On sliding mode and adaptive observers design for multicell converter

Malek Ghanes, Francisco Bejerano, Jean-Pierre Barbot

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***Abstract*— In this paper, a sliding mode and adaptive observers are proposed for multicell converter. The aim is to solve the problem of capacitor's voltages estimation by tacking account the hybrid behavior appearing in the multicell converters. Furthermore, an analysis of convergence for both observers is introduced. Finally some illustrative results of a 3-cell-converter are given in order to show the efficiency of the designed observers. The applicability of the designed observers are emphasized by the robustness test with respect to resistance load variation.**

I. Introduction

The power electronics [5] are well known important technological developments. This is carried out thanks to the developments of the semiconductor of power components and a new systems of energy conversion. Many of those systems present a hybrid dynamics. Among these systems, multicellular converters is based on the association in series of the elementary cells of commutation. This structure, appeared at the beginning of the 90's [6], makes it possible to share the constraints in tension and it also improves the harmonic contents of the wave forms [4]. From a practical point of view, the series of multicell converter, designed by the LEEI (Toulouse-France) [1], leads to a safe series association of components working in a switching mode. This new structure combines additional advantages: reduction of $\frac{dV}{dt}$, this implies possibility of revamping (the revamping allows to avoid the use of classical converters to control electrical motors supporting a high $\frac{dV}{dt}$), better energetically switching and modularity of the topologies. All these qualities make this new topology very attractive in many industrial application. For instance, GEC/ACEC implements this proposal to realize the input converter which supplies their "T13" locomotives in power. Three-phase inverters called "symphony" developed by Alstom for driving electric motors are also based on the same principle. To benefit as much as possible from the large potential of the multicell structure, various research directions were developed in the literature. Furthermore, the normal operation of the series p-cell converter is obtained when the voltages are close to the multiple of $\frac{E}{p}$ where E is the source voltage and p the number of cells. These voltages are generated when a suitable control of switches is applied in order to obtain a specific value. The control inserted by the switches allows cancelling the harmonics at the switching frequency and to reduce the ripple of the chopped voltage. However, these properties are lost if the voltages of these capacitors become far from the desired multiple of $\frac{E}{p}$. Therefore, it is advisable to measure these voltages. But, it is not easy because extra sensors are necessary to measure these voltages, which increases the cost. For this reason, physical sensors for the voltages should be avoided. Hence, the estimation of such voltages by means of an observer becomes an attractive and economical option.

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On the other hand, several approaches have been considered to develop new methods of control and observation of the multicell converter. Initially, models have been developed to describe their instantaneous [4], harmonic [6] or averaging [1] behaviors. These various models were used for the development of control laws in open-loop [12] and in closed loop [2]. Nevertheless, current control algorithms do not take into account the fact that any power converter is a hybrid system. As a consequence, the state vector of a multicell converter is not observable at any time. But, under certain conditions (recently suggested $Z(T_N)$-observability concept [9]), there exists a time sequence (related to the so called hybrid time trajectory [11]), after which we can 'observe' (reconstruct) all the state vector.

In this paper, our goal is to design two observers basing on an instantaneous model describing fully the hybrid behavior of the multicell converter. The first one is based on the super-twisting algorithm ([3], [10]) and the other one is based on an adaptive approach ([7], [8]). Both observers allow to estimate the voltage across the capacitors in a multicell converter using only the current load and the voltage of the source.

II. Multicell converter model

The instantaneous model describing the dynamics of a p-cells converter (Figure 1) reads:

$$\left\{ \begin{array}{rcl} \dot{I}(t) & = & -\frac{R}{L} I(t) + \frac{E}{L} S_p - \sum_{j=1}^{p-1} \frac{v_{c_j}(t)}{L} (S_{j+1} - S_j) \\ \dot{v}_{c_j}(t) & = & \frac{I}{c_j} (S_{j+1} - S_j), \ \ (j = 1, \ldots, p-1) \\ y(t) & = & I(t) \end{array} \right. \quad (1)$$

where I defines the load current, v_{c_j} is the voltage in the j-th capacitor and E represents the voltage of the source. The control $S_j \in \{0, 1\}$ is given by the position of the upper switch on the j-th cell; $S_j = 0$ means an open position of the upper switch and, in the same cell, a closed position of the lower switch. R and L represent the load resistance and inductance, respectively.

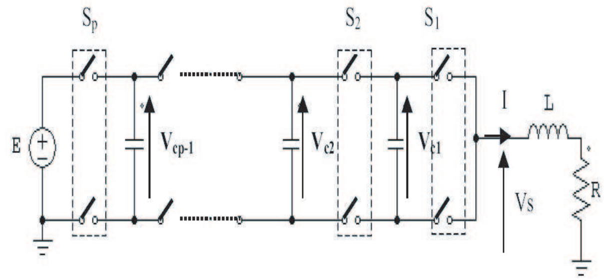

Fig. 1. Multicell converter on RL load.

