Data assimilation in glaciology

Bertrand Bonan, Maëlle Nodet, Olivier Ozenda, Catherine Ritz

Data assimilation in glaciology

B. Bonan
INRIA

M. Nodet
Université Joseph Fourier Grenoble 1, INRIA

O. Ozenda
INRIA

C. Ritz
CNRS

Contents

1 Data assimilation in glaciology 1
1.1 Introduction 1
1.2 Ice-sheet model 1
1.3 Adjoint method and adjoint model 3
1.4 Numerical results on twin experiments 5

References 7

1
Data assimilation in glaciology

1.1 Introduction

In this short paper, we will give one example of an inverse problem in glaciology. This problem is fairly simple to state: how to infer a climatic scenario (i.e. how to reconstruct past polar temperature) from ice volume records?

Ice-sheet dynamics is quite complex. Ice is a non-newtonian fluid, its flow follows a highly non-linear Stokes equation. One key factor of ice dynamics is surface mass balance, that is the amount of ice accumulating at the surface (from snow precipitation) minus the amount of melting ice. This mass budget is related to temperature at the surface of the ice-sheet, depending in particular on temperature. See textbooks (Greve and Blatter, 2009) and (Paterson, 1981) for detailed modelling of ice-sheets.

Ice volume observations are available from oceanic records, that give information about sea level, and therefore about the amount of water stored in ice-sheets, ice-caps and glaciers.

As far as we know, there are very few past works on this subject. (Bintanja, van de Wal and Oerlemans, 2004) and (Bintanja, van de Wal and Oerlemans, 2005) use some a very simple correction method, equivalent to the nudging method, with quite good results.
The idea of this work is to explore the ability of the adjoint method to solve the inverse problem of reconstructing past temperature given all available observations. We start here with a simplified ice-sheet model and perform twin experiments.

We present the ice-sheet model in section 2, the adjoint method in section 3, and give numerical illustrations in section 4.

1.2 Ice-sheet model

1.2.1 Ice dynamics processes

As the full Stokes model is very costful to run over paleoclimatology time scales (thousands of years), large scale ice-sheet models have been developed, using the so-called Shallow Ice Approximation (SIA). Such models are able to reproduce ice-sheet evolution over hundreds of thousands of years, for large ice-sheets such as Antarctica, Greenland, Laurentide and Fennoscandian ice-sheets. One such model, GRISLI, is developed by LGGE (Ritz, Rommelaere and Dumas, 2001). It is a three dimensional thermomechanically coupled model, whose main physical processes are summed up in figure 1.1.

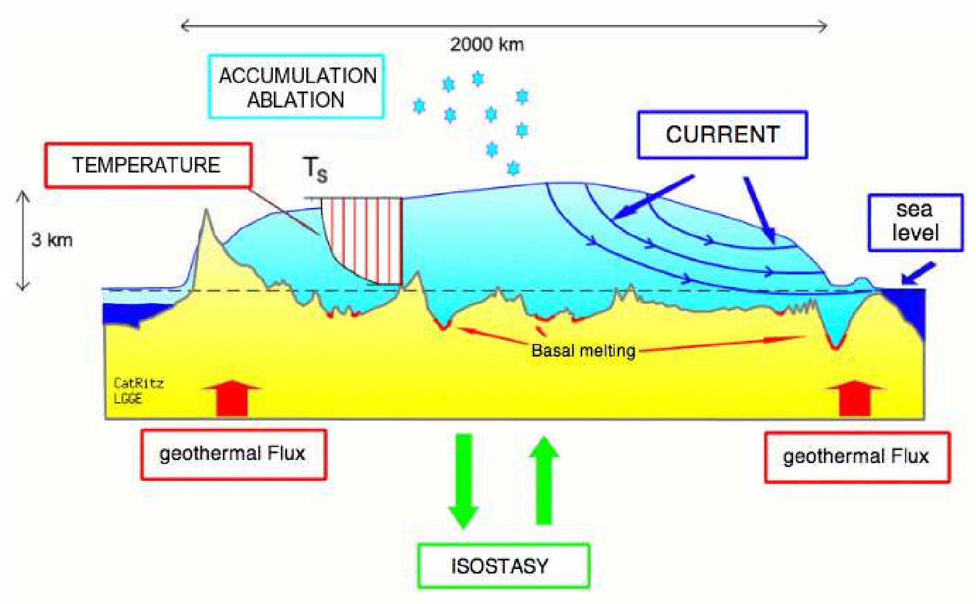

Fig. 1.1 Physical processes impacting ice-sheet dynamics.

