Backstepping Fault Tolerant Control Based on Second Order Sliding Mode Observer : Application to Induction Motors

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Abstract— **In this paper, a fault tolerant control for induction motors based on backstepping strategy is designed. The proposed approach permits to compensate both the rotor resistance variations and the load torque disturbance. Moreover, to avoid the use of speed and flux sensors, a second order sliding mode observer is used to estimate the flux and the speed. The used observer converges in a finite time and permits to give a good estimate of flux and speed even in presence of rotor resistance variations and load torque disturbance. The stability of the closed loop system (controller + observer) is shown in two steps. First, the boundedness of the trajectories before the convergence of the observer is proved. Second, the trajectories convergence is proved after the convergence of the observer. The simulation results show the efficiency of the proposed control scheme.**

I. Introduction

Induction Motors (IM) are widely used in many industrial processes due to their reliability, low cost and high performance. However, because of several stresses (mechanical, environmental, thermal, electrical), IM are subjected to various faults, such as stator short-circuits and rotor failures such as broken bars or rings,...etc. The diagnostic of IM has shown that the presence of faults leads to parameters variations [1]. In this work, we focus on the rotor resistance variations.
Fault Tolerant Control (FTC) systems are able to maintain specific systems performances not only under nominal conditions but also when faults occur (change in system parameters or characteristic properties). There are two types of FTC: active and passive approaches. In the passive approach, the controller is designed to maintain acceptable performances against a set of faults without any change in the control law. In the active approach, first the faults are detected and isolated (fault detection and isolation step), second the control law is changed (control reconfiguration step) to maintain specific performances [2]. This paper is concerned with the passive fault tolerant controller for IM in order to compensate the rotor resistance variations and the load torque disturbance. The proposed approach uses a direct field oriented controller based on backstepping strategy to steer the flux and the speed to their desired references in presence of rotor resistance variations and load torque disturbance. Moreover, sensorless control is considered. This control method avoids the use of the speed sensor [3], [4], [5]. For instance, in [5] the feedback controller uses an adaptive observer in order to estimate the flux and the speed. In [4], the control scheme is based on a first order sliding mode observer. The sliding mode observers are widely used due to their finite time convergence, robustness with respect to uncertainties and the possibility of uncertainty estimation [6]. When we use the first order sliding mode approach the chattering effect appears. To avoid the chattering effect, the high order sliding mode techniques have been developed. In this work, the controller uses a second order sliding mode observer ([7], [8]) to estimate the speed and the flux.
Compared to the existing fault tolerant control schemes reported in the literature ([9]-[12]), the contribution of this paper is first the design of a backstepping controller in presence of rotor resistance variations and load torque disturbance and second is the estimation of the speed and the flux by a second order sliding mode observer which uses only the measured stator currents.

II. Induction motor oriented model

In field oriented control, the flux vector is forced on the d-axis ($\phi_{qr} = \frac{d\phi_{qr}}{dt} = 0$). The resulting induction motor model in the $(d-q)$ reference frame is described by the following state equations ([13]):

$$\begin{aligned}
\frac{di_{ds}}{dt} &= -ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{dr} + \frac{V_{ds}}{\sigma L_s} \\
\frac{di_{qs}}{dt} &= -ai_{qs} - \omega_s i_{ds} - \frac{L_m P}{\sigma L_s L_r}\Omega\phi_{dr} + \frac{V_{qs}}{\sigma L_s} \\
\frac{d\phi_{dr}}{dt} &= \frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r} \\
\frac{d\Omega}{dt} &= \frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega - \frac{T}{J}
\end{aligned} \tag{1}$$

with:

$$\omega_s = P\Omega + \frac{L_m}{\tau_r \phi_{dr}} i_{qs} \tag{2}$$

$$a = \left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma\tau_r}\right)$$

Where σ is the coefficient of dispersion given by:

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}$$

L_s, L_r, L_m are stator, rotor and mutual inductance, respectively. R_s, R_r are respectively stator and rotor resistance. ω_s is the stator pulsation. τ_r is the rotor time constant ($\tau_r = \frac{L_r}{R_r}$). P is the number of pole pairs. V_{ds}, V_{qs} are stator voltage components. ϕ_{dr}, ϕ_{qr} are the rotor flux components. Ω is the mechanical speed. T is the load torque. i_{ds}, i_{qs} are stator current components. J is the moment of inertia of the motor.

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f is the friction coefficient.
In presence of rotor resistance variations, the model (1) becomes([12]):

$$\begin{aligned}\frac{di_{ds}}{dt} =& -ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{dr} + \frac{V_{ds}}{\sigma L_s} + h_1(x)\\ \frac{di_{qs}}{dt} =& -ai_{qs} - \omega_s i_{ds} - \frac{L_m P}{\sigma L_s L_r}\Omega\phi_{dr} + \frac{V_{qs}}{\sigma L_s} + h_2(x)\\ \frac{d\phi_{dr}}{dt} =& \frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r} + h_3(x)\\ \frac{d\Omega}{dt} =& \frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega - \frac{T}{J}\end{aligned} \tag{3}$$

where $x = (i_{ds}, i_{qs}, \phi_{dr}, \Omega)$. $h_1(x)$, $h_2(x)$, $h_3(x)$ represent the fault terms due to rotor resistance variations, they are given by:

$$h_1(x) = \Delta R_r \left(-(\frac{1-\sigma}{\sigma L_r}) i_{ds} + \frac{L_m}{\phi_{dr} L_r} i_{qs}^2 + \frac{L_m}{\sigma L_s L_r^2}\phi_{dr} \right)$$

$$h_2(x) = \Delta R_r \left(-(\frac{1-\sigma}{\sigma L_r}) i_{qs} - \frac{L_m}{\phi_{dr} L_r} i_{ds} i_{qs} \right)$$

$$h_3(x) = \Delta R_r \left(\frac{L_m}{L_r} i_{ds} - \frac{\phi_{dr}}{L_r} \right)$$

Here we introduce some definitions on the practical stability which will be used in the next section (see [14]).