Taking the state vector as $x := \left(I, v_{c_1}, \ldots, v_{c_{p-1}}\right)$ and by the following definition of the vector $q \in \{-1, 0, 1\}^{p-1}$

$$\left\{ \begin{array}{rcl} q_j: & = & S_{j+1} - S_j,\ j = 1, ..., p-1 \\ q: & = & \left[\begin{array}{ccc} q_1 & \cdots & q_{p-1} \end{array}\right], \end{array} \right. \tag{2}$$

the system (1) can be represented by a hybrid matrix state equation, namely:

$$\left\{ \begin{array}{rcl} \dot{x} & = & A(q)\,x + g(S_p) \\ y & = & Cx \end{array} \right. \tag{3}$$

where

$$A(q) = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{q_1}{L} & \cdots & -\dfrac{q_{p-1}}{L} \\ \dfrac{q_1}{c_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{q_{p-1}}{c_{p-1}} & 0 & \cdots & 0 \end{bmatrix},\ g(S_p) = \begin{bmatrix} \dfrac{E}{L}S_p \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array}\right].$$

III. Sliding mode and adaptive observers design

Before describing the Super-Twisting Observer (STO) and the Adaptive Observer (AO), we introduce the following.

A. Definitions, assumptions and remark

Definition 1 ([11]): A hybrid time trajectory is a finite or infinite sequence of intervals $T_N = \{I_i\}_{i=0}^{N}$, such that

- $I_i = [t_{i,0}, t_{i,1}[$, for all $0 \leq i < N$;
- For all $i < N$ $t_{i,1} = t_{i+1,0}$
- $t_{0,0} = t_{ini}$ and $t_{N,1} = t_{end}$

Moreover, we define $\langle T_N \rangle$ as the ordered list of q associated to T_N (i.e. $\{q_0, ..., q_N\}$ with q_i the value of q during the time interval I_I).

From these it is possible to define a new concept of observability [9]:

Definition 2: The function $z := Z(t, x, S_p)$ is said $Z(T_N)$-observable in $\mathcal{U}$ with respect to system (3) and hybrid time trajectory (T_N and $\langle T_N \rangle$) if for any trajectories $\left(t, x^i(t), S_p^i(t)\right)$, $i = 1, 2$, in $\mathcal{U}$ defined in the interval $[t_{\text{ini}}, t_{\text{end}}]$, the equality

$$y^1(t) = y^2(t)\text{, a.e., in } [t_{ini}, t_{end}]$$

implies

$$Z\left(t, x^1(t), S_p^1(t)\right) = Z\left(t, x^2(t), S_p^2(t)\right)\text{, a.e., in } [t_{ini}, t_{end}]$$

Through this paper, it is assumed that:

- A1. The multicell converter is decoupled from any motor, that is, the term $k_{\text{em}}\omega$ is canceled.
- A2. There exists T_N such that $z := x$ is $Z(T_N)$-observable with respect to (1).
- A3. There exists a constant $\tau > 0$ so that the length $(t_{i,1} - t_{i,0})$ of any interval $\mathcal{I}_i$ is greater than τ.

Remark 1: From the state equation (1), it is clear that on an interval of time $\mathcal{I}$ only the sum of voltages $\sum_{j=1}^{p-1} v_{c_j} q_j$ is observable. To overcome such restriction, assumption A2 ensures that the current crosses through all the capacitors, abusing of the words, in a linear independent form. In other words, after the time interval $\mathcal{I}_{i_{p-1}}$, the information from the derivative of the load current allows to obtain a set of $(p-1)$ linearly independent equations with respect to the voltages in the $(p-1)$ capacitors of multicell converter.

B. Main result

Here, we introduce an algorithm which keeps the values of q^{i_j} in such a way that after some sequence switches, the sequence of vectors $\left\{q^{i_1}, ..., q^{i_{p-1}}\right\}$ generates the space $\mathbb{R}^{p-1}$. Thus we can estimate all the voltages capacitor of multicell converter.

Let us define $H_i \in \mathbb{R}^{(p-1)\times(p-1)}$, in the interval $\mathcal{I}_i = [t_{i,0}, t_{i,1})$ (for all $0 \leq i < N$) as a matrix formed by the following algorithm:

- Step 0 $H_{-1} = 0$.
- Step 1 If $q^i \neq 0$, $H_{i,1} = q^i$, where $H_{i,1}$ is the first row of H_i, and set $i_{p-1} = i$. If $q = 0$, $H_i = H_{i-1}$.
- Step k Let $H_{i,k}$ be the k-th row of $H_{i,k}$ for $k \geq 2$. Set $H_{i,k} = q^{i_{p-k}}$, where i_{p-k} is the biggest index such that $i_{p-(k-1)} > i_{p-k}$ and the vectors $\left\{q^{i_{p-1}}, ..., q^{i_{p-k}}\right\}$ are linearly independent. If does not exist an index i_{p-k} such that $\left\{q^{i_{p-1}}, ..., q^{i_{p-k}}\right\}$ are linearly independent, then $H_{i,k} = H_{i,k-1}$.

