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Investments in R&D and Production Capacity with Uncertain Breakthrough Time: Private versus Social Incentives*

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Abstract

The article considers a sequential investment project which starts with a product innovation phase, and subsequently, once R&D is completed, a production phase. The investment decision of the R&D phase involves choosing the time and the size of the R&D investment. The time to breakthrough is stochastic in which the instantaneous probability of innovation is increasing in the R&D investment size. Once R&D is completed the firm starts producing the new product. To do so, the firm first needs to invest in production capacity, the size of which must be determined. We compare the optimal investment decisions of the firm with those of the social planner and conclude that the firm invests too late in R&D and not enough in production capacity. We find that a proper subsidy policy, consisting of an R&D investment and a productive investment subsidy can make up for that. However, taking into account a budget constraint such that subsidy expenses cannot exceed the resulting increase in total surplus, learns that a first-best solution can only be reached if the demand situation is relatively stable, i.e., when growth and demand uncertainty are limited, or when the price elasticity of demand is low.

Keywords: Research and development, welfare, innovation, subsidies, monopolist, government.

JEL Classification Number: D81, G11, 032.

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1 Introduction

The 2021 United Nations Climate Change Conference (COP26) in Glasgow stressed the importance of accelerating the global transition to 100% zero emission cars and vans.[1] In particular, governments of 27 attending countries declared that

> "As governments, we will work towards all sales of new cars and vans being zero emission by 2040 or earlier, or by no later than 2035 in leading markets."

Additionally, eleven automotive manufacturers, such as Ford and Mercedes-Benz, vowed that

> "As automotive manufacturers, we will work towards reaching 100% zero emission new car and van sales in leading markets by 2035 or earlier, supported by a business strategy that is in line with achieving this ambition, as we help build customer demand."

Although these governments and automotive manufacturers find common ground on their goal of 100% zero emission cars and vans, their respective roads towards that goal need not necessarily align. Or worse, the goal need not even be reached — after all, automotive manufacturers failed to meet their own electric vehicle sales targets in 2016 (Hildermeier, 2017). Notwithstanding Germany's decision not to sign the pledge, the new German coalition is committed to have at least 15 million electric vehicles on German roads by 2030 as well as to maintain investments in charging infrastructure for electric vehicles (Posaner & Sugue, 2021). It is evident that the transition to 100% zero emission cars and vans requires substantial investments in production capacity and research and development (R&D).

With the current article we aim to contribute to the understanding of investments in R&D and production capacity, from both a private and social perspective, by employing a real options framework. Here, R&D involves the irreversible investment in an R&D project to develop a new product with the aim of taking it to market. The R&D project is subject to two types of uncertainty, namely technological and market uncertainty. Technological uncertainty arises from the fact that in general it is not known beforehand how much time it takes to achieve the breakthrough in an R&D project. We take this into account in our model in which we impose that the time

[1] The full COP26 declaration can be found here: https://www.gov.uk/government/publications/cop26-declaration-zero-emission-cars-and-vans/cop26-declaration-on-accelerating-the-transition-to-100-zero-emission-cars-and-vans.

to breakthrough is stochastic, such that the instantaneous probability of innovation is increasing in the R&D investment size. Once the breakthrough is accomplished the firm launches the new product on the market and starts selling it. Market uncertainty arises because product demand is uncertain.

We address three investment decisions: when to invest in R&D, how much to invest in R&D, and how much production capacity to acquire once the product innovation has been achieved. The optimal investment decisions of a profit-maximizing firm — the monopolist — and a welfare-maximizing social planner — the government — do not align as they have different optimization incentives. The firm maximizes the producer surplus, whereas the government maximizes the total surplus, which is defined as the sum of the consumer surplus and the producer surplus. The *first-best solution* is the solution that brings about the largest welfare in society and therefore follows from the optimal investment decisions that would arise if the government would have been the decision maker. Because the firm is only concerned about the producer surplus and thus ignores the consumer surplus, the realization of the R&D project according to the optimal investment decisions of the firm will typically result in a *welfare loss*.