III. Preliminary

Consider the following system:

$$\begin{aligned}\dot{x} =& f(t,x)\\ x(t_0) =& x_0,\ t_0 \geq 0\end{aligned} \tag{4}$$

where $x \in R^n$ is the state, $t \in R_{\geq 0}$ is the time and $f : R_{\geq 0} \times R^n \to R^n$ is piecewise continuous in t and locally Lipschitz in x. (t_0, x_0) are the initial conditions. We introduce the following definition in which B_r denotes the closed loop ball in R^n of radius $r > 0$, i.e. : $B_r = \{x \in R^n \text{: } \|x\| \leq r\}$, with $\|.\|$ denotes the Euclidean norm of vectors.

Definition 1: The system (4) is said to be globally uniformly exponentially practically stable (or convergent to a ball B_r with radius $r > 0$), if there exist $\beta > 0$ and $k \geq 0$, such that for all $t_0 \in R_{\geq 0}$ and all $x_0 \in R^n$,

$$\|x\| \leq k\|x_0\| exp(-\beta(t-t_0)) + r,\ \forall t \geq t_0$$

IV. Backstepping control design

This part deals with the speed and flux control by means of backstepping control. This nonlinear control technique can be applied efficiently to linearize a nonlinear system with the existence of uncertainties, it is usually incorporated with the nonlinear damping to enhance robustness ([15], [16]).

A. Step1: Flux control

The objective is to steer the flux ϕ_{dr} to a desired reference ϕ_{dr}^* , let $e_\phi = \phi_{dr} - \phi_{dr}^*$ be the flux tracking error. The dynamic of e_ϕ is:

$$\dot{e}_\phi = \frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r} + h_3(x) - \dot{\phi}_{dr}^* \tag{5}$$

A Lyapunov function is defined as:

$$V_\phi = \frac{1}{2} e_\phi^2 \tag{6}$$

By deriving (6) we obtain:

$$\dot{V}_\phi = e_\phi \dot{e}_\phi = e_\phi \left(\frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r} + h_3(x) - \dot{\phi}_{dr}^* \right) \tag{7}$$

To make $\dot{V}_\phi$ negative definite, i_{ds} is chosen as virtual element of control for stabilizing the flux, its desired value i_{ds}^* is defined as:

$$i_{ds}^* = \frac{\tau_r}{L_m} \left(-k_\phi e_\phi - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) + \frac{\phi_{dr}}{\tau_r} + \dot{\phi}_{dr}^* \right) \tag{8}$$

where $h = 0.2785$ (see [15]). k_1, k_ϕ and ε_1 are positive design parameters.
By setting $i_{ds} = i_{ds}^*$ in (7) we get :

$$\dot{V}_\phi = -k_\phi e_\phi^2 - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) e_\phi + h_3(x) e_\phi \tag{9}$$

for $k_1 > |h_3(x)|_{max}$ we get:

$$\dot{V}_\phi \leq -k_\phi e_\phi^2 - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) e_\phi + k_1 |e_\phi| \tag{10}$$

with:

$$|e_\phi| = e_\phi signe_\phi \tag{11}$$

The derivative of the Lyapunov function (10) becomes:

$$\dot{V}_\phi \leq -k_\phi e_\phi^2 - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) e_\phi + k_1 e_\phi signe_\phi \tag{12}$$

we have (see [16]):

$$0 \leq k_1 e_\phi signe_\phi - k_1 \tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) e_\phi \leq \varepsilon_1 \tag{13}$$

The derivative of the Lyapunov function (12) becomes:

$$\dot{V}_\phi \leq -k_\phi e_\phi^2 + \varepsilon_1 \tag{14}$$

This implies that the variable e_ϕ converges to a ball whose radius can be reduced by making small the tuning parameter ε_1.

B. Step2: Speed control

The objective is to steer the speed Ω to the desired reference Ω^*, let $e_\Omega = \Omega - \Omega^*$ be the speed tracking error. The error dynamic of the speed is:

$$\dot{e}_\Omega = \frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega - \frac{T}{J} - \dot{\Omega}^* \tag{15}$$

