Identification of concentrated structures in slightly compressible flows

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In this paper, different vortex diagnostic methods are compared to obtain a better understanding of boundary layer influence on the transport of vortical structures involving a complete analysis of vorticity, the Vorticity Threshold Criterion (VTC), and the Weiss Criterion (WC). These three techniques are basically confronted to find a suitable understanding of all flow characteristics for a range of laminar to transitional Reynolds numbers. The computations on this dihedral plane are done using a 2D DNS method. The Weiss criterion, coming from the analysis of the incompressible Euler equations is validated and applied to low speed compressible flows (Mach number $= 0\ .2$).

Identification des structures concentrées dans les écoulements faiblement compressibles bidimensionnels

Ce travail est consacré à l'identification des structures cohérentes présentes dans l'écou-lement d'un fluide visqueux au dessus d'un dièdre plan, en régime faiblement compressible ($nombre\ de\ Mach = 0\ ,2$)*. Le critère de Weiss provenant de l'analyse incompressible est validé pour un écoulement faiblement compressible. Nous mettons en évidence que le cri-tère de Weiss, appliqué à notre écoulement, est tout à fait pertinent et complémentaire au critère plus usuel de type « seuil de vorticité ». Plus précisemment, il permet de séparer les structures cohérentes de la couche limite, et ceci indépendemment de tout seuil donné a priori.*

Version française abrégée

L'identification des structures cohérentes représente un élément important dans la dynamique des écoulements réels. En effet, ces structures sont principalement convectées par l'écoulement, et constituent des entités qui demeurent quasi intactes au cours du temps. Elles transportent de plus une bonne partie de l'enstrophie [2]. Plusieurs stratégies d'identification existent. La plus naturelle consiste en la visualisation des isovaleurs de la vorticité. Si cette première méthode permet de se faire une bonne idée de la localisation des tourbillons, elle n'est toutefois pas suffisamment précise dès qu'il s'agit d'isoler ces tourbillons. Un critère de détection utilisant un seuil de vorticité fixé a priori ne donne pas des résultats fiables. En effet, si par définition, il permet d'isoler les parties de l'écoulement dans lesquelles la vorticité est la plus forte (en valeur absolue), il n'arrive pas toujours à distinguer les zones cohérentes des zones non cohérentes. Weiss a alors proposé [1] une méthode plus satisfaisante pour un fluide incompressible, qui consiste à identifier aux zones cohérentes la partie de l'écoulement dans laquelle la vorticité domine la déformation, et aux zones non cohérentes le reste de l'écoulement. Cela revient à considérer la nature des valeurs propres du tenseur des gradients de vitesse : si celles-ci sont complexes, le mouvement est localement elliptique et correspond à une zone cohérente ; si celles-ci sont réelles, le mouvement est localement hyperbolique et correspond à une zone non cohérente. Dans ce travail, on se propose de généraliser l'utilisation du critère de Weiss au cas d'un fluide faiblement compressible, et de comparer ce critère aux deux autres critères précités. Pour cela, on utilise une simulation numérique directe d'un fluide visqueux faiblement compressible (nombre de Mach $= 0{,}2$) [3] au dessus d'un dièdre plan (*figure 1*). On explique tout d'abord en quoi le critère de Weiss peut être appliqué au cas faiblement compressible, par un argument basé sur des constatations numériques (*figures 2* et *3*), puis par une justification théorique à partir des équations générales de Navier–Stokes. On constate alors sur l'exemple les défauts des deux critères utilisant la vorticité. Les isolignes de vorticité englobent tout le domaine rotationnel et empêchent d'isoler rigoureusement les tourbillons (*figure 4a*). L'utilisation du critère seuil de vorticité avec un seuil trop grand empêche de détecter les structures cohérentes (*figure 4c*), alors que l'utilisation de ce même critère avec un seuil trop petit détecte également des zones de cisaillement non cohérentes (*figure 4d*). Cela est dû au fait que l'analyse exclusive de la vorticité oublie un élément clé à prendre en compte, qui est le comportement quasi idéal du fluide à l'intérieur de la structure cohérente, caractérisé par une déformation faible par rapport à la rotation. C'est aussi pourquoi le critère de Weiss donne des résultats utiles (*figure 4b*). Enfin, même si le critère de Weiss ne nécessite aucun seuil théorique donné a priori, il est cependant nécessaire de spécifier un seuil numérique ε pour sa mise en oeuvre. La détermination de ce paramètre est effectuée en considérant l'intersection des tangentes à l'origine et à l'infini à la courbe donnant le pourcentage d'enstrophie contenue dans les zones cohérentes détectées par le critère de Weiss en fonction du paramètre ε choisi (*figure 5*). Ce critère de détection, ayant finalement été validé, pourrait être utilisé de façon pertinente afin d'établir certaines propriétés intrinsèques aux tourbillons, en fonction du nombre de Reynolds : nombre et taille des tourbillons, pourcentage d'enstrophie de l'écoulement qu'ils transportent, etc.