Let us define H_i^+ as the pseudo-inverse of H_i. It is well-known that for the case when H_i is non-singular $H_i^+ \equiv H_i^{-1}$.

Now we can introduce the observers: STO and AO.

1) Description of a Super-Twisting Observer: The STO is given by the following set of equations.

$$
\begin{cases}
\dot{x}_{\mathrm{a}} = -\dfrac{R}{L}I + \dfrac{E}{L}S_p - \dfrac{1}{L}\sum\limits_{j=1}^{p-1}\left(\bar{v}_{\mathrm{c}_j} + \tilde{v}_{\mathrm{c}_j}\right)q_j \\
+\sum\limits_{j=1}^{p-1}\lambda\left|q_j\right|\left|I - x_a\right|^{1/2}\operatorname{sign}\left(I - x_a\right) \\
\dot{\bar{v}}_{\mathrm{c}_j} = \dfrac{I}{c_j}q_j,\ \bar{V}_{\mathrm{c}}^T = \left[\begin{array}{ccc} \bar{v}_{\mathrm{c}_1} & \cdots & \bar{v}_{\mathrm{c}_{p-1}} \end{array}\right] \\
\dot{\tilde{v}}_{\mathrm{c}_j} = -\alpha q_j \operatorname{sign}\left(I - x_a\right), \quad (j = 1, \ldots, p-1) \\
\tilde{V}_{\mathrm{c}}^T = \left[\begin{array}{ccc} \sum\limits_{j=1}^{p-1} q_j^{i_{p-1}}\tilde{v}_{\mathrm{c}_j}^{i_{p-1}} & \cdots & \sum\limits_{j=1}^{p-1} q_j^{i_1}\tilde{v}_{\mathrm{c}_j}^{i_1} \end{array}\right]; \\
\quad \tilde{v}_{\mathrm{c}_j}^{i_k} \equiv \tilde{v}_{\mathrm{c}_j} \text{on } \mathrm{I}_{i_k} = [t_{i_k,0}, t_{i_k,1}) \\
\quad \tilde{v}_{\mathrm{c}_j}^{i_k}(t) = \tilde{v}_{\mathrm{c}_j}^{i_k}(t_{i_k,1}) \text{ for } t \geq t_{i_k,1} \\
\hat{V}_{\mathrm{c}}(t) = \bar{V}_{\mathrm{c}}(t) + H_i^{+}\tilde{V}_{\mathrm{c}}(t) \\
\hat{V}_{\mathrm{c}}^T =: \left[\begin{array}{ccc} \hat{v}_{\mathrm{c}_1} & \cdots & \hat{v}_{\mathrm{c}_{p-1}} \end{array}\right]
\end{cases}
\tag{4}
$$

with α et λ ($j = 1, \ldots, p-1$) satisfying

$$
\alpha > 0,\ \lambda > \frac{(1+\theta)}{(1-\theta)}\sqrt{\frac{2\alpha}{L}},\ 0 < \theta < 1 \tag{5}
$$

Defining $e_1 = I - x_{\mathrm{a}}$ and $e_{\mathrm{c}_j} = v_{\mathrm{c}_j} - \hat{v}_{\mathrm{c}_j}$ ($j = 1, ..., p-1$), we obtain

$$
\begin{array}{l}
\dot{e}_1 = -\dfrac{1}{L}\sum\limits_{j=1}^{p-1}\left(v_{\mathrm{c}_j} - \bar{v}_{\mathrm{c}_j} + \tilde{v}_{\mathrm{c}_j}\right)q_j - \lambda\left|e_1\right|^{1/2}\operatorname{sign} e_1 \sum\limits_{j=1}^{p-1}\left|q_j\right| \\
\dfrac{d}{dt}\sum\limits_{j=1}^{p-1}\left(v_{\mathrm{c}_j} - \bar{v}_{\mathrm{c}_j} + \tilde{v}_{\mathrm{c}_j}\right)q_j = -(p-1)\,\alpha \operatorname{sign}\left(e_1\right)
\end{array}
\tag{6}
$$

