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► **To cite this version:**

Gabriel Synnaeve, Pierre Bessi re. A Bayesian Model for Plan Recognition in RTS Games applied to StarCraft. Artificial Intelligence and Interactive Digital Entertainment Conference (AIIDE 2011), Oct 2011, Palo Alto, United States. pp.79–84. hal-00641323

HAL Id: hal-00641323

<https://hal.archives-ouvertes.fr/hal-00641323>

Submitted on 16 Nov 2011

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A Bayesian Model for Plan Recognition in RTS Games applied to StarCraft

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Abstract

The task of keyhole (unobtrusive) plan recognition is central to adaptive game AI. “Tech trees” or “build trees” are the core of real-time strategy (RTS) game strategic (long term) planning. This paper presents a generic and simple Bayesian model for RTS build tree prediction from *noisy* observations, which parameters are learned from *replays* (game logs). This *unsupervised* machine learning approach involves minimal work for the game developers as it leverage players’ data (common in RTS). We applied it to StarCraft¹ and showed that it yields high quality and robust predictions, that can feed an adaptive AI.

Introduction

In a RTS, players need to gather resources to build structures and military units and defeat their opponents. To that end, they often have *worker units* than can gather resources needed to build *workers*, *buildings*, *military units* and *research upgrades*. Resources may have different uses, for instance in StarCraft: minerals are used for everything, whereas gas is only required for advanced buildings or military units, and technology upgrades. The military units can be of different types, any combinations of ranged, casters, contact attack, zone attacks, big, small, slow, fast, invisible, flying... Units can have attacks and defenses that counter each others as in rock-paper-scissors. Buildings and research upgrades define technology trees (precisely: directed acyclic graphs). Tech trees are tied to strategic planning, because they put constraints on which units types can be produced, when and in which numbers, which spells are available and how the player spends her resources.

Most real-time strategy (RTS) games AI are either not challenging or not fun to play against. They are not challenging because they do not adapt well dynamically to different strategies (long term goals and army composition) and tactics (army moves) that a human can perform. They are not fun to play against because they cheat economically, gathering resources faster, and/or in the intelligence war, bypassing the fog of war. We believe that creating AI that adapt to the

strategies of the human player would make RTS games AI much more interesting to play against.

We worked on StarCraft: Brood War, which is a canonical RTS game, as Chess is to board games. It had been around since 1998, it has sold 10 millions licenses and was the best competitive RTS for more than a decade. There are 3 factions (Protoss, Terran and Zerg) that are totally different in terms of units, tech trees and thus gameplay styles. StarCraft and most RTS games provide a tool to record game logs into *replays* that can be re-simulated by the game engine and watched to improve strategies and tactics. All high level players use this feature heavily either to improve their play or study opponents style. Observing replays allows players to see what happened under the fog of war, so that they can understand timing of technologies and attacks and find clues/evidences leading to infer the strategy as well as weak points (either strategic or tactical). We used this replay feature to extract players actions and learn the probabilities of tech trees to happen at a given time.

In our model, we used the buildings part of tech trees because buildings can be more easily viewed than units when fog of war is enforced, and our main focus was our StarCraft bot implementation (see Figure 1), but nothing hinders us to use units and upgrades as well in a setting without fog of war (commentary assistant or game AI that cheat).

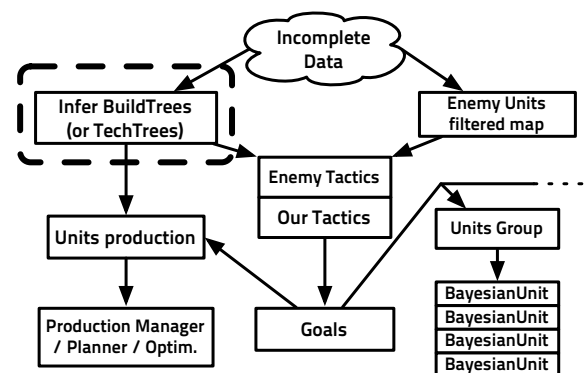


Figure 1: Data flow of the free software StarCraft robotic player BROODWARBOTQ. In this paper, we only deal with the upper left part (in a dotted line).

