Dynamic Mesh Optimization for Free Surfaces in Fluid Simulation

David Lopez, Bruno Lévy

Dynamic Mesh Optimization for Free Surfaces in Fluid Simulation

David Lopez[1] and Bruno Lévy[1]

Project ALICE, INRIA Nancy Grand-Est and LORIA
(Bruno.Levy | David.Lopez)@inria.fr

We propose a new free surface fluid front tracking algorithm. Based on Centroidal Voronoi Diagram optimizations, our method creates a serie of either isotropic or anisotropic meshes that confroms with an evolving surface.

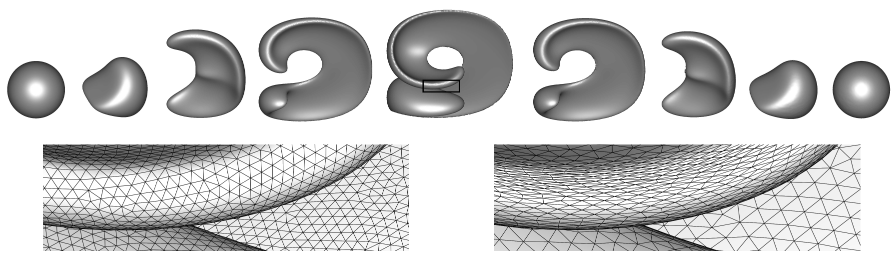

Fig. 1. Enright's test at steps 0, 50, 75, 100, 150, 200, 225, 250 and 300 (top) and details of the isotropic and anisotropic 150th step mesh (resp. bottom right and left).

1 Introduction

We consider free-surface fluid simulations that operate on a per time-step basis. Each step requires to compute the velocity field governing the fluid motion, to track its surface and to accurately represent it. We do not discuss how to simulate the physics and we assume that we have an everywhere and anytime known velocity field to move surface vertices according with an accurate advection method, eg Runge Kutta.

In this paper, we focus on accurately sampling and optimizing the advected surface regarding its geometry and combinatorics. Assuming curl-free velocity fields, like incompressible fluid flows, we will also minimize volume variations due to polygonal approximation. This, in a direct manner, without any other representation like tetrahedral mesh or sub-grid.

After a short review of previous work (section 2), we explain our remeshing operations (section 3) and discuss first results and perspectives (section 4).

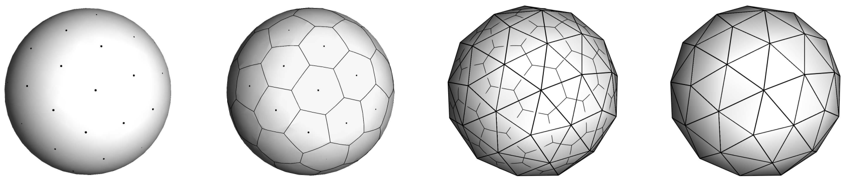

Fig. 2. from left to right : samples P on a surface S, $\mathrm{Vor}_{P|S}$, duals and $\mathrm{Del}_{P|S}$.

2 previous work and foundations

Dynamic interfaces : Eulerian methods such as Level Set [5] or Volume of Fluid [6] are populars because they implicitly handle topological changes like merging or breaking of droplets. On the other hand, Lagrangian methods like ours or "El Topo" [2] provide accurate explicit surfaces that could be directly used (rendering...). We refer to the reviews in [2] and [10].

Evaluation : In order to evaluate the accuracy of our front tracker, we use velocity fields that are defined in closed form by an explicit formula. They give anywhere and anytime known velocity fields and do not suffer of physics simulation errors. Enright's [5] and Curl-noise [2] tests are commonly used (see respectively figures 1 and 5).

Surface meshing : Our goal is to construct a mesh of good quality at each time step. We propose to optimize the placement of the mesh vertices.

Our optimization is based on the Restricted Voronoi Diagram, see figure 2 and [11, 7] for details. Given a 2-manifold surface $S \in \mathbb{R}^3$and a sample set $P \in S$, the Voronoi Diagram of P restricted to S is defined as $\mathrm{Vor}_{P|S} = \{\Omega_{i|S}\}$ with :

$$\Omega_{i|S} = \{p \in S \mid d(p, p_i) < d(p, p_j)\ \forall i, j \in P\}$$

where $d(a, b)$ denotes the euclidean distance between a and b.

