A Bayesian Jamming Game in an OFDM Wireless Network

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***Abstract*—The goal of this paper is to investigate how incomplete information on the fading channel gains impacts transmission parameters. We consider in an OFDM network with transmitters and jammers. To deal with this situation we employ a Bayesian approach by introducing different type of user and jammer corresponding to their knowledge of the network environment. To get an insight of the problem, the signal to interference and noise ratio (SINR) is considered as the main metric to optimize. First, equilibrium are found in closed form expressions. Second, we show interesting results saying that incomplete information on the jammer channel gains leads to equilibrium strategies which correspond to utilization of the same channels by the different types of the jammer. Meanwhile incomplete information about the transmitter leads to channels sharing transmission equilibrium strategies employed by different types of users.**

I. Introduction

In this paper we deal with two types of transmitters: one transmitter (normal) would like to transmit a signal and the other transmitter (jammer) would like to jam this transmission. From one hand jammer can be considered as a natural noise and it means that we look for determining the optimal transmitter's strategy under the worst environment condition. For which we take into account jamming power as well as where jamming source could be. From the other hand, jammer can be consider as an active agent. For example in military or police special forces operations, this type of jammer appears. According to [4], the US military routinely uses jammers to protect secure military areas from electronic surveillance. Jammers can also be used to protect travelling convoys from cell phone triggered roadside bombs. To deal with jamming problems quite often game theoretical-model are applied, say, for examples [2], [3], [6], [7].

Here we deal with the concept of jamming with incomplete information about either jammer or transmitter, namely, about their fading channel gains. This situation is modeled using Bayesian approach. Over the last ten years, Bayesian game-theoretic tools have been used to design distributed resource allocation strategies only in a few contexts, e.g., CDMA networks [13], [14], multicarrier interference networks [15], [5], fading multiple access channels (MAC) where transmitters have incomplete information about the channel state information (CSI), i.e., each transmitter knows his own channel state, but does not know the states of other transmitters was studied in [8]. Our motivation is therefore to study how Bayesian games can be applied to the context of fading OFDM. The goal of this paper is to get in closed form the Bayesian equilibrium strategies and investigate how incomplete information impact on both side of the transmission: the transmitter and the jammer. It is worth to note that [6] studied the scenario where the transmitter as well as the jammer does not no the state of all the channels without employing Bayesian approach. In our situation Bayesian approach seems more realistic and it could supply more intellectual algorithms of rivals' behaviour.

II. OFDM jamming model with complete information

In this scenario one transmitter should assign different power levels over n channels in order to maximize his throughput under worst nature condition. This interference signal is described as a jammer distributing an extra noise of the total power $\bar{J}$ among the channels. The strategy of the transmitter is the vector $T = (T_1, \dots, T_n)$ with $T_i \geq 0, i \in [1, n]$ such that $\sum_{i=1}^n T_i = \bar{T}$, where $\bar{T} > 0$ is the total available power for the transmitter. Let T_i is the power level assigned for channel i. The strategy of jammer is the vector $J = (J_1, \dots, J_n)$ with $J_i \geq 0, i \in [1, n]$ such that $\sum_{i=1}^n J_i = \bar{J}$, where $\bar{J} > 0$ is the total jamming power. The transmitter's payoff is defined as the signal to interference and noise ratio (SINR):

$$v_T(T, J) = \sum_{i=1}^n \frac{\alpha_i T_i}{\sigma^2 + \beta_i J_i},$$

where α_i (resp. β_i) are the fading channel gains of the transmitter (resp. the jammer) on channel i and σ^2 is the background noise level. Note that in the regime of low SINR the present objective can serve as an approximation of the Shannon capacity. A central motivation to consider SINR as an objective function and not Shannon capacity, is that current technology for voice over wireless does not try to achieve Shannon capacity but rather uses given codecs that can adapt the transmission rate to the SINR; these turns out to adapt the rate in a way that it is linear in the SINR over a wide range of throughput. The SINR has therefore been used very often to represent directly an approximation of the throughput [10], [11]. The validity of this can be seen e.g. in [9, p. 151, 222, 239]. As we see from [9, Fig. 10.4, p. 222], the ratio between the throughput and the SINR is close to a constant throughout long range of bit rates. For example, between 16Kbps and

256Kbps, the maximum variation around the median value is less than 20%.

The jammer objective is to minimize the transmitter's payoff, i.e. to maximize the function $-v_T(T, J)$. We assume that all the fading channel gains α_i and β_i, the noise level σ^2, the total powers $\bar{T}$ and $\bar{J}$ are known to both rivals. The following result gives an important result in the literature on our game problem but with complete information. This result will serve as a benchmark for our analysis.

Theorem 1 ([6]): In the game with complete information about fading channel gains the equilibrium (saddle point) $(T, J) = (T(\omega), J(\omega))$ is given as follows:

$$J_i(\omega) = \frac{\alpha_i}{\beta_i}\left[\frac{1}{\omega} - \frac{\sigma^2}{\alpha_i}\right]_+, \quad i \in [1, n],$$

and

$$T_i(\omega) = \begin{cases} \dfrac{(\beta_i/\alpha_i)}{\sum\limits_{k \in I^J(\omega)} (\beta_k/\alpha_k)}\bar{T}, & i \in I^J(\omega), \\ 0, & i \notin I^J(\omega), \end{cases}$$

where $I^J(\omega) := \{i \in [1, n] : J_i(\omega) > 0\}$ is the set of jammed channels, and ω is the unique root of the following water-filling equation:

$$\sum_{i=1}^{n} \frac{\alpha_i}{\beta_i}\left[\frac{1}{\omega} - \frac{\sigma^2}{\alpha_i}\right]_+ = \bar{J}.$$

III. Incomplete information on the jammer's fading channel gains

In this section we consider the scenario where the transmitter does not know exactly the jammer's fading channel gains β_i for all channels i. The transmitter has only statistical knowledge about the jammer's fading channel gains but the jammer knows them. For ease of understanding we assume that each channel gain could be in two states β_i^1 (good state) and β_i^2 (bad state). The transmitter knows the following probabilities: $p^1 := I\!P(\beta_i = \beta_i^1)$ and $p^2 := I\!P(\beta_i = \beta_i^2)$. We assume that the channels are symmetric, i.e. the probabilities do not depend on the channel. Note that dealing with networks that can be either in two states (good or bad) or in three states (good, medium or bad) are quite typical for studying in literature ([17]).

