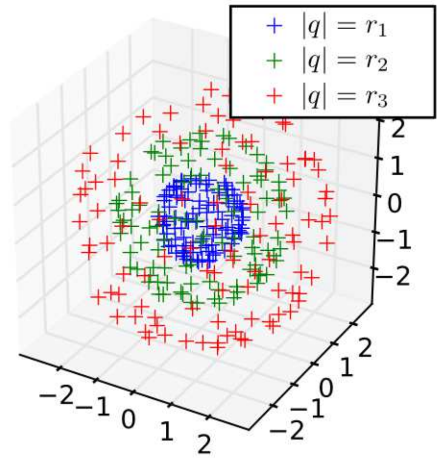

Fig. 1 - 300 points optimized for 3 shells

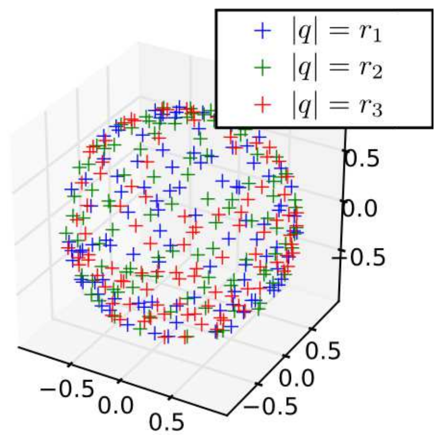

Fig. 2 - 300 points optimized for 3 shells, reprojected on the unit sphere.

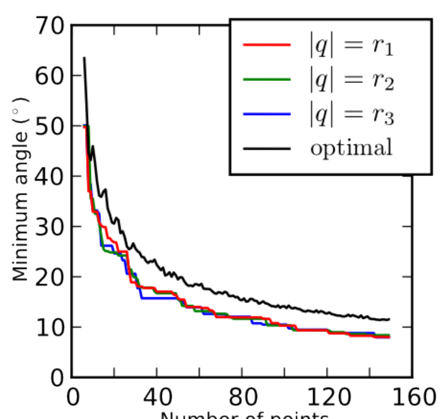

Fig. 3 - Minimum angle between any two points

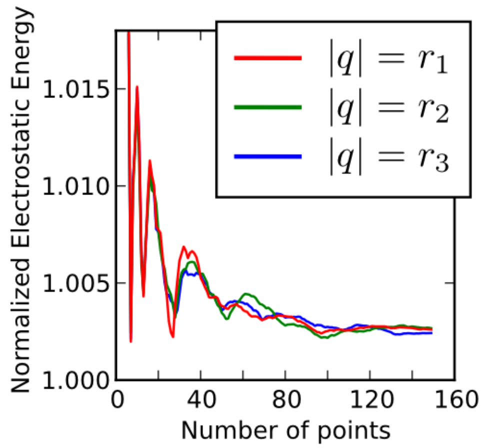

Fig.4 - Normalized Electrostatic Energy

Conclusions

We have proposed a method to generate sampling schemes suitable for multiple Q-shells acquisitions, for any number of shells, and provided that their relative importance is known. The energy function can be approximately minimized incrementally, thus providing a very efficient way to draw diffusion acquisition schemes, with the property of being approximately optimal whenever they are stopped.

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