We find that taking investment decisions from a private perspective results in two welfare disruptions. The first disruption is one on the product market in the sense that the firm installs a production capacity that is too small. The second welfare disruption is one that is the result of the firm investing too late in R&D. In other words, from a social perspective the firm should install a larger production capacity and invest in R&D earlier. There is, however, no welfare disruption with respect to the R&D investment size, in the sense that the firm and social planner invest the same amount in R&D at their respective optimal investment times.

The article investigates whether the welfare disruptions can be reduced by a proper subsidy policy, consisting of an *R&D investment* and a *productive investment subsidy*. Indeed, the production capacity of the firm can be moved towards that of the social planner by a *productive investment subsidy*, because this incentivizes the firm to install a larger production capacity. The productive investment subsidy is determined such that it equals the amount required to compensate the loss in producer surplus, which is the result of installing a production capacity larger than what is optimal for the firm. Typically, if the firm waits for a higher demand level before it invests, for instance due to the fact that there is a lot of demand uncertainty, the productive investment subsidy needs to be larger to compensate the loss in producer surplus. Furthermore, to bring the R&D investment timing of the firm towards that of the social planner, the government provides the *R&D investment subsidy*, which is a one-time capital injection to reduce the R&D investment costs for the firm.

We impose that the provision of subsidies is such that subsidy expenses cannot exceed the resulting increase in total surplus. We denote this constraint by *budget constraint*. The need to satisfy this constraint implies that the government is not always able to subsidize the firm in such a way that first-best solution behavior results. In fact, we find that when product demand is stable over time, i.e., when uncertainty and market trend are both limited or when the price elasticity of demand is low, the increase in total surplus exceeds the necessary government expenses on R&D investment and productive investment subsidies, so then it works. However, it is more expensive for the government to compensate the firm's loss in producer surplus if demand uncertainty goes up, if there is more growth in demand, and/or if the price elasticity of demand is high. The result is that the productive investment subsidy needed to cover the loss in producer surplus becomes too expensive in case of high uncertainty and/or considerable market growth, and then violation of the budget constraint prevents the first-best solution to occur.

We apply this analysis within a case study on the European and American electric vehicle market, in which the demand is characterized by a negligible drift rate and low volatility. There our conclusion is confirmed in the sense that the welfare disruptions could be completely repaired by introducing the R&D investment and the productive investment subsidies.

An important signal pointing to the robustness of our results is that we obtain the same conclusion under two very different demand specifications, namely linear and isoelastic demand. In particular, we show that under both specifications proper subsidization can result in the firm investing according to the first-best solution when demand is sufficiently stable over time. In addition, all previously mentioned results, such as the two types of welfare disruptions, remain valid under both demand functions.

The article is organized as follows. The remainder of this introductory section discusses related relevant literature. In Section 2 we introduce the model with the linear demand function. We derive the optimal investment decisions, both from a private and social perspective, and pinpoint the two welfare disruptions. We move towards the socially-desirable outcome in Section 3 by introducing the two types of subsidies and the budget constraint. In Section 4 we analyze our framework with respect to the isoelastic demand function. Section 5 discusses our model in relation to the European and American electric vehicle market. Section 6 concludes. The proofs of the propositions are provided in the appendices.

1.1 Related literature

The R&D model in this article puts the monopoly model of Huisman and Kort (2015) in an R&D framework. In the model of Huisman and Kort (2015), the monopolist is considering entering a market. To this end, it determines the optimal investment timing as well as the optimal quantity or capacity level. Here, on the other hand, the firm has yet to develop the product and thus has to go through a research and development phase before it can enter a market.