A Lyapunov function is defined as:

$$V_\Omega = \frac{1}{2} e_\Omega^2 \tag{16}$$

By deriving (16) we obtain:

$$\dot{V}_\Omega = e_\Omega \dot{e}_\Omega = e_\Omega(\frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega - \frac{T}{J} - \dot{\Omega}^*) \quad (17)$$

i_{qs} is chosen as virtual element of control for stabilizing the speed, its desired value i^*_{qs} is defined as:

$$i^*_{qs} = \frac{JL_r}{L_m P\phi_{dr}}(-k_\Omega e_\Omega - k_2 \tanh\frac{k_2 h}{\varepsilon_2} e_\Omega + \frac{f}{J}\Omega + \dot{\Omega}^*), \ \phi_{dr} \neq 0 \quad (18)$$

where k_2 and k_Ω and ε_2 are positive design parameters. By setting $i_{qs} = i^*_{qs}$ in (17) we get:

$$\dot{V}_\Omega = e_\Omega(-k_\Omega e_\Omega - k_2 \tanh\frac{k_2 h}{\varepsilon_2} e_\Omega - \frac{T}{J}) \quad (19)$$

For $k_2 > |\frac{T}{J}|_{max}$ we obtain:

$$\dot{V}_\Omega \leq -k_\Omega e^2_\Omega - k_2 \tanh(\frac{k_2 h}{\varepsilon_2} e_\Omega)e_\Omega + k_2|e_\Omega| \leq -k_\Omega e^2_\Omega + \varepsilon_2 \quad (20)$$

This implies that the variable e_Ω converges to a ball whose radius can be reduced by making small the tuning parameter ε_2.

C. Step3: Currents control

The objective is to steer the currents i_{ds} and i_{qs} to their desired references i^*_{ds} and i^*_{qs}, respectively. Let $e_d = i_{ds} - i^*_{ds}$ and $e_q = i_{qs} - i^*_{qs}$ be the tracking errors of the currents, then the dynamics of the tracking errors are:

$$\begin{aligned}
\dot{e}_d =& -ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{dr} + \frac{V_{ds}}{\sigma L_s} \\
& - \frac{\tau_r}{L_m} F_1(e_\phi)\left(\frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r}\right) - \frac{\tau_r}{L_m}\ddot{\phi}^*_{dr} \\
& + \frac{\tau_r}{L_m}\left(F_1(e_\phi) - \frac{1}{\tau_r}\right)\dot{\phi}^*_{dr} + h_1(x) - \frac{\tau_r}{L_m} F_1(e_\phi) h_3(x) \\
\dot{e}_q =& -ai_{qs} - \omega_s i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\phi_{dr} + \frac{V_{qs}}{\sigma L_s} \\
& - F_3(e_\Omega, \Omega, \phi_{dr}) - \frac{JL_r}{L_m P\phi_{dr}}\ddot{\Omega}^* \\
& - \frac{JL_r}{L_m P\phi_{dr}} F_2(e_\Omega)\left(\frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega\right) \\
& - \frac{JL_r}{L_m P\phi_{dr}}\left(\frac{f}{J} - F_2(e_\Omega)\right)\dot{\Omega}^* \\
& + h_2(x) + \frac{L_r F_2(e_\Omega)}{PL_m\phi_{dr}} T - F_4 h_3(x) \\
\dot{e}_\phi =& -k_\phi e_\phi - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) + \frac{L_m}{\tau_r} e_d + h_3(x) \\
\dot{e}_\Omega =& \frac{PL_m}{L_r J} e_q \phi_{dr} - k_\Omega e_\Omega - k_2 tanh(\frac{k_2 h}{\varepsilon_2} e_\Omega) - \frac{T}{J}
\end{aligned} \quad (21)$$

where:

$$F_1(e_\phi) = -k_\phi - \frac{k_1^2 h}{\varepsilon_1}\left(1 - tanh(\frac{k_1 h}{\varepsilon_1} e_\phi)^2\right) + \frac{1}{\tau_r}$$

$$F_2(e_\Omega) = -k_\Omega - \frac{k_2^2 h}{\varepsilon_2}\left(1 - tanh(\frac{k_2 h}{\varepsilon_2} e_\Omega)^2\right) + \frac{f}{J}$$

$$F_3(e_\Omega, \Omega, \phi_{dr}) = \left(\frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r}\right) F_4(e_\Omega, \Omega, \phi_{dr})$$

$$F_4(e_\Omega, \Omega, \phi_{dr}) = \frac{JL_r}{PL_m\phi^2_{dr}}\left(k_\Omega e_\Omega + k_2 tanh(\frac{k_2 h}{\varepsilon_2} e_\Omega) - \frac{f}{J}\Omega - \dot{\Omega}^*\right)$$

The actual control inputs are chosen as follows:

$$\begin{aligned}
V_{ds} =& \sigma L_s \Bigg(-k_d e_d - k_3 tanh\left(\frac{k_3 h}{\varepsilon_3} e_d\right) + ai_{ds} - \frac{L_m}{\tau_r} e_\phi \\
& - \omega_s i_{qs} - \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{dr} + \frac{\tau_r}{L_m} F_1\left(\frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r}\right) \\
& - \frac{\tau_r}{L_m}\left(F_1 - \frac{1}{\tau_r}\right)\dot{\phi}^*_{dr} + \frac{\tau_r}{L_m}\ddot{\phi}^*_{dr}\Bigg)
\end{aligned} \quad (22)$$

$$\begin{aligned}
V_{qs} =& \sigma L_s \Bigg(-k_q e_q - k_4 tanh(\frac{k_4 h}{\varepsilon_4} e_q) + ai_{qs} + \omega_s i_{ds} \\
& + \frac{L_m}{\sigma L_s L_r} P\Omega\phi_{dr} - \frac{PL_m}{JL_r} e_\Omega \phi_{dr} \\
& + \frac{JL_r}{L_m P\phi_{dr}} F_2(e_\Omega)(\frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega) + F_3(e_\Omega, \Omega, \phi_{dr}) \\
& + \frac{JL_r}{L_m P\phi_{dr}}(\frac{f}{J} - F_2(e_\Omega))\dot{\Omega}^* + \frac{JL_r}{L_m P\phi_{dr}}\ddot{\Omega}^*\Bigg)
\end{aligned} \quad (23)$$

