# A historical survey of geometric computer vision

Peter Sturm

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Peter Sturm, INRIA

# Introduction

Three examples of today's state of the art in multi-view 3D modeling:

- **Highly detailed and accurate** shape estimation, Furukawa and Ponce
- **Real-time** shape and motion estimation, e.g. Davison et al.
- Shape estimation and geo-localization from **very many images,** Snavely et al.

# Very Brief (and Partial) History of Geometric Computer Vision

Geometrical works ("structure-from-motion"):

- 1980's: Emergence as independent discipline, from AI and robotics
- 1982: Epipolar geometry [Longuet-Higgins]
- 1992: 3D reconstruction from uncalibrated images (projective reconstruction) [Faugeras, Hartley]
- 1992: Self-calibration [Faugeras and Maybank]
- 1990's: Trifocal geometry of points and lines [Aloimonos, Hartley, Shashua, Faugeras, Triggs]
- since the 1990's: Many advances in structure-from-motion, including non-rigid objects

Other crucial works:

- since ca. 1995: systematic use of robust statistics
- since ca. 1995: use of better numerical algorithms
- since ca. 1998: true multi-view 3D surface modeling
- since ca. 2000: wide-baseline matching
- Strategies to tackle scale and/or speed requirements:
  - memory management
  - parallelization
  - hierarchical usage of images
  - etc.

# Outline

- Introduction

- **1st Flashback: Invention of Photography and Photogrammetry**
- **2nd Flashback: Surveying / Topography**

- 3rd Flashback: General Orientation Procedures for Perspectives
- 4th Flashback: Discovery and Study of Perspective

- Other early examples of scientific applications of photography

# 1st Flashback: Invention of Photography and Photogrammetry

- 1839: After decades of research, usable photography was achieved [Daguerre and Niepcé]
- The use of photographs for measuring objects and landscapes was proposed immediately
- 1850's: Laussedat first used perspective drawings, then photographs, for measurements
  - His approach was inspired directly from surveying / topography

Aimé Laussedat (1819-1907)

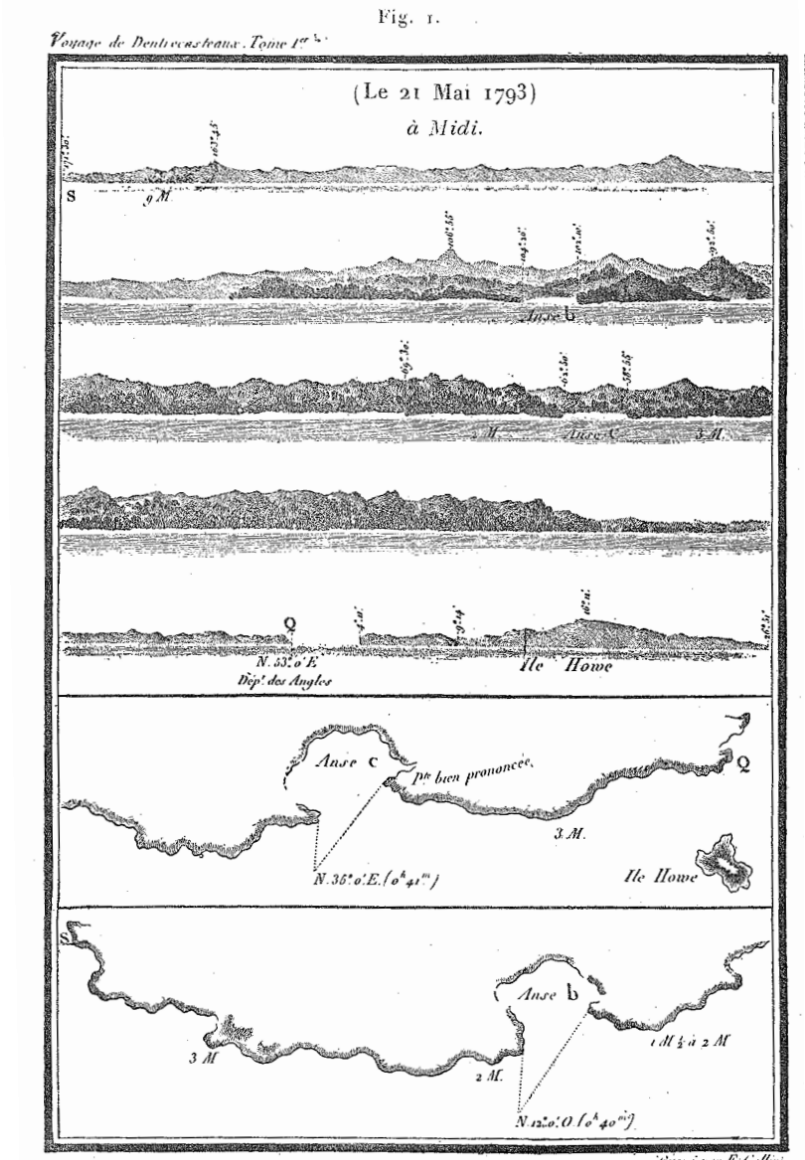
# 2nd Flashback: Surveying / Topography

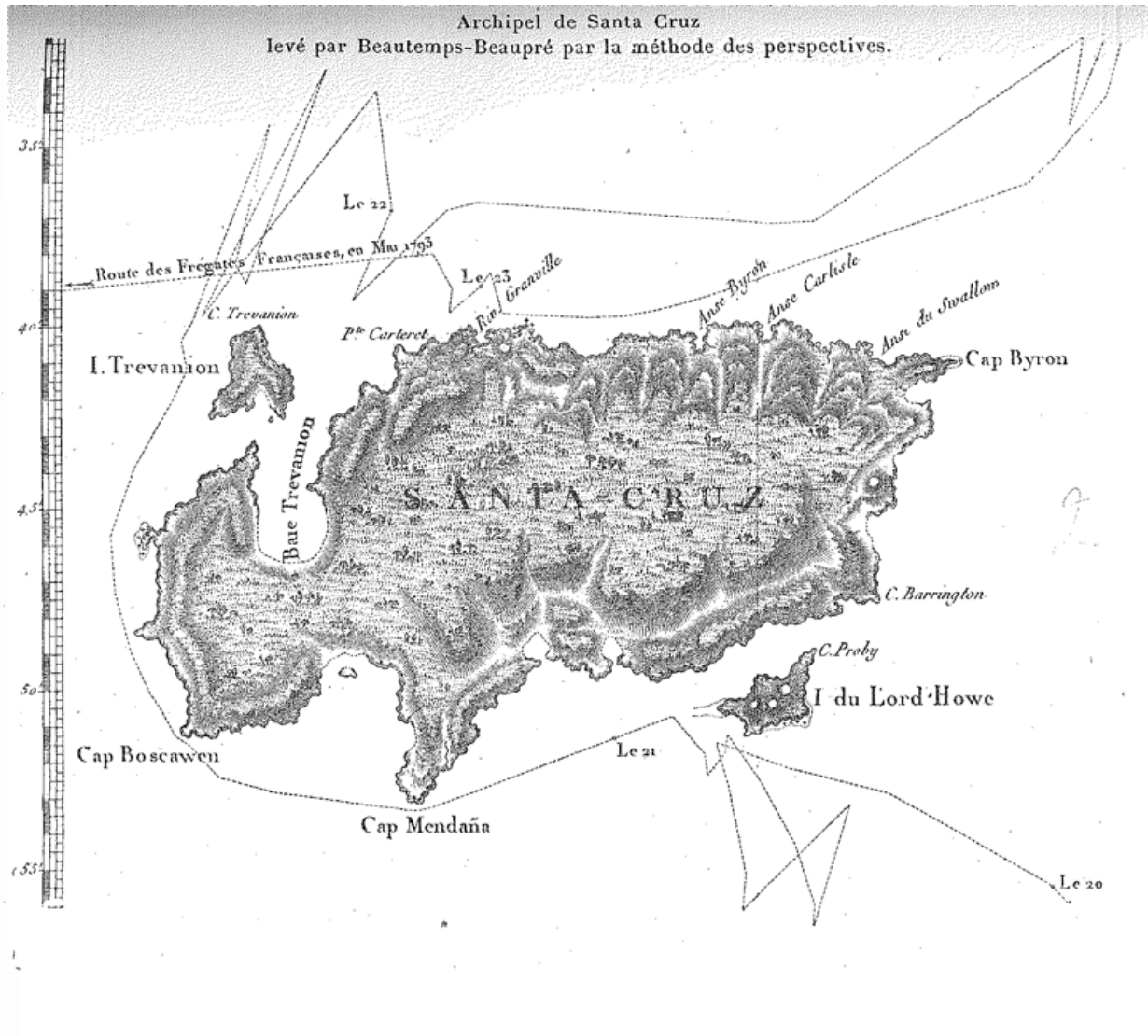
- Surveying procedures are in use since at least ca. 5000 years (Egypt: 3000 BC)
- Laussedat's inspiration:
  - From ca. 1793: Beautemps-Beaupré created maps of coastlines during expeditions, from angular and distance measurements reported on drawings

# 2nd Flashback: Surveying / Topography

- Laussedat's inspiration:
  - From ca. 1793: Beautemps-Beaupré created maps of coastlines during expeditions, from angular and distance measurements reported on drawings

    (contrary to later interpretations, the drawings only seemed to serve as support to report measurements, not to make measurements from them)

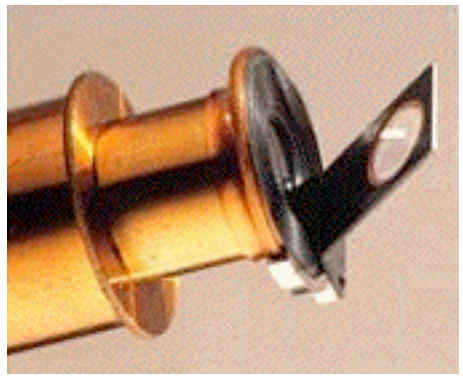


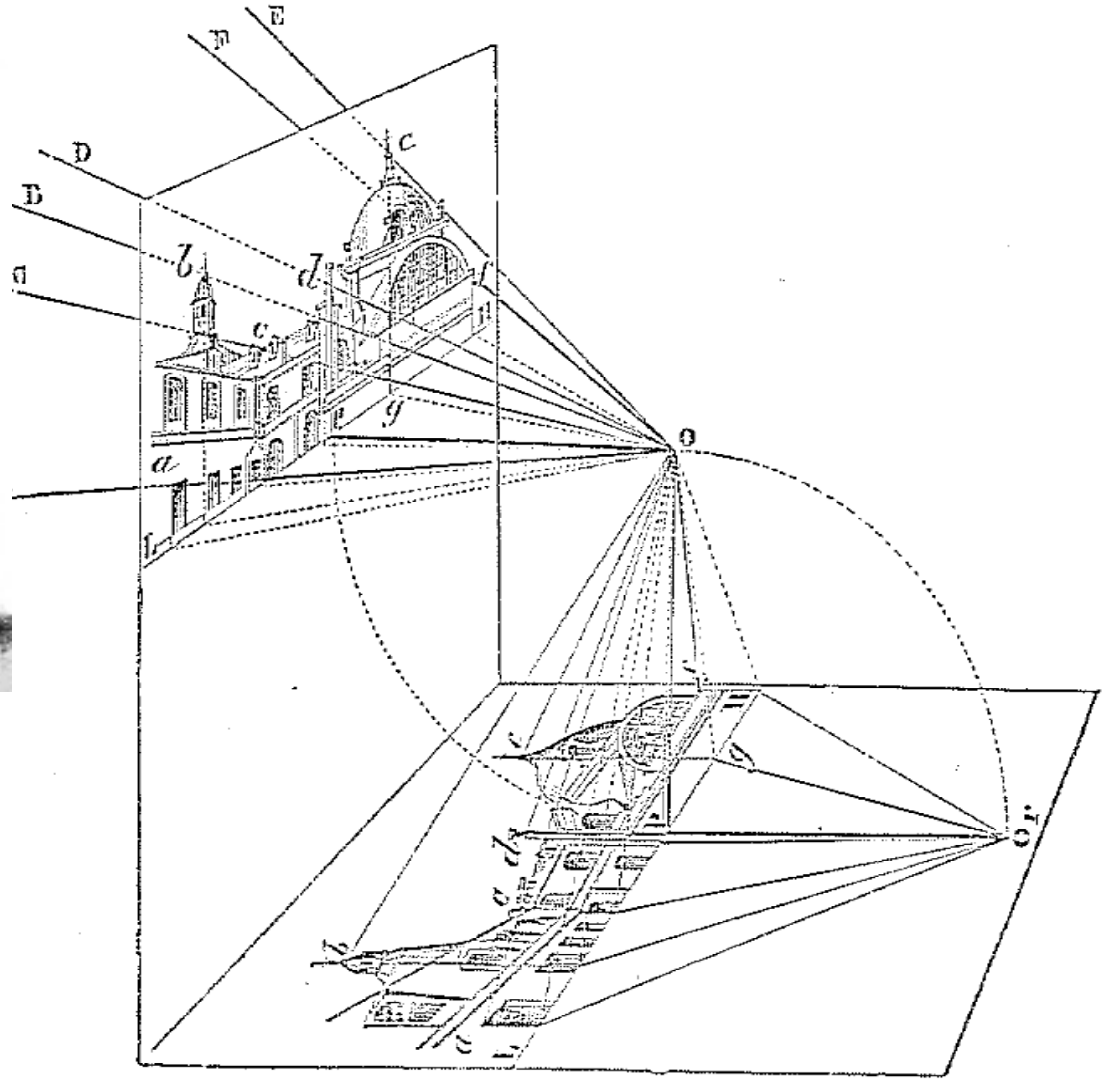
# 1st Flashback: Invention of Photography and Photogrammetry

- Laussedat's approach, demonstrated for a camera lucida (used in 1854):

Camera lucida: Wollaston, 1807
(Kepler, 1611)



Perspective drawing made by Laussedat



# 1st Flashback: Invention of Photography and Photogrammetry

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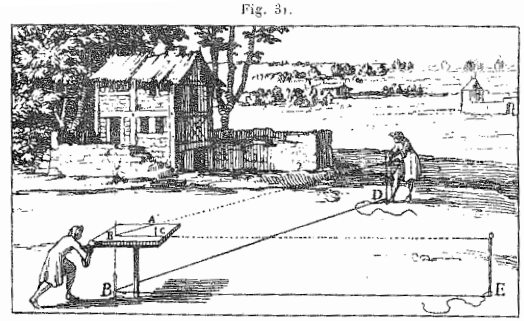
Fig. 19.

Échelle de $\frac{1}{3,5}$ de l'original.

Fig. 31.



Emploi de la planchette simple sur le terrain.

# 1st Flashback: Invention of Photography and Photogrammetry

- 1850's: Photogrammetry was born
- 1726: Kappeler created a topographic map of a Swiss mountain range, apparently with the aide of perspective drawings. However, no clear description of his approach is known and he is thus not commonly considered as inventor of "photo"grammetry.

Some next steps over the decades:

- Improved equipment, for example:
  - Portable equipment

PORTABLE PHOTOGRAPHIC APPARATUS.