Background

Related Works

This work was encouraged by the reading of Weber and Mateas (2009) Data Mining Approach to Strategy Prediction and the fact that they provided their dataset. They tried and evaluated several machine learning algorithms on replays that were labeled with strategies (supervised learning).

There are related works in the domains of opponent modeling (Hsieh and Sun 2008; Schadd, Bakkes, and Spronck 2007; Kabanza et al. 2010). The main methods used to these ends are case-based reasoning (CBR) and planning or plan recognition (Aha, Molineaux, and Ponsen 2005; Ontañón et al. 2008; Ontañón et al. 2007; Hoang, Lee-Urban, and Muñoz-Avila 2005; Ramírez and Geffner 2009). There are precedent works of Bayesian plan recognition (Charniak and Goldman 1993), even in games with (Albrecht, Zukerman, and Nicholson 1998) using dynamic Bayesian networks to recognize a user’s plan in a multi-player dungeon adventure. Also, Chung, Buro, and Schaeffer (2005) describe a Monte-Carlo plan selection algorithm applied to Open RTS.

Aha, Molineaux, and Ponsen (2005) used CBR to perform dynamic plan retrieval extracted from domain knowledge in Wargus (Warcraft II clone). Ontañón et al. (2008) base their real-time case-based planning (CBP) system on a plan dependency graph which is learned from human demonstration. In (Ontañón et al. 2007; Mishra, Ontañón, and Ram 2008), they use CBR and expert demonstrations on Wargus. They improve the speed of CPB by using a decision tree to select relevant features. Hsieh and Sun (2008) based their work on CBR and Aha, Molineaux, and Ponsen (2005) and used StarCraft replays to construct states and building sequences. Strategies are choices of building construction order in their model.

Schadd, Bakkes, and Spronck (2007) describe opponent modeling through hierarchically structured models of the opponent behaviour and they applied their work to the Spring RTS (Total Annihilation clone). Hoang, Lee-Urban, and Muñoz-Avila (2005) use hierarchical task networks (HTN) to model strategies in a first person shooter with the goal to use HTN planners. Kabanza et al. (2010) improve the probabilistic hostile agent task tracker (PHATT (Geib and Goldman 2009), a simulated HMM for plan recognition) by encoding strategies as HTN.

The work described in this paper can be classified as probabilistic plan recognition. Strictly speaking, we present model-based machine learning used for prediction of plans, while our model is not limited to prediction. The plans are build trees directly learned from the replays (unsupervised learning).

Bayesian Programming

Probability is used as an alternative to classical logic and we transform incompleteness (in the experiences, the perceptions or the model) into uncertainty (Jaynes 2003). We introduce Bayesian programs (BP), a formalism that can be used to describe entirely any kind of Bayesian model, subsuming Bayesian networks and Bayesian maps, equivalent to probabilistic factor graphs (Diard, Bessière, and Mazer

2003). There are mainly two parts in a BP, the **description** of how to compute the joint distribution, and the **question(s)** that it will be asked.

The description consists in explaining the relevant *variables* $\{X^1, \dots, X^n\}$ and explain their dependencies by *decomposing* the joint distribution $P(X^1 \dots X^n | \delta, \pi)$ with existing preliminary knowledge π and data δ . The *forms* of each term of the product specify how to compute their distributions: either parametric forms (laws or probability tables, with free parameters that can be learned from data δ) or recursive questions to other Bayesian programs.