A [restricted] Voronoi tessellation is said to be centroidal ($[R]CVT$ for short) if each seed corresponds to it's cell centroid. A simple way to compute

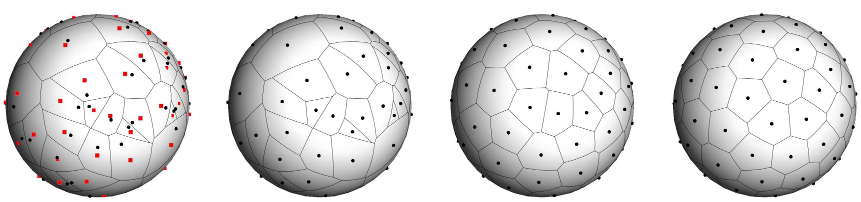

Fig. 3. on the left, the Voronoi Tessellation of a point set (dots) restricted to a sphere, cells centroids are represented with squares. The three others images are respectively the result of 2, 5 and 20 Lloyd iterations.

such distribution from a random sampling is to iteratively replace samples by their cell centroids as shown in figure 3 (Lloyd algorithm [9]).

CVT-computing can also be viewed as an optimization problem [3] of a C^2 function [8] for which quasi-newton solvers could be used to converge faster.

3 Algorithm :

Each advected surface is remeshed with the following algorithm. Some details are given below.

Algorithm 1: simulation step

Input: a 2-manifold surface S_t representing the fluid surface at time t
Data: min_lg, max_lg : edge length bounds computed from S_0,
$algo$, nb_iter, dim : RCVT parameters,
$nb\_volum\_optim\_iter$
Result: a 2-manifold surface S_{t+1} representing the fluid surface at time $t+1$

```
    // surface vertices advection
S ← advect(S_t, velocity_field)
    // surface adaptive sampling
P ← Vertices(S)
for edge(pi, pj) ∈ edges(S) do
    if length(edge) < min_length or length(edge) > max_length then
        add (p_i + p_j) * 0.5 to P
        if length(edge) < min_length then remove p_i and p_j from P
    end
end
    // sampling optimization
P ← Compute_RCVT(P, S, dim, algo, nb_iter)    // algo={Lloyd|q-newton}
    // surface building
check and fix Vor_{P|S} cell conformations    // detailed below
S_{t+1} ← duals(Vor_{P|S}) ;
    // surface optimization
minimize local volume differences between S_t^{adv} and S_{t+1} // detailed below
```

check and fix Vor$_{P|S}$ cell conformations : we need to build a valid surface (S_{t+1}) from Vor$_{P|S}$. As explained in [4], some conditions are required to ensure that there exists an homeomorphism between S_{t+1} and the advected surface.

In addition, for free-surface fluid simulation, we need to detect merging and splitting events where the field is discontinuous and modify the surface topology accordingly. The management of the topology is realized on special Vor$_{P|S}$ cell configurations, with combinatorial corrections and/or local refinements (see [11]).

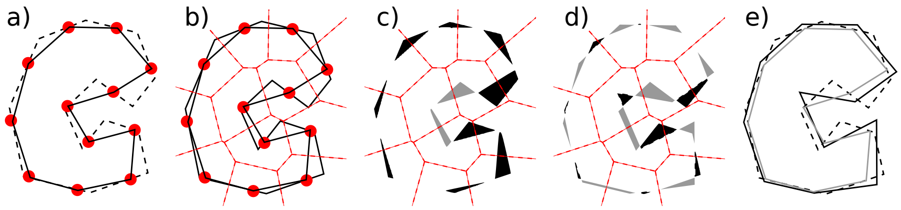

Fig. 4. Local volume difference minimization : (a) a sampled surface (dotted line) and its remeshing (plain line), (b) surfaces with sample voronoi diagram superimposed, (c) highlighted volume loss and gains (resp. black and grey areas), (d) minimized volume differences, (e) original, remeshed and optimized surfaces (resp. dotted, grey and black plain lines)

Surface volumetric optimization : For curl-free velocity fields, the fluid volume must remain constant throughout the simulation. We assume that S_{t+1} and S_t^{adv} are smooth and geometrically near to each other.