1.2.2 Model equations

As GRISLI model is still costful to run, we will use a simplified, flowline, isothermal prototype of this model, on a flat bedrock topography, without floating shelves, which is faster to run, in order to study the paleoclimate inverse problem. Flowline means that the model is one-dimensional in space.
The first equation is the mass balance equation:

$$\frac{\partial H}{\partial t} = \dot{b}_m - \frac{\partial \left(\overline{U} H\right)}{\partial x}, \quad H|_{t=0} = H_0$$

with

- t time, x latitude
- $H(t,x)$ ice thickness, $H_0(x)$ initial ice thickness
- $\overline{U}(t,x)$ Eulerian velocity averaged over the ice thickness
- $\dot{b}_m(t,x)$ surface mass balance rate

Vertically averaged ice velocity $\overline{U}(t)$ is a diagnostic variable, that is to say it depends only on geometry variables (S, H) at time t, and it is given by the following equation:

$$\begin{aligned}
&\overline{U} = u_B - a_1 \frac{\partial S}{\partial x}\frac{H^2}{3} - a_2 \left(\frac{\partial S}{\partial x}\right)^3 \frac{H^4}{3} \\
&S = B + H \\
&H \geq 0
\end{aligned}$$

with

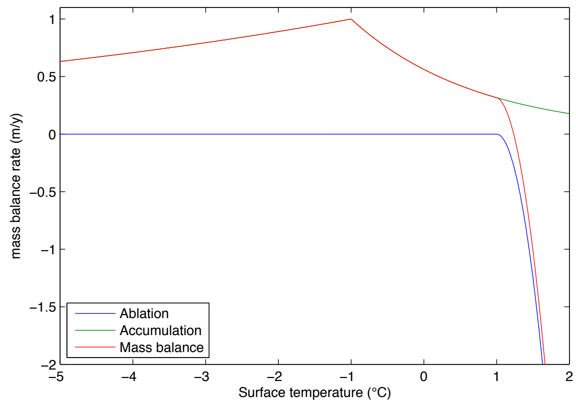

Fig. 1.2 Accumulation, ablation and surface mass balance as a function of surface temperature.

- $u_B(t, x)$ basal velocity
- $S(t, x)$ ice surface elevation
- $B(t, x)$ ice base (bedrock elevation, constant in this study)
- a_1, a_2 given coefficients

1.2.3 Climatic scenario and surface mass balance

The surface mass balance is given as a function of the ice surface temperature:

$$\begin{cases} \dot{b}_m = \mathrm{Acc} + \mathrm{Abl} \\ \mathrm{Acc} = f(T_S) \\ \mathrm{Abl} = g(T_S) \end{cases}$$

with

- $\mathrm{Acc}(t, x)$ accumulation rate
- $\mathrm{Abl}(t, x)$ ablation rate
- $T_S(t, x)$ surface temperature
- f, g given functions

See figure 1.2 to see how accumulation and ablation depend on surface temperature.

And finally, the surface temperature itself depends on latitude x, on altitude S, and on the (poorly known) temperature at the northern point at sea-level height $T_{\mathrm{clim}}(t)$:

$$T_S(t, x) = T_{\mathrm{clim}}(t) + b\, x + c\, S(t, x)$$

with b, c given coefficients.
The aim of the problem is to infer $T_{\mathrm{clim}}(t)$ from volume measurements.

1.3 Adjoint method and adjoint model

We refer to main and specialized lectures regarding variational data assimilation and adjoint code construction.

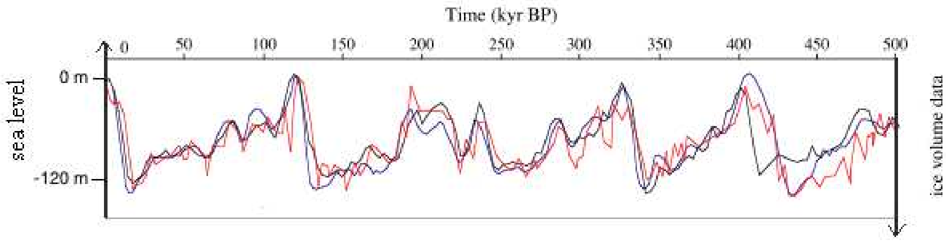

Fig. 1.3 Past ice volume records. black: mesured data from (Raymo, 1997); blue and red : different models