**TOPOGRAPHER and ASSISTANT,**
**Showing mode of carrying instruments.**

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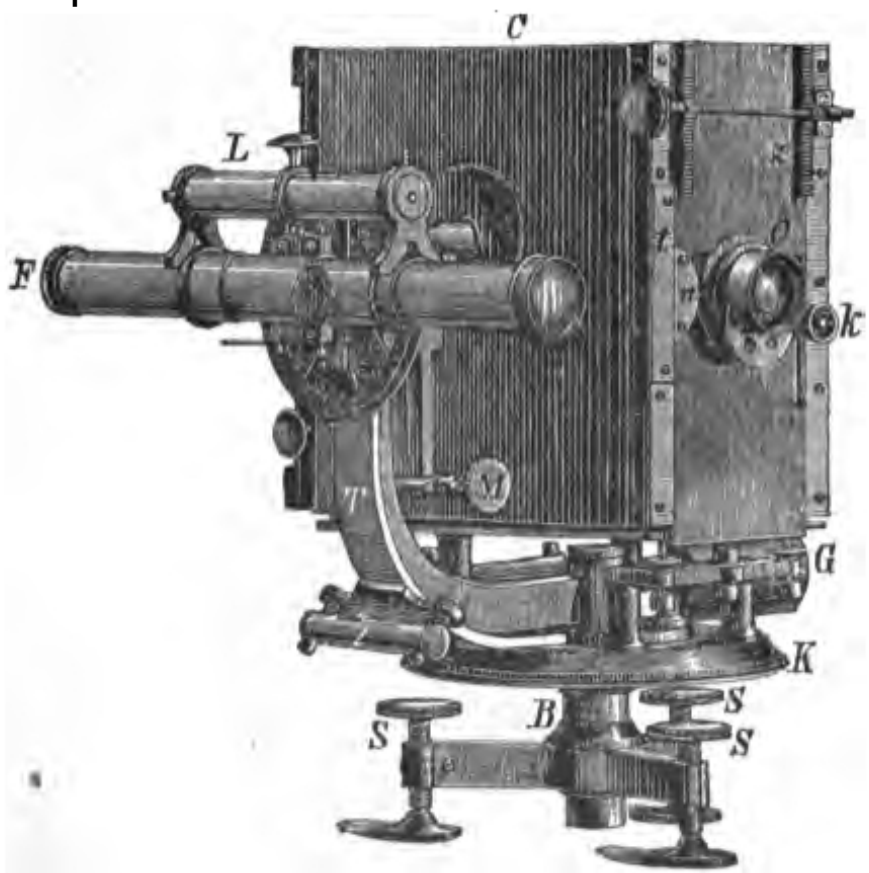

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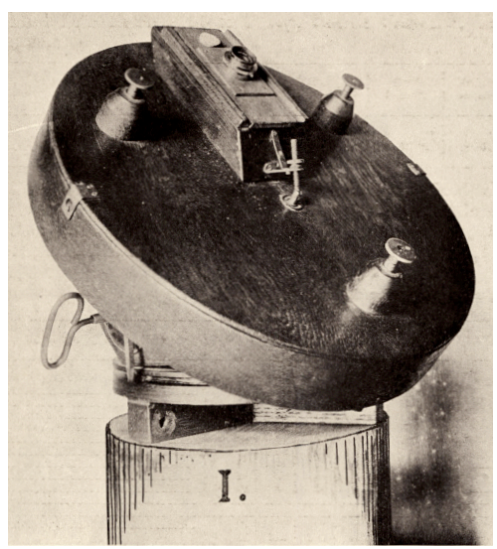
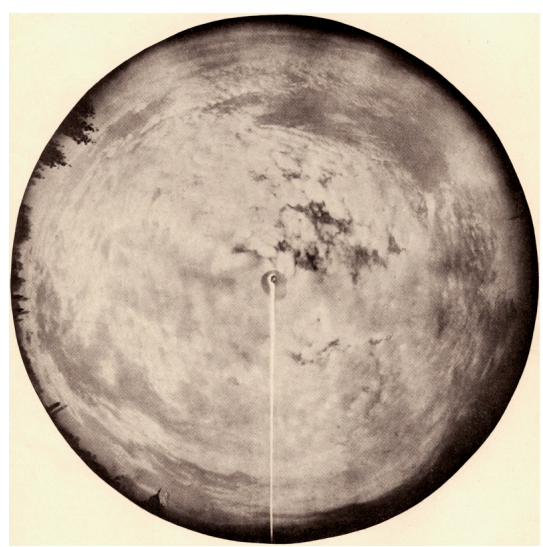
  - Panoramic photography (from 1843!)

Cylindrograph, Moëssard 1884



"Cloud camera", 190?

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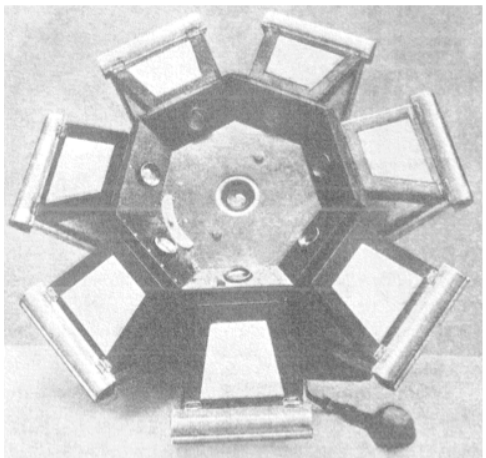
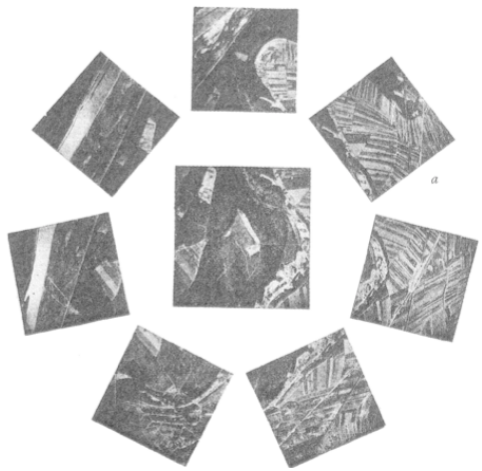
  - Multi-camera systems (from 1870's or 1880's)



Scheimpflug, 190?



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- Improved equipment, for example:
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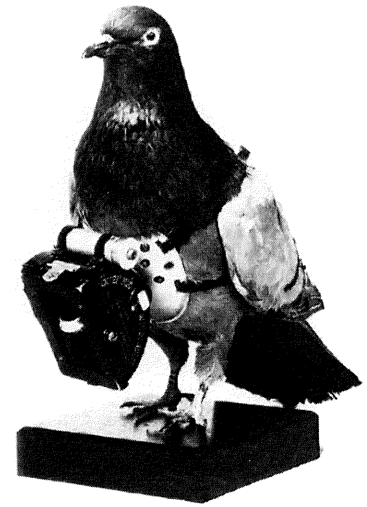
  - Multi-camera systems (from 1870's or 1880's)
- Orientation procedures for more general image acquisition procedures, e.g. for aerial imaging



# Outline

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- 1st Flashback: Invention of Photography and Photogrammetry
- 2nd Flashback: Surveying / Topography

- **3rd Flashback: General Orientation Procedures for Perspectives**
- **4th Flashback: Discovery and Study of Perspective**

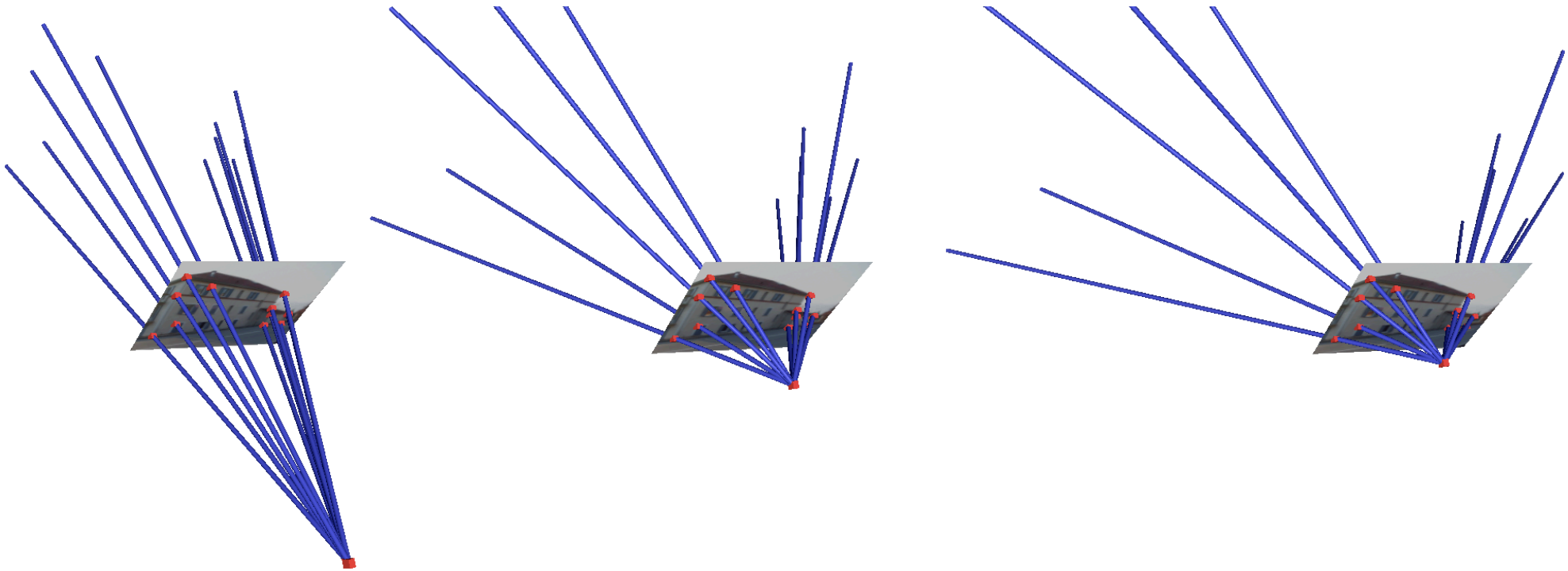
- Other early examples of scientific applications of photography

# 3rd Flashback: General Orientation Procedures for Perspectives

Goal: enable to work with images acquired in general position

Four basic procedures are:

- **Calibration** (inner orientation)
- Triangulation of 3D points (intersection)
- Pose estimation (resection): estimate position and orientation from image of known object
- Motion estimation (relative orientation): relative position/orientation of two images



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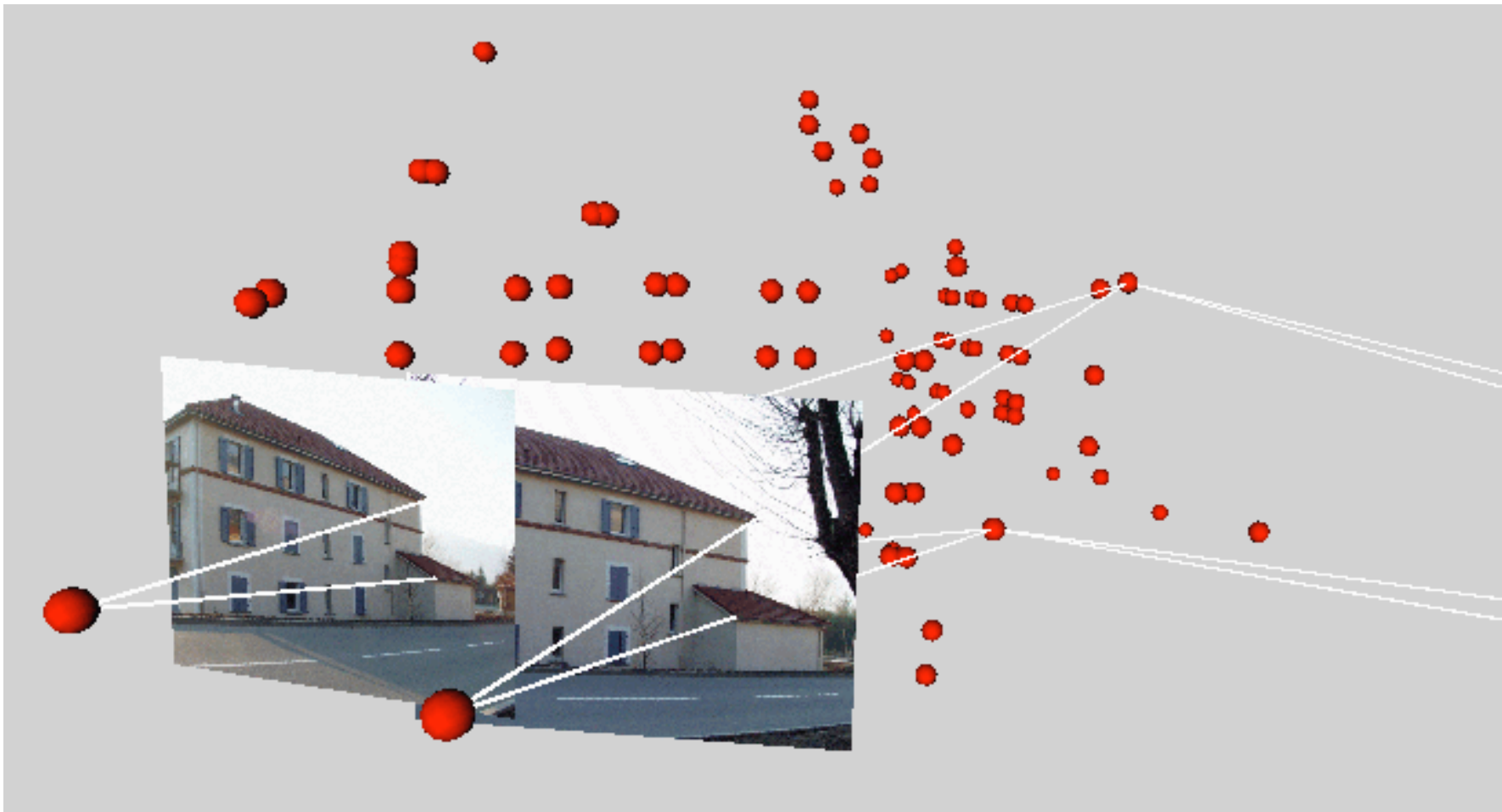

n object

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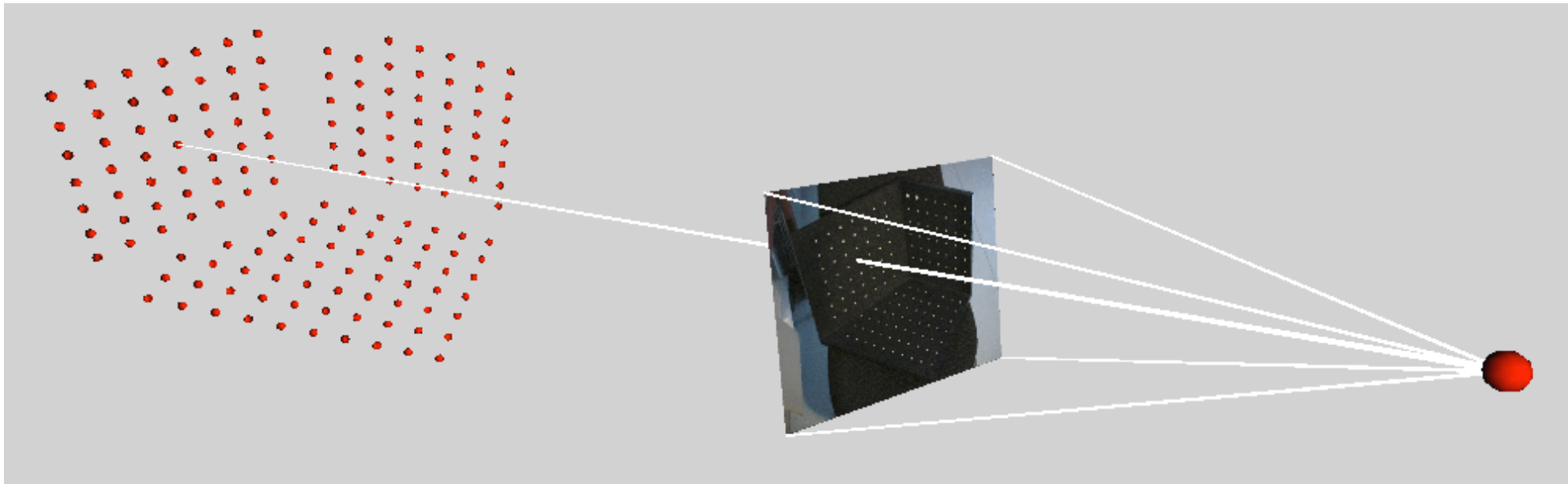