For q constant, and α and λ satisfying (5), there exists $T_i > t_{i,0}$, such that

$$
e_1(t) = 0, \dot{e}_1(t) = 0 \text{ for } t \geq T_i \tag{7}
$$

The equality (7) is assured only if q stays fixed. Therefore, since q is constant on $t \in [t_{i,0}, t_{i,1}[$, it must be ensured T_i to be smaller than $t_{i,1}$. From the proof of convergence given in [3], one can obtain the time of converge to the sliding mode, that is,

$$
T_i - t_{i,0} \leq \frac{1+\theta}{2\theta\alpha}\sum_{j=1}^{p-1}\left|v_{\mathrm{c}_j}(0) - \bar{v}_{\mathrm{c}_j}(0)\right| \tag{8}
$$

Thus, choosing α enough big so that $T_i - t_{i,0} < \tau$ and according to assumption A3, one gets $T_i < t_{i,1}$. Hence, from (7) and the equation for $\dot{e}_1$ in (6), one deduce

$$
\sum_{j=1}^{p-1}\left(v_{\mathrm{c}_j} - \bar{v}_{\mathrm{c}_j} - \tilde{v}_{\mathrm{c}_j}\right)q_j = 0. \text{ for } t_{i,1} \geq t \geq T_i \tag{9}
$$

Theorem 1: Under assumption A1 and A2, the identities

$$
\hat{v}_{\mathrm{c}_i} \equiv v_{\mathrm{c}_i},\ i = 1, \ldots, p-1 \tag{10}
$$

are achieved, for all $t > T_{i_{p-1}} \geq t_{i_{p-1},0}$, where $T_{i_{p-1}}$ is the reaching time to the i_{p-1}-th sliding mode.

Proof: A1 implies that there exist $(p-1)$ indexes i_k ($k = 1, ..., p-1$) ($i_{k+1} > i_k$) so that $\left\{q^{i_1}, ..., q^{i_{p-1}}\right\}$ is a set of $(p-1)$ linearly independent vectors, then, from (9), the set of equalities (11) holds.

$$
\begin{cases}
q^{i_{p-1}}v_{\mathrm{c}}(t) = q^{i_{p-1}}\bar{v}_{\mathrm{c}}(t) + q^{i_{p-1}}\tilde{v}_{\mathrm{c}}^{i_{p-1}}(t) \\
t \in \left[T_{i_{p-1}}, t_{i_{p-1},1}\right] \\
\vdots \\
q^{i_1}v_{\mathrm{c}}(t) = q^{i_1}\bar{v}_{\mathrm{c}}(t) + q^{i_1}\tilde{v}_{\mathrm{c}}^{i_1}(t),\ t \in \left[T_{i_1}, t_{i_1,1}\right]
\end{cases}
\tag{11}
$$

Since $\dot{v}_{\mathrm{c}}(t) - \dot{\bar{v}}_{\mathrm{c}}(t) = 0$ for all $t \geq 0$, $q^{i_j}v_{\mathrm{c}}(t) \equiv q^{i_j}\bar{v}_{\mathrm{c}}(t) + q^{i_j}\tilde{v}_{\mathrm{c}}^{i_j}\left(T_{i_j}\right)$ for all $t \geq T_{i_j}$. Obviously $\tilde{v}_{\mathrm{c}}^{i_j}(t)$ stays constant on $\left[T_{i_j}, t_{i_j,1}\right[$. Rearranging (11), into a matrix equation, we have

$$
\bar{V}_{\mathrm{c}} + \left(H_i\right)^{-1}\tilde{V}_{\mathrm{c}} \equiv V_{\mathrm{c}} \text{ for all } t \geq T_{i_{p-1}} \tag{12}
$$

A comparison between (12) and the formula that defines $\hat{v}_{\mathrm{c}}$ in (4) finishes the proof. ■

2) Description of an Adaptive Observer: Consider the multicell converter model (3). As mentioned in remark (1), on an interval of time $\mathcal{I}$ only the sum of voltages $\sum\limits_{j=1}^{p-1} v_{c_j}q_j$ is observable. From this point on view let us define the quantity $\sum\limits_{j=1}^{p-1} v_{c_j}q_j = b$ as a state variable. Then the multicell converter model (1) can be rewritten as:

$$
\begin{cases}
\dot{X} & = & AX + G(u, y) \\
y & = & CX
\end{cases}
\tag{13}
$$

where

$$
\begin{array}{rcl}
A & = & \left[\begin{array}{cc} 0 & -\dfrac{1}{L} \\ 0 & 0 \end{array}\right],\ G(u, y) = \left[\begin{array}{c} \dfrac{E}{L}S_p - \dfrac{R}{L}I \\ \sum\limits_{j=1}^{p-1}\dfrac{\left|q_j\right|}{c_j}I \end{array}\right] \\
C & = & \left[\begin{array}{cc} 1 & 0 \end{array}\right],\ X^T = \left[\begin{array}{cc} I & b \end{array}\right].
\end{array}
$$

We can remark that (13) is on the form of affine systems ([7], [8]) where the matrix $A(u) = A$ is constant.