1.3.1 Observations and control variables

As we said before, we will use ice volume observations, available from sea level records, see figure 1.3 from (Paillard and Parrenin, 2004) as an example. As we do twin experiments, we will generate ourselves our data, from a reference "true state".
In the framework of four-dimensional variational assimilation, we write our inverse problem as a minimization problem: we want to find the "best" temperature $T_{\text{clim}}(t)$, sot that the misfit between modeled and observed volume is smallest. This means that our control variable is the climatic temperature $T_{\text{clim}}(t)$, and our cost function writes as follows:

$$\mathcal{J}(T_{\text{clim}}) = \mathcal{J}^b + \int_{t_0}^{t_f} \|\text{Vol}(t) - \text{Vol}^{\text{obs}}(t)\|^2 \, dt, \quad \text{Vol}(t) = \int_x H(t,x)\, dx$$

where the background term $\mathcal{J}^b$ is a classical regularising term, involving a background temperature and background error covariance matrix:

$$\mathcal{J}^b(T_{clim}) = \|T_{clim} - T^b_{clim}\|^2_k$$

The norm $\|.\|_k$ is classicaly given by:

$$\|X\|^2_k = \int_{t=t_0}^{t_f} \int_{s=t_0}^{t_f} k(s,t) X(s) X(t) \, ds \, dt$$

with k a regularizing kernel. In discrete form, this would writes:

$$\|X\|^2_{B^{-1}} = X^T B^{-1} X$$

where B is the background error covariance matrix, and X contains $T_{clim}(t_i) - T^b_{clim}(t_i)$ for all discrete controle times t_i.

1.3.2 Adjoint code validation

We minimize our cost function using quasi-Newton descent algorithm. This requires the gradient of $\mathcal{J}$, obtained thanks to the adjoint model. As our flowline model is simple and in Matlab, we implemented by hand the adjoint code using classical recipes

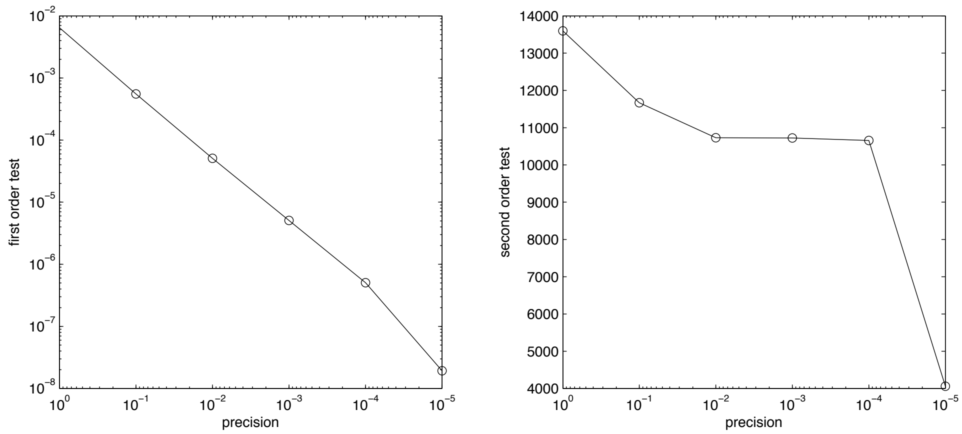

Fig. 1.4 First- (left) and second- (right) order gradient tests.

for adjoint code construction, see e.g. (Giering and Kaminski, 1998). To validate our gradient, we perform first- and second-order tests, that is we check that the 1st and 2nd order Taylor approximation formula

$$\mathcal{J}(x+\alpha z)=\mathcal{J}(x)+\alpha\nabla\mathcal{J}(x)z+\frac{\alpha^2}{2}z^T\nabla^2\mathcal{J}(x)z+o(\alpha^2)$$

is valid. To do so we form

$$\delta=\nabla\mathcal{J}(x)z,\quad \tau=\frac{\mathcal{J}(x+\alpha z)-\mathcal{J}(x)}{\alpha},\quad \varepsilon=\frac{\tau(\alpha,z)-\delta(z)}{|\delta(z)|},\quad r=\frac{\tau(\alpha,z)-\delta(z)}{\alpha}$$

and we check that ε tends to zero as α tends to zero, and r tends to a constant depending only on z, see figure 1.4 for an illustration with our model.

1.4 Numerical results on twin experiments

In order to validate our method and ajoint model, we present a first illustration on twin experiments (see figures 1.5 and 1.6). The principle of such experiment is as follows: first we choose a reference climatic scenario T_{clim}, called the "true" temperature forcing. Second we perform a direct run of the model with the "true" forcing and we generate synthetic observations (figure 1.5). And finally, we forget that we know the true temperature, we start with a constant first guess for T_{clim}, and we perform data assimilation thanks to the adjoint method, in order to get an assimilated T_{clim}. We can then compare our result with the true T_{clim} (figure 1.6).