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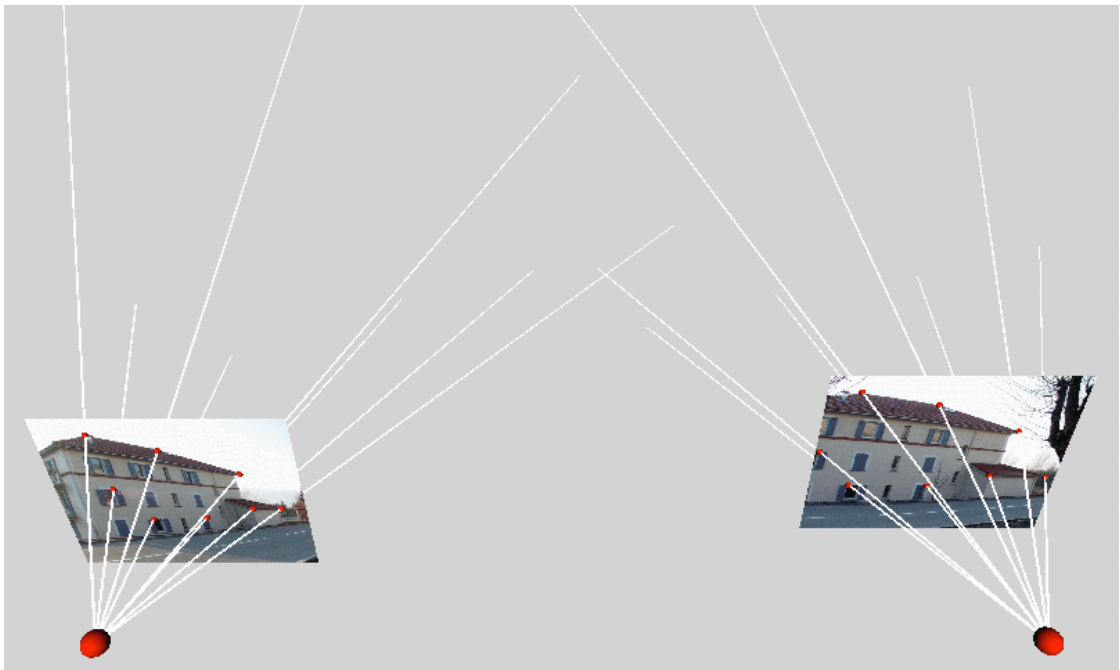

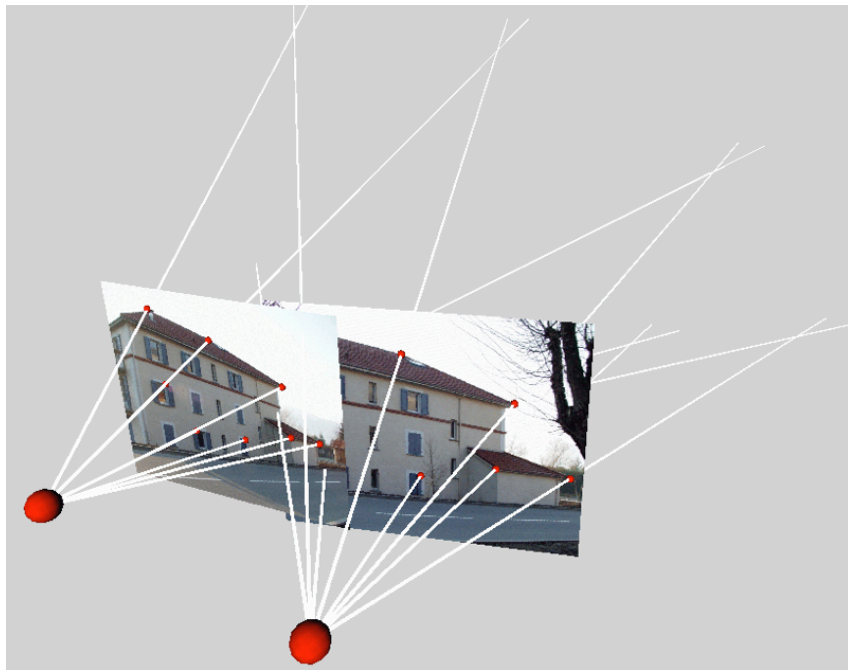

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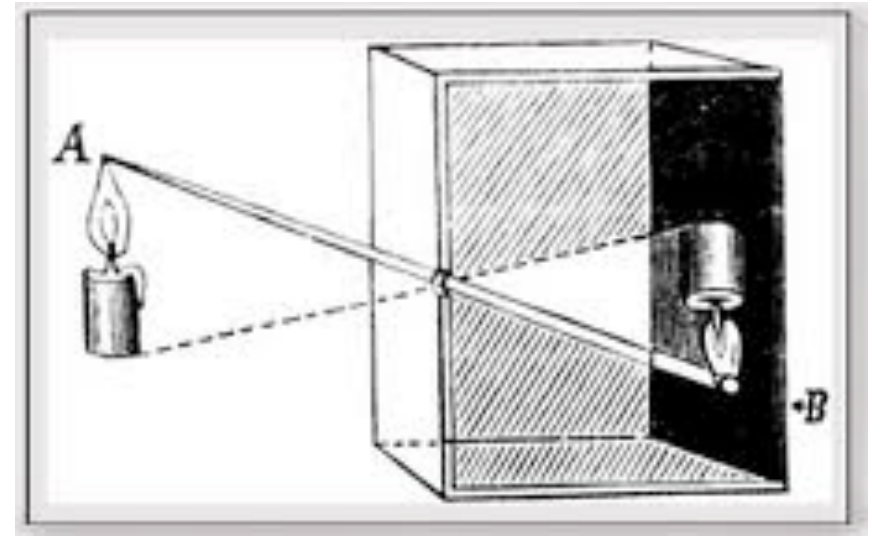
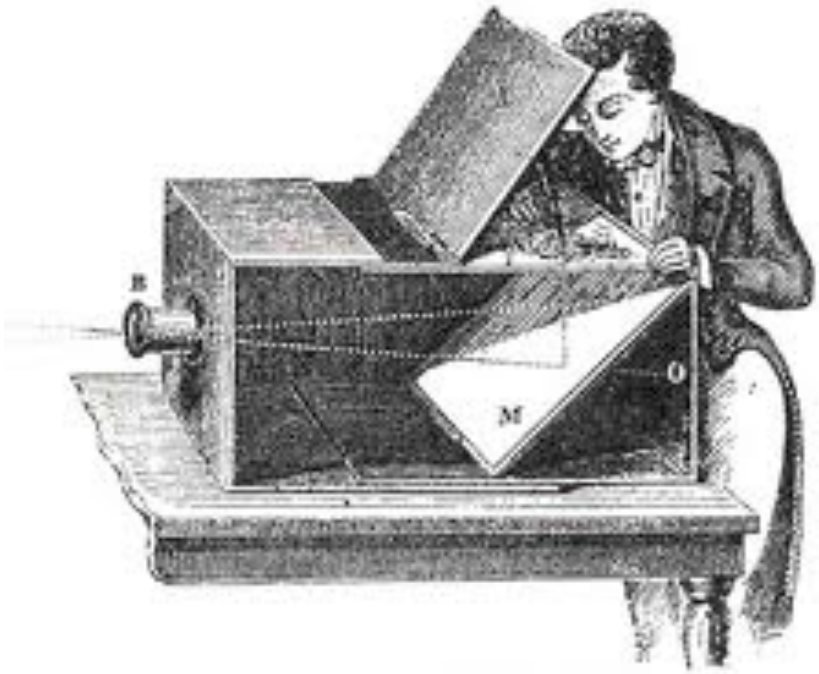
These were achieved in full generality by exploiting the principles of perspective projection and sometimes, using projective geometry.

# 4th Flashback: Discovery and Study of Perspective

Camera obscura and pinhole camera: principles (partly) known since over 2400 years [wikipedia]:

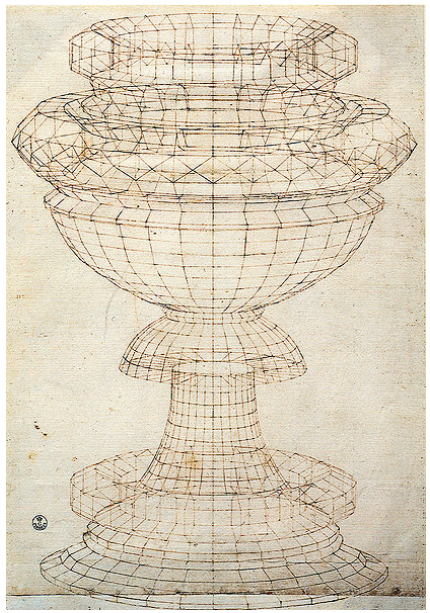
- China: 5th century BC
- Greece: 4th century BC
- Egypt: 11th century
- Throughout Europe: from 11th century onwards



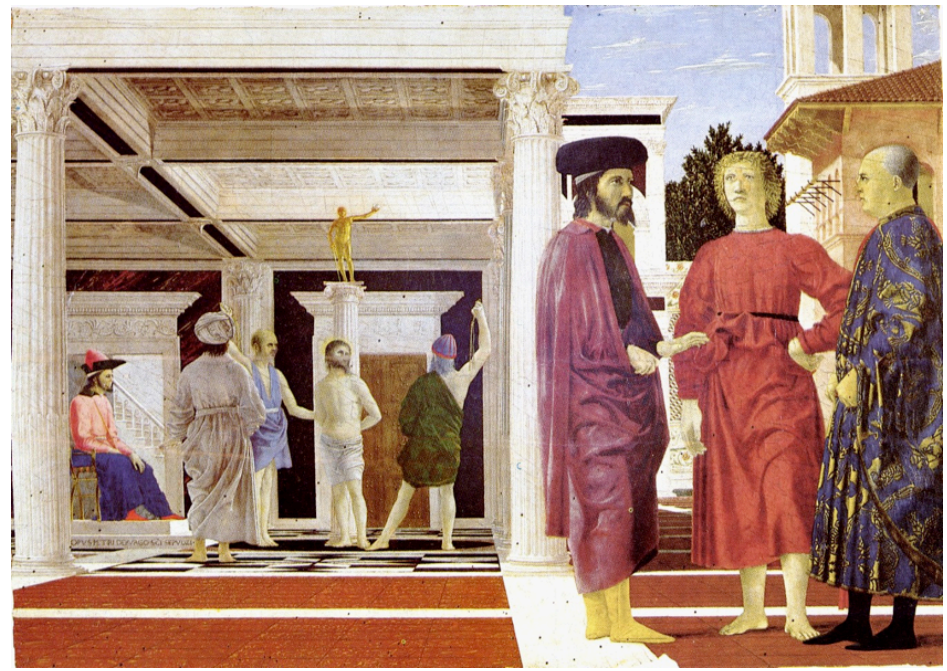



# 4th Flashback: Discovery and Study of Perspective

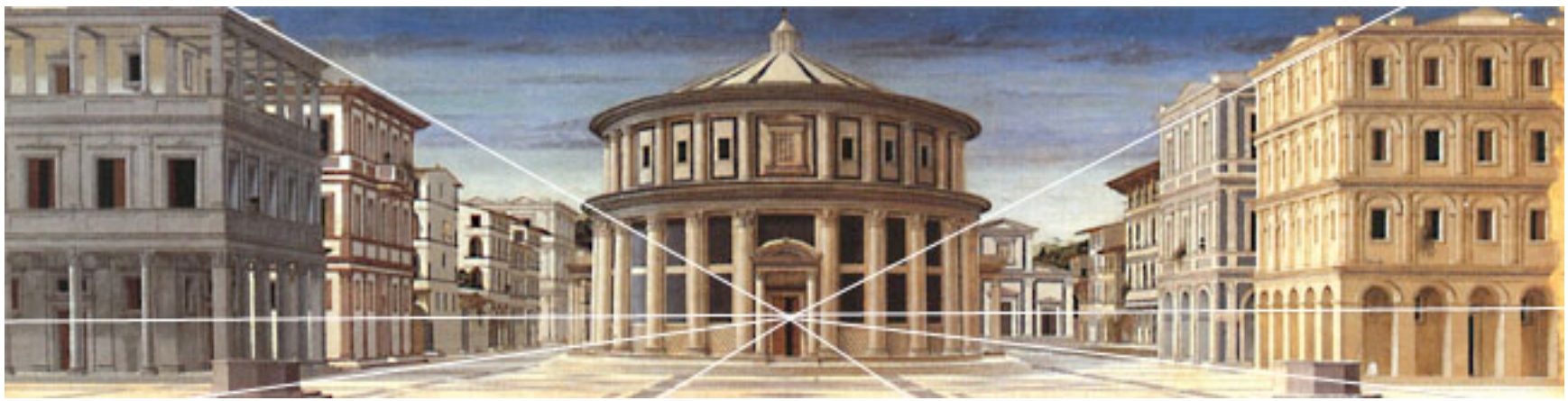
Full discovery and mathematical treatment seems to have started with the **Italian renaissance** in the 15th century and lasted till the 19th century



Perspective study of a chalice, Uccello, ca. 1450



The flagellation of Christ, della Francesca, ca. 1460

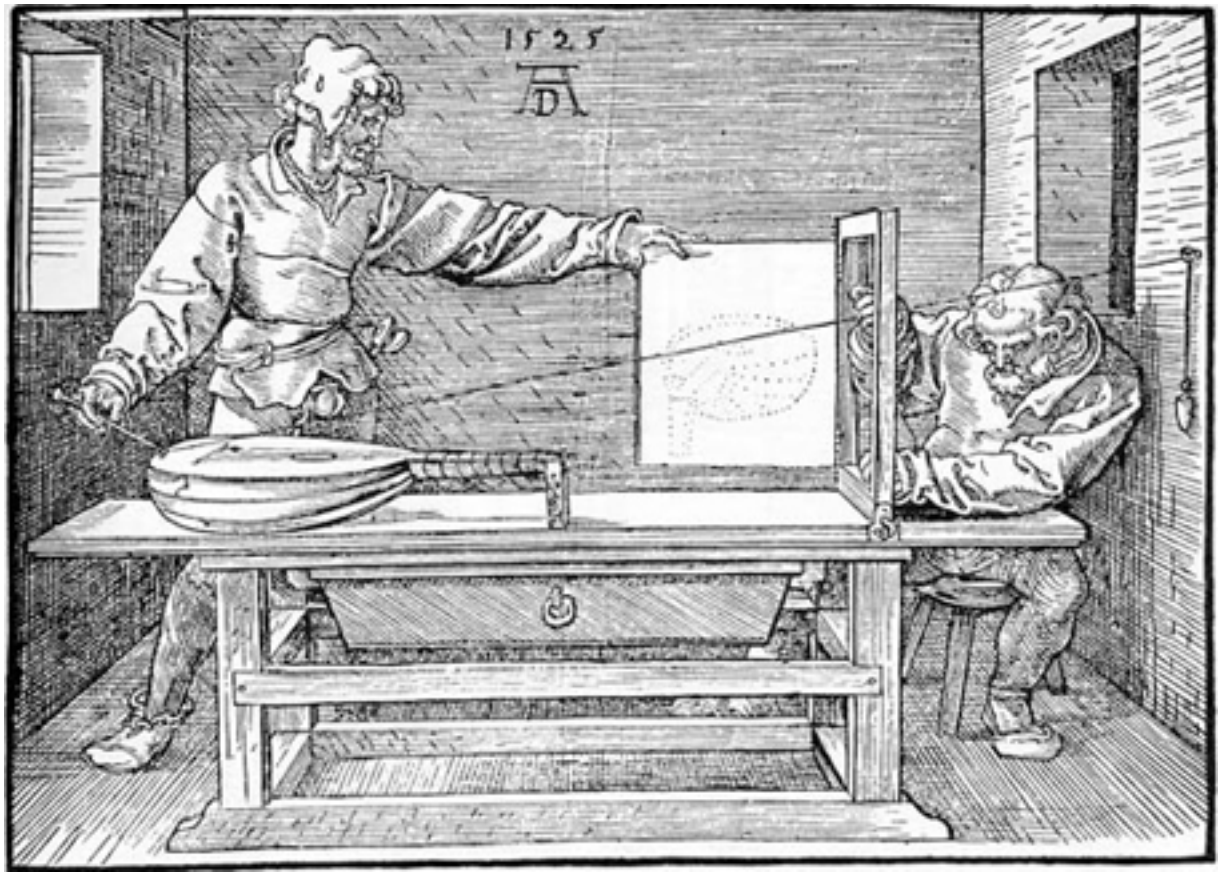


Ideal City, della Francesca, ca. 1470

# 4th Flashback: Discovery and Study of Perspective

Full discovery and mathematical treatment seems to have started with the Italian renaissance in the 15th century and lasted till the 19th century