Proposition 1: Consider the system (3) and the control inputs (22) and (23) where : k_d, k_q, k_3, k_4 are positive design parameters. ε_1, ε_2, ε_3 and ε_4 are positive and arbitrary small parameters. Then, if $k_3 > \left|h_1(x) - \frac{\tau_r}{L_m} F_1 h_3(x)\right|_{max}$ and $k_4 > \left|h_2(x) - F_4 h_3(x) + \frac{L_r F_2(e_\Omega)}{PL_m\phi_{dr}} T\right|_{max}$, the error variables e_ϕ, e_Ω, e_d and e_q are globally uniformly exponentially practically stable.

Proof: By substituting the control laws (22) and (23) in the error system (21) we get:

$$\begin{aligned}
\dot{e}_d =& -k_d e_d - k_3 tanh(\frac{k_3 h}{\varepsilon_3} e_d) - \frac{L_m}{\tau_r} e_\phi + h_1(x) - \frac{\tau_r}{L_m} F_1 h_3(x) \\
\dot{e}_q =& -k_q e_q - k_4 tanh(\frac{k_4 h}{\varepsilon_4} e_q) - \frac{PL_m}{JL_r} e_\Omega \phi_{dr} + h_2(x) \\
& - F_4 h_3(x) + \frac{L_r F_2(e_\Omega)}{PL_m \phi_{dr}} T \\
\dot{e}_\phi =& -k_\phi e_\phi - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\phi) + \frac{L_m}{\tau_r} e_d + h_3(x) \\
\dot{e}_\Omega =& \frac{PL_m}{L_r J} e_q \phi_{dr} - k_\Omega e_\Omega - k_2 tanh(\frac{k_2 h}{\varepsilon_2} e_\Omega) - \frac{T}{J}
\end{aligned} \quad (24)$$

Consider the following Lyapunov function:

$$V = \frac{1}{2}(e^2_d + e^2_q + e^2_\phi + e^2_\Omega) \quad (25)$$

From the step 1 and 2 we have $k_1 > |h_3(x)|_{max}$ and $k_2 > |\frac{T}{J}|_{max}$. Then, for $k_3 > \left|h_1(x) - \frac{\tau_r}{L_m} F_1 h_3(x)\right|_{max}$ and $k_4 > \left|h_2(x) - F_4 h_3(x) + \frac{L_r F_2(e_\Omega)}{PL_m\phi_{dr}} T\right|_{max}$ we get:

$$\dot{V} \leq -k_\phi e^2_\phi - k_\Omega e^2_\Omega - k_d e^2_d - k_q e^2_q + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 \quad (26)$$

This implies that the error variables e_ϕ, e_Ω, e_d and e_q converge to a ball whose radius can be reduced by making small the tuning parameters ε_1, ε_2, ε_3 and ε_4. This means that the error variables are globally uniformly exponentially practically stable (see the definition 1). ∎

In order to implement the control laws (22) and (23) without flux and speed sensors, a second order sliding mode observer is used to estimate the speed Ω and the flux ϕ_{dr}.

V. SECOND ORDER SLIDING MODE OBSERVER DESIGN

The IM model in $(\alpha-\beta)$ reference frame is given by:

$$\begin{aligned}
\dot{i}_{\alpha s} &= -ai_{\alpha s} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{\alpha r} + \frac{L_m P}{\sigma L_s L_r}\Omega\phi_{\beta r} + \frac{V_{\alpha s}}{\sigma L_s} \\
\dot{i}_{\beta s} &= -ai_{\beta s} - \frac{L_m P}{\sigma L_s L_r}\Omega\phi_{\alpha r} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{\beta r} + \frac{V_{\beta s}}{\sigma L_s} \\
\dot{\phi}_{\alpha r} &= -P\Omega\phi_{\beta r} + \frac{L_m}{\tau_r}i_{\alpha s} - \frac{1}{\tau_r}\phi_{\alpha r} \\
\dot{\phi}_{\beta r} &= P\Omega\phi_{\alpha r} + \frac{L_m}{\tau_r}i_{\beta s} - \frac{1}{\tau_r}\phi_{\beta r} \\
\dot{\Omega} &= \frac{PL_m}{L_r J}(i_{\beta s}\phi_{\alpha r} - i_{\alpha s}\phi_{\beta r}) - \frac{f}{J}\Omega - \frac{T}{J}
\end{aligned} \tag{27}$$

with $V_{\alpha s}$, $V_{\beta s}$ are stator voltage components. $\phi_{\alpha r}$, $\phi_{\beta r}$ are the rotor flux components. Ω is the mechanical speed. T is the load torque. $i_{\alpha s}, i_{\beta s}$ are stator current components. The currents $i_{\alpha s}$, $i_{\beta s}$ are assumed to be measured.
By applying the following change of variable:

$$\begin{aligned}
z_1 &= i_{\alpha s} \\
z_2 &= i_{\beta s} \\
z_3 &= \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{\alpha r} + \frac{L_m P}{\sigma L_s L_r}\Omega\phi_{\beta r} \\
z_4 &= -\frac{L_m P}{\sigma L_s L_r}\Omega\phi_{\alpha r} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{\beta r} \\
z_5 &= \dot{z}_3 \\
z_6 &= \dot{z}_4
\end{aligned} \tag{28}$$

the system (27) becomes as follows:

$$\begin{aligned}
\dot{z}_1 &= -az_1 + z_3 + \frac{V_{\alpha s}}{\sigma L_s} \\
\dot{z}_2 &= -az_2 + z_4 + \frac{V_{\beta s}}{\sigma L_s} \\
\dot{z}_3 &= z_5 \\
\dot{z}_4 &= z_6 \\
\dot{z}_5 &= z_7 \\
\dot{z}_6 &= z_8
\end{aligned} \tag{29}$$

A second order sliding mode observer is defined as [8]:

$$\begin{aligned}
\dot{\hat{z}}_1 &= -az_1 + \tilde{z}_3 + \lambda_1|z_1-\hat{z}_1|^{0.5}sign(z_1-\hat{z}_1) + \frac{V_{\alpha s}}{\sigma L_s} \\
\dot{\tilde{z}}_3 &= \alpha_1 sign(z_1-\hat{z}_1) \\
\dot{\hat{z}}_2 &= -az_2 + \tilde{z}_4 + \lambda_2|z_2-\hat{z}_2|^{0.5}sign(z_2-\hat{z}_2) + \frac{V_{\beta s}}{\sigma L_s} \\
\dot{\tilde{z}}_4 &= \alpha_2 sign(z_2-\hat{z}_2) \\
\dot{\hat{z}}_3 &= E_1E_2\left(\tilde{z}_5 + \lambda_3|\tilde{z}_3-\hat{z}_3|^{0.5}sign(\tilde{z}_3-\hat{z}_3)\right) \\
\dot{\tilde{z}}_5 &= E_1E_2\alpha_3 sign(\tilde{z}_3-\hat{z}_3) \\
\dot{\hat{z}}_4 &= E_1E_2\left(\tilde{z}_6 + \lambda_4|\tilde{z}_4-\hat{z}_4|^{0.5}sign(\tilde{z}_4-\hat{z}_4)\right) \\
\dot{\tilde{z}}_6 &= E_1E_2\alpha_4 sign(\tilde{z}_4-\hat{z}_4) \\
\dot{\hat{z}}_5 &= E_1E_2E_3E_4\left(\tilde{z}_7 + \lambda_5|\tilde{z}_5-\hat{z}_5|^{0.5}sign(\tilde{z}_5-\hat{z}_5)\right) \\
\dot{\tilde{z}}_7 &= E_1E_2E_3E_4\alpha_5 sign(\tilde{z}_5-\hat{z}_5) \\
\dot{\hat{z}}_6 &= E_1E_2E_3E_4\left(\tilde{z}_8 + \lambda_6|\tilde{z}_6-\hat{z}_6|^{0.5}sign(\tilde{z}_6-\hat{z}_6)\right) \\
\dot{\tilde{z}}_8 &= E_1E_2E_3E_4\alpha_6 sign(\tilde{z}_6-\hat{z}_6)
\end{aligned} \tag{30}$$

where $E_i = 1$ if $\tilde{z}_i - \hat{z}_i = 0$ else $E_i = 0$ for i=1,...,n. with $\tilde{z}_1 = z_1$, $\tilde{z}_2 = z_2$. For a suitable choice of the parameters λ_i and α_i: $\alpha_1 > z_{5max}$, $\lambda_1 > (\alpha_1 + z_{5max})\sqrt{\frac{2}{\alpha_1 - z_{5max}}}$, $\alpha_2 > z_{6max}$, $\lambda_2 > (\alpha_2 + z_{6max})\sqrt{\frac{2}{\alpha_2 - z_{6max}}}$,etc (for proof see [8]), the observation errors $(\tilde{z}_i - \hat{z}_i)$ tend to zero in finite time. Then, the speed and the flux are estimated as follows:
From equations (28) we have:

$$\begin{aligned}
z_3 &= b\phi_{\alpha r} + c\Omega\phi_{\beta r} \\
z_4 &= -c\Omega\phi_{\alpha r} + b\phi_{\beta r}
\end{aligned} \tag{31}$$

where: $b = \frac{L_m}{\sigma L_s L_r \tau_r}$, $c = \frac{L_m P}{\sigma L_s L_r}$.
By solving the above equations we get:

$$\phi_{\alpha r} = \frac{bz_3 - c\Omega z_4}{b^2 + c^2\Omega^2}$$

$$\phi_{\beta r} = \frac{c\Omega z_3 + bz_4}{b^2 + c^2\Omega^2}$$

Substituting z_3 and z_4 by their estimates $\hat{z}_3$ and $\hat{z}_4$ we obtain the flux estimates as follows:

$$\hat{\phi}_{\alpha r} = \frac{b\hat{z}_3 - c\hat{\Omega}\hat{z}_4}{b^2 + c^2\hat{\Omega}^2}$$

$$\hat{\phi}_{\beta r} = \frac{c\hat{\Omega}\hat{z}_3 + b\hat{z}_4}{b^2 + c^2\hat{\Omega}^2}$$

