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Thermally damped linear compressional waves in a 2D solar coronal model

A. Marcu[1], I. Ballai[2] and B. Orza[1]

[1]Department of Theoretical and Computational Physics, Babes-Bolyai University, 1, M.Kogalniceanu, Cluj-Napoca, Romania
[2]SP[2]RC, Department of Applied Mathematics, University of Sheffield, Sheffield S3 7RH, UK
email: `amarc@phys.bbcluj.ro, i.ballai@sheffield.ac.uk`

Abstract. The high resolution observations (TRACE and SOHO) of waves in coronal structures have revealed a rapid damping of modes, sometimes their damping length being of the same order as their wavelength. The rapid damping of modes in coronal loops permits us to derive values for magnetic field and transport coefficients. In this contribution we study the damping of linear compressional waves considering a two-dimensional propagation in gravitationally stratified plasma in the presence of thermal conduction. By considering this 2D model, we show that the presence of an additional transversal motion has an important effect on the damping of the waves. This theoretical model allows as to conclude that the main effects influencing the damping of the waves are the degree of the transversal structuring and temperature.

Keywords. waves, damping, thermal conduction, Sun: atmosphere, Sun: Corona

1. Introduction

The launch of the high resolution satellites and the study of data provided by these space telescopes revealed the existence of waves propagating in the solar corona in the magnetohydrodynamic (MHD) frequency domain. The common feature of the observed waves and oscillations in the solar corona is their rapid damping. Mendoza-Briceno (2004), De Moortel and Hood (2003 and 2004) studied the damping of coronal compressional waves supposing an 1D model. Comparing the observational damping values with theoretical results they found that slow MHD modes can be damped efficiently and there is a minimum damping length (or time) that can be obtained by thermal conduction alone. Combining the effects of gravitational stratification, thermal conduction and optical thick radiation, De Moortel and Hood (2004) and Mendoza-Briceno *et al.* (2004) showed that the stratification increases the damping length considerably. Their numerical analysis proved that the combined action of thermal conduction and loop cross-area divergence yields damping length values in good agreement with TRACE observations.

The damping of waves is a very popular area of study of modern solar physics since it is one of the key ingredients of *coronal seismology*. Damping of modes is studied in coronal loops to derive information about the magnetic field, transport coefficients, stratification, and heating functions (Nakariakov et al. 1999, Ruderman and Roberts 2002, Banerjee *et al.* 2007, Dymova and Ruderman 2007, Verth *et al.* 2007). The damping of global waves is used to obtain maps of the magnetic field in the quiet Sun, or information about the source of these waves, e.g. coronal mass ejections (Ballai *et al.* 2005, Ballai 2007)

In this paper we investigate the combined effect of an additional transversal motion and gravitational stratification on thermally damped linear compressional waves propagating in coronal loops in a two dimensional equilibrium.

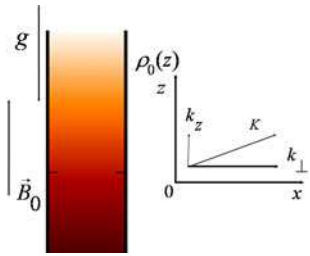

Figure 1. A schematic picture of the working model.

2. Model and Governing Equations

The proposed 2D dimensional working model is shown in Figure 1. The atmosphere is stratified under gravity in the z direction and waves propagate upwardly in low-beta, completely ionized gas. In order to simplify the treatment we restrict our model only to those waves which have wavelengths comparable to the size of the loop (waveguide). In this way we neglect any dispersive effect which would appear due to the geometrical transversal size of the coronal loop. It was shown by Edwin and Roberts (1983) that waves corresponding to this limit (wide tube limit) are weakly dispersive, i.e. they propagate similar to waves in an unbounded plasma. We suppose vertical wavelengths comparable to the gravitational wavelength, so the effect of gravitational stratification must be taken into account. In order to describe non-ideal effects, which may act to damp the considered modes, we consider thermal conduction as one of the most plausible mechanisms that can affect the propagation of MHD waves (certainly for slow MHD modes). Under coronal conditions, thermal conductivity is a tensorial quantity with the parallel component to the ambient magnetic field being much larger than the perpendicular one (Ruderman *et al.* 2000). We consider small but finite amplitude perturbations about the equilibrium. Observations show that the amplitudes of the modes are small (De Moortel *et al.* 2002), therefore we limit ourself to linear waves only.