As in 2D example given in fig. 3, for each voronoi cell we define a polygon (polyhedron in 3D) with $\mathrm{Vor}_{P|S_{t+1}}$ and $\mathrm{Vor}_{P|S_t^{adv}}$ cell facets and Voronoi planes. Hence we can compute a local signed volume attached to each Voronoi seed. By minimizing the sum of squared local volumes (shaded triangles in figure 4), we improve the accuracy of the surface tracking.

4 Results and future work

Enright's and Curlnoise test screenshots are shown in figures 1 and 5 respectively. The mesh details in the first figure show isotropic and anisotropic meshes.

Anisotropic meshes are more suitable because they allow to represent very thin and sharp features with a small number of points. In addition, our local volumetric optimization further reduces physical errors involved by remeshing.

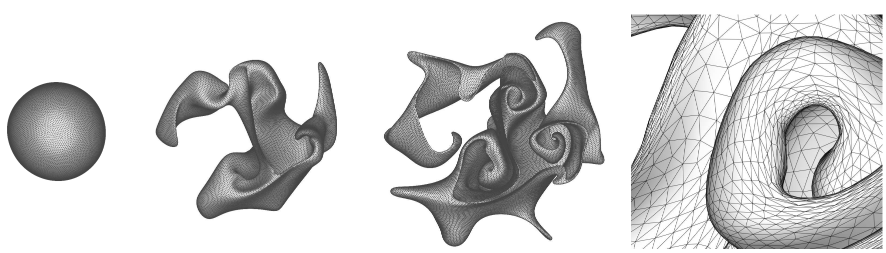

Fig. 5. Curl-noise test screenshots.

Volume is well preserved throughout the simulation. Less than 0.2% of volume is lost during 3D Enright's test (same running conditions than in [2]).

Merging and splitting should also be improved to complete intricate fluid simulations. We plan to couple our front tracker with a free surface fluid simulator [1].

Acknowledgments

This work is partly supported by the European Research Council (project GOODSHAPE, ERC-StG-205693) and the ANR (project MORPHO).

References

1. Tyson Brochu, Christopher Batty, and Robert Bridson. Matching fluid simulation elements to surface geometry and topology. *ACM Trans. Graph.*, 29(4):47:1–47:9, July 2010.
2. Tyson Brochu and Robert Bridson. Robust topological operations for dynamic explicit surfaces. *SIAM J. Sci. Comput.*, 31(4):2472–2493, 2009.
3. Qiang Du, Vance Faber, and Max Gunzburger. Centroidal voronoi tessellations: Applications and algorithms. *SIAM Rev.*, 41(4):637–676, dec 1999.
4. H. Edelsbrunner and N.R. Shah. Triangulating Topological Spaces. *International Journal of Computational Geometry and Applications*, 7(4):365–378, 1997.
5. Douglas Enright, Duc Nguyen, Frédéric Gibou, and Ronald Fedkiw. Using the particle level set method and a second order accurate pressure boundary condition for free surface flows. In *Proc. of the 4th ASME-JSME Joint Fluids Eng. Conf.*, jun 2003.
6. C. W. Hirt and B. D. Nichols. Volume of fluid /vof/ method for the dynamics of free boundaries. *Journal of Computational Physics*, 39:201–225, jan 1981.
7. Bruno Lévy and Nicolas Bonnel. Variational anisotropic surface meshing with voronoi parallel linear enumeration. In *International Meshing roundtable*, 2012.
8. Yang Liu, Wenping Wang, Bruno Lévy, Feng Sun, Dong-Ming Yan, Lin Lu, and Chenglei Yang. On centroidal voronoi tessellation - energy smoothness and fast computation. *ACM Trans. Graph.*, 28(4):101:1–101:17, sep 2009.
9. Stuart P. Lloyd. Least squares quantization in PCM. *IEEE Transactions on Information Theory*, IT-28(2):129–137, mar 1982.
10. Chris Wojtan, Matthias Müller-Fischer, and Tyson Brochu. Liquid simulation with mesh-based surface tracking. In *ACM SIGGRAPH 2011 Courses*, SIGGRAPH '11, pages 8:1–8:84, New York, NY, USA, 2011. ACM.
11. Dong-Ming Yan, Bruno Lévy, Yang Liu, Feng Sun, and Wenping Wang. Isotropic remeshing with fast and exact computation of restricted voronoi diagram. In *Proceedings of the Symposium on Geometry Processing*, SGP '09, pages 1445–1454, Aire-la-Ville, Switzerland, Switzerland, 2009. Eurographics Association.