Then an A0 to estimate the capacitors voltages of the multicellular by using only the current measurement is described by following set of equations:

$$
\begin{cases}
\dot{Z} = AZ + G(u,y) + P^{-1}C^T(y-\hat{y}) \\
\dot{P} = -\rho P - A^T P - PA + 2C^T C \\
\hat{y} = CZ \\
Z^T = \begin{bmatrix} \hat{I} & \hat{b} \end{bmatrix} \\
\dot{\bar{v}}_{cj} = \frac{q_j}{c_j} I \; ; \bar{V}_c = \begin{bmatrix} \bar{v}_{c_1} & \cdots & \bar{v}_{c_{p-1}} \end{bmatrix}^T \\
\Lambda = \hat{b} - \sum_{j=1}^{p-1} \bar{v}_{cj} q_j \\
\tilde{V}_c = \begin{bmatrix} \Lambda^{i_{p-1}} & \cdots & \Lambda^{i_1} \end{bmatrix}^T \\
\quad \Lambda^{i_k} \equiv \Lambda \text{ in } \mathrm{I}_{i_k} = [t_{i_k,0}, t_{i_k,1}) \\
\quad \Lambda^{i_k}(t) = \Lambda^{i_k}(t_{i_k,1}) \text{ for } t \geq t_{i_k,1} \\
\hat{V}_c = \bar{V}_c + H_i^+ \tilde{V}_c \\
\hat{V}_c = \begin{bmatrix} \hat{v}_{c_1} & \cdots & \hat{v}_{c_{p-1}} \end{bmatrix}^T
\end{cases}
\tag{14}
$$

with $\rho > 0$.

Defining the estimation error $e = X - Z$ and consider the Lyapunov function candidate $V(e) = e^T P e$. By taking the time derivative of V along the trajectory of the estimation error dynamics $\dot{e} = (A - P^{-1}C^T C)e$ we get $\dot{V}(e) = -\rho V(e)$. The solution of $V(e)$ is given by $V(e) = V(e(0))e^{-\rho(t-t_0)}$. Then it is easy to see that

$$
\|e(t)\| \leq K \|e(t_0)\| e^{-\rho(t-t_0)} \tag{15}
$$

For q constant only and ρ sufficiently large, it exists $\Gamma_i > t_{i,0}$ such that

$$
\|e(t)\| \leq \gamma, \forall t \geq t_{i,1} \tag{16}
$$

where γ is an acceptable constant smaller error after convergence.

Since q is constant on $t \in [t_{i,0}, t_{i,1}]$, Γ_i must be smaller that $t_{i,1}$. From (15), the time of convergence Γ_i is given by $\Gamma_i - t_{i,0} \leq \dfrac{logK\|e(t_0)\| - log\gamma}{\rho}$. Hence, by choosing ρ sufficiently large so that $\Gamma_i - t_{i,0} < \tau$ according to assumption A3, it follows that $\Gamma_i < t_{i,1}$. Now from (16) and equation of Λ in (14), the following is deduced

$$
\begin{aligned}
\Lambda &= b + \delta^{i_{p-1}} - \sum_{j=1}^{p-1} \bar{v}_{cj} q_j \\
\Lambda &= \sum_{j=1}^{p-1} v_{cj} q_j + \delta^{i_{p-1}} - \sum_{j=1}^{p-1} \bar{v}_{cj} q_j \\
\Lambda &= q(V_c - \bar{V}_c) + \delta^{i_{p-1}}
\end{aligned}
\tag{17}
$$

where $\delta^{i_{p-1}} = e(t_{i,1})$ is a constant small (acceptable) error.

Lemma 2: Under assumptions A1 and A2, by choosing ρ sufficiently large, the identities

$$
\hat{V}_c = V_c + H_i^+ \delta \tag{18}
$$

are achieved, for all $t \geq t_{i_{p-1},1}$, where $\delta = \begin{bmatrix} \delta^{i_{p-1}} & \ldots & \delta^{i_1} \end{bmatrix}^T$ is a constant smaller (acceptable) error.