Since the transmitter does not know about the jammer's fading channel gains, we can think that in the environment two types of jammer are possible with strategies $J^t = (J_1^t, \ldots, J_n^t)$ such that $J_i^t \geq 0, i \in [1, n]$ and $\sum_{i=1}^n J_i^t = \bar{J}$ with $\bar{J} > 0$ is the total jamming power and $t = 1, 2$. The transmitter's payoff is defined as the expected SINR:

$$v_T(T, J^1, J^2) = \sum_{t=1}^{2} p^t \sum_{i=1}^{n} \frac{\alpha_i T_i}{\sigma^2 + \beta_i^t J_i^t}. \tag{1}$$

The payoff of the type t jammer is the following:

$$v_J^t(T, J^k) = -\sum_{i=1}^{n} \frac{\alpha_i T_i}{\sigma^2 + \beta_i^t J_i^t}. \tag{2}$$

We are faced to a Bayesian game and we look for a Bayesian equilibrium, that is, we want to find (T^*, J^{1*}, J^{2*}) such that for any (T, J^1, J^2) the following inequality holds

$$\begin{aligned} v_T(T, J^{1*}, J^{2*}) &\leq v_T(T^*, J^{1*}, J^{2*}), \\ v_J^t(T^*, J^t) &\leq v_J^t(T^*, J^{t*}), \quad t = 1, 2. \end{aligned}$$

Of course, the Bayesian game is equivalent to the zero-sum game with transmitter's payoff given by $v_T(T, (J^1, J^2)) = v_T(T, J^1, J^2)$ where (J^1, J^2) is the jammer's strategy. This equivalence means that all equilibria of the Bayesian game and the corresponding zero-sum game coincide.

Theorem 2: The game with uncertainty about jammer fading channel gains has the unique equilibrium $(T, J^1, J^2) = (T(\omega, \tau), J^1(\omega, \tau), J^2(\omega, \tau))$, where

$$J_i^1 = \begin{cases} \dfrac{1}{\beta_i^1}\left(\dfrac{\alpha_i}{\omega}\left(p^1 + p^2\sqrt{\dfrac{\beta_i^1 \tau}{\beta_i^2}}\right) - \sigma^2\right), & i \in I_{11}, \\ \dfrac{1}{\beta_i^1}\left(\dfrac{p^1 \alpha_i}{\omega - p^2 \alpha_i/\sigma^2} - \sigma^2\right), & i \in I_{10}, \\ 0, & i \in \bar{I}_{10}, \end{cases}$$

$$J_i^2 = \begin{cases} \dfrac{1}{\beta_i^2}\left(\dfrac{\alpha_i}{\omega}\left(p^2 + p^1\sqrt{\dfrac{\beta_i^2}{\beta_i^1 \tau}}\right) - \sigma^2\right), & i \in I_{11}, \\ \dfrac{1}{\beta_i^2}\left(\dfrac{p^2 \alpha_i}{\omega - p^1 \alpha_i/\sigma^2} - \sigma^2\right) & i \in I_{01}, \\ 0, & i \in \bar{I}_{01}, \end{cases}$$

$$T_i = \frac{\bar{T}}{\bar{H}_T(\omega, \tau)} \begin{cases} \dfrac{\alpha_i}{\omega^2}\left(p^1\sqrt{\dfrac{1}{\beta_i^1}} + p^2\sqrt{\dfrac{\tau}{\beta_i^2}}\right)^2, & i \in I_{11}, \\ \dfrac{\tau}{\alpha_i \beta_i^2}\left(\dfrac{p^2 \alpha_i}{\omega - p^1 \alpha_i/\sigma^2}\right)^2 & i \in I_{01}, \\ \dfrac{1}{\alpha_i \beta_i^1}\left(\dfrac{p^1 \alpha_i}{\omega - p^2 \alpha_i/\sigma^2}\right)^2 & i \in I_{10}, \\ 0, & \text{otherwise} \end{cases}$$

with

$$\begin{aligned} I_{00}(\omega, \tau) &= \left\{i \in [1, n] : \frac{\alpha_i}{\sigma^2} \leq \omega\right\}, \\ I_{11}(\omega, \tau) &= \left\{i \in [1, n] : \omega < \left(p^1 + p^2\sqrt{\frac{\beta_i^1 \tau}{\beta_i^2}}\right)\frac{\alpha_i}{\sigma^2}, \right. \\ &\qquad \left. \omega < \left(p^2 + p^1\sqrt{\frac{\beta_i^2}{\beta_i^1 \tau}}\right)\frac{\alpha_i}{\sigma^2}\right\}, \\ I_{10}(\omega, \tau) &= \left\{i \in [1, n] : \left(p^2 + p^1\sqrt{\frac{\beta_i^2}{\beta_i^1 \tau}}\right)\frac{\alpha_i}{\sigma^2} \leq \omega < \frac{\alpha_i}{\sigma^2}\right\}, \\ \bar{I}_{10} &= [1, n] \backslash (I_{10} \cup I_{11}), \\ I_{01}(\omega, \tau) &= \left\{i \in [1, n] : \left(p^1 + p^2\sqrt{\frac{\beta_i^1 \tau}{\beta_i^2}}\right)\frac{\alpha_i}{\sigma^2} \leq \omega < \frac{\alpha_i}{\sigma^2}\right\}, \\ \bar{I}_{01} &= [1, n] \backslash (I_{01} \cup I_{11}), \end{aligned}$$

where

$$\tau := \tau_*, \quad \omega := \omega_* = \omega^1(\tau_*)$$

with τ_* is the unique root of the equation

$$\omega^1(\tau_*) = \omega^2(\tau_*)$$

where $\omega^t(\tau)$ is given as the unique solution for a fixed τ of the equation

$$\bar{H}_J^t(\omega^t(\tau), \tau) = \bar{J},$$

and

$$\begin{aligned}
\bar{H}_J^1(\omega,\tau) &= \sum_{i\in I_{10}} \frac{1}{\beta_i^1}\left(\frac{p^1\alpha_i}{\omega - p^2\alpha_i/\sigma^2} - \sigma^2\right) \\
&\quad + \sum_{i\in I_{11}} \frac{1}{\beta_i^1}\left(\frac{\alpha_i}{\omega}\left(p^1 + p^2\sqrt{\frac{\beta_i^1\tau}{\beta_i^2}}\right) - \sigma^2\right), \\
\bar{H}_J^2(\omega,\tau) &= \sum_{i\in I_{01}} \frac{1}{\beta_i^2}\left(\frac{p^2\alpha_i}{\omega - p^1\alpha_i/\sigma^2} - \sigma^2\right) \\
&\quad + \sum_{i\in I_{11}} \frac{1}{\beta_i^2}\left(\frac{\alpha_i}{\omega}\left(p^2 + p^1\sqrt{\frac{\beta_i^2}{\beta_i^1\tau}}\right) - \sigma^2\right), \quad (3) \\
\bar{H}_T(\omega,\tau) &= \sum_{i\in I_{10}} \frac{1}{\alpha_i\beta_i^1}\left(\frac{p^1\alpha_i}{\omega - p^2\alpha_i/\sigma^2}\right)^2 \\
&\quad + \tau\sum_{i\in I_{01}} \frac{1}{\alpha_i\beta_i^2}\left(\frac{p^2\alpha_i}{\omega - p^1\alpha_i/\sigma^2}\right)^2 \\
&\quad + \sum_{i\in I_{11}} \frac{\alpha_i}{\omega^2}\left(p^1\sqrt{\frac{1}{\beta_i^1}} + p^2\sqrt{\frac{\tau}{\beta_i^2}}\right)^2.
\end{aligned}$$

In the next section, the impact of the uncertainty on the transmitter's fading channel gains are studied.