Our article models technological uncertainty by a Poisson process. This has its origins in R&D models by Loury (1979) and Lee and Wilde (1980). Their aim is to study the influence of market structure on R&D performance. The article has in common that it also assumes a positive relationship between the R&D investment size and the instantaneous probability that innovation occurs. Although both Loury (1979) and Lee and Wilde (1980) determine the optimal R&D investment size of a firm, neither of them determines the optimal R&D investment timing; the present article determines both. More related to our work is Weeds (2002). She, in fact, optimizes the R&D investment timing but assumes a fixed R&D investment cost and a constant arrival rate of the Poisson process. In the existing literature the assumption of a constant arrival rate is prevalent among (game-theoretic) models of innovation and R&D, see, e.g., Farzin, Huisman, and Kort (1998), Huisman and Kort (2004) and Nishihara and Ohyama (2008). More recent works, such as Hagspiel, Huisman, and Nunes (2015) and Deeney, Cummins, Heintz, and Pryce (2021), allow for a changing arrival rate, which depends on the state of the R&D phase; however, still the size of investment is fixed. To the best of our knowledge, there does not exist a model of R&D that, in addition to computing the optimal R&D investment timing, computes the optimal R&D investment size in which an increased investment accelerates the expected time to breakthrough of the R&D project. This is what the present article does. In this way, our model also captures the element of quantity inherent in investment projects. In accordance with contributions like Dangl (1999), Huisman and Kort (2015), Hagspiel, Huisman, and Kort (2016) and Hagspiel, Huisman, Kort, and Nunes (2016), we find that increased demand uncertainty leads to larger R&D projects, though they are initiated at a later point in time.

In our model, disruptions to welfare stem from the the firm investing too late in R&D and installing too little production capacity once the R&D stage is completed. The contribution of our model is the joint analysis of these two types of welfare disruptions. A monopolist restricting supply, thereby deviating from the social optimum, is also observed in other real option models, for instance in Novy-Marx (2007) and Huisman and Kort (2015). A monopolist deviating from the social optimum by delaying R&D can be interpreted as R&D being less attractive for a monopolist. Economic models of R&D by Lee (1983), and Fishman and Rob (2000) support this notion. On the other hand, a monopolist can have a stronger incentive to innovate because it acts as a preemptive activity, for instance through patents (Gilbert & Newbery, 1982). Nevertheless, as Weeds (2002) argues, firms can delay R&D investment for fear of starting a patent race, which is socially inefficient. Moreover, Cortazar, Schwartz, and Salinas (1998) observe that in markets with highly volatile prices it is less attractive to invest in environmental protection technologies, and that an increased discount rate reduces optimal environmental investment levels. We obtain the same comparative statics results with respect to R&D in our model.

In our article a government can stimulate investments in production capacity and R&D through subsidization. Romano (1989) argues that subsidization as a policy instrument for R&D is particularly useful in a monopolistic environment — in fact, even more so than in a competitive environment. We mention some contributions to the literature that study a monopolist's investment decisions in relation to those of a social planner. Evans and Guthrie (2012) focus on price-cap and quantity regulation with respect to economies of scale. Willems and Zwart (2018) analyze the optimal regulation of capacity investments subject to a budget constraint and a participation constraint. These two works are concerned with regulation of a monopolist rather than subsidization. There is nonetheless ample literature on subsidization and its effect on a firm's investment decisions, for instance in relation to renewable energy. Bigerna, Wen, Hagspiel, and Kort (2019) is a related work to ours that studies subsidization of green energy to achieve renewable energy consumption targets set by the European Union. Subsidization takes place in the form of a fixed feed-in premium which increases the net price the monopolist receives for one unit of renewable energy. An increase of the feed-in premium accelerates investment, but also decreases the installed capacity level. This complicates things because the feed-in premium should serve two purposes, namely to meet the production capacity target and to invest before a certain deadline. It turns out that the feed-in premium does not always suffice, which calls for an additional subsidy that induces immediate investment in the right capacity size. This confirms Dobbs (2004) and Willems and Zwart (2018), in which it is concluded that uncertainty of demand implies that one policy instrument cannot be used to achieve two goals. Instead, one should rely on several policy instruments to attain the social optimum. This is also what we have in our article: introducing subsidies on both R&D and production capacity makes that the social optimum may be reached in our sequential monopoly investment model.

2 R&D from a private perspective

A typical timeline for the firm is given in Figure 2.1. Currently, the firm has not yet started its R&D project. In this waiting region the firm has the option to commence its R&D project. If the firm exercises the option, it undertakes an irreversible R&D investment. Upon completion of the R&D phase, which is of random duration, the firm can launch its product to receive a revenue stream.