This included the development of processes for generation of perspective drawings, e.g.:




Dürer, 1525

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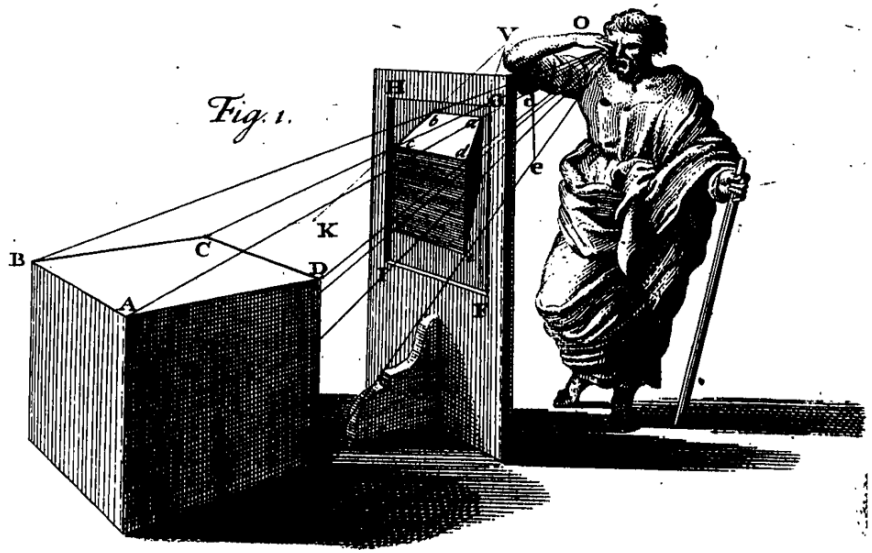
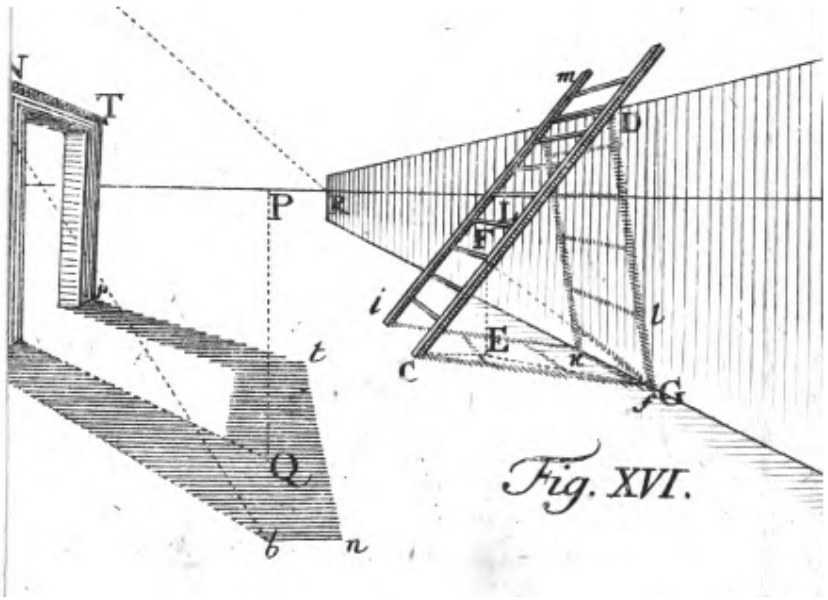
First complete theories and recipes of perspective principles:

- Taylor, 1715




Brook Taylor (1685-1731)

- Lambert, 1759




Johann Heinrich Lambert (1728-1777)

# 4th Flashback: Discovery and Study of Perspective

Full discovery and mathematical treatment seems to have started with the Italian renaissance in the 15th century and lasted till the 19th century

**Inverse problems of perspective:**

- Leonardo da Vinci seems to have been one of the first to wonder how to determine the painter's eye point from a perspective painting. He does not seem to have investigated this further though.
- Others studied two types of inverse problems (for single perspectives):
  - determine the painter's eye point (pose estimation...)
  - determine an object's shape
- In both cases, assumptions need to be made on the object (shape/position/orientation)
- The following scientists proposed solutions for various special cases:

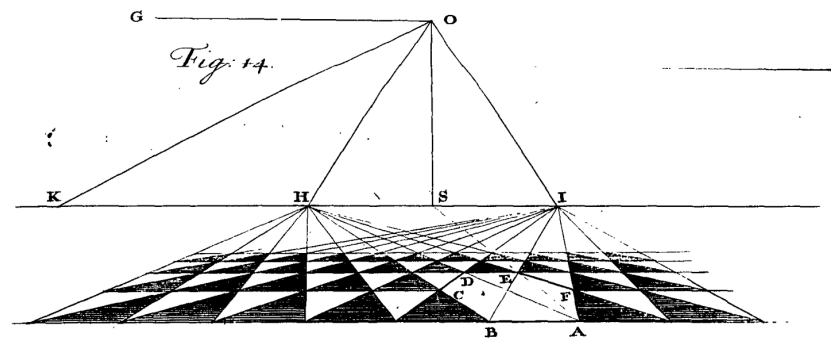
  Guidobaldo, 1600

  Stevin, 1605

  Marolois, 1614

  Taylor, 1715

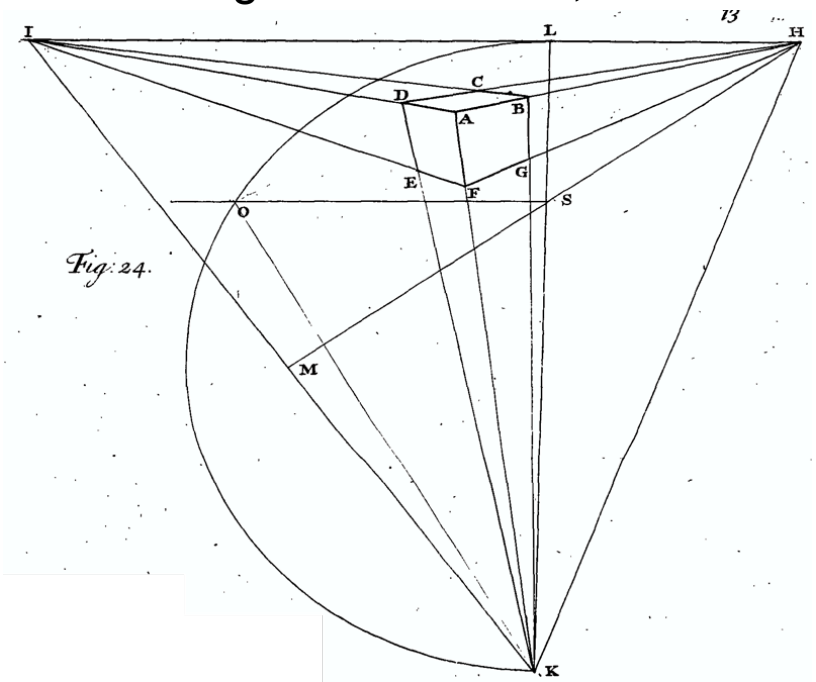
  Lambert, 1759



# 4th Flashback: Discovery and Study of Perspective

Two **pre-photography** discoveries that are still used/relevant today,
expressed in “computer vision language”:

1715, Taylor: an algorithm for estimating the focal length and principal point of a camera, as well as the side lengths of a cuboid, from an image thereof




1773, Lagrange: full sketch of a solution to the general 3-point **pose** problem

- earliest other known reference: Grunert, 1841
- planar 3-point pose problem was solved ca. 1615 by Snellius

Joseph-Louis Lagrange (1736-1813)

# 3rd Flashback: General Orientation Procedures for Perspectives

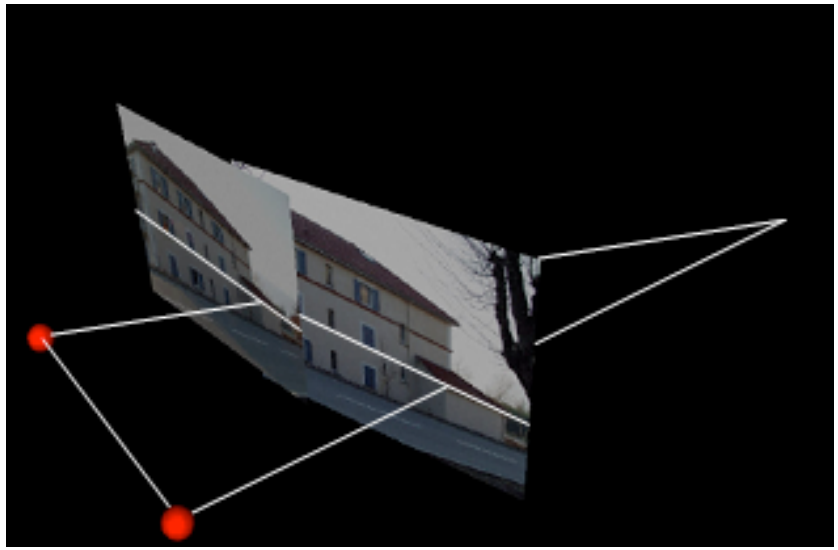
Results after the invention of photography:

**Epipolar geometry**

1862, Terrero

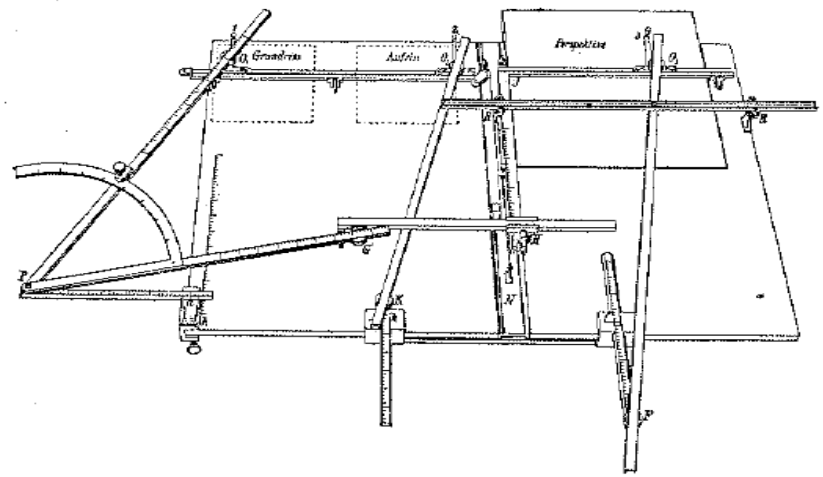
1883, Hauck

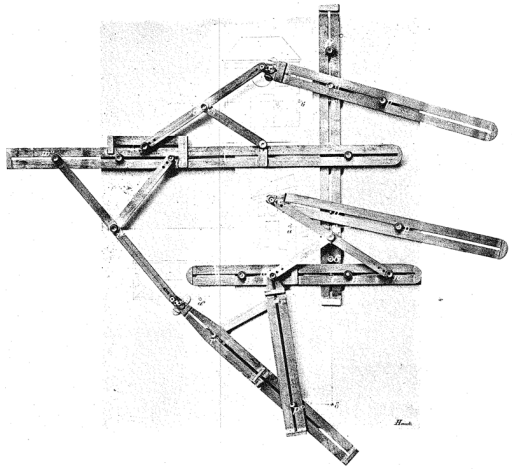
1899, Finsterwalder: solution for 4 coplanar and 2 other points



**Trifocal geometry** (trilinear matching constraints, for points and lines)

1883, Hauck: besides deriving the theory, Hauck also designed and built a device allowing to draw a perspective image from two orthographic ones





Perspectographe de Guido Hauck

# 3rd Flashback: General Orientation Procedures for Perspectives

Results after the invention of photography:

**Motion estimation**

1880, Schröter: 4-point problem for coplanar object points

1913, Kruppa: 5-point problem

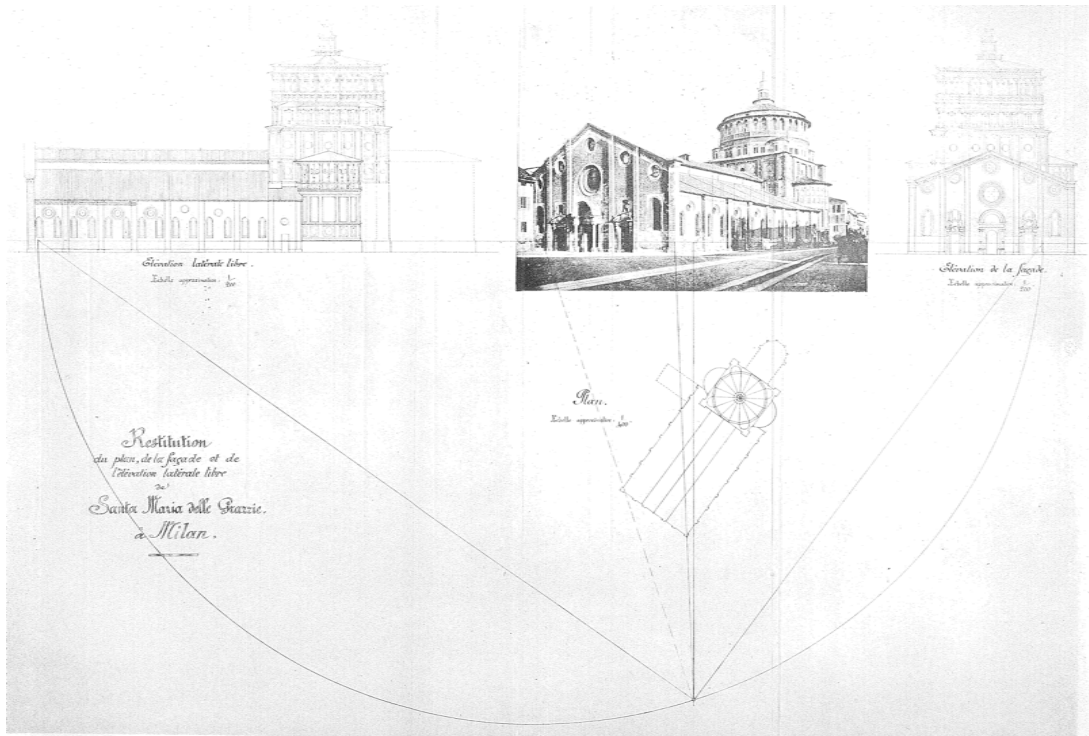
2003, Nistér: first algorithm for the 5-point problem that gives at most 10 solutions

**3D modeling from a single image using vanishing points and lines**

1883, Hauck

1892, Meydenbauer

18??, Laussedat



# 3rd Flashback: General Orientation Procedures for Perspectives

Results after the invention of photography:

**3D reconstruction from uncalibrated images (projective reconstruction)**

1883, Hauck

1899, Finsterwalder

**Camera self-calibration**

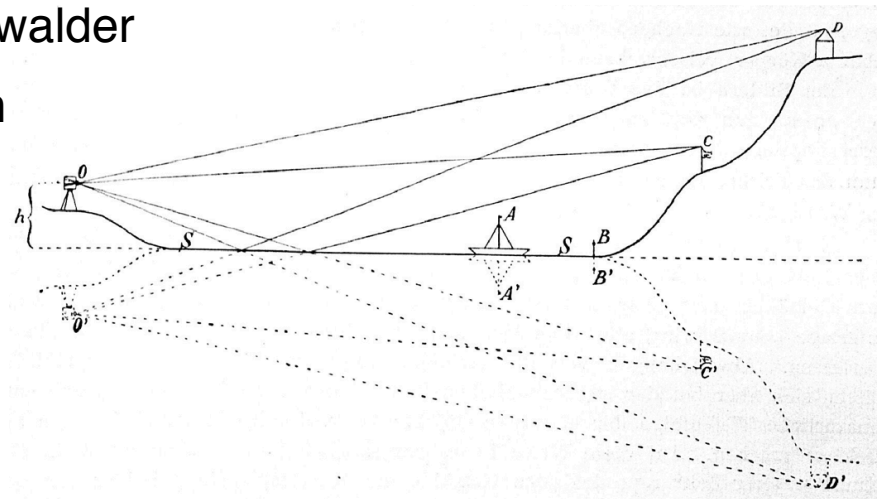
1892, Meydenbauer: mentions that self-calibration of a rotating camera is possible

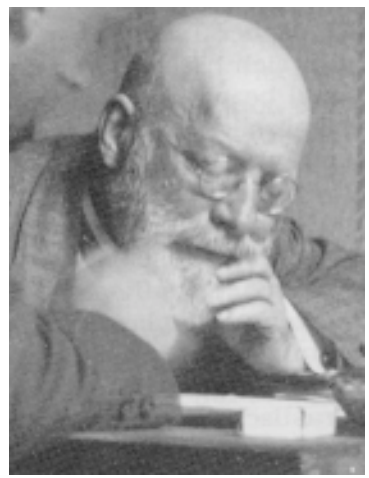
1899, Finsterwalder: self-calibration approach for general motion, based on the concepts of absolute conic and circular points

**3D modeling using mirror images or exploiting symmetries of objects**

1899, Finsterwalder

1905, Pulfrich



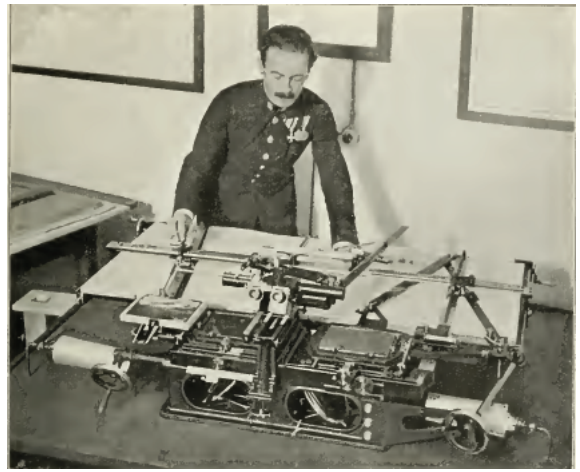



Sebastian Finsterwalder (1862-1951)

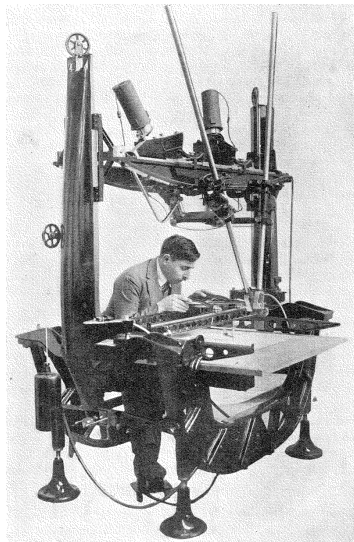
# 3rd Flashback: General Orientation Procedures for Perspectives

Examples of photogrammetric and photographic equipment:

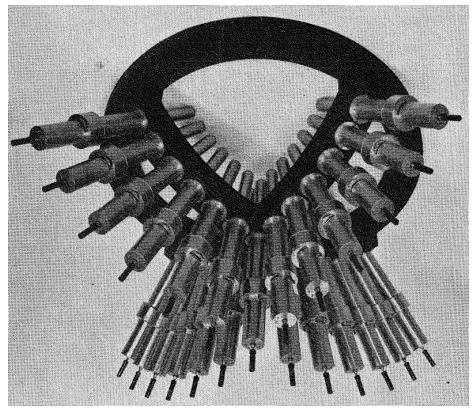
Stereoautograph, around 1908

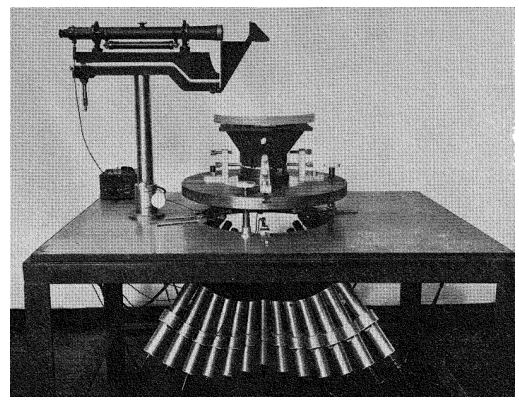


ca. 1930

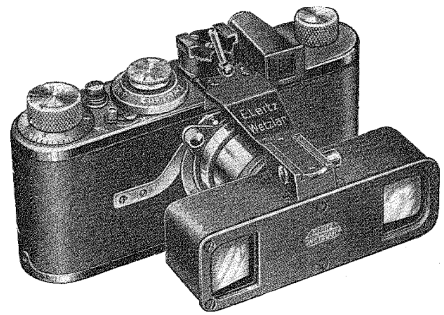


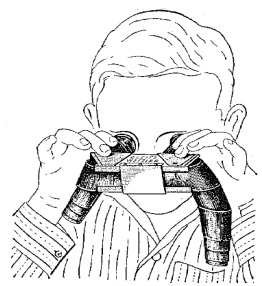
Collimator-based camera calibration





Mirror-based stereo camera
and stereo viewing, 1930





# Outline

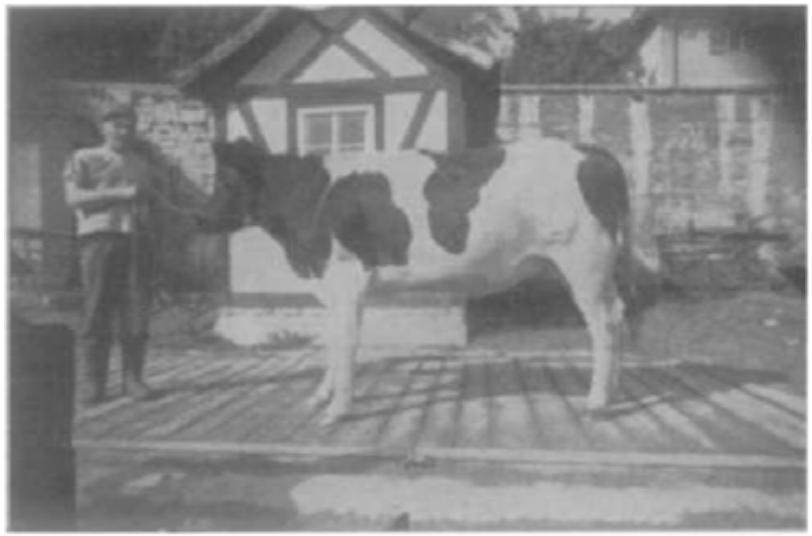
- Introduction

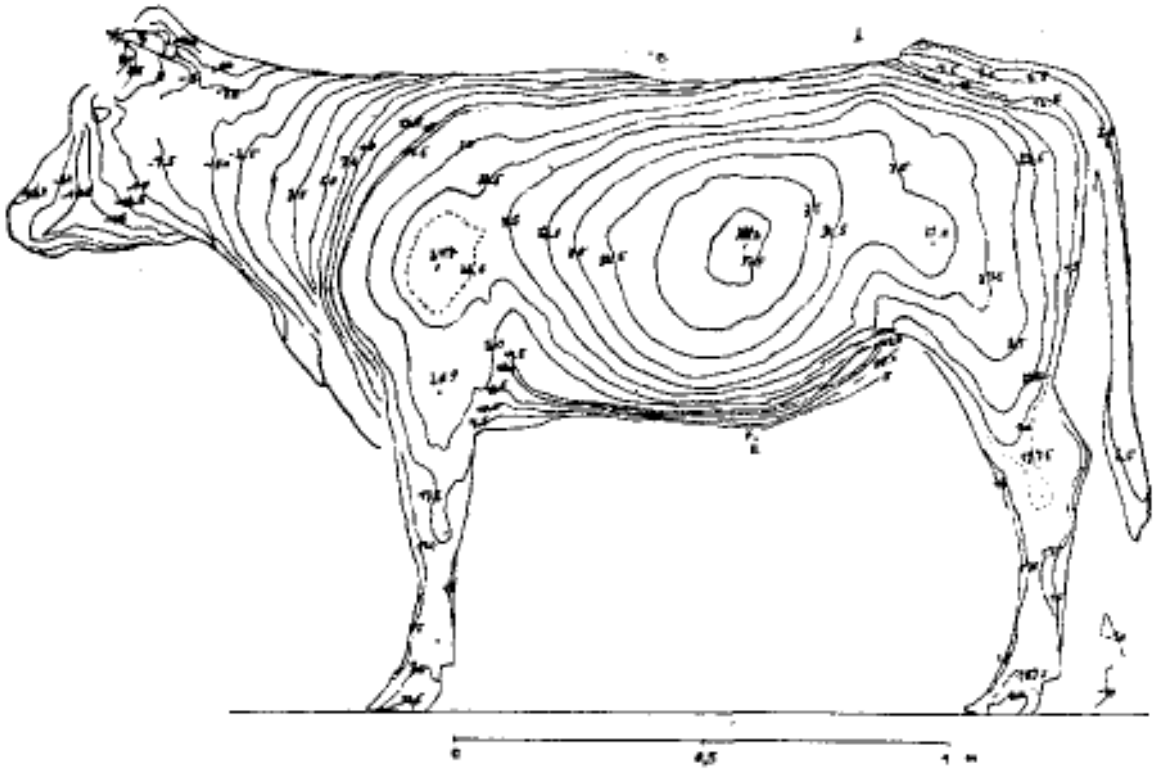
- 1st Flashback: Invention of Photography and Photogrammetry
- 2nd Flashback: Surveying / Topography

- 3rd Flashback: General Orientation Procedures for Perspectives
- 4th Flashback: Discovery and Study of Perspective

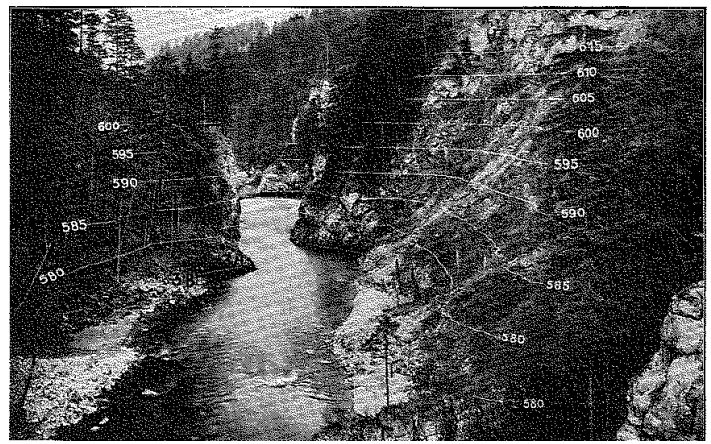
- **Other early examples of scientific applications of photography**

# Cow Topography





Leydolph, around 1938



# Underwater photography/photogrammetry

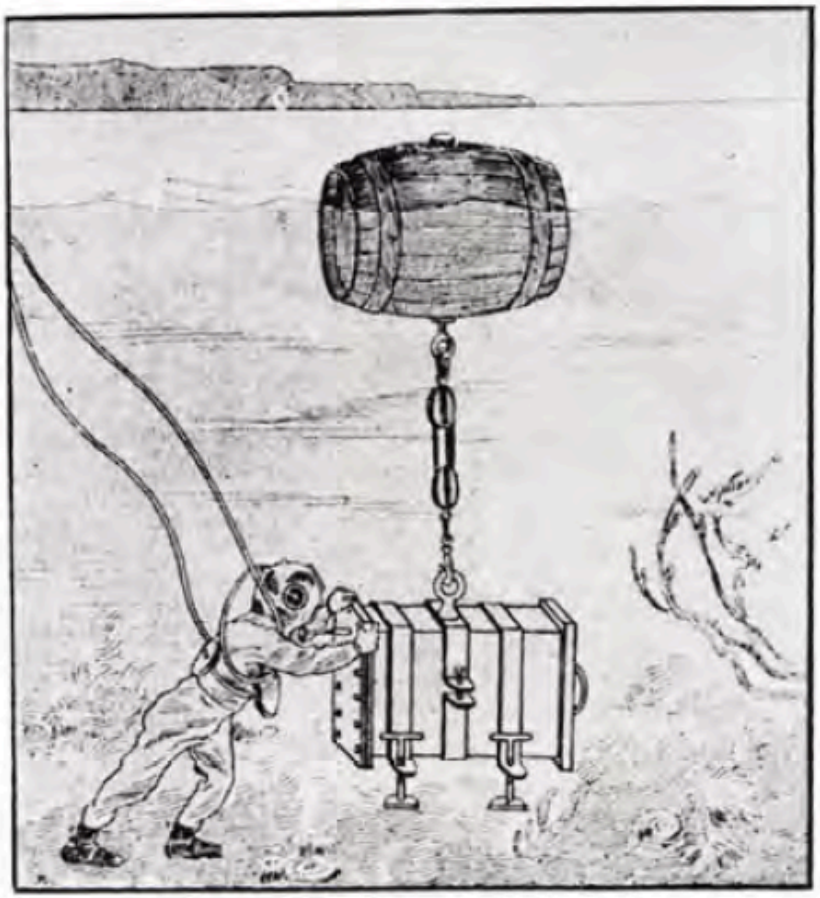
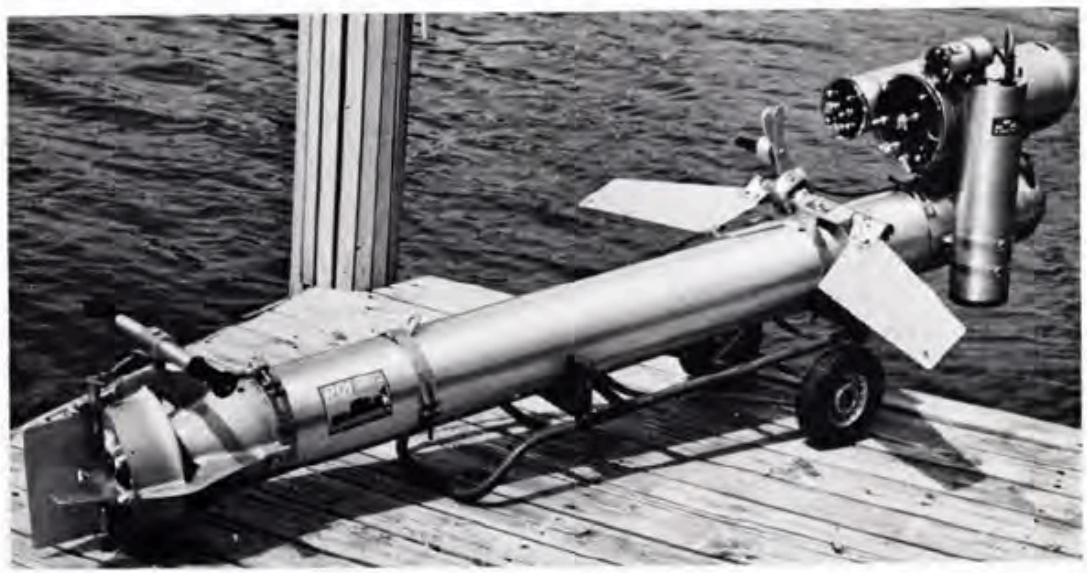


FIG. 1. Louis Boutan pushing his 8×10-inch underwater plate camera into position about 1894.