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1. Introduction

In this work, three different vortex diagnostic methods are compared to obtain a better understanding of boundary layer influence on the flow dynamics involving a complete analysis of vorticity, the Vorticity Threshold Criterion (VTC), and the Weiss Criterion (WC). These three techniques are compared to find a suitable understanding of all flow characteristics (the swirling dynamics produced downstream of the discontinuity point of the dihedral) for a range of laminar to transitional Reynolds numbers. All tests are performed over a dihedral plane.

The computations on this dihedral plane are performed using a 2D DNS method. The Weiss criterion, arising from the analysis of the incompressible Euler equations ([1] and [2]), is adapted to this case study (compressible viscous flow, with Mach number equal to 0.2). This adaptation is permitted due to low compressibility of the flow (there is no shock). This criterion is consistent with the more classical vorticity threshold criterion, and therefore prooves an appropriate tool to isolate coherent vortices.

The flow domain is shown in *figure 1*. The computational domain is plotted in dotted lines. ν is the kinematic viscosity; $u,\ v,\ p$ and ρ represent respectively the two velocity coordinates on (Ax, Ay), the pressure and the density.

The characteristic Reynolds number is then defined by $\mathrm{Re}_\delta = \delta u_\infty/\nu$. δ is the thickness of the boundary layer at the entry of the computational domain. The nondimensional values are $u_{\mathrm{ref}} = u_\infty$ and $l_{\mathrm{ref}} = \delta_{200}$, where δ_{200} is the thickness of the boundary layer at the entry of the simulation domain for $\mathrm{Re}_\delta = 200$. All

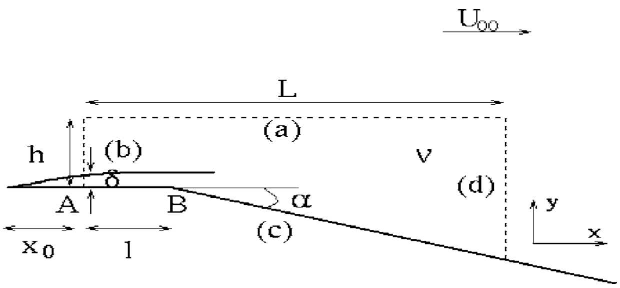

Figure 1. Dihedral configuration.

calculations are performed in this paper using these nondimensional parameters. We choose $L = 48\delta_{200}$, $h = 24\delta_{200}$, and $l = 8\delta_{200}$.

2. Governing equations and numerical scheme

The Navier–Stokes equations are solved using a DNS method based on unstructured triangular grids with a good refinement for high gradient regions. The numerical scheme, that is based on a classical finite volume – finite element method, is fully explicit in time [3].

As the domain is bounded, it is necessary to specify appropriate boundary conditions. In this work, four boundary conditions are applied. On the boundary (a), the nonreflecting boundary condition with a pressure extrapolation is used. On the boundary (b), which is a subsonic inflow, the two components of the velocity as well as the temperature are strongly imposed using the Blasius profile: $u = u_{\mathrm{imp}}(y)$, $v = v_{\mathrm{imp}}(y)$, $T = T_0$. On the boundary (c), which is an isothermal no slip wall, the velocity is canceled and the temperature specified as: $u = 0$, $v = 0$, $T = T_0$. Each time step, the density is deduced from the continuity equation, respecting the previous boundary conditions [4]. The pressure is then obtained using the state equation. On the boundary (d), Hernandez [5] pointed out that the nonreflecting boundary condition with a pressure recall, similar to the one used for the boundary (a), didn't give physical results. To overcome this difficulty, Bruneau and Creusé proposed a new way to evaluate successfully the amplitude of the entering characteristic waves across the outflow boundary [3].

3. Identification of coherent structures

The first step of this work is to define a Coherent Structure (or Concentrated Structure). A defintion is given by Lugt as 'many rotating particles turning around a common center' [6]. Jeong and Hussain [7] have generalised the Weiss criterion to three dimensions and have compared the efficiency of different detection techniques. In this work, we compare the complete vorticity field representation to the vortex threshold identification method as well as to the Weiss criterion, and we try to find the best compromise to study and visualise the dynamics of vorticity. The flow is two-dimensional and slightly compressible.