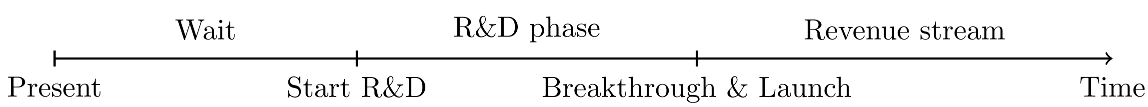


Figure 2.1: A typical timeline for the firm's R&D project.

The R&D project is completed at the moment that the breakthrough arrives. Occurrence of this breakthrough is modeled by a Poisson process with one jump, given by

$$dq(t) = \begin{cases} u & \text{with probability } \lambda dt, \\ 0 & \text{with probability } 1 - \lambda dt, \end{cases}$$

in which $q(0) = 0$. A jump in the Poisson process indicates that there has been a breakthrough so that we have $q(t) = u$ forever. The arrival rate is given by λ and the probability of innovation in time interval dt is equal to λdt. In this article, we assume that the arrival rate λ is positively influenced by the R&D investment size. Let R denote the fixed sunk costs of the R&D project. These costs include, but are not limited to, the cost of a new laboratory or research facility, equipment costs as well as possible licensing costs. We assume that $\lambda\colon [0,\infty) \to [0,\infty)$ is a real-valued continuous function that has the following three properties. First, $\lambda(0) = 0$, so no investment implies that a breakthrough cannot take place. Second, for all $R \geq 0$, $\lambda'(R) > 0$, which means that the probability of a breakthrough is larger if the firm invests more in R&D. Third, for all $R \geq 0$, $\lambda''(R) < 0$, which means that the R&D investment exhibits decreasing returns to scale.

Let T denote the stochastic innovation time. The random variable T is exponentially distributed with mean $\frac{1}{\lambda(R)}$ and its probability density function, for all $t \geq 0$, is given by $f(t) = \lambda(R)e^{-\lambda(R)t}$.

The R&D breakthrough results in the invention of a new product of which the demand is uncertain. This implies that the firm faces demand uncertainty in addition to technological uncertainty. The product price is governed by a geometric Brownian motion with drift. A geometric Brownian motion is a continuous-time stochastic process $X(t)$ in which the rate of change in $X(t)$ is given by

$$dX(t) = \mu X(t)dt + \sigma X(t)dZ(t), \tag{2.1}$$

in which μ is the drift rate, $\sigma > 0$ is a variance parameter, and $dZ(t)$ is the increment of a Wiener process, i.e., $dZ(t) = \varepsilon_t\sqrt{dt}$, with $\varepsilon_t \sim N(0,1)$ and $\mathbb{E}[\varepsilon_t\varepsilon_s] = 0$ for $t \neq s$. The starting value, $X(0)$, of the geometric Brownian motion is henceforth denoted by X and is strictly positive.

The market of the new product is assumed to be homogeneous with linear demand. The price at time t is given by

$$P(t) = (1 - \alpha K)X(t), \tag{2.2}$$

in which K is an endogenous quantity we want to optimize, and α is a positive constant. The firm can become active on the product market when the newly-developed product is available. We impose that the firm produces up to capacity, which, in our model, is also optimal. The price of the product is subject to stochastic shocks, $X(t)$, that follow a geometric Brownian motion with drift as given in (2.1). The instantaneous profit is given by

$$\pi(t) = KP(t) = K(1 - \alpha K)X(t).$$

The firm is risk neutral and discounts against rate r (> 0). It is assumed that $\mu < r$, otherwise the problem does not make sense as the firm will wait indefinitely and thus will never undertake the R&D investment.