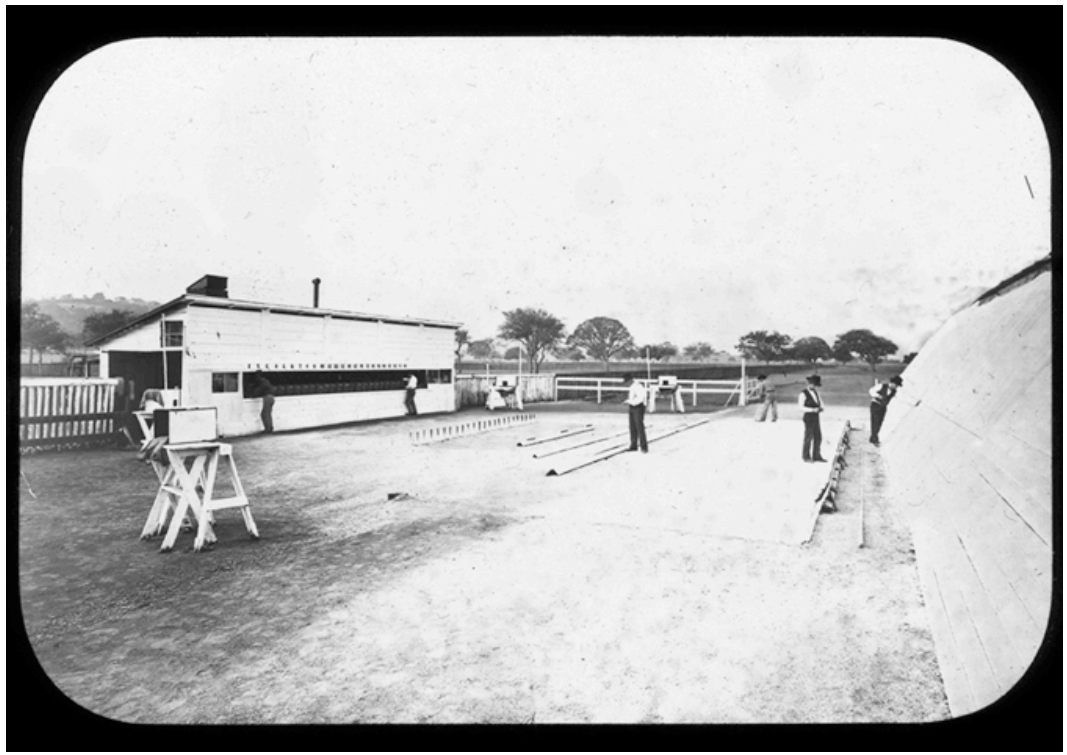


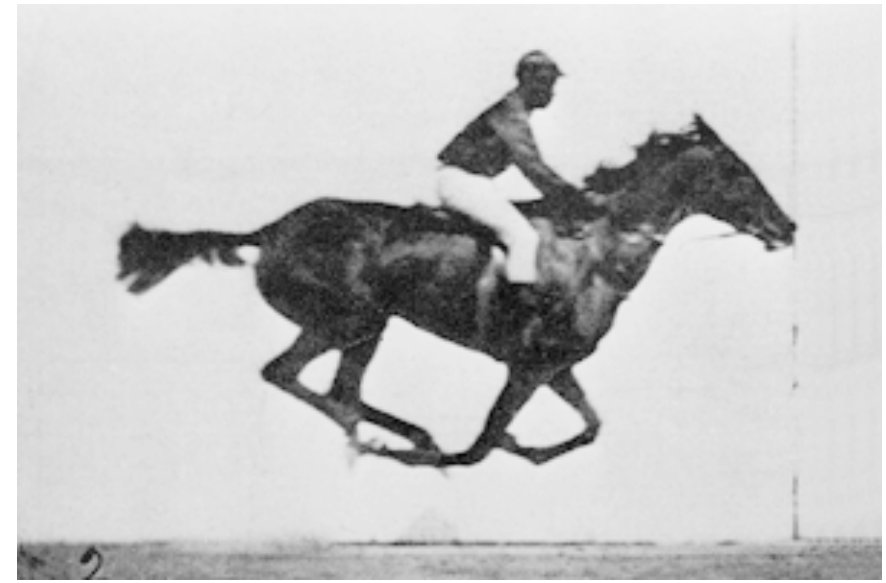
FRONTISPIECE. The latest *Pegasus* 70 mm. underwater photogrammetric system.

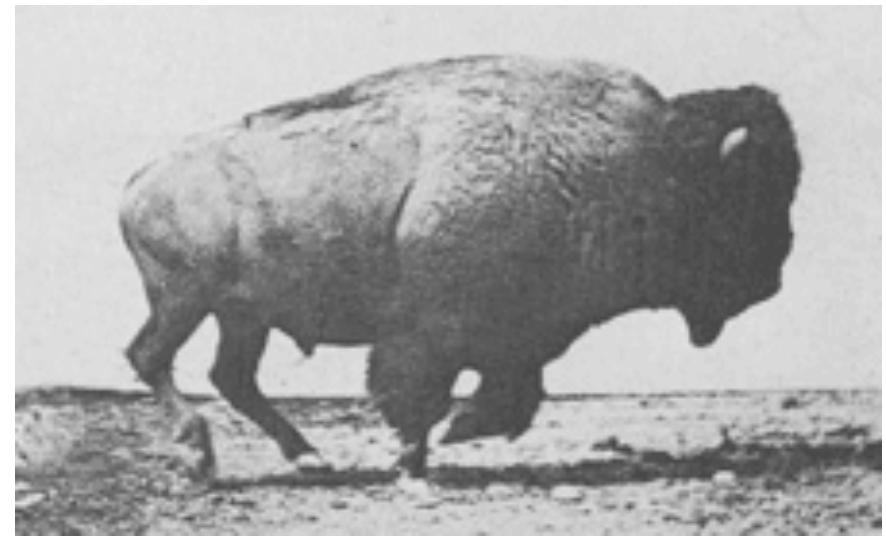
1967

# Studies of moving objects

Muybridge, 1879 (before invention of motion picture)

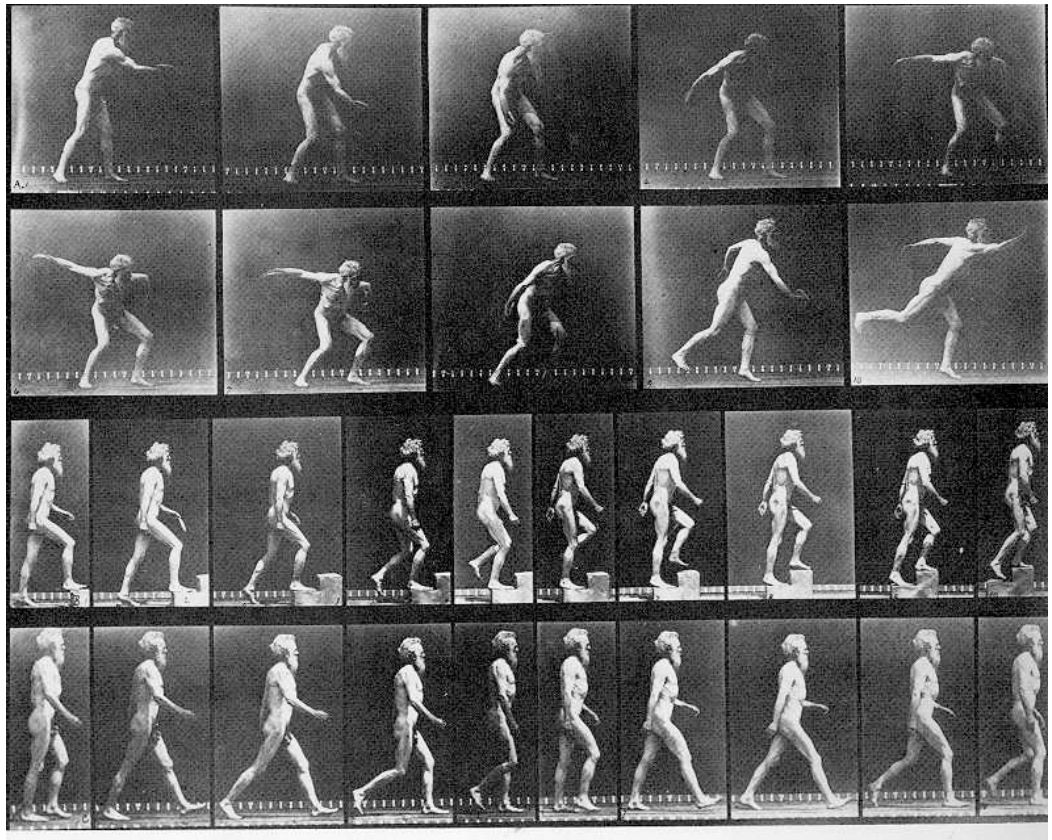


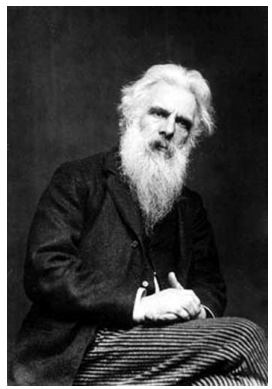




# Studies of moving objects

Muybridge, 1879



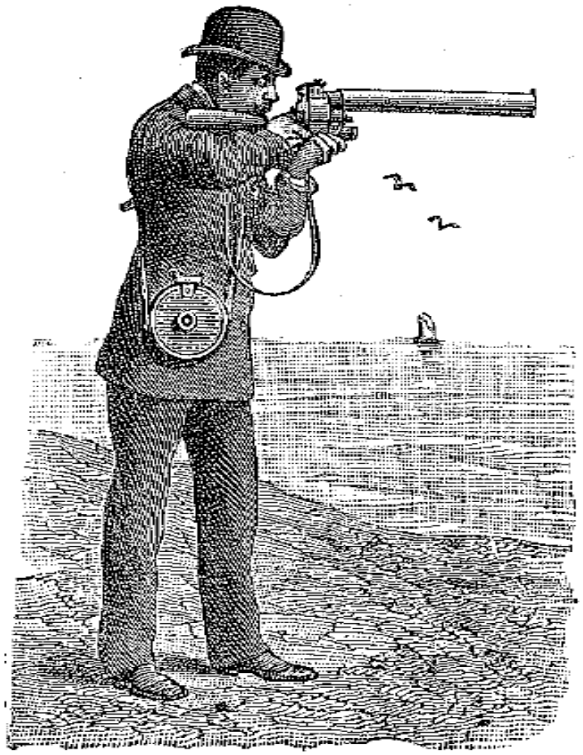


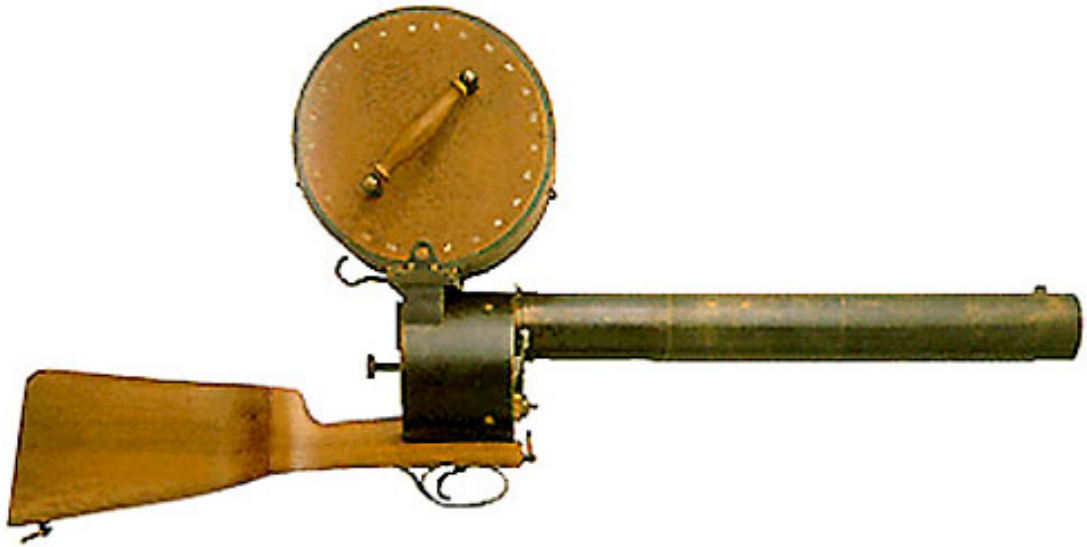
Eadweard Muybridge (1830-1904)
(born Edward James Muggeridge,
then Eduardo Santiage Muggeridge,
then Eadweard Muybridge,
pseudonym Helios)

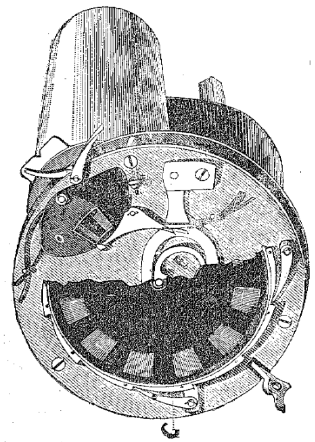
Zoopraxiscope

# Studies of moving objects

Marey, 1882: “photographic gun”