Proof: A2 implies that there exist $(p-1)$ indexes i_k ($k = 1, ..., p-1$) ($i_{k+1} > i_k$) so that $\left\{q^{i_1}, ..., q^{i_{p-1}}\right\}$ is a set of $(p-1)$ linearly independent vectors, then, from (17), the set of equalities (19) holds.

$$
\begin{aligned}
\Lambda^{i_{p-1}} &= q^{i_{p-1}}(V_c - \bar{V}_c) + \delta^{i_{p-1}},\ t \geq t_{i_{p-1},1} \\
&\ \vdots \\
\Lambda^{i_1} &= q^{i_1}(V_c - \bar{V}_c) + \gamma_b^{i_1},\ t \geq t_{i_1,1}
\end{aligned}
\tag{19}
$$

Since $\dot{V}_c(t) - \dot{\bar{V}}_c(t) = 0$ for all $t \geq 0$, $(V_c - \bar{V}_c)$ stays constant when $t \geq t_{i_j,1}$. By rewritten (19), into a matrix equation, one has

$$
\begin{bmatrix} \Lambda^{i_{p-1}} \\ \vdots \\ \Lambda^{i_1} \end{bmatrix} = \tilde{V}_c = H_i(V_c - \bar{V}_c) + \delta \text{ for all } t \geq t_{i_{p-1},1}
$$

Now, by replacing (18) in the equation of $\hat{V}_c$ given by (14), one gets

$$
\begin{aligned}
\hat{V}_c &= \bar{V}_c + H_i^+ H_i (V_c - \bar{V}_c) + H_i^+ \delta \\
\hat{V}_c &= V_c + H_i^+ \delta \text{ for all } t \geq t_{i_{p-1},1}.
\end{aligned}
$$

This end the proof. ■

IV. Simulations for a 3 cells converter

To illustrate the performance of both proposed observers, we consider here a 3 cells converter connected to RL load. This converter is governed by the state equations (1) where $p = 3$. The voltage of the source is: $E = 120$V and the parameters of load are: $R = 33\Omega$, $L = 50 \cdot 10^{-3}$H, $c_1 = c_2 = 33 \cdot 10^{-6}$F. Thus, by using the definition $q_1 = S_2 - S_1$ and $q_2 = S_3 - S_2$, the 3 cells converter can be described by the following hybrid representation:

$$
\begin{cases}
\dot{x} = A(q)x + g(S_3) \\
y = Cx
\end{cases}
\tag{20}
$$

where $x = \begin{bmatrix} I & v_{c_1} & v_{c_2} \end{bmatrix}^T$, $g(S_3) = \begin{bmatrix} \frac{E}{L} S_3 \\ 0 \\ 0 \end{bmatrix}$,

$$
A(q) = \begin{bmatrix} -\frac{R}{L} & -\frac{q_1}{c_1} & -\frac{q_2}{c_2} \\ \frac{q_1}{c_1} & 0 & 0 \\ \frac{q_2}{c_2} & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T.
$$

Remark 2: The inputs of switches S_j, $j = 1, ..., 3$ are generated by a simple PWM where the sampling frequency is chosen equal to $800Hz$. The voltage bandwidth of load is equal to $\dfrac{R}{L} = 660Hz$. Thus the sampling frequency may be appears insufficient for PWM proposed. But due to multicell structure converter, the voltage frequency applied to the load is multiplied by the number of cells (here $p = 3$). Thus, the voltage load frequency is equal to $3 * 600 = 1,8kHz$ which is enough for PWM proposed.

Now, for this hybrid representation (20), the proposed super twisting and adaptive observers (4) and (14) respectively are applied and designed as follows:

- Super twisting observer:

$$\begin{cases} \dot{x}_{\mathrm{a}} = -\frac{R}{L}I + \frac{E}{L}S_3 - \frac{1}{L}[(\bar{v}_{\mathrm{c}_1} + \tilde{v}_{\mathrm{c}_1})\,q_1 + (\bar{v}_{\mathrm{c}_2} + \tilde{v}_{\mathrm{c}_2})\,q_2] \\ +\lambda(|q_1| + |q_2|)\,|I - x_a|^{1/2}\,\mathrm{sign}\,(I - x_a) \\ \dot{\bar{v}}_{\mathrm{c}_1} = \frac{I}{c_1}q_1, \quad \dot{\bar{v}}_{\mathrm{c}_2} = \frac{I}{c_2}q_2, \quad \bar{V}_{\mathrm{c}}^T = \left[\begin{array}{cc} \bar{v}_{\mathrm{c}_1} & \bar{v}_{\mathrm{c}_2} \end{array}\right] \\ \dot{\tilde{v}}_{\mathrm{c}_1} = -\alpha q_1\,\mathrm{sign}\,(I - x_a)\,, \quad \dot{\tilde{v}}_{\mathrm{c}_2} = -\alpha q_2\,\mathrm{sign}\,(I - x_a) \\ \tilde{V}_{\mathrm{c}}^T = \left[\begin{array}{cc} q_1^{i_2}\tilde{v}_{\mathrm{c}_1}^{i_2} + q_2^{i_2}\tilde{v}_{\mathrm{c}_2}^{i_2} & q_1^{i_1}\tilde{v}_{\mathrm{c}_1}^{i_1} + q_2^{i_1}\tilde{v}_{\mathrm{c}_2}^{i_1} \end{array}\right]; \\ \hat{V}_{\mathrm{c}}\,(t) = \bar{V}_{\mathrm{c}}\,(t) + H_i^+\tilde{V}_{\mathrm{c}}\,(t) \\ \hat{V}_{\mathrm{c}}^T =: \left[\begin{array}{cc} \hat{v}_{\mathrm{c}_1} & \hat{v}_{\mathrm{c}_2} \end{array}\right] \end{cases}$$

with α et λ satisfying $\alpha > 0$, $\lambda > \frac{(1+\theta)}{(1-\theta)}\sqrt{\frac{2\alpha}{L}}$, $0 < \theta < 1$.