Upon innovation the firm launches the product with optimal capacity size $K^* = \frac{1}{2\alpha}$, so that its corresponding expected value at X is given by[2]

$$\Omega(X) = \frac{X}{4\alpha(r-\mu)}. \tag{2.3}$$

The investment problem the firm is facing boils down to an optimal stopping problem:

$$V(X) = \max_{\tau \geq 0, R \geq 0} \mathbb{E}\left[\int_{t=\tau}^{\infty} \lambda(R) e^{-\lambda(R)t} e^{-rt} \Omega(X(t)) dt - e^{-r\tau} R \,\middle|\, X(0) = X\right], \tag{2.4}$$

in which the expectation is conditional on $X = X(0)$, which is the current level of the geometric Brownian motion, $\Omega(X(t))$ is given by (2.3), τ is the time at which the firm starts R&D, and R is the irreversible R&D investment.

We let X^* denote the optimal R&D investment trigger. This is the minimal value of the geometric Brownian motion whereupon the firm starts its R&D project. In fact, it is the value for which the firm is indifferent between investing and not investing. It follows that it is optimal for the firm to wait with investment if $X < X^*$, whereas it is optimal for the firm to undertake the investment immediately if $X \geq X^*$. Therefore, if $X < X^*$, the optimal investment timing τ is the moment in time at which the level of the geometric Brownian motion hits X^*; τ equals zero when $X \geq X^*$.

The optimal investment size, $R^*(X)$, follows from solving

$$\max_{R \geq 0} \mathbb{E}\left[\int_{t=0}^{\infty} \lambda e^{-\lambda t} e^{-rt} \Omega(X(t)) dt - R \,\middle|\, X(0) = X\right],$$

[2]These expressions follow from equation (5) and Proposition 1 in Huisman and Kort (2015). It is optimal for the firm to launch its product immediately upon breakthrough if $\delta = 0$, in which δ represents the cost of one unit of capacity. The case $\delta > 0$ is considerably more intricate and as a consequence cannot be analyzed analytically. As the main aim of our article is to provide an analytically tractable R&D framework, we leave the case $\delta > 0$ as future research. Nevertheless, the case with positive unit cost of capacity is still taken into account in Section 4, where we consider our framework with respect to an isoelastic demand function.

which is equivalent to

$$\max_{R\geq 0}\left\{\Omega(X)\frac{\lambda(R)}{\lambda(R)+r-\mu}-R\right\}. \tag{2.5}$$

The optimal $R^*(X)$ follows from the first-order condition[3] and solves

$$\Omega(X)\frac{\lambda'(R^*(X))(r-\mu)}{(\lambda(R^*(X))+r-\mu)^2}=1. \tag{2.6}$$

If no positive solution to (2.6) exists, then it means that investing is suboptimal, i.e., $R^*(X)=0$. From (2.6) it follows that the optimal investment size is increasing in X, which is also observed in Huisman and Kort (2015) with respect to optimal production capacity, $K^*(X)$. The following proposition presents the optimal investment decisions and the corresponding project value.

Proposition 2.1. *The project value of the firm is equal to*

$$V(X)=\begin{cases} AX^{\beta} & \text{if } X<X^*,\\ \dfrac{X}{4\alpha(r-\mu)}\dfrac{\lambda(R^*(X))}{\lambda(R^*(X))+r-\mu}-R^*(X) & \text{if } X\geq X^*, \end{cases} \tag{2.7}$$

in which

$$\beta=\frac{1}{2}-\frac{\mu}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}>1,$$

in which the constant A is given by

$$A=\frac{(X^*)^{-\beta}R^*}{\beta-1}, \tag{2.8}$$

and in which $R^*(X)$ is implicitly determined by (2.6).