IV. Incomplete information on the transmitter's fading channel gains

In this section we consider the scenario with unknown transmitter's fading channel gains α_i for the jammer. We assume, as in the previous section, that it can take two discrete values $\{\alpha_i^t\}$ with probability q^t ($t = 1, 2$). To deal with it, we introduce two types of transmitter where the strategy of type t transmitter is $T^t = (T_1^t, \dots, T_n^t)$ with $\sum_{i=1}^n T_i^t = \bar{T}$ and $T_i^t \geq 0$ for $i \in [1, n]$, $t = 1, 2$. The payoff v_T^t for type t transmitter's and v_J for the jammer are given as follows:

$$v_T^t(T^t, J) = \sum_{i=1}^n \frac{\alpha_i^t T_i^t}{\sigma^2 + \beta_i J_i}, \quad t = 1, 2 \quad (4)$$

and

$$v_J(T^1, T^2, J) = -\sum_{i=1}^n \frac{q^1\alpha_i^1 T_i^1 + q^2\alpha_i^2 T_i^2}{\sigma^2 + \beta_i J_i}. \quad (5)$$

We assume that all the fading channel gains α_i^t and β_i, the noise level σ^2, the total powers $\bar{T}^1$, $\bar{T}^2$ and $\bar{J}$, the probabilities q^t the transmitter of type t comes into action are known to both players.

This particular game is similar to previous one in which one of the player has incomplete information on the type of the other player. We shall look also for a Bayesian equilibrium, which is in this context, a strategy profile (T^{1*}, T^{2*}, J^*) such that for any (T^1, T^2, J) the following inequalities hold:

$$\begin{aligned}
v_T^t(T^t, J^*) &\leq v_T^t(T^{t*}, J^*), \quad t = 1, 2, \\
v_J(T^{1*}, T^{2*}, J) &\leq v_J(T^{1*}, T^{2*}, J^*).
\end{aligned}$$

Since the transmitter's payoff is linear on T^k and the jammer's payoff is concave on J, existence of equilibrium is obvious [16]. Here we focus on getting equilibrium in closed form. Assume that there is no proportional correlation between state of channels, namely, $\alpha_s^1/\alpha_s^2 \neq \alpha_r^1/\alpha_r^2$ for any $s \neq r$. Then without loss of generality we can rank the channels such that

$$\frac{\alpha_1^1}{\alpha_1^2} > \frac{\alpha_2^1}{\alpha_2^2} > \dots > \frac{\alpha_n^1}{\alpha_n^2}. \quad (6)$$

In the following auxiliary proposition a switching point for channels is defined.

Proposition 1: There exists the unique integer k in $[1, n]$ such that

$$\text{either } \varphi_{k-1} \leq A_k \leq \varphi_k, \quad (7)$$

$$\text{or } A_{k+1} < \varphi_k \leq A_k \quad (8)$$

with

$$A_s = \left(\frac{q^1}{q^2}\right)\left(\frac{\alpha_s^1}{\alpha_s^2}\right)^2, \quad s \in [1, n], \quad A_{n+1} = 0$$

and

$$\begin{aligned}
\varphi_s &= \left(\sum_{i=1}^s \frac{\alpha_i^1}{\beta_i}\right) \Big/ \left(\sum_{i=s+1}^n \frac{\alpha_i^2}{\beta_i}\right), s \in [1, n-1], \\
\varphi_0 &= 0, \quad \varphi_n = \infty.
\end{aligned} \quad (9)$$

First we investigate the basic situation where the total jamming power $\bar{J}$ is large enough to jam all the channels. Then $J_i > 0$ for any i and we show in the following theorem, that the transmitter's equilibrium strategies have a channel sharing structure.

Theorem 3: The total jamming power $\bar{J}$ is large enough to jam all the channels at the equilibrium if and only if

$$\omega^1 \leq \frac{1}{\sigma^2}\begin{cases} \min\left\{\min\limits_{i\in[1,k]} \alpha_i^1, \frac{\alpha_k^1}{\alpha_k^2}\min\limits_{i\in[k+1,n]} \alpha_i^2\right\}, & \text{for (7)}, \\ \min\left\{\min\limits_{i\in[1,k]} \alpha_i^1, \sqrt{\frac{q^2\varphi_k}{q^1}}\min\limits_{i\in[k+1,n]} \alpha_i^2\right\}, & \text{for (8)}, \end{cases} \quad (10)$$

where k is defined in Proposition 1 and

$$\omega^1 = \begin{cases} \dfrac{\sum\limits_{i=1}^{k-1}(\alpha_i^1/\beta_i) + (\alpha_k^1/\alpha_k^2)\sum\limits_{i=k}^n(\alpha_i^2/\beta_i)}{\bar{J} + \sigma^2\sum\limits_{i=1}^n(1/\beta_i)} & \text{for (7)}, \\ \dfrac{\sum\limits_{i=k+1}^n (\alpha_i^2/\beta_i)\left(\varphi_k + \sqrt{(q^2\varphi_k)/q^1}\right)}{\bar{J} + \sigma^2\sum\limits_{i=1}^n(1/\beta_i)} & \text{for (8)}. \end{cases}$$

(a) If condition (7) holds then the transmitter's equilibrium strategies (T^1, T^2) corresponds to sharing the channel k and the unique equilibrium (T^1, T^2, J) is given as follows:

$$T_i^1 = \begin{cases} \dfrac{\alpha_i^1}{\beta_i}\dfrac{\Omega}{q^1}, & i \le k-1, \\ \bar{T} - \dfrac{\Omega}{q^1}\sum_{i=1}^{k-1}\dfrac{\alpha_i^1}{\beta_i}, & i = k, \\ 0, & i \ge k+1, \end{cases}$$

$$T_i^2 = \begin{cases} 0, & i \le k-1, \\ \bar{T} - \dfrac{\Omega}{q^2}\left(\dfrac{\alpha_k^1}{\alpha_k^2}\right)^2 \sum_{i=k+1}^{n}\dfrac{\alpha_i^2}{\beta_i}, & i = k, \\ \dfrac{\alpha_i^2}{\beta_i}\dfrac{\Omega}{q^2}\left(\dfrac{\alpha_k^1}{\alpha_k^2}\right)^2, & i \ge k+1, \end{cases}$$