By deriving the equations (31) we get:

$$z_5 = \dot{z}_3 = -\frac{1}{\tau_r}z_3 - P\Omega z_4 + b\frac{L_m}{\tau_r}i_{\alpha s} + c\frac{L_m}{\tau_r}\Omega i_{\beta s} + c\phi_{\beta r}\dot{\Omega} \tag{32}$$

$$z_6 = \dot{z}_4 = -\frac{1}{\tau_r}z_4 + P\Omega z_3 + b\frac{L_m}{\tau_r}i_{\beta s} - c\frac{L_m}{\tau_r}\Omega i_{\alpha s} - c\phi_{\alpha r}\dot{\Omega} \tag{33}$$

The estimates of the speed and its derivative $\hat{\Omega}$ and $\dot{\hat{\Omega}}$ can be obtained from (32) and (33) where the variables z_3, z_4, z_5, z_6, $\phi_{\alpha r}$ and $\phi_{\beta r}$ must be replaced by their estimates $\hat{z}_3$,

$\hat{z}_4$, $\hat{z}_5$, $\hat{z}_6$, $\hat{\phi}_{\alpha r}$ and $\hat{\phi}_{\beta r}$, respectively.
In the $(d-q)$ reference frame the estimated flux and currents are given as follows:

$$\hat{i}_{ds} = \cos(\hat{\rho}) i_{\alpha s} + \sin(\hat{\rho}) i_{\beta s}$$
$$\hat{i}_{qs} = -\sin(\hat{\rho}) i_{\alpha s} + \cos(\hat{\rho}) i_{\beta s}$$
$$\hat{\rho} = \arctan \frac{\hat{\phi}_{\beta r}}{\hat{\phi}_{\alpha r}}$$
$$\hat{\phi}_{dr} = \sqrt{\hat{\phi}_{\alpha r}^2 + \hat{\phi}_{\beta r}^2}$$

VI. Stability analysis of the closed loop system

To implement the control laws (22) and (23), the speed and the flux and the currents must be replaced by their estimates as follows:

$$\begin{aligned} V_{ds} =& \sigma L_s \left(-k_d \hat{e}_d - k_3 tanh\left(\frac{k_3 h}{\varepsilon_3}\hat{e}_d\right) + a\hat{i}_{ds} - \frac{L_m}{\tau_r}\hat{e}_\phi - \hat{\omega}_s \hat{i}_{qs} \right. \\ & - \frac{L_m}{\sigma L_s L_r \tau_r}\hat{\phi}_{dr} + \frac{\tau_r}{L_m} F_1(\hat{e}_\phi)\left(\frac{L_m}{\tau_r}\hat{i}_{ds} - \frac{\hat{\phi}_{dr}}{\tau_r}\right) \\ & \left. - \frac{\tau_r}{L_m}\left(F_1(\hat{e}_\phi) - \frac{1}{\tau_r}\right)\dot{\phi}^*_{dr} + \frac{\tau_r}{L_m}\ddot{\phi}^*_{dr} \right) \end{aligned} \tag{34}$$

$$\begin{aligned} V_{qs} =& \sigma L_s \left(-k_q \hat{e}_q - k_4 tanh(\frac{k_4 h}{\varepsilon_4}\hat{e}_q) + a\hat{i}_{qs} + \hat{\omega}_s \hat{i}_{ds} \right. \\ & + \frac{L_m}{\sigma L_s L_r} P\hat{\Omega}\hat{\phi}_{dr} - \frac{PL_m}{JL_r}\hat{e}_\Omega \hat{\phi}_{dr} \\ & + \frac{JL_r}{L_m P \hat{\phi}_{dr}} F_2(\hat{e}_\Omega)(\frac{PL_m}{L_r J}\hat{i}_{qs}\hat{\phi}_{dr} - \frac{f}{J}\hat{\Omega}) + F_3(\hat{e}_\Omega, \hat{\Omega}, \hat{\phi}_{dr}) \\ & \left. + \frac{JL_r}{L_m P \hat{\phi}_{dr}}(\frac{f}{J} - F_2(\hat{e}_\Omega))\dot{\Omega}^* + \frac{JL_r}{L_m P\hat{\phi}_{dr}}\ddot{\Omega}^* \right) \end{aligned} \tag{35}$$

where: $\hat{e}_d = \hat{i}_{ds} - \hat{i}^*_{ds}$, $\hat{e}_q = \hat{i}_{qs} - \hat{i}^*_{qs}$, $\hat{e}_\Omega = \hat{\Omega} - \Omega^*$, $\hat{e}_\phi = \hat{\phi}_{dr} - \phi^*_{dr}$.

$$\hat{\omega}_s = P\hat{\Omega} + \frac{L_m}{\tau_r \hat{\phi}_{dr}}\hat{i}_{qs}$$
$$\hat{i}^*_{ds} = \frac{\tau_r}{L_m}\left(-k_\phi \hat{e}_\phi - k_1 tanh(\frac{k_1 h}{\varepsilon_1}\hat{e}_\phi) + \frac{\hat{\phi}_{dr}}{\tau_r} + \dot{\phi}^*_{dr} \right)$$
$$\hat{i}^*_{qs} = \frac{JL_r}{L_m P \hat{\phi}_{dr}}(-k_\Omega \hat{e}_\Omega - k_2 \tanh\frac{k_2 h}{\varepsilon_2}\hat{e}_\Omega + \frac{f}{J}\hat{\Omega} + \dot{\Omega}^*)$$