Answering a question is computing the distribution $P(\text{Searched} | \text{Known})$, with *Searched* and *Known* two disjoint subsets of the variables. $P(\text{Searched} | \text{Known})$

$$\begin{aligned} &= \frac{\sum_{Free} P(\text{Searched}, \text{Free}, \text{Known})}{P(\text{Known})} \\ &= \frac{1}{Z} \times \sum_{Free} P(\text{Searched}, \text{Free}, \text{Known}) \end{aligned}$$

General Bayesian inference is practically intractable, but conditional independence hypotheses and constraints (stated in the description) often simplify the model. Also, there are different well-known approximation techniques, for instance Monte Carlo methods and variational Bayes (Beal 2003). In this paper, we will use only simple enough models that allow complete inference to be computed in real-time.

$$BP \left\{ \begin{array}{l} Desc. \left\{ \begin{array}{l} Spec.(\pi) \left\{ \begin{array}{l} Variables \\ Decomposition \\ Forms (Parametric or Program) \end{array} \right. \\ Identification (based on \delta) \end{array} \right. \\ Question \end{array} \right.$$

For the use of Bayesian programming in sensory-motor systems, see (Bessière, Laugier, and Siegwart 2008). For its use in cognitive modeling, see (Colas, Diard, and Bessière 2010). For its first use in video games (first person shooter gameplay, Unreal Tournament), see (Le Hy et al. 2004).

Methodology

Build/Tech Tree Prediction Model

The outline of the model is that it infers the distribution on (probabilities for each of) our opponent’s build tree from observations, which tend to be very partial due to the fog of war. From what is common (conditionally of observations) and the hierarchical structure of a build tree, it diminishes or raises their probabilities. Our predictive model is a Bayesian program, it can be seen as the “Bayesian network” represented in Figure 2. It is a generative model and this is of great help to deal with the parts of the observations’ space where we do not have too much data (RTS games tend to diverge from one another as the number of possible actions grow exponentially). Indeed, we can model our uncertainty by putting a large standard deviation on too rare observations and generative models tend to converge with fewer observations than discriminative ones (Ng and Jordan 2001). Here is the description of our Bayesian program:

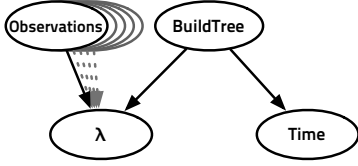


Figure 2: Graphical representation of the build tree prediction Bayesian model

Variables

- $BuildTree \in \{\emptyset, \{building_1\}, \{building_2\}, \{building_1 \wedge building_2\}, \dots\}$: all the possible building trees for the given race. For instance $\{pylon, gate\}$ and $\{pylon, gate, core\}$ are two different $BuildTrees$.
- Observations: $O_{i \in [1 \dots N]} \in \{0, 1\}$, O_k is 1/true if we have seen (observed) the k th building (it can have been destroyed, it will stay “seen”).
- $\lambda \in \{0, 1\}$: coherence variable (restraining $BuildTree$ to possible values with regard to $O_{1:N}$)
- $Time: T \in [1 \dots P]$, time in the game (1 second resolution).

At first, we generated all the possible (according to the game rules) $BuildTree$ values (in StarCraft, between ≈ 500 and 1600 depending on the race without the same building twice). We observed that a lot of possible $BuildTree$ values are too absurd to be performed in a competitive match and were never seen during the learning. So, we restricted $BuildTree$ to have its value in all the build trees encountered in our replays dataset and we added multiple instances of the basic unit producing buildings (gateway, barracks), expansions and supply buildings (depot, pylon, “overlord” as a building). This way, there are 810 build trees for Terran, 346 for Protoss and 261 for Zerg (learned from ≈ 3000 replays for each race).

Decomposition The joint distribution of our model is the following:

$$P(T, BuildTree, O_1 \dots O_N, \lambda) = P(T|BuildTree) \cdot P(BuildTree) \cdot P(\lambda|BuildTree, O_{1:N}) \cdot P(O_{1:N})$$

This can also be see as Figure 2.