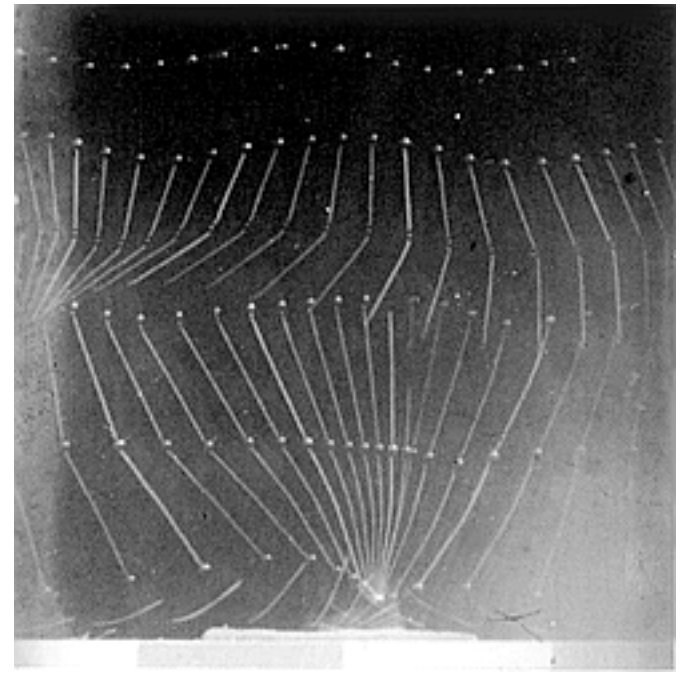




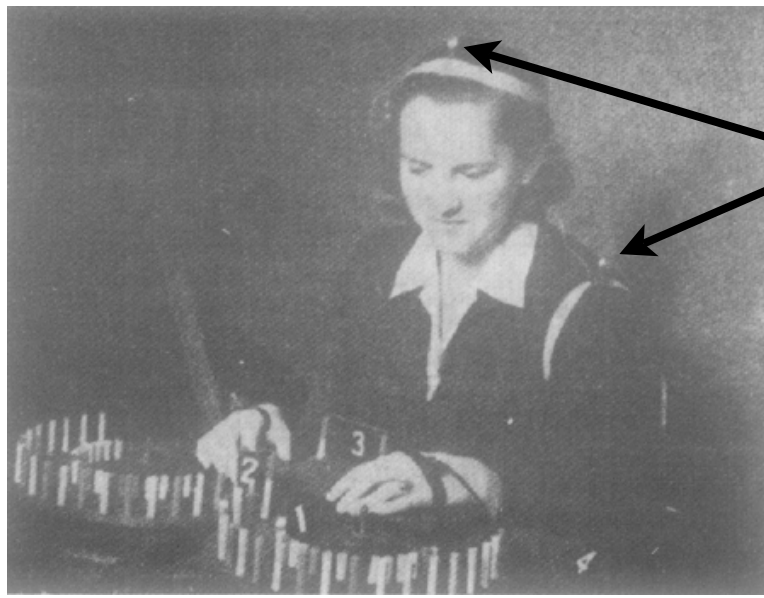


# Studies of moving objects

Marey, end 19th century

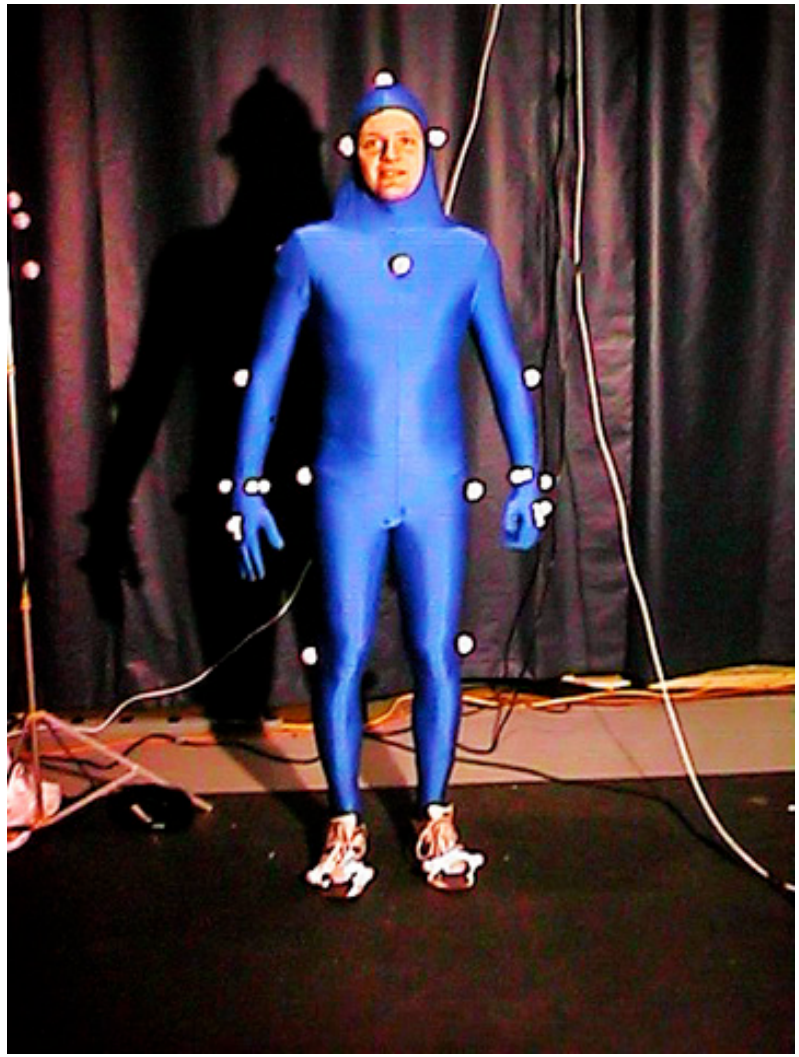


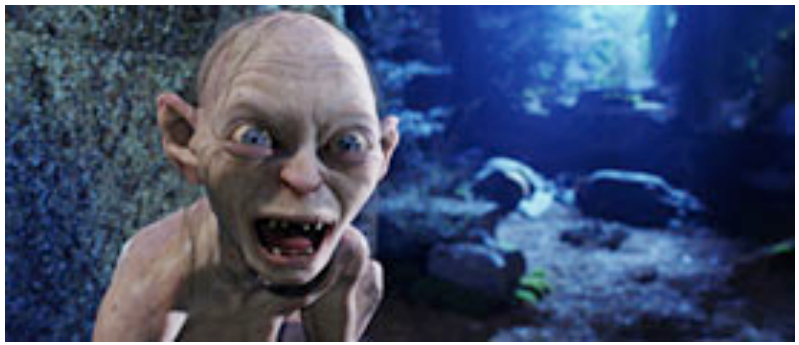
Zeller, 1952



# Studies of moving objects

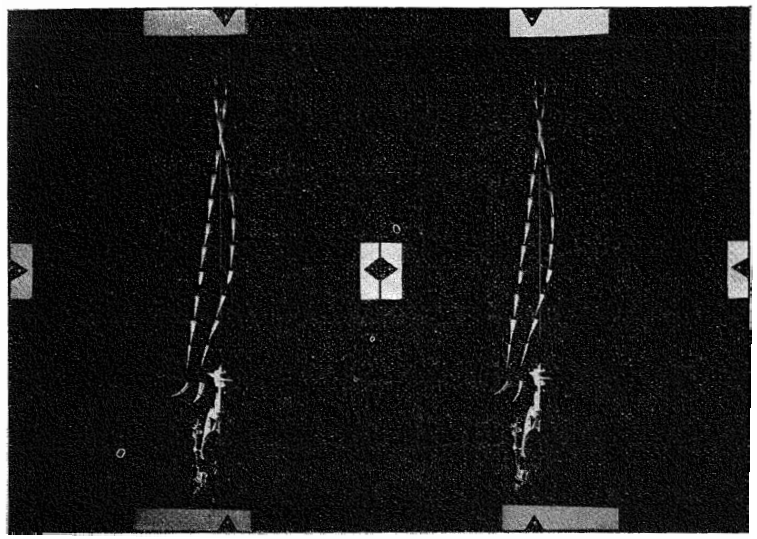
Today's motion capture

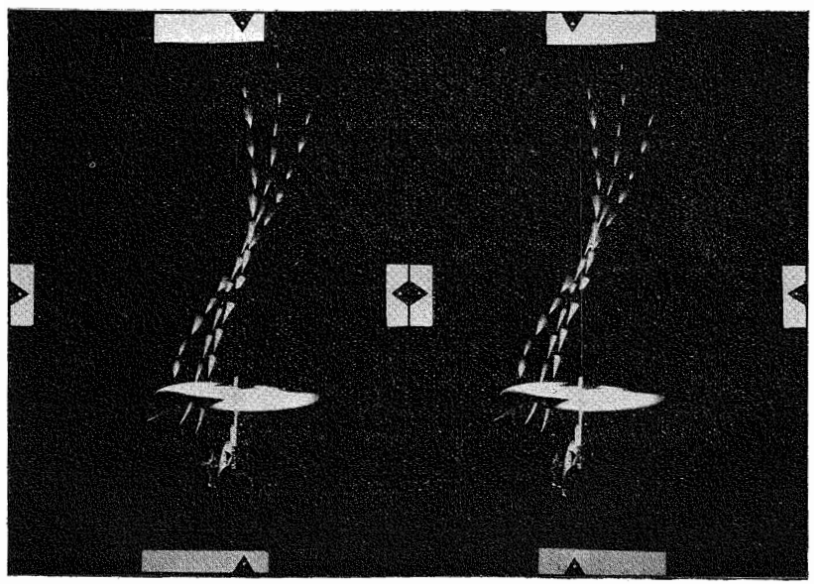




# Studies of moving objects

Katzmayr, 1914: streamlines of turbulent gaz





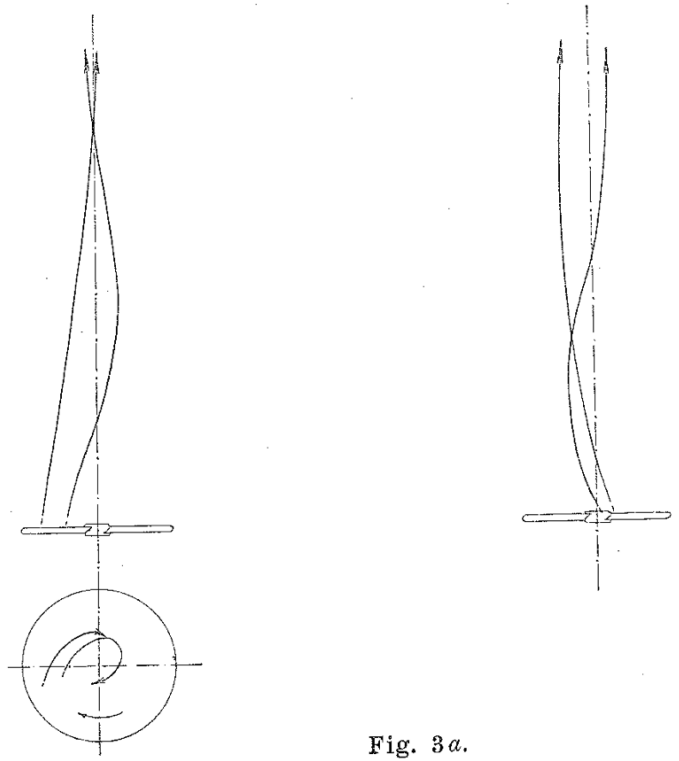


Fig. 3*a*.

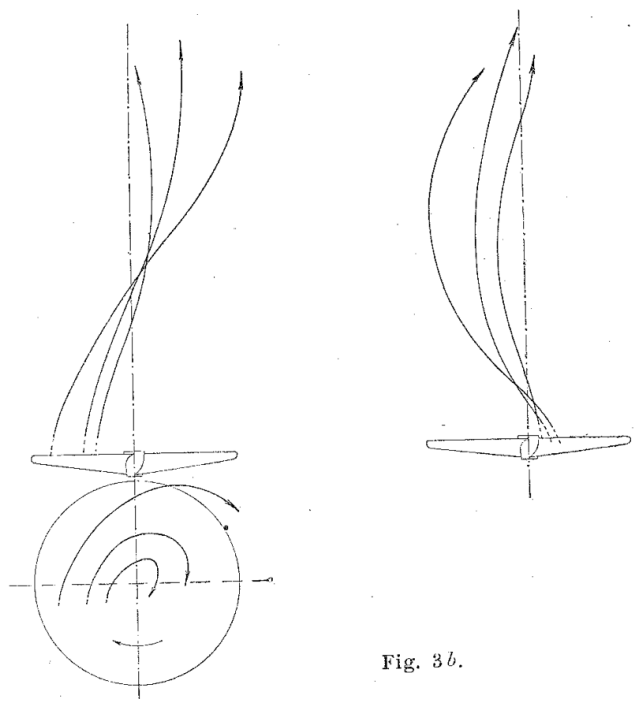
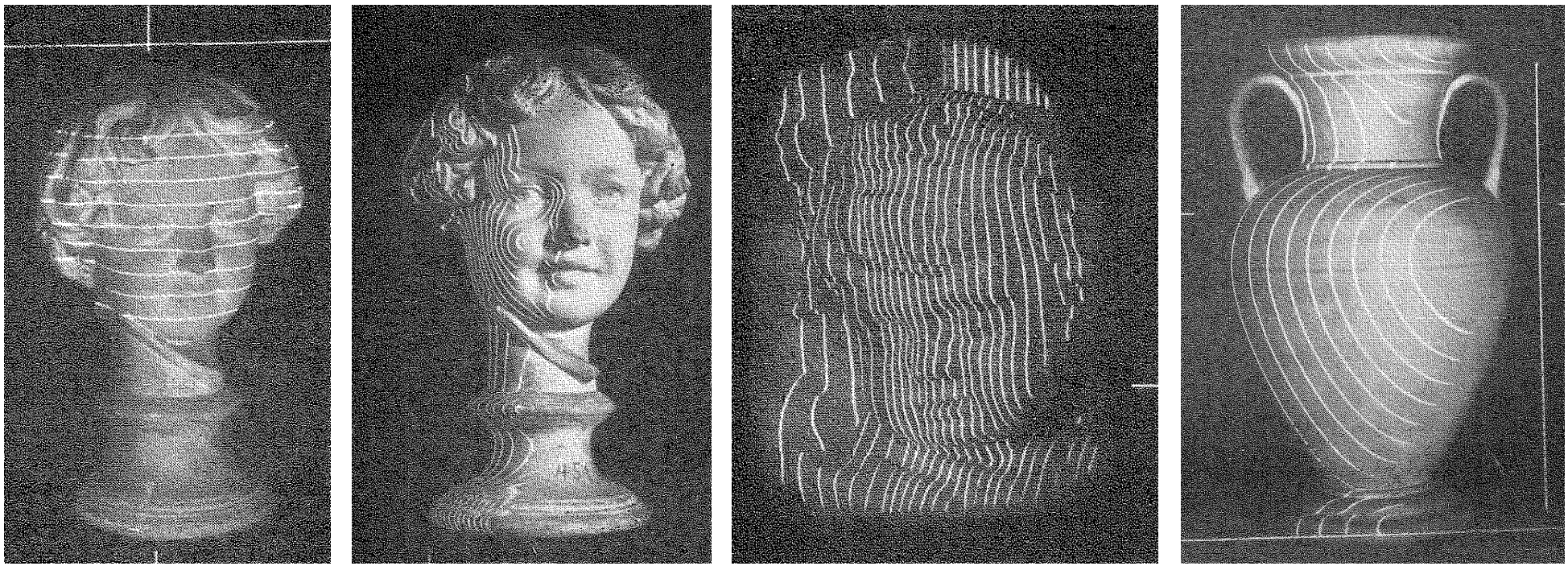


Fig. 3*b*.

# Structured Light for 3D modeling

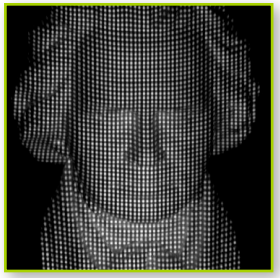
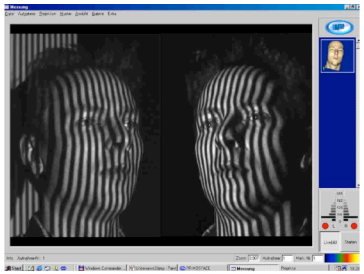
Zaar, 1914

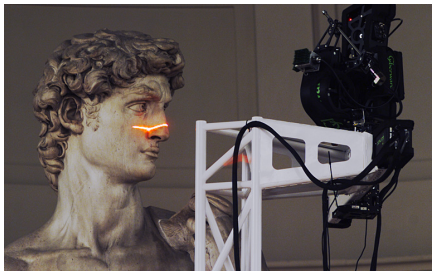


Mentions also exploitation of rotational symmetry in 3D modeling and 3D modeling from shadows.