$$J_i = \begin{cases} \dfrac{1}{\beta_i}\left[\dfrac{\alpha_i^1}{\omega^1} - \sigma^2\right], & i \le k-1, \\ \dfrac{1}{\beta_i}\left[\dfrac{\alpha_i^2}{\omega^1}\dfrac{\alpha_k^1}{\alpha_k^2} - \sigma^2\right], & i \ge k \end{cases} \tag{11}$$

with

$$\Omega = \frac{\dfrac{\beta_k}{\alpha_k^1}q^1 + \dfrac{\beta_k}{\alpha_k^2}\left(\dfrac{\alpha_k^2}{\alpha_k^1}\right)^2 q^2}{1 + \dfrac{\beta_k}{\alpha_k^1}\displaystyle\sum_{i=1}^{k-1}\dfrac{\alpha_i^1}{\beta_i} + \dfrac{\beta_k}{\alpha_k^2}\displaystyle\sum_{i=k+1}^{n}\dfrac{\alpha_i^2}{\beta_i}}\bar{T}.$$

(b) If condition (8) holds then the transmitter's equilibrium strategies (T^1, T^2) corresponds not sharing any channel and the unique equilibrium (T^1, T^2, J) is given as follows:

$$T_i^1 = \begin{cases} \dfrac{\dfrac{\alpha_i^1}{\beta_i}\bar{T}}{\displaystyle\sum_{j=1}^{k}\dfrac{\alpha_j^1}{\beta_j}}, & i \le k, \\ 0, & i \ge k+1, \end{cases} \qquad T_i^2 = \begin{cases} 0, & i \le k, \\ \dfrac{\dfrac{\alpha_i^2}{\beta_i}\bar{T}}{\displaystyle\sum_{j=k+1}^{n}\dfrac{\alpha_j^2}{\beta_j}}, & i \ge k+1, \end{cases}$$

$$J_i = \begin{cases} \dfrac{1}{\beta_i}\left[\dfrac{\alpha_i^1}{\omega^1} - \sigma^2\right], & i \le k, \\ \dfrac{1}{\beta_i}\left[\dfrac{\alpha_i^2}{\omega^1}\sqrt{\dfrac{q^2}{q^1}\varphi_k} - \sigma^2\right], & i \ge k+1. \end{cases} \tag{12}$$

In the case where the jamming power is not large enough to jam all the channels, we have to apply the previous theorem by considering a subset of channels. This result is described in the following theorem.

If the total jamming power $\bar{J}$ is not large enough to jam all the channels, i.e. the condition (10) does not hold, then the problem is to determine the subset of channels to transmit on. Namely, let

$$i_0 = \text{argmin}_i \left\{\alpha_i^1, i \in [1,k], \frac{\alpha_k^1}{\alpha_k^2}\alpha_i^2, i \in [k+1, n]\right\} \text{ for (7)}$$

and

$$i_0 = \text{argmin}_i \left\{\alpha_i^1, i \in [1,k], \sqrt{\frac{q^2\varphi_k}{q^1}}\alpha_i^2, i \in [k+1, n]\right\} \text{ for (8).}$$

Then instead of all the channel spectrum $\{1, \ldots, n\}$, the jammer considers the subset $\{1, \ldots, n\}\backslash\{i_0\}$ of channels and so on.

Due to the existence of the equilibrium in the original game [16] and the finite number of steps of induction supplies the equilibrium strategies, i.e. the maximal channel spectrum subset for which the corresponding reduced condition (10) holds supplies the equilibrium strategies.

V. Discussion on the uniqueness

The assumption about proportionality of these gains is essential for the uniqueness. Namely, let $\alpha_i^1/\alpha_i^2 = \xi$ for $i \in [1, n]$. Then the game has the unique jamming equilibrium strategy J and continuum of the transmitter's equilibrium strategies T^1 and T^2. Namely, the equilibrium jamming strategy has the following water-filling form

$$J_i = J_i(\omega) := (1/\beta_i)\left[\alpha_i^1/\omega - \sigma^2\right]_+, i \in [1, n]$$

with ω is the unique root of the equation

$$\sum_{i=1}^{n}(1/\beta_i)\left[\alpha_i^1/\omega - \sigma^2\right]_+ = \bar{J}.$$

The transmitter's equilibrium strategies are any T^1 and T^2 such that

$$q^1T_i^1 + q^2\xi T_i^2 = \begin{cases} \dfrac{(q^1 + q^2\xi)\bar{T}(\beta_i/\alpha_i^1)}{\displaystyle\sum_{k\in I(\omega)}(\beta_k/\alpha_k^1)}, & i \in I(\omega), \\ 0, & i \notin I(\omega) \end{cases}$$

$$\sum_{i=1}^{n}T_i^t = \bar{T}, t = 1, 2, I(\omega) = \left\{i \in [1, n] : \omega \le \frac{\alpha_i^1}{\sigma^2}\right\}.$$

In particular, such equilibrium strategies can be the following ones

$$T_i^1 = \begin{cases} \dfrac{\beta_i/\alpha_i^1}{\displaystyle\sum_{k\in I(\omega)}\beta_k/\alpha_k^1}\bar{T}, & i \in I(\omega)\backslash\{s,t\}, \\ \dfrac{\beta_s/\alpha_s^1}{\displaystyle\sum_{k\in I(\omega)}\beta_k/\alpha_k^1}\bar{T} - \dfrac{\epsilon}{q^1}, & i = s, \\ \dfrac{\beta_t/\alpha_t^1}{\displaystyle\sum_{k\in I(\omega)}\beta_k/\alpha_k^1}\bar{T} + \dfrac{\epsilon}{q^1}, & i = t, \\ 0, & \text{otherwise} \end{cases}$$

$$T_i^2 = \begin{cases} \dfrac{\beta_i/\alpha_i^1}{\displaystyle\sum_{k\in I(\omega)}\beta_k/\alpha_k^1}\bar{T}, & i \in I(\omega)\backslash\{s,t\}, \\ \dfrac{\beta_s/\alpha_s^1}{\displaystyle\sum_{k\in I(\omega)}\beta_k/\alpha_k^1}\bar{T} + \dfrac{\epsilon}{\xi q^2}, & i = s, \\ \dfrac{\beta_t/\alpha_t^1}{\displaystyle\sum_{k\in I(\omega)}\beta_k/\alpha_k^1}\bar{T} - \dfrac{\epsilon}{\xi q^2}, & i = t, \\ 0, & \text{otherwise} \end{cases}$$

for any $s, t \in I(\omega)$ and any enough small $\epsilon > 0$.