By substituting the control laws (34) and (35) in the system of the tracking errors (21) we get:

$$\begin{aligned} \dot{e}_d =& -k_d e_d - k_3 tanh\left(\frac{k_3 h}{\varepsilon_3}(e_d + \varepsilon_d + i^*_{ds} - \hat{i}^*_{ds})\right) \\ & - \frac{L_m}{\tau_r}e_\phi + h_1(x) - \frac{\tau_r}{L_m}F_1(e_\phi)h_3(x) + d_1(\varepsilon, x, \hat{x}) \\ \dot{e}_q =& -k_q e_q - k_4 tanh\left(\frac{k_4 h}{\varepsilon_4}(e_q + \varepsilon_q + i^*_{qs} - \hat{i}^*_{qs})\right) + d_2(\varepsilon, x, \hat{x}) \\ & - \frac{PL_m}{JL_r}\phi_{dr} e_\Omega + h_2(x) + \frac{L_r F_2(e_\Omega)}{PL_m \phi_{dr}}T - F_4 h_3(x) \\ \dot{e}_\phi =& -k_\phi e_\phi - k_1 tanh(\frac{k_1 h}{\varepsilon_1}e_\phi) + \frac{L_m}{\tau_r}e_d + h_3(x) \\ \dot{e}_\Omega =& \frac{PL_m}{L_r J}e_q \phi_{dr} - k_\Omega e_\Omega - k_2 tanh(\frac{k_2 h}{\varepsilon_2}e_\Omega) - \frac{T}{J} \end{aligned} \tag{36}$$

with: $\varepsilon = (\varepsilon_d, \varepsilon_q, \varepsilon_\phi, \varepsilon_\Omega)$ denote the vector of the estimation errors, $\varepsilon_d = i_{ds} - \hat{i}_{ds}$, $\varepsilon_q = i_{qs} - \hat{i}_{qs}$, $\varepsilon_\phi = \phi_{dr} - \hat{\phi}_{dr}$, $\varepsilon_\Omega = \Omega - \hat{\Omega}$, $x = (i_{ds}, i_{qs}, \phi_{dr}, \Omega)$, $\hat{x} = (\hat{i}_{ds}, \hat{i}_{qs}, \hat{\phi}_{dr}, \hat{\Omega})$. The expression of the perturbation terms $d_1(\varepsilon, x, \hat{x})$ and $d_2(\varepsilon, x, \hat{x})$ can be easly obtained and are omitted for limited space.

The stability of the system (36) will be shown in two steps. First, we prove the boundedness of the trajectories before the convergence of the observer. Second, we prove the trajectories convergence after the convergence of the observer.

Lemma 1: Consider the system (36). If $k_3 > \left|h_1(x) - \frac{\tau_r}{L_m}F_1(e_\phi)h_3(x) + d_1(\varepsilon, x, \hat{x})\right|_{max}$, $k_4 > \left|h_2(x) + \frac{L_r F_2(e_\Omega)}{PL_m \phi_{dr}}T - F_4 h_3(x) + d_2(\varepsilon, x, \hat{x})\right|_{max}$, $k_1 > |h_3(x)|_{max}$ and $k_2 > \left|\frac{T}{J}\right|_{max}$, then the states of system (36) are uniformly bounded before the convergence of the observer.
To study the boundedness of the system (36) we use the following definition (see [17]).

Definition 2: The system (4) is globally uniformly bounded, if there exists a continuous positive definite function $W_3(x)$ such that the derivative of the Lyapunov function V along the trajectories of the system (4) satisfies:

$$\dot{V} \leq -W_3(x),\ \forall \|x\| \geq \mu > 0,\ \forall t \geq t_0 \tag{37}$$

i.e for every $a > 0$ there exists $b = b(a) > 0$ such that, for all $t_0 \geq 0$,

$$\|x(t_0)\| \leq a \Rightarrow \|x(t)\| \leq b(a),\ \forall t \geq t_0 \tag{38}$$

Proof: To show the boundedness of the system (36) before the convergence of the observer we use the following Lyapunov function:

$$V = \frac{1}{2}(e_d^2 + e_q^2 + e_\phi^2 + e_\Omega^2) \tag{39}$$

with $|tanh(x)| \leq 1$ and for: $k_3 > \left|h_1(x) - \frac{\tau_r}{L_m}F_1(e_\phi)h_3(x) + d_1(\varepsilon, x, \hat{x})\right|_{max}$, $k_4 > \left|h_2(x) + \frac{L_r F_2(e_\Omega)}{PL_m \phi_{dr}}T - F_4 h_3(x) + d_2(\varepsilon, x, \hat{x})\right|_{max}$, $k_1 > |h_3(x)|_{max}$ and $k_2 > \left|\frac{T}{J}\right|_{max}$ we get:

$$\dot{V} \leq -k_d e_d^2 - k_q e_q^2 - k_\phi e_\phi^2 - k_\Omega e_\Omega^2 + 2k_3|e_d| + 2k_4|e_q| + \varepsilon_1 + \varepsilon_2 \tag{40}$$