The vortex threshold criterion

To isolate the eddy structures, one idea should come from focusing in areas with a definite (or higher) value of vorticity $|\omega| = |\partial v/\partial x - \partial u/\partial y| > \varepsilon$, where ε is a parameter depending on the studied flow. This criterion is based on the fact that the intensity of the vorticity inside the eddy structures is higher than in the outside. One possibility to define ε in internal flows is to choose it equivalent to the average enstrophy of the flow [8]. This choice can reflect in a basic way the flow dynamics, but needs a finite bounded domain terminology and shouldn't be used in our case with open boundaries.

Weiss criterion and variable density problems

Here we consider an inviscid incompressible fluid, with corresponding 2D Euler equations:

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} &= -\vec{\nabla} p \\ \nabla \cdot \vec{u} &= 0 \end{aligned} \tag{1}$$

where $\vec{u} = (u, v)$ is the velocity vector and p is the pressure. To study the vorticity field deformations, Weiss proposes to take successively the curl and the gradient of (1) to obtain:

$$\frac{\mathrm{d}\vec{\nabla}\omega}{\mathrm{d}t} + A^t \cdot \vec{\nabla}\omega = 0$$

where A is the velocity gradient tensor:

$$A = \begin{pmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{pmatrix}$$

Considering that this tensor varies slowly compared to the vorticity gradient, this equation can be taken as a linear equation with constant coefficients. Then the behaviour of the vorticity gradient is locally determined by the nature of the eigenvalues of A. These eigenvalues are simply the roots of the equation $4\lambda^2 = \sigma^2 - \omega^2$, where σ corresponds to the deformation:

$$\sigma^2 = \frac{1}{2}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + 4\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]$$

The Weiss criterion is then defined. In the areas where $\sigma^2 - \omega^2$ is positive, the eigenvalues are real and the motion is principally hyperbolic, then the rotation is dominated by the deformation. These areas are not considered as vortical regions. In the areas where $\sigma^2 - \omega^2$ is negative, the eigenvalues are imaginary and the motion is principally elliptic, then the deformation is dominated by the rotation. These regions are considered as vortical regions. In fact, the condition $\omega^2 > \sigma^2$ may be considered as a 'vorticity threshold criterion', with ε depending essentially on the local deformation.

Let now justify and validate this method to a weakly compressible flow with a variable ρ. Combining the continuity equation with the compressible Euler equations, we obtain:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho}\vec{\nabla} p$$

Then taking the curl and the gradient of this equation, the following relationship is achieved:

$$\frac{\mathrm{d}\vec{\nabla}\omega}{\mathrm{d}t} + A^t \cdot \vec{\nabla}\omega = -\frac{2}{\rho^3}\left(\vec{\nabla}\rho \times \vec{\nabla}p\right)_s \vec{\nabla}\rho + \frac{1}{\rho^2}\vec{\nabla}\left(\vec{\nabla}\rho \times \vec{\nabla}p\right)_s \tag{2}$$

where $(a \times b)_s$ is the unique non-zero component of the vector product of vectors a and b.

Nevertheless, the right term of equation (2) may be neglected for the two following reasons. First, the flow studied in this work is slightly compressible, then the values of $\vec{\nabla}\rho$ are small. Furthermore, if p could be considered as a unique function of ρ, we would write $p = f(\rho)$, and:

$$\vec{\nabla} p = \frac{\partial p}{\partial \rho}\vec{\nabla}\rho$$

So, vector products in (2) would vanish.

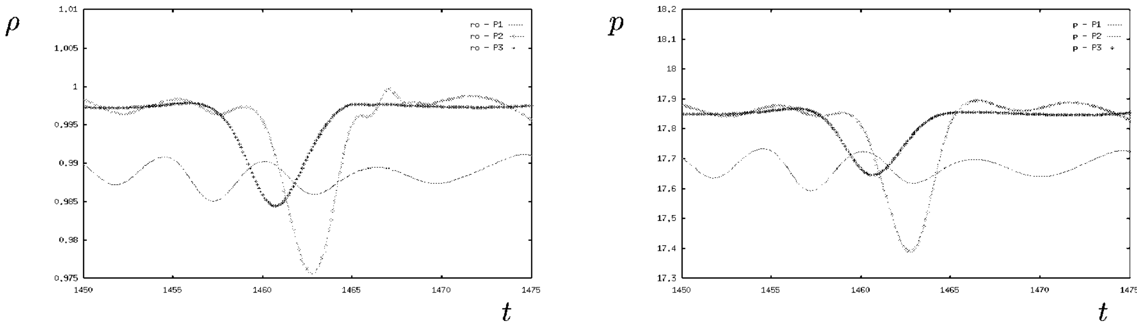

Figure 2. Density and pressure signals at three points in the flow.