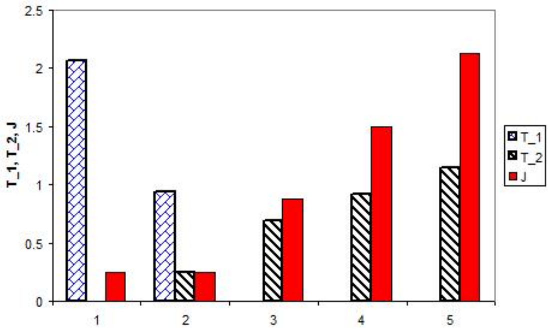

Fig. 1. The equilibrium strategies (T^1, T^2, J)

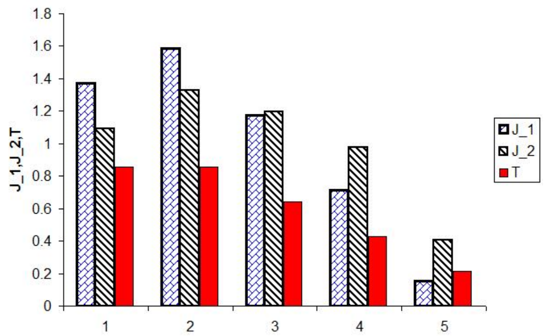

Fig. 2. The equilibrium strategies (T, J^1, J^2)

VI. Numerical illustration

As numerical illustration of difference in strategies we consider five channels network ($n = 5$) and that the total transmission and jamming power are $\bar{T} = 3$, $\bar{J} = 5$ and the background noise is $\sigma^2 = 1$.

Figure 1 illustrates the structure of the equilibrium (T^1, T^2, J) for probabilities $q^1 = 0.1$, $q^2 = 0.9$, and fading channel gains $\beta = (1, 1, 1, 1, 1)$, $\alpha^1 = (4, 4, 3, 2, 1)$ and $\alpha^2 = (1, 2, 3, 4, 5)$. Thus, strategy T^1 employs the channel spectrum $[1, 2]$ meanwhile strategy T^2 uses the channel spectrum $[2, 5]$ and they share the only channel 2.

Figure 2 illustrates the structure of the equilibrium (T, J^1, J^2) for probabilities $p^1 = 0.1$, $p^2 = 0.9$, and fading channel gains $\alpha = (4, 4, 3, 2, 1)$, $\beta^1 = (1, 1, 1, 1, 1)$, $\beta^2 = (5, 4, 3, 2, 1)$. In this case both jamming strategies jam all the channel spectrum employed by the transmitter, so there is no channel sharing at all. Intensity of jamming attack of both type of jammer reduce along with decreasing of the transmitting power, but relation between jamming powers at each channel is defined by quality of the jamming fading channel gains.

VII. Possible generalizations

The obtained result is quite robust to the case where total power of the rival is unknown either to the transmitter or the jammer, in the sense that the structure of the equilibrium strategies keeps on being the same. Also it is easy to generalize the result for the situation where both players, the transmitter and jammer, does not know type of its rival. Just to keep the model for the convenient of the readers as transparent as possible we do not include it into the paper.

The situation with more than two states of channels could drastically change the structure of the equilibrium as one can see from the proof of Theorem 3 since it essentially has to depend on mutual ratio between each pair of fading channel gains. Finally we note that an subject of our future work is to extend the suggested closed form equilibrium approach for others user's utility, for example, such that Shannon capacity.

VIII. Conclusion

We have investigated in this paper how incomplete information on the fading channel gains impacts on both side the transmitter and the jammer in an OFDM network. In particular we have theoretically demonstrated interesting results saying that incomplete information about jammer channel gains leads to using the same channels for the different type of jammer at equilibrium. Meanwhile, incomplete information about the transmitter channel gains leads to an equilibrium in which the transmitters share their power between several channels. Moreover, we have prove the existence and uniqueness of the equilibrium in both uncertainty framework (transmitter or jammer channel gains) and also demonstrate under which condition the uniqueness can be crashed.

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IX. Appendix

Proof of Theorem 2. Since the payoff (1) is the linear on T and the payoffs (2) is concave on J^k existence of equilibrium is obvious [16]. Here we focus on getting equilibrium in closed form and investigating its uniqueness. To do so we can apply a mix of linear and non-linear optimization approaches to get the following result. (T, J^1, J^2) is an equilibrium if and only if there are ω (the minimal expected induced noise), and ν^1 and ν^2 (the Lagrangian multipliers) such that

$$T_i \begin{cases} \geq 0, & p^1 \frac{\alpha_i}{\sigma^2 + \beta_i^1 J_i^1} + p^2 \frac{\alpha_i}{\sigma^2 + \beta_i^2 J_i^2} = \omega, \\ = 0, & p^1 \frac{\alpha_i}{\sigma^2 + \beta_i^1 J_i^1} + p^2 \frac{\alpha_i}{\sigma^2 + \beta_i^2 J_i^2} < \omega, \end{cases}$$
$$\frac{\alpha_i \beta_i^t T_i}{(\sigma^2 + \beta_i^t J_i^t)^2} \begin{cases} = \nu^t, & J_i^t > 0, \\ \leq \nu^t, & J_i^t = 0, \end{cases} t = 1, 2. \tag{13}$$

By (13) we express equilibrium strategies $(T, J^1, J^2) = (T(\omega, \nu^1, \nu^2), J^1(\omega, \nu^1, \nu^2), J^2(\omega, \nu^1, \nu^2))$ as function on the minimal expected induced noise ω, and Lagrangian multipliers ν^1 and ν^2 in closed form as follows

$$J_i^1 = \begin{cases} \frac{1}{\beta_i^1}\left(\frac{\alpha_i}{\omega}\left(p^1 + p^2\sqrt{\frac{\nu^2\beta_i^1}{\nu^1\beta_i^2}}\right) - \sigma^2\right), & i \in I_{11}, \\ \frac{1}{\beta_i^1}\left(\frac{p^1\alpha_i}{\omega - p^2\alpha_i/\sigma^2} - \sigma^2\right), & i \in I_{10}, \\ 0, & \text{otherwise}, \end{cases}$$
$$J_i^2 = \begin{cases} \frac{1}{\beta_i^2}\left(\frac{\alpha_i}{\omega}\left(p^2 + p^1\sqrt{\frac{\nu^1\beta_i^2}{\nu^2\beta_i^1}}\right) - \sigma^2\right), & i \in I_{11}, \\ \frac{1}{\beta_i^2}\left(\frac{p^2\alpha_i}{\omega - p^1\alpha_i/\sigma^2} - \sigma^2\right) & i \in I_{01}, \\ 0, & \text{otherwise}, \end{cases} \tag{14}$$
$$T_i = \begin{cases} \frac{\alpha_i}{\omega^2}\left(p^1\sqrt{\frac{\nu^1}{\beta_i^1}} + p^2\sqrt{\frac{\nu^2}{\beta_i^2}}\right)^2, & i \in I_{11}, \\ \frac{\nu^2}{\alpha_i\beta_i^2}\left(\frac{p^2\alpha_i}{\omega - p^1\alpha_i/\sigma^2}\right)^2 & i \in I_{01}, \\ \frac{\nu^1}{\alpha_i\beta_i^1}\left(\frac{p^1\alpha_i}{\omega - p^2\alpha_i/\sigma^2}\right)^2 & i \in I_{10}, \\ 0, & \text{otherwise}, \end{cases}$$