Let $0 < \theta < 1$. Then $\dot{V}$ can be written as follows:

$$\begin{aligned} \dot{V} \leq& -k_d(1-\theta)e_d^2 - k_q(1-\theta)e_q^2 - k_\phi(1-\theta)e_\phi^2 \\ & - k_\Omega(1-\theta)e_\Omega^2 - k_d\theta e_d^2 + 2k_3|e_d| - k_q\theta e_q^2 + 2k_4|e_q| \\ & - k_\phi\theta e_\phi^2 + \varepsilon_1 - k_\Omega\theta e_\Omega^2 + \varepsilon_2 \end{aligned} \tag{41}$$

If: $-k_d\theta e_d^2 + 2k_3|e_d| \leq 0$, $-k_q\theta e_q^2 + 2k_4|e_q| \leq 0$, $-k_\phi\theta e_\phi^2 + \varepsilon_1 \leq 0$ and $k_\Omega\theta e_\Omega^2 + \varepsilon_2 \leq 0$ i.e.: $|e_q| \geq \frac{2k_4}{k_q\theta}$, $|e_d| \geq \frac{2k_3}{k_d\theta}$, $|e_\phi| \geq \sqrt{\frac{\varepsilon_1}{k_\phi\theta}}$ and $|e_\Omega| \geq \sqrt{\frac{\varepsilon_2}{k_\Omega\theta}}$, $\dot{V}$ becomes:

$$\begin{aligned} \dot{V} \leq& -k_d(1-\theta)e_d^2 - k_q(1-\theta)e_q^2 - k_\phi(1-\theta)e_\phi^2 \\ & - k_\Omega(1-\theta)e_\Omega^2 \end{aligned} \tag{42}$$

This means that the variables e_d, e_q, e_ϕ and e_Ω are uniformly bounded before the convergence of the observer (see the definition 2). ■

Proposition 2: Consider the system (36) and the observer (30), at $t = tf$ the observer converges i.e. $\varepsilon \to 0$. Then the variables e_d, e_q, e_ϕ and e_Ω are globally uniformly exponentially practically stable.

Proof: When the observer converges ($\varepsilon = 0$), the perturbation terms vanish ($d_1(0,x,\hat{x}) = 0$, $d_2(0,x,\hat{x}) = 0$), then the system (36) is equal to the system (24) whose stability is proved by the Lyapunov function (25). ■

VII. Simulation Results

Numerical simulations have been performed to validate the proposed control scheme. The IM parameters are given in the appendix.The controller parameters are chosen as follows: $k_\Omega = 0.5$, $k_\phi = 10$, $k_1 = 10$, $k_2 = 300$, $k_3 = 500$, $k_4 = 1000$, $k_d = 100$ and $k_q = 100$. The speed and flux references are fixed at $\Omega_* = 100rd/s$ and $\phi^*_{dr} = 0.9Wb$, respectively, also a load disturbance $T = 3N.m$ is applied.Figure 1 and 2 show the responses of the IM with rotor resistance variations of $+50\%R_r$ and $+100\%R_r$, respectively. It can be seen that the controller rejects the rotor resistance variations.

VIII. Conclusion

In this paper a sensorless fault tolerant controller for IM has been presented. First, a field oriented controller based on backstepping strategy is designed to steer the flux and the speed to their desired references in presence of rotor resistance variations and load torque disturbance. Second, to achieve the sensorless fault tolerant control, a second order sliding mode observer is used to estimate the speed and the flux from the stator currents measurements. The simulation results show the robustness of the proposed control scheme.

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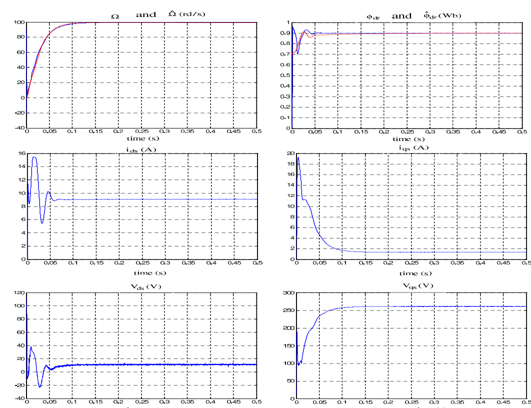

Fig. 1. Responses of the IM with rotor resistance variation of +50%Rr

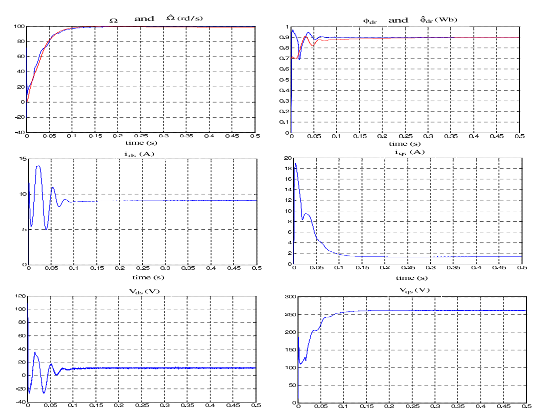

Fig. 2. Responses of the IM with rotor resistance variation of +100%Rr

Appendix

The induction motor used in this work is a $1.5KW$, $U = 220V$, $50Hz$, $I_n = 7.5A$. The parameters are: $R_s = 1.633\Omega$, $R_r = 0.93\Omega$, $L_r = 0.076H$, $L_s = 0.142H$, $L_m = 0.099H$, $J = 0.0111Kg.m^2$, $f = 0.0018N.m/rd/s$ and $P = 2$.