The plasma is in a hydrostatic and isothermal equilibrium which means that the density and Alfvén wave profiles are given by $\rho_0(z) = \rho_0(0)e^{-z/H}$ and $v_A(z) = v_{A0}(0)e^{z/2H}$, where $\rho_0(0)$ and $v_{A0}(0)$ are the density and Alfvén speed at $z = 0$ and $H(= c_S^2/\gamma g)$ is the isothermal scale-height. All perturbations oscillate in phase, so we write all perturbations proportional to $\exp[i(\omega t - k_\perp x)]$, where ω is the frequency of the wave and $k_\perp$ is the component of the wave vector in the x direction. The linearized system of equation describing the motion in a stratified and thermally conductive medium is given by

$$i\omega\rho - \frac{\rho_0}{H}v_z + \rho_0(ik_\perp v_x + \frac{\partial v_z}{\partial z}) = 0, \quad i\omega v_x = -ik_\perp p + \frac{B_0}{\mu}(\frac{\partial b_x}{\partial z} - ik_\perp b_z), \tag{2.1}$$

$$i\omega b_x = B_0\frac{\partial v_x}{\partial z}, \quad b_z = -\frac{B_0 k_\perp}{\omega}v_x, \quad i\omega\rho_0 v_z = -\frac{\partial p}{\partial z} - \rho\frac{c_S^2}{\gamma H}, \tag{2.2}$$

$$i\omega(p - \rho c_S^2) + \frac{\rho_0 c_S^2(\gamma-1)}{H\gamma}v_z + \frac{\chi c_S^2}{(\gamma-1)}\left[\frac{\gamma}{c_S^2}\left(\frac{1}{H}\frac{dp}{dz} + \frac{d^2p}{dz^2}\right) - \left(\frac{1}{H}\frac{d\rho}{dz} + \frac{d^2\rho}{dz^2}\right)\right] = 0 \tag{2.3}$$

where $\kappa_\parallel = \frac{3\rho_0 k_B^2 T_0 \tau_e}{m_p m_e}$ is the parallel component of the thermal conductivity (see, e.g.

Braginskii 1965) and we introduced the notation

$$\chi = \frac{m_p(\gamma-1)^2\kappa_\|}{2\gamma k_B\rho_0}.$$

The equations given by the system (2.1)–(2.3) will be reduced to a dispersion relation which will allow us calculating the damping length of waves.

3. Results and Discussion

In the particular case of waves in unstratified plasma under the effect of thermal conduction, the system of equations describing the evolution of waves is simplified by considering $H \to \infty$, i.e. the Alfvén speed becomes height-independent. Now all perturbations can be Fourier-analyzed in the z direction and write all quantities proportional to $\exp(-ik_z z)$. The system of equations describing the dynamics of waves can easily derived to lead to the dispersion relation

$$\omega^5 + Q_1\omega^4 + Q_2\omega^3 + Q_3\omega^2 + Q_4\omega + Q_5 = 0, \tag{3.1}$$

where the coefficients Q_i, $(i = 1\dots5)$ are defined as

$$Q_1 = -\frac{i\chi\gamma k_z^2}{\gamma-1},\;\; Q_2 = -K^2(v_A^2 + c_S^2),\;\; Q_3 = \frac{i\chi k_z^2 K^2(c_S^2+\gamma v_A^2)}{\gamma-1},$$

$$Q_4 = k_z^2K^2c_S^2v_A^2,\;\; Q_5 = -\frac{i\chi c_S^2 v_A^2 K^2 k_z^4}{\gamma-1},$$