where $I_{ts} = I_{ts}(\omega, \nu^1, \nu^2)$, $s, t = 0, 1$ are subsets of $[1, n]$ and they are given as

$$\begin{aligned} I_{00}(\omega, \nu^1, \nu^2) &= \left\{i : \frac{\alpha_i}{\sigma^2} \leq \omega\right\}, \\ I_{10}(\omega, \nu^1, \nu^2) &= \left\{i : \left(p^2 + p^1\sqrt{\frac{\nu_1\beta_i^2}{\nu_2\beta_i^1}}\right)\frac{\alpha_i}{\sigma^2} \leq \omega < \frac{\alpha_i}{\sigma^2}\right\}, \\ I_{01}(\omega, \nu^1, \nu^2) &= \left\{i : \left(p^1 + p^2\sqrt{\frac{\nu_2\beta_i^1}{\nu_1\beta_i^2}}\right)\frac{\alpha_i}{\sigma^2} \leq \omega < \frac{\alpha_i}{\sigma^2}\right\}, \\ I_{11}(\omega, \nu^1, \nu^2) &= \left\{i : \omega < \left(p^1 + p^2\sqrt{\frac{\nu_2\beta_i^1}{\nu_1\beta_i^2}}\right)\frac{\alpha_i}{\sigma^2}, \right. \\ & \left. \omega < \left(p^2 + p^1\sqrt{\frac{\nu_1\beta_i^2}{\nu_2\beta_i^1}}\right)\frac{\alpha_i}{\sigma^2}\right\}. \end{aligned} \tag{15}$$

Then, to find the optimal ω, ν^1 and ν^2 we have to solve the following equations meaning that all power, the user and the jammer have in their disposition, they have to distribute among n channels:

$$\begin{aligned} H_J^t(\omega, \nu^1, \nu^2) &:= \sum_{i=1}^n J_i^k(\omega, \nu^1, \nu^2) = \bar{J}, \quad t = 1, 2, \\ H_T(\omega, \nu^1, \nu^2) &:= \sum_{i=1}^n T_i(\omega, \nu^1, \nu^2) = \bar{T}. \end{aligned} \tag{16}$$

Closed form of $J_i^t(\omega, \nu^1, \nu^2)$, $t = 1, 2$ and $T_i(\omega, \nu^1, \nu^2)$ given by (14) allows to specify structure of $H_J^k(\omega, \nu^1, \nu^2)$ and $H_T(\omega, \nu^1, \nu^2)$ as following functions depending on ω and ratio ν^2/ν^1:

$$\begin{aligned} H_J^r(\omega, \nu^1, \nu^2) &= \bar{H}_J^r(\omega, \nu^2/\nu^1), \quad r = 1, 2 \\ H_T(\omega, \nu^1, \nu^2) &= \nu^1 \bar{H}_T(\omega, \nu^2/\nu^1), \end{aligned}$$

where $\bar{H}_T(\omega, \nu^2/\nu^1)$ and $\bar{H}_J^r(\omega, \nu^2/\nu^1)$, $r = 1, 2$ are given by (3). Introduce a new variable $\tau = \frac{\nu^2}{\nu^1}$. In this notation the equations (16) turn into:

$$\bar{H}_J^1(\omega, \tau) = \bar{J}, \tag{17}$$

$$\bar{H}_J^2(\omega, \tau) = \bar{J}, \tag{18}$$

$$\nu^1 \bar{H}_T(\omega, \tau) = \bar{T}. \tag{19}$$

Function $\bar{H}_J^1(\omega, \tau)$ has the following properties. It is continuous on ω and τ, it is decreasing on ω and increasing on τ. For a fixed $\tau > 0$ $\bar{H}_J^1(\omega, \tau) = 0$ for $\omega \geq \frac{1}{\sigma^2}\max_i \alpha_i$ and $\bar{H}_J^1(0+, \tau) = \infty$. Thus, for a fixed $\tau > 0$ there is $\omega^1(\tau)$ such that

$$\bar{H}_J^1(\omega^1(\tau), \tau) = \bar{J}.$$

It is clear that $\omega^1(\tau)$ is continuous increasing function such that

$$\omega^1(\infty) = \infty, \omega^1(0) = \bar{\omega}^1,$$

where $\omega = \bar{\omega}^1$ is the unique root of the following water-filling equation

$$\sum_{i=1}^n \frac{\alpha_i}{\beta_i^1}\left[\frac{p^1}{\omega} - \frac{\sigma^2}{\alpha_i}\right]_+ = \bar{J}. \tag{20}$$

Similarly, for a fixed $\tau > 0$ there is $\omega^2(\tau)$ such that

$$\bar{H}_J^2(\omega^2(\tau), \tau) = \bar{J}.$$

Also, $\omega^2(\tau)$ is continuous decreasing function such that

$$\omega^2(0) = \infty, \omega^2(\infty) = \bar{\omega}^2,$$

where $\omega = \bar{\omega}^2$ is the unique root of the following water-filling equation

$$\sum_{i=1}^n \frac{\alpha_i}{\beta_i^2}\left[\frac{p^2}{\omega} - \frac{\sigma^2}{\alpha_i}\right]_+ = \bar{J}.$$

Then, we can define

$$\nu^2/\nu^1 = \tau_*, \tag{21}$$

where $\tau = \tau_*$ is the unique root of the equation

$$\omega^1(\tau) = \omega^2(\tau)$$

It implies that the optimal $\omega = \omega_*$ is given as follows

$$\omega_* = \omega^1(\tau_*). \tag{22}$$

By (19) the optimal $\nu^1 = \nu_*^1$ can be obtained from the following relation:

$$\nu_*^1 = \frac{\bar{T}}{\bar{H}_T(\omega_*, \tau_*)}.$$

Then, that is clear that

$$H_T(\omega_*, \nu_*^1, \nu_*^2) = \bar{T}$$

with by (21)

$$\nu_*^2 = \nu_*^1 \tau_*$$

and the result follows. ■

Proof of Proposition 1: Since φ_s is increasing from 0 to ∞ and A_s is decreasing from A_1 to 0 there exists at most one k such that there exists at most one k such that (7) holds and there exists at most one k such that (8) holds.