Forms

- $P(BuildTree)$ is the prior distribution on the build trees. It can either be learned from the labeled replays (histograms) or set to the uniform distribution, as we did.
- $P(O_{1:N})$ is unspecified, we put the uniform distribution (we could use a prior over the most frequent observations).
- $P(\lambda|BuildTree, O_{1:N})$ is a functional Dirac that restricts $BuildTree$ values to the ones than can co-exist with the

observations.

$$\begin{aligned} P(\lambda = 1|buildTree, o_{1:N}) \\ &= 1 \text{ if } buildTree \text{ can exist with } o_{1:N} \\ &= 0 \text{ else} \end{aligned}$$

A $BuildTree$ value ($buildTree$) is compatible with the observations if it covers them fully. For instance, $BuildTree = \{pylon, gate, core\}$ is compatible with $o_{\#core} = 1$ but it is not compatible with $o_{\#forge} = 1$. In other words, $buildTree$ is incompatible with $o_{1:N}$ iff $\{o_{1:N} \setminus \{o_{1:N} \wedge buildTree\}\} \neq \emptyset$.

- $P(T|BuildTree)$ are “bell shape” distributions (discretized normal distributions). There is one bell shape over $Time$ per $buildTree$. The parameters of these discrete Gaussian distributions are learned from the replays.

Identification (learning) The learning of the $P(T|BuildTree)$ bell shapes parameters takes into account the uncertainty of the $buildTrees$ for which we have few observations. Indeed, the normal distribution $P(T|buildTree)$ begins with a high σ^2 , and **not** a “Dirac” with μ on the seen T value and $sigma = 0$. This accounts for the fact that the first(s) observation(s) may be outlier(s). This learning process is independent on the order of the stream of examples, seeing point A and then B or B and then A in the learning phase produces the same result.

Questions The question that we will ask in all the benchmarks is:

$$\begin{aligned} P(BuildTree|T = t, O_{1:N} = o_{1:N}, \lambda = 1) \\ \propto P(t|BuildTree) \cdot P(BuildTree) \\ P(\lambda|BuildTree, o_{1:N}) \cdot P(o_{1:N}) \end{aligned}$$

Note that if we see $P(BuildTree, Time)$ as a plan, asking $P(BuildTree|Time)$ for ourselves boils down to use our “plan recognition” mode as a planning algorithm, which could provide good approximations of the optimal goal set (Ramírez and Geffner 2009), or build orders.

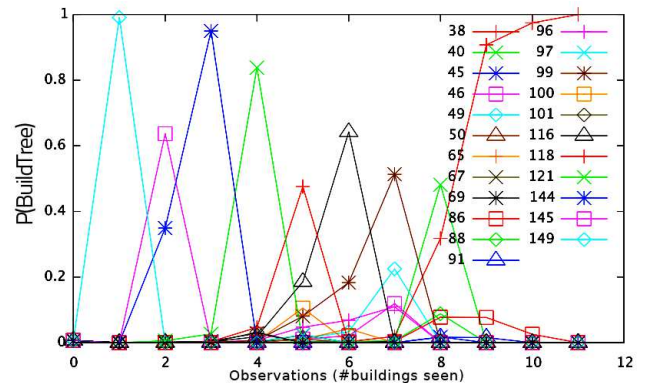


Figure 3: Evolution of $P(BuildTree|Observations\dots)$ in Time (seen/observed buildings on the x-axis). Only $BuildTrees$ with a probability > 0.01 are shown.

Results

All the results presented in this section represents the nine match-ups (races combinations) in 1 versus 1 (duel) of StarCraft. We worked with a data-set of 8806 replays (≈ 1000 per match-up) of highly skilled human players and we performed cross-validation with 9/10th of the dataset used for learning and the remaining 1/10th of the dataset used for evaluation. Performance wise, the learning part (with ≈ 1000 replays) takes around 0.1 second on a 2.8 Ghz Core 2 Duo CPU (and it is serializable). Each inference (question) step takes around 0.01 second. The memory footprint is around 3 Mb on a 64 bits machine.