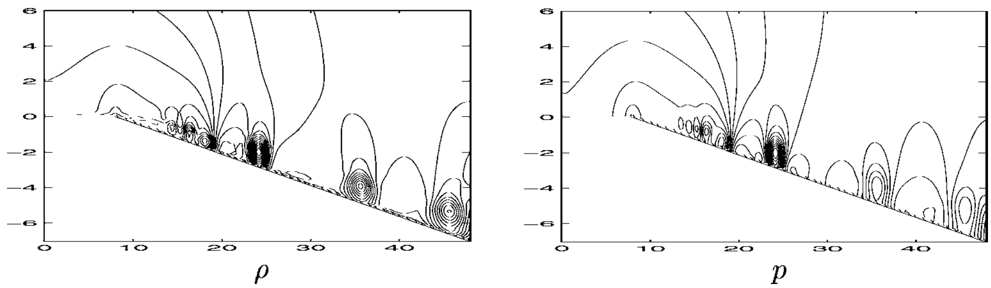

Figure 3. Density and pressure isolines.

This numerical hypothesis is verified by observing the time evolution of the density and the pressure on three monitoring points of the flow located in the zone where vortices evolve (*figure 2*), as well as their instantaneous isolines on the whole flow domain (*figure 3*). Both figures (*figures 2* and *3*) confirm the similarity of the ρ and p behaviour either in time and in space. This numerical observation displays a good correlation between the ρ and p values. Using these considerations, we can validate and apply the Weiss criterion to slightly compressible flows. Now, to have a better idea about the complementarity of these techniques, every criterion is verified for the flow over a dihedral plane.

The case study corresponds to a compressible viscous flow on a dihedral plane with quite low Reynolds numbers ($\mathrm{Re}_\delta = 400$). The vorticity field plot confirms the creation of some vortical zones downstream of the discontinuity point B (*figure 4a*). The vorticity detected by the VTC is observed for two different values of ε, one small and the other one large. For the larger value of ε, the concentrated vortices are merely identified except of the boundary layer vorticity with its high shear properties (and therefore high rotational behaviour) (*figure 4c*). For a smaller ε, the structures are present but mainly covered inside the boundary layer extension, and VTC does not isolate them from each other which makes the detection task quite difficult (*figure 4d*). However, the WC distinguishes and separates several CS in the dihedral slopping transport domain (*figure 4b*).

Obviously, one should ask: why not use directly the vorticity isolines (vorticity field)? To answer this question, it should be mentioned that the vorticity field is a natural and important source of information on the rotational flow dynamics. Nevertheless, sometimes we need to isolate coherent structures and to determine not only their size and their number in the flow, but also their individual characteristics like enstrophy, circulation etc. For these studies, the vortex detection criteria like WC, and its extensions [2, 9], offer more efficient tools. They permit detection of CS in a wide range of individual enstrophy values (small or large even in the boundary layers) and define their size independently from any cutoff ε that is quite difficult to define in such flows like the second case study (because of the arbitrary choice of

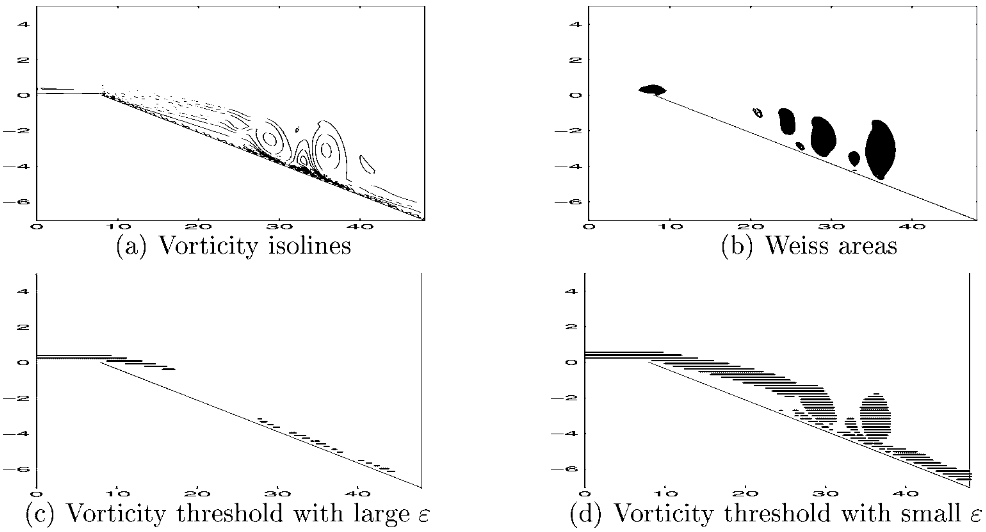

Figure 4. Comparison of the different criteria on the dihedral.