Also, (7) and (8) cannot hold simultaneously. Assume that it is not true and there are k_1 and k_2 such that (7) and (8) hold for them correspondingly, i.e.

$$\varphi_{k_1-1} \le A_{k_1} \le \varphi_{k_1} \tag{23}$$

and

$$A_{k_2+1} < \varphi_{k_2} \le A_{k_2} \tag{24}$$

Then since φ_s is increasing and A_s is decreasing we have that $k_2 < k_1$. By (24) $A_{k_2+1} < \varphi_{k_2}$. Thus, also $A_{k_1-1} < \varphi_{k_1}$ what contradicts to (23).

Finally, the conditions (7) and (8) cover all possible situation, namely, either (7) or (8) have to hold.

(a) Let (7) do not hold for any k. Then for the minimal k such that the following inequality holds

$$A_k \le \varphi_k \tag{25}$$

also the following inequality has to hold:

$$A_k < \varphi_{k-1}. \tag{26}$$

since otherwise (7) holds.

For this k also we have to have

$$A_{k-1} < \varphi_{k-1} \tag{27}$$

since otherwise $\varphi_{k-1} \ge A_{k-1}$. So, by (26), (25) holds for $k-1$ what contradicts the assumption that k is the minimal such that (25) holds.

Then by (26) and (27) we have that

$$A_k < \varphi_{k-1} < A_{k-1}$$

and (8) holds for $k := k-1$.

(b) Let (8) do not hold for any k. Then for the maximal k such that the following inequality holds

$$A_k \le \varphi_k \tag{28}$$

also the following inequality has to hold:

$$\varphi_k \le A_{k+1} \tag{29}$$

since otherwise (8) holds.

For this k also we have to have

$$A_{k+1} < \varphi_{k+1} \tag{30}$$

since otherwise $\varphi_{k+1} \le A_{k+1}$. So, by (29), (28) holds for $k+1$ what contradicts the assumption that k is the maximal such that (8) holds.

Then by (29) and (30) we have that

$$\varphi_k \le A_{k+1} < \varphi_{k+1}$$

and (8) holds for $k := k+1$. ■

Proof of Theorem 3. Since the payoff (4) is the linear on T^k and the payoffs (5) is concave on J existence of equilibrium is obvious [16]. Here we focus on getting equilibrium in closed form and investigating its uniqueness. To do so we can apply a mix of linear and non-linear optimization approaches to get the following result. Namely, (T^1, T^2, J) is an equilibrium if and only if there are ω^1 and ω^2 (the minimal induced noises), and ν (the Lagrangian multiplier) such that

$$T_i^t \begin{cases} \ge 0, & \dfrac{\alpha_i^t}{\sigma^2 + \beta_i J_i} = \omega^t, \\ = 0, & \dfrac{\alpha_i^k}{\sigma^2 + \beta_i J_i} \le \omega^t, \end{cases} \quad t = 1, 2, \tag{31}$$

$$\frac{(q^1 \alpha_i^1 T_i^1 + q^2 \alpha_i^2 T_i^2)\beta_i}{(\sigma^2 + \beta_i J_i)^2} \begin{cases} = \nu, & J_i > 0, \\ \le \nu, & J_i = 0. \end{cases} \tag{32}$$

Then by (31) there is at most one channel k where $T_k^1 > 0$ and $T_k^2 > 0$. Then for such k the following linear correlation between ω^1 and ω^2 holds:

$$\frac{\alpha_k^1}{\alpha_k^2} = \frac{\omega^1}{\omega^2}. \tag{33}$$

For the rest channels $i \ne k$ we have either $T_i^1 > 0$ and $T_i^2 = 0$, or $T_i^1 = 0$ and $T_i^2 > 0$. The case $T_i^1 = 0$ and $T_i^2 = 0$ is impossible since, if we assume $T_i^1 = 0$ and $T_i^2 = 0$, then by (32) $J_i = 0$, what contradicts the assumption that the total power is enough to jam all the channels.

(a) *First* we deal with situation where there exists a channel k such that both type of the user employ it, i.e. $T_k^1 > 0$ and $T_k^2 > 0$. Then, by (6) the user's strategies T^1 and T^2 have to have the following channels sharing structure:

$$T_i^1 \begin{cases} > 0, & i \in [1, k], \\ = 0, & i \in [k+1, n], \end{cases} \qquad T_i^2 \begin{cases} = 0, & i \in [1, k-1], \\ > 0, & i \in [k, n]. \end{cases} \tag{34}$$

Then, by (31)

$$J_i = \begin{cases} \dfrac{1}{\beta_i} \left[\dfrac{\alpha_i^1}{\omega^1} - \sigma^2 \right]_+, & i \in [1, k-1], \\ \dfrac{1}{\beta_i} \left[\dfrac{\alpha_i^2}{\omega^2} - \sigma^2 \right]_+, & i \in [k, n]. \end{cases} \tag{35}$$

By the assumption that $\bar{J}$ is large enough to jam all the channels, $J_i > 0$ for any i. Then, taking into account (33) and (35), the equation $\sum_{i=1}^n J_i = \bar{J}$ turns into

$$\bar{J} = \sum_{i=1}^{k-1} \frac{1}{\beta_i} \left(\frac{\alpha_i^1}{\omega^1} - \sigma^2 \right) + \sum_{i=k}^{n} \frac{1}{\beta_i} \left(\frac{\alpha_i^2}{\omega^1} \frac{\alpha_k^1}{\alpha_k^2} - \sigma^2 \right). \tag{36}$$

Solving the last equation by ω^1 implies (3).

Also, by (34), (33), (35) and (32) the user's equilibrium strategies have the form:

$$T_i^1=\begin{cases}\dfrac{\alpha_i^1}{\beta_i}\dfrac{\nu}{q^1(\omega^1)^2}, & i\in[1,k-1],\\ T_k^1, & i=k,\\ 0, & i\in[k+1,n],\end{cases}$$
$$T_i^2=\begin{cases}0, & i\in[1,k-1],\\ T_k^2, & i=k,\\ \dfrac{\alpha_i^2}{\beta_i}\dfrac{\nu}{q^2(\omega^1)^2}\left(\dfrac{\alpha_k^1}{\alpha_k^2}\right)^2, & i\in[k+1,n],\end{cases}\tag{37}$$

where

$$\frac{\beta_k}{\alpha_k^1}q^1T_k^1+\frac{\beta_k}{\alpha_k^2}\left(\frac{\alpha_k^2}{\alpha_k^1}\right)^2q^2T_k^2=\frac{\nu}{(\omega^1)^2}.\tag{38}$$