# Structured Light for 3D modeling

## Modern examples

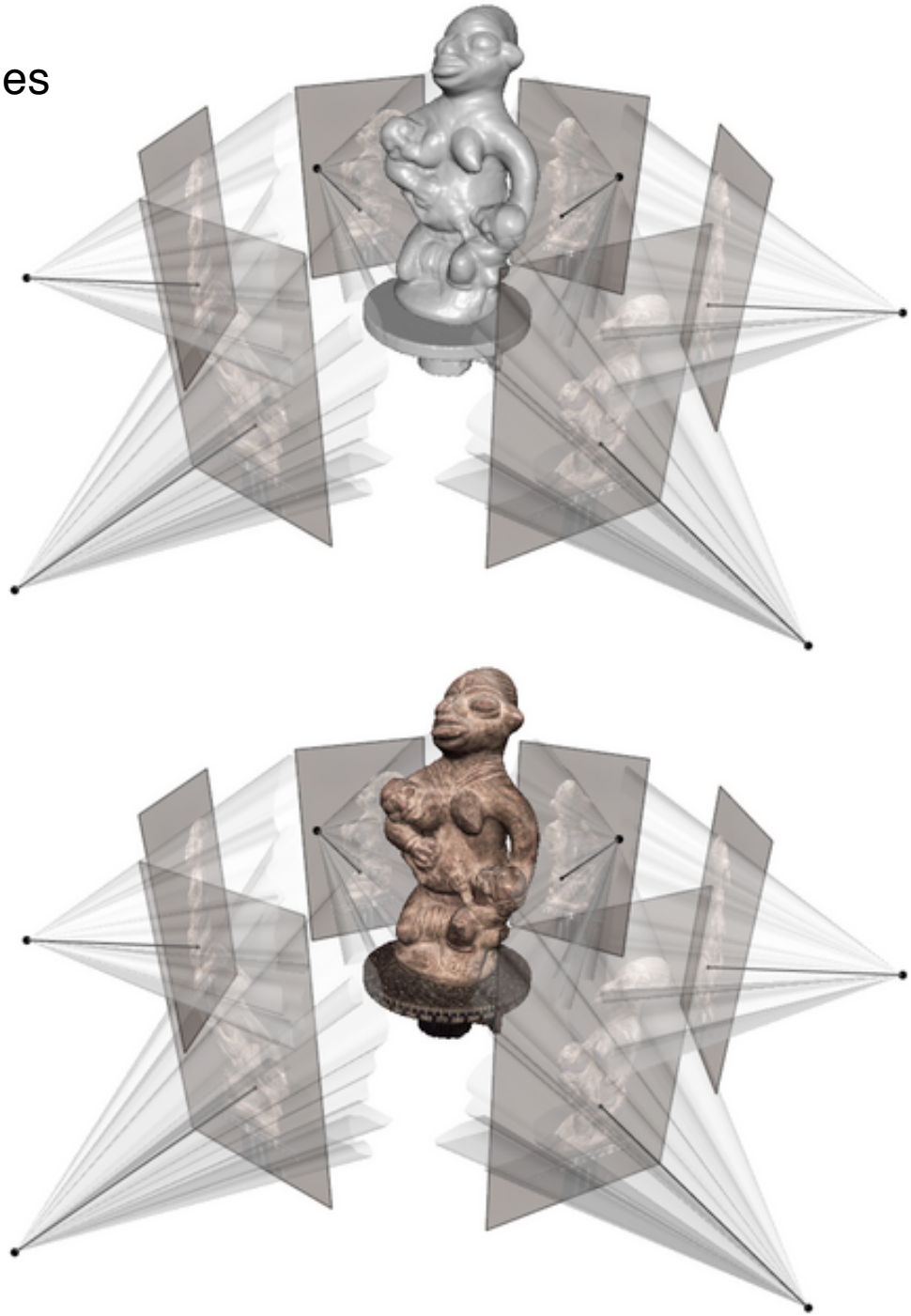



## Kinect


# Shape from Silhouettes / Visual Hull

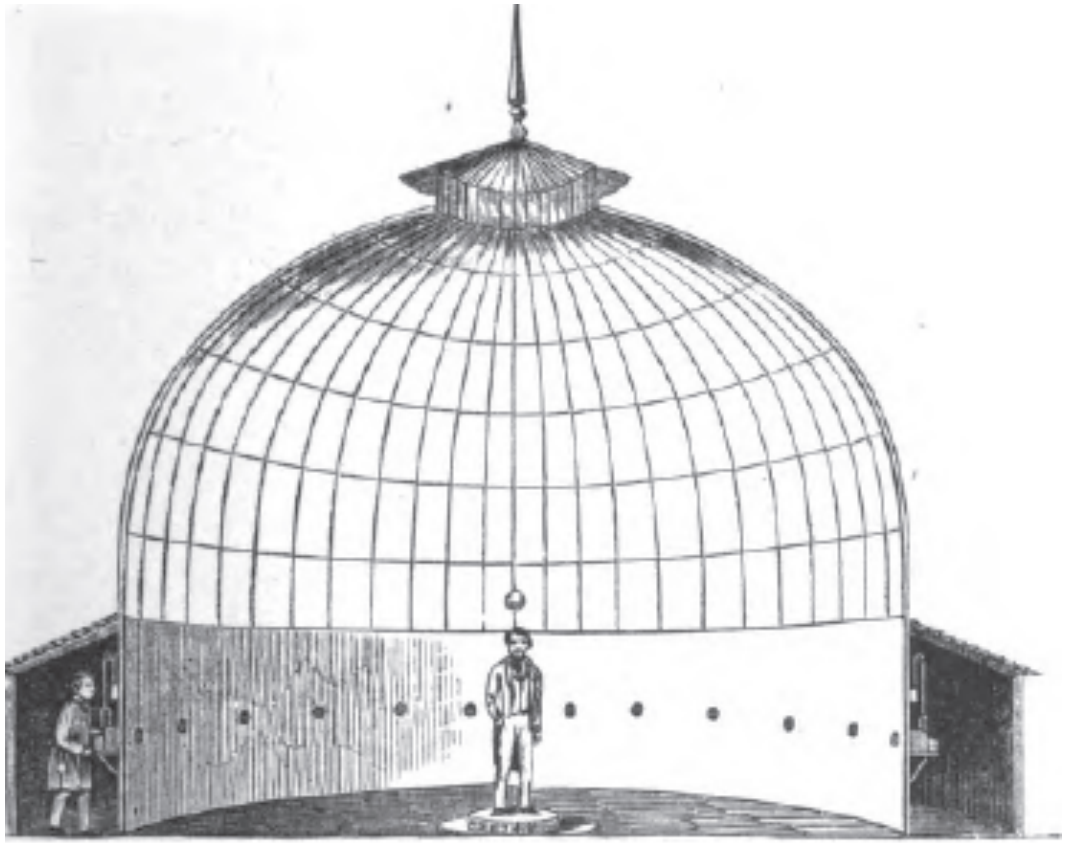
Volumetric modeling by intersecting silhouette cones

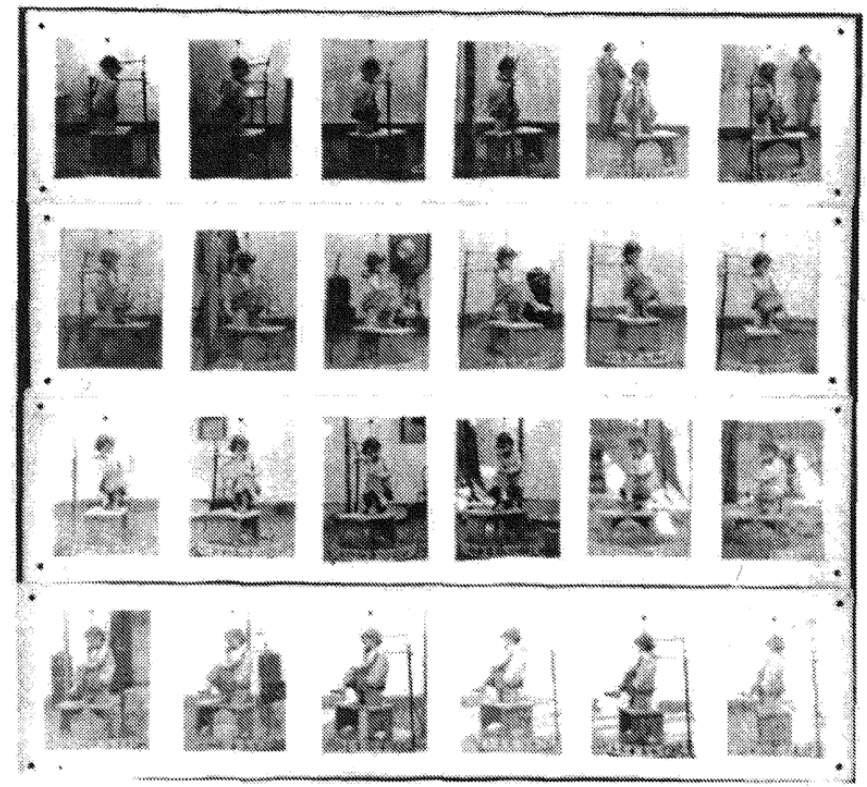
courtesy of Carlos Hernández Esteban



# Shape from Silhouettes / Visual Hull

A “mechanical” version was developed by Willème around 1860:

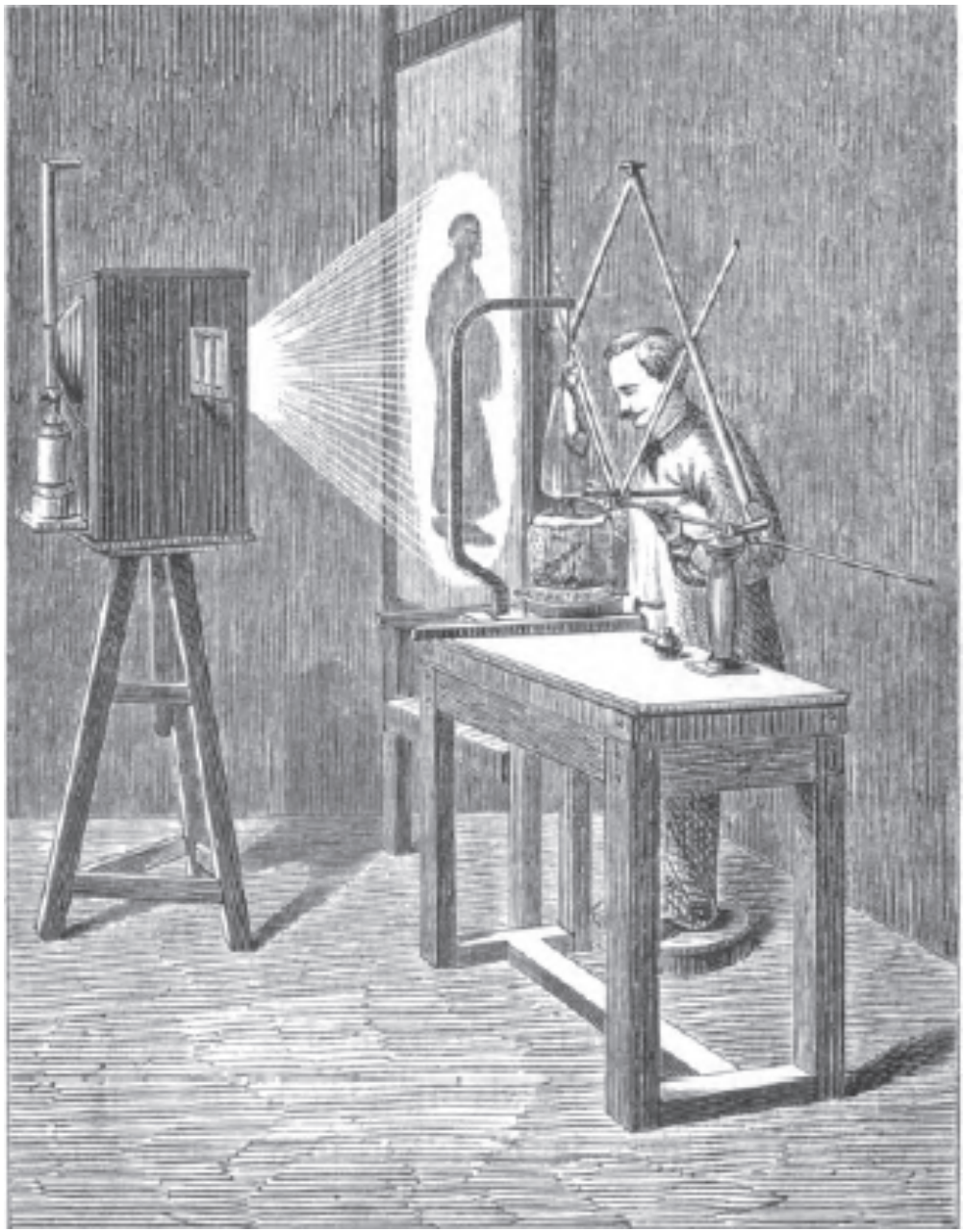


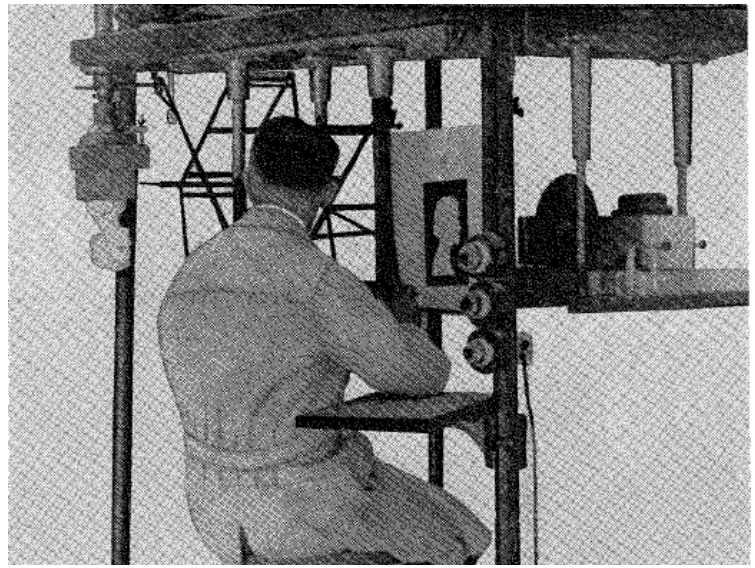


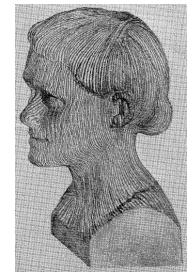
6 Willème, twenty-four sequential photographs of child model, ca. 1863-67, albumen prints. Rochester, GEH

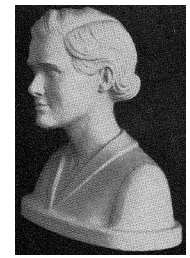
# Shape from Silhouettes / Visual Hull

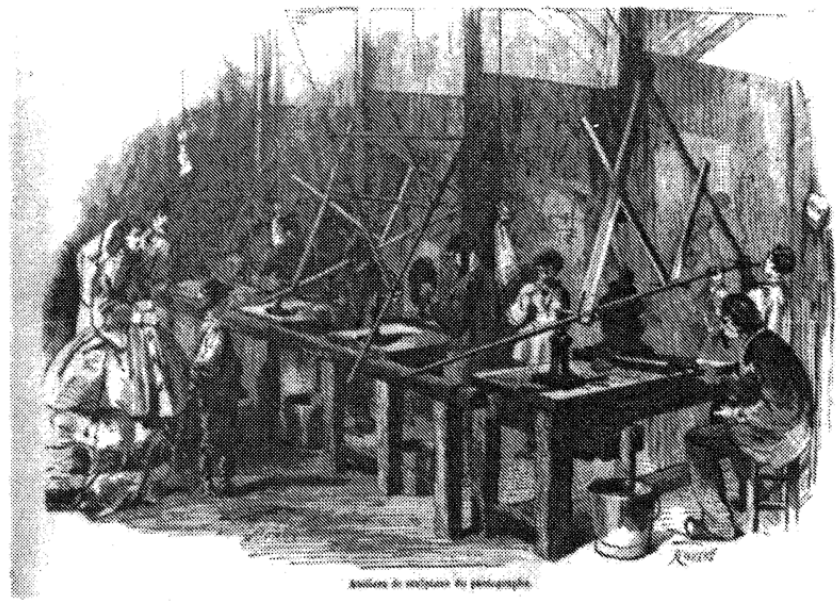
A mechanical version was developed by Willème around 1860:











10 E. Morin and E. Rovins, pantographic studio (from *Le Monde illustré*, December 17, 1864)

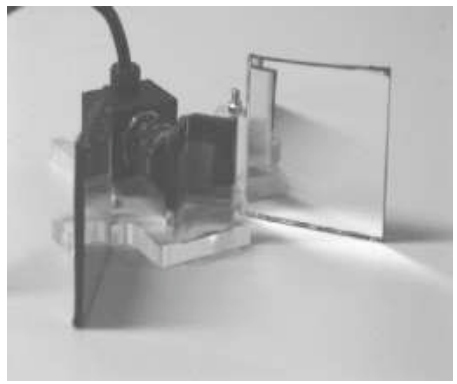
# Conclusions

Historical study of structure-from-motion has become a hobby of mine ;-)

What was shown is only a preliminary overview (more to come, one day...)
(I apologize for any inaccuracy)

Science and technology are often re-invented, increasingly frequently, e.g. in our area:

- panoramic image acquisition devices
- 3D modelling approaches
- analytical camera models
- structure from motion theory and algorithms

Potential reasons for this:

- sometimes because literature is spread across communities, more and more abundant and difficult to browse
- sometimes because publication pressure and frequency increases and less time is spent on literature research
- often because an application of a new theory is not known or not mature yet