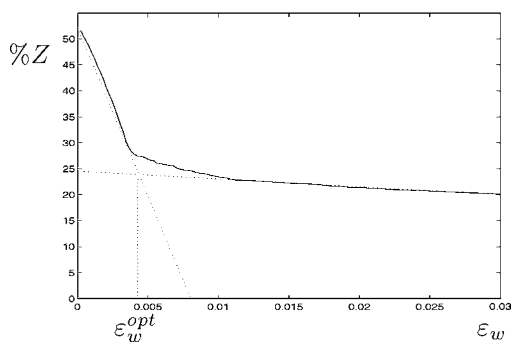

Figure 5. The $\varepsilon_{\mathrm{opt}}$ determination.

the computational domain). Therefore, the WC (and its extensions) are recommanded to identify CS. It should be noted that the WC and the VTC are very complementary. The VTC compares the vorticity to an experimentally chosen value. Such as the average enstrophy intensity for flows within a bounded domain. However this 'cutoff' value is globaly defined in the flow domain. The global choice is arbitrary when compared to high vorticity domains. A global cutoff of the form $|\omega| > \varepsilon$ means that $|\omega|$ varies linearly versus ε (or ω^2 versus ε^2), and is not adjusted to local non-linearities and variations. But the extended WC locally determines domains where ω^2 dominates σ^2 ($\omega^2 > \sigma^2$). The new cutoff respects local physical properties of the flow related to the choice of separate CS.

However, numerically it's necessary to determine a 'tolerance gap', to improve the verification $\omega^2 - \sigma^2 > \varepsilon_w$, where ε_w is a small positive value, that should be chosen asymptotically. Consider either a laminar Poiseuille flow, or a laminar boundary layer over a flat plate. These flows which can be considered as the 'background' flow of the previous test cases, have a value of $\omega^2 - \sigma^2$ which is roughly zero. However, a small peturbation can push $\omega^2 - \sigma^2$ to a locally positive value mimicking a vortical behaviour with no real vortex structure presence. For the dihedral geometry, the proportion of the CS enstrophy (using WC) to the global flow is plotted versus the parameter ε_w (*figure 5*). As the figure shows, two different behaviours are distinguished at the right and the left-hand side of $\varepsilon_w^{\mathrm{opt}} = 0.004$, defined as the cross-section of tangents to the curve at 0 and infinity. For $0 < \varepsilon_w < \varepsilon_w^{\mathrm{opt}}$, the high variation of the enstrophy value corresponds to shear zones of the boundary layer that can be identified by the WC only if ε_w is very small (when ω^2

is very close to σ^2). These zones disappear when ε_w is increasing, even very slightly. For $\varepsilon_w > \varepsilon_w^{\text{opt}}$, an asymptotically state is acheived. That means: when ε is increasing, the regions identified by the WC stay almost unchanged. They correspond, therefore, to the definition of CS. In these regions, ω^2 is sensibly higher than σ^2.

To verify these conditions, many numerical experiments have been performed for a wide range of ε_w values. They fulfilled entirely the above mentioned remarks concerning the choice of $\varepsilon_w^{\text{opt}}$. This value of $\varepsilon_w^{\text{opt}}$ is large enough to remove the background flow, and small enough to distinguish the ensemble of vortical structures.

4. Conclusion

This work was devoted to identify coherent vortical structures in slightly compressible flows. The geometry of the study was a dihedral plane where the shear effect of the separation point leads to generate regularly concentrated eddies. At the beginning we analysed different Coherent Structure detection methods (Weiss Criterion and Vorticity Threshold Criterion). We generalised the WC for low Mach number compressible flows and demonstrated its capability to be applied to this kind of flows. Then, we compared two criteria to the general vorticity field representation and showed that, if in internal flows with a suitable choice of the threshold cutoff (average enstrophy ε^2) two criteria give similar results, in arbitrary domains the WC presents a better and more natural alternative to capture CS (avoiding not only the background flow but also other non-coherent rotational zones). Then, we presented a methodology to choose an 'optimal tolerance gap' which permits one to smooth approximately coherent zones. This smoothing value was chosen when the global enstrophy evolution curve began to achieve a plateau. We determined that the WC permits us to extract the 'only-CS' regions capturing principally points corresponding to them. Once the vortex identification task is fulfilled, this work is being complemented to analyse and study the eddy dynamics in a full length paper.

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