- Adaptive observer:

$$\begin{cases} \dot{Z} = AZ + G(u,y) + P^{-1}C^T(y - \hat{y}) \\ \dot{P} = -\rho P - A^T P - PA + 2C^T C \\ \hat{y} = CZ \\ Z^T = \left[\hat{I} \quad \hat{b};\right]; \quad \hat{b} = \hat{v}_{c_1}q_1 + \hat{v}_{c_2}q_2 \\ \dot{\bar{v}}_{c1} = \frac{q_1}{c_1}I, \quad \dot{\bar{v}}_{c1} = \frac{q_2}{c_2}I; \\ \bar{V}_c = \left[\bar{v}_{c_1} \quad \bar{v}_{c_2}\right]^T \\ \Lambda = \hat{b} - \bar{v}_{c_1}q_1 - \bar{v}_{c_2}q_2 \\ \tilde{V}_c = \left[\Lambda^{i_2} \quad \Lambda^{i_1}\right]^T \\ \hat{V}_c = \bar{V}_c + H_i^+\tilde{V}_c; \quad \hat{V}_c = \left[\hat{v}_{c_1} \quad \hat{v}_{c_2}\right]^T \end{cases}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & -\frac{1}{L} \\ 0 & 0 \end{bmatrix}, \; G\,(u,y) = \begin{bmatrix} \frac{E}{L}S_3 - \frac{R}{L}I \\ \frac{|q_1|}{c_1}I + \frac{|q_2|}{c_2}I \end{bmatrix} \\ C &= \left[\begin{array}{cc} 1 & 0 \end{array}\right] \text{ and } \rho > 0. \end{aligned}$$

- Pseudo inverse H_i^+:

The pseudo inverse H_i^+ used by both observers is constructed by the following algorithm (see (III-B) for more details):

Step 0 $H_{-1} = 0$.

Step 1 If $q^i \neq 0$, $H_{i,1} = q^i$, where $H_{i,1}$ is the first row of H_i, and set $i_2 = i$. If $q = 0$, $H_i = H_{i-1}$.

Step 2 Let $H_{i,2}$ be the 2-th row of $H_{i,2}$. Set $H_{i,2} = q^{i_1}$, where i_1 is the biggest index such that $i_2 > i_1$ and the vectors $\left\{q^{i_2}, q^{i_1}\right\}$ are linearly independent. If does not exist an index i_2 such that $\left\{q^{i_2}, q^{i_1}\right\}$ are linearly independent, then $H_{i,2} = H_{i,1}$.

Thus, for the case when H_i is non-singular $H_i^+ \equiv H_i^{-1}$.

The parameters of the super twisting observer are $\alpha = 15000$ and $\lambda = 5000$. The parameter ρ of the adaptive observer is chosen as follows $\rho = 1500$.

The voltage and its estimation of the first and second capacitors are illustrated respectively in figures 2 and 3. Simulation results show that the trajectories of both observers converge to the ones of the measured capacitor's voltages (at steady conditions $\frac{E}{3} = 40V$ for v_{c_1} and $\frac{2E}{3} = 80V$ for v_{c_2}). The convergence of the super twisting observer is faster than the one of the adaptive observer. However, the convergence rate of both observers can be modified by tuning their respective parameters.

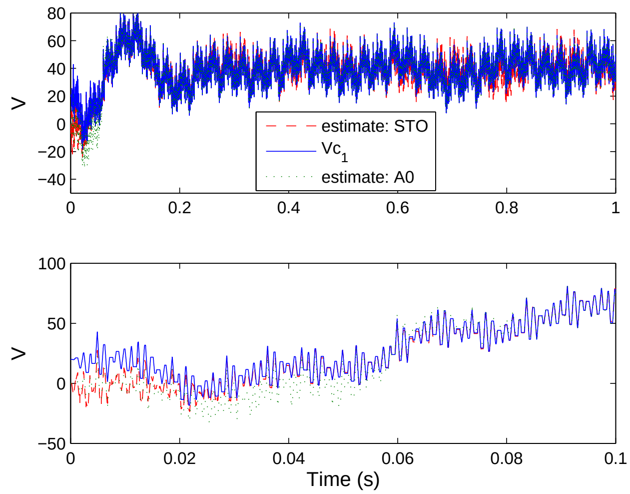

Fig. 2. v_{c_1} (—), $\hat{v}_{c_1}$, STO (- - -) and AO (...).