where $K^2 = k_\perp^2 + k_z^2$. The solutions of Eq.(3.1) describe the frequency of slow, fast MHD and thermal modes. Under coronal conditions $c_S^2 \ll v_A^2$ and the slow waves represent acoustic waves modified by the presence of the magnetic field, while fast waves can be seen as Alfvén waves modified by the compressibility of the plasma. Under the same considerations, the frequency of waves can be approximated by $\omega^2 \approx k_z^2c_S^2$ for slow waves and $\omega^2 \approx K^2v_A^2$ for fast waves. In order to study the dependence of the damping length of these modes we suppose a real frequency and a complex longitudinal wavenumber, i.e. $k_z = k_r + ik_i$. The wavelength of waves is given by $\lambda = 2\pi/k_r$, while the damping length by $L_d = 1/|k_i|$. We also suppose that waves will have a weak damping, expressed mathematically by the inequality $|k_r| \gg |k_i|$. If this form of the wavenumber is inserted back into Eq. (3.1) from, then we can isolate the imaginary part of the wavenumber giving us the damping length of waves. Solved for slow waves, the damping length is

$$L_{d0}^{slow} \approx \frac{6\chi\alpha_1}{3w^2(\gamma-1)c_S^2 + \alpha_2}, \tag{3.2}$$

where

$$w = k_\perp/k_z,\;\; [\alpha_1 = [(2w^2 - 5\gamma + 5)c_S^2 + (\gamma-1)(2w^2+5)v_A^2],$$

$$\alpha_2 = \sqrt{9w^4(\gamma-1)^2c_S^6 + 12k_r^2\chi^2\alpha_1[(w^2-\gamma+1)c_S^2 + (w^2+1)(\gamma-1)v_A^2]}.$$

Further analytical progress can be made supposing a situation when the Alfvén speed (i.e. equilibrium density and pressure) is locally constant (local analysis). In this approximation the dispersion relation will have a similar form as Eq. (3.1), but now the coefficients will be modified as

$$\tilde{Q_1} = -\frac{i\chi\gamma k_z^2}{\gamma-1},\;\; \tilde{Q_2} = -K^2(v_A^2+c_S^2) - \frac{ic_S^2k_z}{H\gamma},\;\; \tilde{Q_3} = \frac{\chi k_z^2}{\gamma-1}[iK^2(c_S^2+\gamma v_A^2) - \frac{c_S^2k_z}{H}],$$

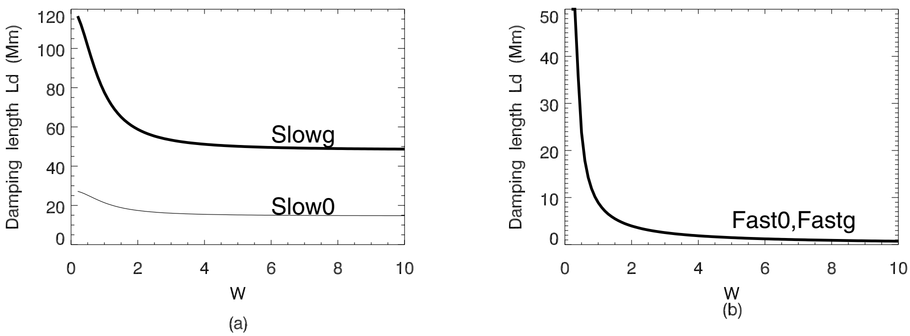

Figure 2. The variation of the damping lengths of slow and fast waves with the ratio $w = k_\perp/k_z$ in a homogeneous and in a stratified coronal loop in the presence of thermal conduction $(k_r = 1.4 \times 10^{-7}m^{-1}, \chi = 10^{11}m^2s^{-1}, c_S = 152kms^{-1}, v_A = 900kms^{-1}, H = 50Mm)$

$$\tilde{Q_4} = k_z K^2 c_S^2 v_A^2 (k_z + \frac{i}{H\gamma}), \quad \tilde{Q_5} = \frac{\chi c_S^2 v_A^2 K^2 k_z^4 3}{\gamma - 1}(ik_z - \frac{1}{H}).$$

Using the same technique as before, we can obtain the damping length of slow waves in the presence of stratification. The results are shown in Figure 2.

The damping length of fast waves has been studied numerically by solving the dispersion relation for k_i. The variation of the damping length of slow and fast waves is studied with respect to the ratio $k_\perp/k_r (= w)$ in two cases (Figure 2). While for slow waves there is a clear difference between the homogeneous and stratified case, in the case of fast waves the two solutions coincide meaning that stratification (at least in the 'local' sense) has no effect on the damping of these waves. For both types of waves large changes in the damping length occur for small values of w. For larger values the damping length seems not to be sensitive to the existence of a transversal scale.

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