Predictive Power

The predictive power of our model is measured by the $k > 0$ next buildings for which we have “good enough” prediction of future build trees in:

$$P(\text{BuildTree}^{t+k} | T = t, O_{1:N} = o_{1:N}, \lambda = 1)$$

“Good enough” is measured by a distance d to the actual build tree of the opponent that we tolerate. We used a set distance: $d(bt_1, bt_2) = \text{card}(bt_1 \Delta bt_2) = \text{card}((bt_1 \cup bt_2) \setminus (bt_1 \cap bt_2))$. One less or more building in the prediction is at a distance of 1 from the actual build tree, the same buildings except for one difference is at a distance of 2 (that would be 1 is we used tree edit distance with substitution). We call $d(\text{best}, \text{real})$ = “best” the distance between the most probable build tree and the one that actually happened. We call $d(bt, \text{real}) * P(bt)$ = “mean” the marginalized distance between what was inferred balanced (variable bt) by the probability of inferences ($P(bt)$). Note that this distance is always over **all** the build tree (and not only the next inference). This distance was taken into account only after the fourth (4th) building so that the first buildings would not penalize the prediction metric (the first building can not be predicted 4 buildings in advance).

We used $d = 1, 2, 3$, with $d = 1$ we have a very strong sense of what the opponent is doing or will be doing, with $d = 3$, we may miss one key building or have switched a tech path. We can see in Table 1 that with $d = 1$ and without noise, our model predict in average more than one building in advance what the opponent will build next if we use only its best prediction, and almost **four** buildings in advance if we marginalize over all the predictions. Of course, if we accept more error, the predictive power (number of buildings ahead that our model is capable to predict) increases, up to 6.12 for $d = 3$ without noise.

Robustness to Noise

The robustness of our algorithm is measured by the quality of the predictions of the build trees for $k = 0$ (reconstruction) or $k > 0$ (prediction) with missing observations in:

$$P(\text{BuildTree}^{t+k} | T = t, O_{1:N} = \text{partial}(o_{1:N}), \lambda = 1)$$

The “reconstructive” power (infer what has not been seen) ensues from the learning of our parameters from real data: even in the set of build trees that are possible, with regard to the game rules, only a few will be probable at a given time

and/or with some key structures. Abiding by probability theory gives us consistency with regard to concurrent build tree. This “reconstructive” power of our model is shown in Table 1 with d (distance to actual building tree) for increasing noise at fixed $k = 0$.

Figure 4 displays first (on top) the evolution of the error rate (distance to actual building) with increasing random noise (from 0% to 80%, no missing observations to 8 missing observations over 10). We consider that having an average distance to the actual build tree a little over 1 for 80% missing observations is a success. We think that this robustness is due to $P(T | \text{BuildTree})$ being precise with the amount of data that we used. Secondly, Figure 4 displays (at the bottom) the evolution of the predictive power (number of buildings ahead from the build tree that it can predict) with the same increase of noise.

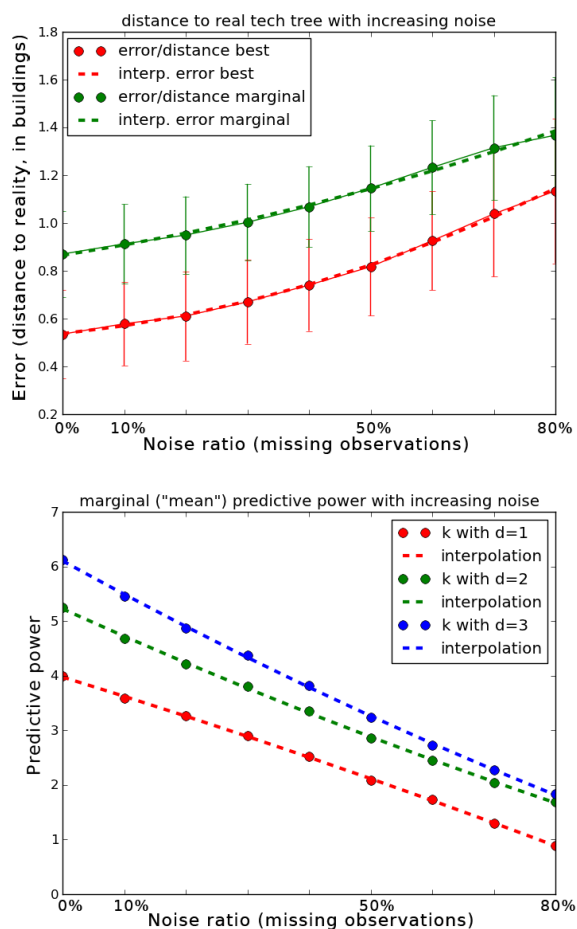


Figure 4: Evolution of our metrics with increasing noise, from 0 to 80%. The top graphic shows the increase in distance between the predicted build tree, both most probable (“best”) and marginal (“mean”) and the actual one. The bottom graphic shows the decrease in predictive power: numbers of buildings ahead (k) for which our model predict a build tree closer than a fixed distance/error (d).