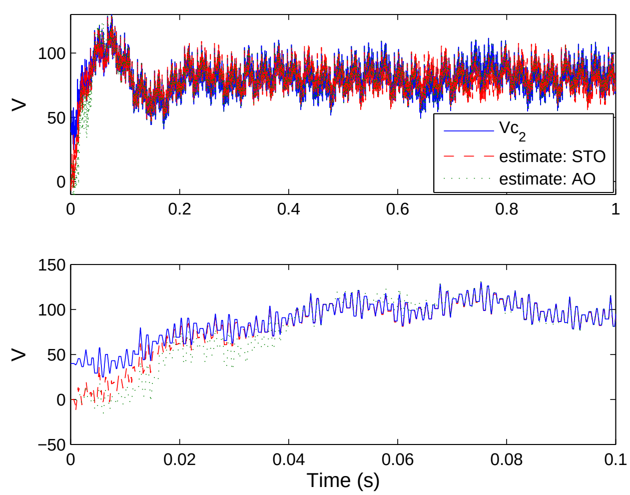

Fig. 3. v_{c_2} (—), $\hat{v}_{c_2}$, STO (- - -) and AO (...).

The robustness of both observers was checked with a load resistance variation of $+20\%$ according to the value of the first case. The simulation results that we have obtained are depicted in figures 4 and 5. It can be noticed that the performances of the adaptive observer are more affected compared to the ones of the super twisting observer.

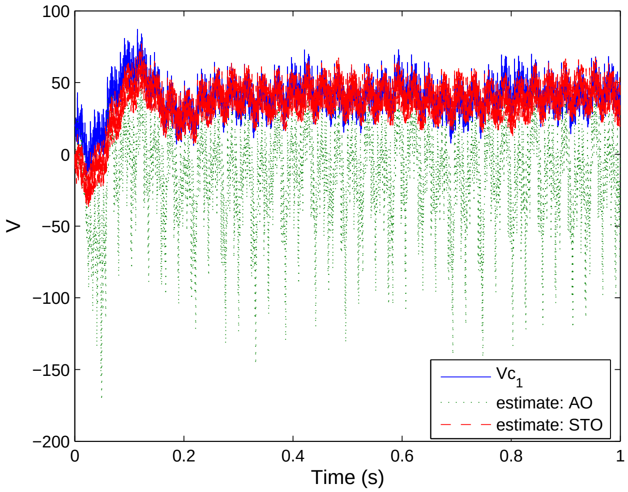

Fig. 4. v_{c_1} (—), $\hat{v}_{c_1}$, STO (- - -) and AO (...).

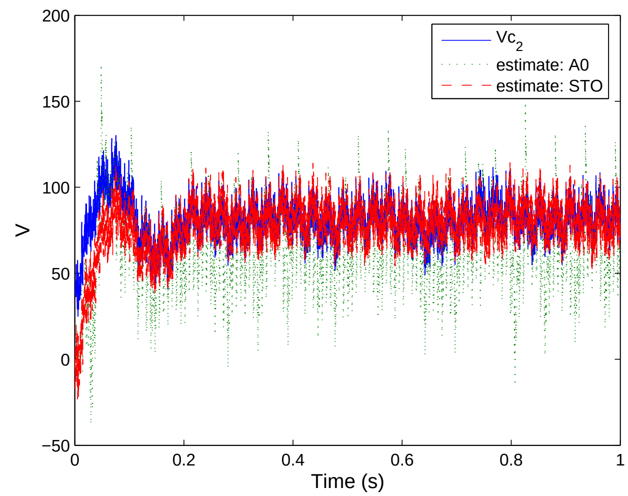

Fig. 5. v_{c_2} (—), $\hat{v}_{c_2}$, STO (- - -) and AO (...).

V. Conclusion

In this paper, a sliding mode (super twisting) and adaptive observers were proposed for the estimation problem of the voltages across the capacitors in the multicell converter. To observer the voltages across the capacitors it was assumed the multicell converter to be $Z(T_n)$-observable. It was shown that even when, on an interval of time when all switches do not change its position (the control stays constant), the multicell converter is non-observable in the classical sense, it is still possible to estimate the voltage on every capacitor after some time. That is, theoretically the voltages on the capacitors can not be estimated instantaneously after any time greater than zero (observability in the classical sense), but they can be estimated after some time by saving the information recovered before a switching occurs. The robustness of both observers was verified by the resistance variation of load. It was found that the super twisting observer is more robust than the adaptive observer.

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