Parametric Dictionary Learning in Diffusion MRI

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computational effort, we split our data set in 5 partitions, i.e. $K = 5$. Our previous experiments showed that it is sufficient to obtain a close approximation of the optimal λ.

IV. A POLYNOMIAL APPROACH FOR MAXIMA EXTRACTION

The HARDI contest is based on fiber orientation estimation via the extraction of ODF maxima. After computing the ODF, we extract its maxima using a polynomial approach that can analytically bracket and numerically refine with high precision *all* maxima, ensuring that none are missed [4]. First, leveraging the linear bijection between the spherical harmonic basis and the homogeneous polynomial basis of the same order and degree, we rewrite the ODF as a polynomial, $P(\mathbf{x} = [x_1, x_2, x_3])$, with $\|\mathbf{x}\|_2 = 1$. Second we formulate the optimization problem for computing the ODF extrema:

$$\partial F/\partial x_1 = \partial F/\partial x_2 = \partial F/\partial x_3 = \|\mathbf{x}\|_2^2 - 1 = 0, \quad (3)$$

where using Lagrange multipliers $F(\mathbf{x}, \Lambda) = P(\mathbf{x}) - \Lambda\left(\|\mathbf{x}\|_2^2 - 1\right)$. However, instead of optimizing, which is inherently a local approach dependent on initializations, we solve the polynomial system Eq. 3 using a polynomial system solver that can analytically bracket all roots – missing none and refine them numerically to high precision. Finally, we identify the maxima from the extrema using the Bordered Hessian, which is the generalization of the Hessian.

V. EXPERIMENTAL PARAMETERS

For the HARDI contest, we choose $\mathbf{H}$ to be constituted of the continuous SPF basis functions [1], [2] and constrain our dictionary $\mathbf{D}$ to represent a combination of the SPF functions. We choose the SPF basis because it facilitates the ODF estimation [3]. We learn our dictionary on the training data set given for the HARDI contest. Then we probe the testing data set on 15 measurements uniformly spread on a single shell at b value $b = 2000\text{s} \cdot \text{mm}^{-2}$.

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