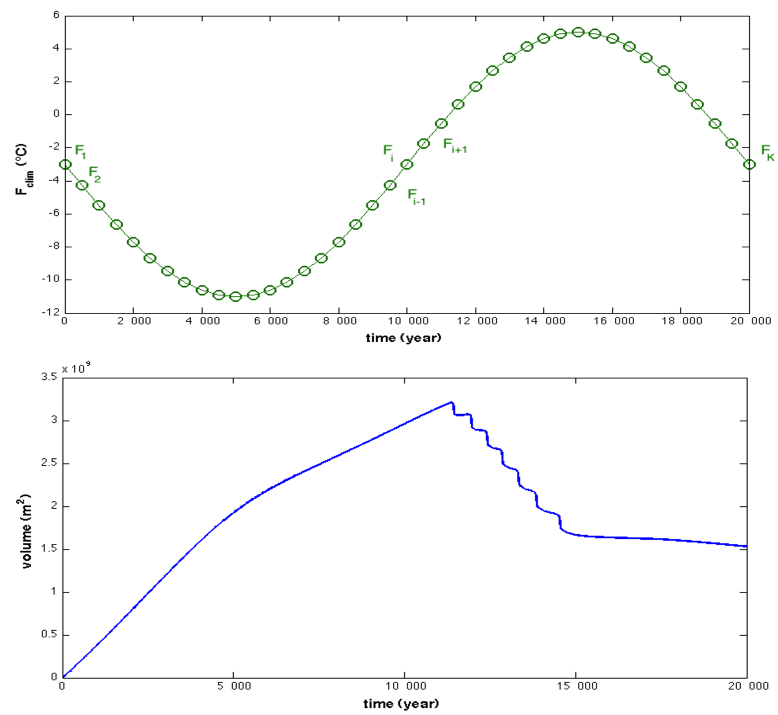

Fig. 1.5 Twin assimilation results: True climatic scenario T_{clim} (top), generated volume data (bottom).

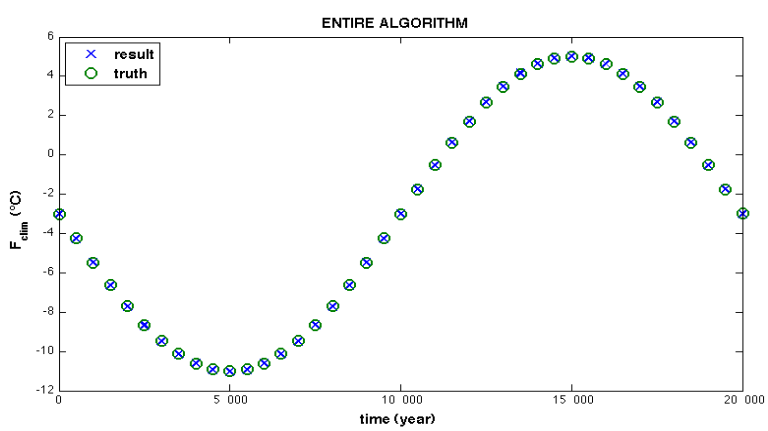

Fig. 1.6 Twin assimilation results: T_{clim} recovered thanks to data assimilation.

References

Bintanja, Richard, van de Wal, Roderik S W, and Oerlemans, Johannes (2004, December). A new method to estimate ice age temperatures. *Clim Dyn*, **24**(2-3), 197–211.

Bintanja, Richard, van de Wal, Roderik S W, and Oerlemans, Johannes (2005, September). Modelled atmospheric temperatures and global sea levels over the past million years. *Nature*, **437**(7055), 125–128.

Giering, R. and Kaminski, T. (1998). Recipes for adjoint code construction. *ACM Transactions on Mathematical Software (TOMS)*, **24**(4), 437–474.

Greve, Ralph and Blatter, Heinz (2009, August). *Dynamics of ice-sheets and glaciers*. Springer.

Paillard, Didier and Parrenin, Frédéric (2004, November). The Antarctic ice sheet and the triggering of deglaciations. *Earth and Planetary Science Letters*, **227**(3-4), 263–271.

Paterson, W.S.B. (1981). *The physics of glaciers (2nd edition)*. Pergamon Press.

Raymo, M. E. (1997). The timing of major climate terminations. *Paleoceanography*, **12**(4), 577–585.

Ritz, C., Rommelaere, V., and Dumas, C. (2001). Modeling the evolution of Antarctic ice sheet over the last 420, 000 years- Implications for altitude changes in the Vostok region. *Journal of Geophysical Research*, **106**, 31.