Table 1: Summarization of the main results/metrics, one full results set for 10% noise

noise	measure	d for $k = 0$		k for $d = 1$		k for $d = 2$		k for $d = 3$	
		$d(best, real)$	$d(bt, real) * P(bt)$	best	"mean"	best	"mean"	best	"mean"
0%	average	0.535	0.870	1.193	3.991	2.760	5.249	3.642	6.122
	min	0.313	0.574	0.861	2.8	2.239	3.97	3.13	4.88
	max	1.051	1.296	2.176	5.334	3.681	6.683	4.496	7.334
10%	PvP	0.397	0.646	1.061	2.795	2.204	3.877	2.897	4.693
	PvT	0.341	0.654	0.991	2.911	2.017	4.053	2.929	5.079
	PvZ	0.516	0.910	0.882	3.361	2.276	4.489	3.053	5.308
	TvP	0.608	0.978	0.797	4.202	2.212	5.171	3.060	5.959
	TvT	1.043	1.310	0.983	4.75	3.45	5.85	3.833	6.45
	TvZ	0.890	1.250	1.882	4.815	3.327	5.873	4.134	6.546
	ZvP	0.521	0.933	0.89	3.82	2.48	4.93	3.16	5.54
	ZvT	0.486	0.834	0.765	3.156	2.260	4.373	3.139	5.173
	ZvZ	0.399	0.694	0.9	2.52	2.12	3.53	2.71	4.38
	average	0.578	0.912	1.017	3.592	2.483	4.683	3.213	5.459
	min	0.341	0.646	0.765	2.52	2.017	3.53	2.71	4.38
max	1.043	1.310	1.882	4.815	3.45	5.873	4.134	6.546	
20%	average	0.610	0.949	0.900	3.263	2.256	4.213	2.866	4.873
	min	0.381	0.683	0.686	2.3	1.858	3.25	2.44	3.91
	max	1.062	1.330	1.697	4.394	3.133	5.336	3.697	5.899
30%	average	0.670	1.003	0.747	2.902	2.055	3.801	2.534	4.375
	min	0.431	0.749	0.555	2.03	1.7	3	2.22	3.58
	max	1.131	1.392	1.394	3.933	2.638	4.722	3.176	5.268
40%	average	0.740	1.068	0.611	2.529	1.883	3.357	2.20	3.827
	min	0.488	0.820	0.44	1.65	1.535	2.61	1.94	3.09
	max	1.257	1.497	1.201	3.5	2.516	4.226	2.773	4.672
50%	average	0.816	1.145	0.493	2.078	1.696	2.860	1.972	3.242
	min	0.534	0.864	0.363	1.33	1.444	2.24	1.653	2.61
	max	1.354	1.581	1	2.890	2.4	3.613	2.516	3.941
60%	average	0.925	1.232	0.400	1.738	1.531	2.449	1.724	2.732
	min	0.586	0.918	0.22	1.08	1.262	1.98	1.448	2.22
	max	1.414	1.707	0.840	2.483	2	3.100	2.083	3.327
70%	average	1.038	1.314	0.277	1.291	1.342	2.039	1.470	2.270
	min	0.633	0.994	0.16	0.79	1.101	1.653	1.244	1.83
	max	1.683	1.871	0.537	1.85	1.7	2.512	1.85	2.714
80%	average	1.134	1.367	0.156	0.890	1.144	1.689	1.283	1.831
	min	0.665	1.027	0.06	0.56	0.929	1.408	1.106	1.66
	max	1.876	1.999	0.333	1.216	1.4	2.033	1.5	2.176