Thus, since $\sum_{i=1}^n T_i^s=\bar{T}$, $s=1,2$ by (37), the equation (38) is equivalent to

$$\frac{\beta_k}{\alpha_k^1}q^1\left(\bar{T}^1-\frac{\nu}{q^1(\omega^1)^2}\sum_{i=1}^{k-1}\frac{\alpha_i^1}{\beta_i}\right)$$
$$+\frac{\beta_k}{\alpha_k^2}\left(\frac{\alpha_k^2}{\alpha_k^1}\right)^2q^2\left(\bar{T}-\frac{\nu}{q^2(\omega^1)^2}\left(\frac{\alpha_k^1}{\alpha_k^2}\right)^2\sum_{i=k+1}^{n}\frac{\alpha_i^2}{\beta_i}\right)=\frac{\nu}{(\omega^1)^2}.$$

Solving it by $\nu/(\omega^1)^2$ implies

$$\frac{\nu}{(\omega^1)^2}=\frac{\dfrac{\beta_k}{\alpha_k^1}q^1+\dfrac{\beta_k}{\alpha_k^2}\left(\dfrac{\alpha_k^2}{\alpha_k^1}\right)^2q^2}{1+\dfrac{\beta_k}{\alpha_k^1}\displaystyle\sum_{i=1}^{k-1}\frac{\alpha_i^1}{\beta_i}+\dfrac{\beta_k}{\alpha_k^2}\displaystyle\sum_{i=k+1}^{n}\frac{\alpha_i^2}{\beta_i}}\bar{T}.\tag{39}$$

Also, by (37), T^1 and T^2 are strategies if $T_k^1\ge 0$ and $T_k^2\ge 0$ which are equivalent to

$$\frac{\nu}{(\omega^1)^2}\le\frac{q^1\bar{T}}{\sum\limits_{i=1}^{k-1}\dfrac{\alpha_i^1}{\beta_i}},\quad \frac{\nu}{(\omega^1)^2}\le\left(\frac{\alpha_k^2}{\alpha_k^1}\right)^2\frac{q^2\bar{T}}{\sum\limits_{i=k+1}^{n}\dfrac{\alpha_i^2}{\beta_i}}.\tag{40}$$

By (39), the conditions (40) turn into the following ones:

$$\varphi_{k-1}\le\frac{q^1}{q^2}\left(\frac{\alpha_k^1}{\alpha_k^2}\right)^2\le\varphi_k\tag{41}$$

with φ_s given by (9).
Note that

$$\varphi_{s+1}-\varphi_s=\frac{\sum\limits_{i=1}^{s+1}\dfrac{\alpha_i^1}{\beta_i}}{\sum\limits_{i=s+2}^{n}\dfrac{\alpha_i^2}{\beta_i}}-\frac{\sum\limits_{i=1}^{s}\dfrac{\alpha_i^1}{\beta_i}}{\sum\limits_{i=s+1}^{n}\dfrac{\alpha_i^2}{\beta_i}}$$
$$=\frac{\dfrac{\alpha_{s+1}^2}{\beta_{s+1}}\displaystyle\sum_{i=1}^{s}\frac{\alpha_i^1}{\beta_i}+\dfrac{\alpha_{s+1}^1}{\beta_{s+1}}\displaystyle\sum_{i=s+2}^{n}\frac{\alpha_i^2}{\beta_i}+\dfrac{\alpha_{s+1}^1\alpha_{s+1}^2}{(\beta_{s+1})^2}}{\left(\displaystyle\sum_{i=s+2}^{n}\frac{\alpha_i^2}{\beta_i}\right)\left(\sum\limits_{i=s+1}^{n}\dfrac{\alpha_i^2}{\beta_i}\right)}>0.$$

So, φ_s is increasing. Since, by the assumption (6), $\dfrac{\alpha_s^1}{\alpha_s^2}$ is decreasing on s and it is clear that k given by (41) uniquely defined and (a) follows.

(b) *Second* we deal with the situation where the user's equilibrium strategies (T^1,T^2) do not share any cannel at all, so there is no s such that $T_s^1>0$ and $T_s^2>0$. Then, by (6), there is a k such that

$$T_i^1\begin{cases}>0, & i\in[1,k],\\ =0, & i\in[k+1,n],\end{cases}\qquad T_i^2\begin{cases}=0, & i\in[1,k],\\ >0, & i\in[k+1,n]\end{cases}\tag{42}$$

and

$$\frac{\alpha_k^1}{\alpha_k^2}\ge\frac{\omega^1}{\omega^2}>\frac{\alpha_{k+1}^1}{\alpha_{k+1}^2}.\tag{43}$$

Then (32) and (42) yield

$$T_i^1=\begin{cases}\dfrac{\alpha_i^1}{\beta_i}\dfrac{\nu}{q^1(\omega^1)^2}, & i\in[1,k],\\ 0, & i\in[k+1,n],\end{cases}$$
$$T_i^2=\begin{cases}0, & i\in[1,k],\\ \dfrac{\alpha_i^2}{\beta_i}\dfrac{\nu}{q^2(\omega^1)^2}\left(\dfrac{\alpha_k^1}{\alpha_k^2}\right)^2, & i\in[k+1,n].\end{cases}\tag{44}$$

Substituting T_i^s from (44) into $\sum_{i=1}^n T_i^s=\bar{T}$ and solving the obtained equation on $(\omega^s)^2$ implies

$$(\omega^1)^2=\frac{\nu}{q^1\bar{T}}\sum_{i=1}^{k}\frac{\alpha_i^1}{\beta_i}\text{ and }(\omega^2)^2=\frac{\nu}{q^2\bar{T}}\sum_{i=k+1}^{n}\frac{\alpha_i^2}{\beta_i}.$$

Thus,

$$\left(\frac{\omega^1}{\omega^2}\right)^2=\frac{q^2}{q^1}\frac{\sum\limits_{i=1}^{k}\dfrac{\alpha_i^1}{\beta_i}}{\sum\limits_{i=k+1}^{n}\dfrac{\alpha_i^2}{\beta_i}}=(\text{by (9)})=\frac{q^2}{q^1}\varphi_k.\tag{45}$$

Then, by (43), the condition (8) holds.
The equilibrium strategy J has the form given by (35). Thus, the condition $\sum_{i=1}^n J_i=\bar{J}$ which is equivalent to (36) can be presented in the form

$$\varphi_k\frac{1}{\omega^1}+\frac{1}{\omega^2}=\frac{\bar{J}+\sigma^2\displaystyle\sum_{i=1}^{n}\frac{1}{\beta_i}}{\displaystyle\sum_{i=k+1}^{n}\frac{\alpha_i^2}{\beta_i}}.\tag{46}$$

By (45)

$$\frac{1}{\omega^2}=\sqrt{\frac{q^2}{q^1}\varphi_k}\frac{1}{\omega^1}.\tag{47}$$

Then (46) and (47) imply that ω^1 is given by (3).
Finally note that the jamming power is enough to jam all the channels if $J_i>0$ for $i\in[1,n]$. Then (10) follows from (11) and (12). ■