Conclusions

Discussion and Perspectives

Developing beforehand a RTS game AI that specifically deals with whatever strategies the players will come up is very hard. And even if game developers were willing to patch their AI afterwards, it would require a really modular design and a lot of work to treat each strategy. With our model, the AI can adapt to the evolutions in play by learning its parameters from the replay, it can dynamically adapt during the games by asking $P(\text{BuildTree}|\text{Observations}, \text{Time}, \lambda = 1)$ and even $P(\text{TechTree}|\text{Observations}, \text{Time}, \lambda = 1)$ if we add units and technology upgrades to buildings. This would allow for the bot to dynamically choose/change build orders and strategies.

This work can be extended by have a model for the two players (the bot/AI and the opponent):

$$P(\text{BuildTree}_{bot}, \text{BuildTree}_{op}, \text{Obs}_{op,1:N}, \text{Time}, \lambda)$$

So that we could ask this (new) model:

$$P(\text{BuildTree}_{bot}|\text{obs}_{op,1:N}, \text{time}, \lambda = 1)$$

This would allow for simple and dynamic build tree adaptation to the opponent strategy (dynamic re-planning), by the inference path:

$$\begin{aligned} & P(\text{BuildTree}_{bot}|\text{obs}_{op,1:N}, \text{time}, \lambda = 1) \\ & \propto \sum_{\text{BuildTree}_{op}} P(\text{BuildTree}_{bot}|\text{BuildTree}_{op}) \text{ (learned)} \\ & \quad \times P(\text{BuildTree}_{op}) \cdot P(o_{op,1:N}) \text{ (priors)} \\ & \quad \times P(\lambda|\text{BuildTree}_{op}, o_{op,1:N}) \text{ (consistency)} \\ & \quad \times P(\text{time}|\text{BuildTree}_{op}) \text{ (learned)} \end{aligned}$$

That way, one can ask “what build/tech tree should I go for against what I see from my opponent”, which tacitly seeks the distribution on BuildTree_{op} to break the complexity of the possible combinations of $\text{Obs}_{1:N}$. It is possible to *not* marginalize over BuildTree_{op} , but consider only the most probable(s) BuildTree_{op} , for computing efficiency. A filter on BuildTree_{bot} (as simple as

$P(\text{BuildTree}_{bot}^t | \text{BuildTree}_{bot}^{t-1})$ can and should be added to prevent switching build orders or strategies too often.

The Bayesian model presented in this paper for opponent build tree prediction can be used in two main ways:

- as the corner stone of adaptive (to the opponent’s dynamic strategies) RTS game AI:
 - without noise in the case of built-in game AI (cheat).
 - with noise in the case of RTS AI tournaments (as AI-IDE’s) of matches against human players.
- as a commentary assistant (null noise, prediction of tech trees), showing the probabilities of possible strategies as Poker commentary software do.

Finally, a hard problem is detecting the “fake” builds of very highly skilled players. Indeed, some pro-gamers have build orders which purposes are to fool the opponent into thinking that they are performing opening A while they are doing B. For instance they could “take early gas” leading the opponent to think they are going to do tech units, not gather gas and perform an early rush instead.

Conclusion

We presented a probabilistic model computing the distribution over build (or tech) trees of the opponent in a RTS game. The main contributions (with regard to Weber and Mateas) are the ability to deal with *partial* observations and *unsupervised* learning. This model yields high quality prediction results (up to 4 buildings ahead with a total build tree distance less than 1, see Table 1) and shows a strong robustness to noise with a predictive power of 3 buildings ahead with a build tree distance less than 1 under 30% random noise (a quality that we need for real setup/competitive games). It can be used in production thanks to its low computational (CPU) and memory footprint. Our implementation is free software and can be found online². We will use this model (or an upgraded version of it) in our StarCraft AI competition entry bot as it enables it to deal with the incomplete knowledge gathered from scouting.

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²<https://github.com/SnippyHollow/OpeningTech/>

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