The optimal investment size at the trigger, $R^*\equiv R^*(X^*)$, is implicitly determined by

$$\beta=\frac{\lambda(R^*)+r-\mu}{\lambda(R^*)+(1-\frac{R^*\lambda'(R^*)}{\lambda(R^*)})(r-\mu)}, \tag{2.9}$$

[3]The second-order condition is always satisfied because $\lambda''(R)<0$ for all $R\geq 0$, see also (A.4) in Appendix A.

and the optimal investment trigger X^* is given by

$$X^* = \left(\frac{\beta}{\beta - 1}\right) \frac{4\alpha(r-\mu)R^*(\lambda(R^*) + r - \mu)}{\lambda(R^*)}. \tag{2.10}$$

If (2.9) *and* (2.10) *give no positive solution, then the firm will not undertake the R&D investment.*

The term AX^β in the value function (2.7) is the value of the option to start the R&D project. Given R, the value function (2.7) for $X \geq X^*$ equals the expected project value if the firm starts its R&D project at $X \geq X^*$, which is given by

$$\mathbb{E}\left[e^{-rT}\Omega(X(T)) - R \,|\, X(0) = X\right] = \frac{X}{4\alpha(r-\mu)} \frac{\lambda(R)}{\lambda(R) + r - \mu} - R,$$

in which T is the stochastic innovation time, and $\Omega(X(T))$ is given by (2.3).

If we specify the functional form of the arrival rate, $\lambda(R)$, we can obtain R^* and X^* explicitly.

Corollary 2.1. *Let $\lambda(R) = cR^\gamma$ with $c > 0$ and $\gamma \in (0, 1)$. If $\beta \leq \frac{1}{1-\gamma}$, then*

$$R^* = \left(\frac{(r-\mu)(1-\beta(1-\gamma))}{c(\beta-1)}\right)^{\frac{1}{\gamma}}, \tag{2.11}$$

and

$$X^* = \frac{\beta^2\gamma 4\alpha(r-\mu)^{1+\frac{1}{\gamma}}(1-\beta(1-\gamma))^{\frac{1}{\gamma}-1}}{(\beta-1)^{1+\frac{1}{\gamma}}c^{\frac{1}{\gamma}}}; \tag{2.12}$$

otherwise the firm will not undertake the R&D investment.

Under the functional form in Corollary 2.1, a 1% increase in the investment size results in a γ% increase in the arrival rate — that is, $\frac{d\lambda(R)}{\lambda} = \gamma\frac{dR}{R}$. In this way γ reflects the efficiency of the technological progress. The firm will undertake the R&D project if its R&D process is sufficiently efficient, i.e., if γ is large enough. Furthermore, the firm invests more in R&D but delays investment if γ increases.[4]

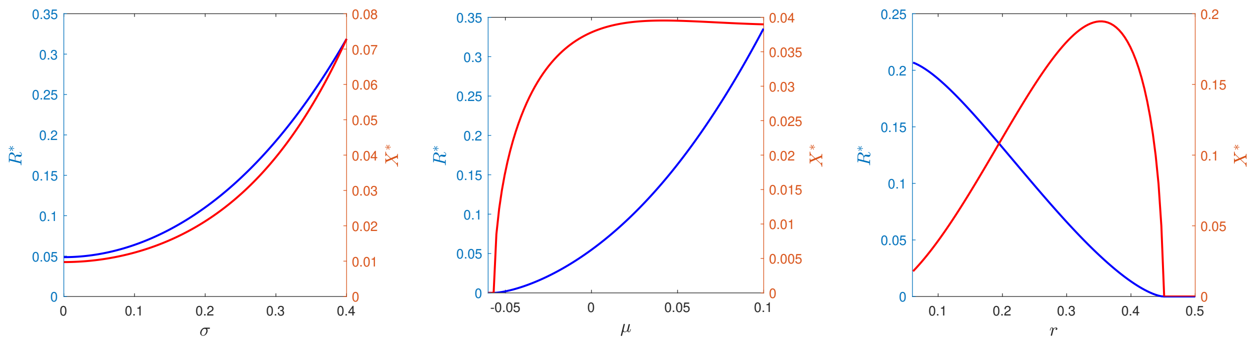
Figures 2.2a, 2.2b, and 2.2c show the optimal investment size, R^*, and the optimal investment threshold, X^*, as a function of σ, μ and r, respectively, for a particular specification of the parameter

[4]Under the functional form $\lambda(R) = cR^\gamma$ we require that $c \geq \frac{r-\mu}{e^1(\beta-1)}$, which, from a practical perspective, is not a strong requirement. For example, $c \geq 0.04415$ if $\sigma = 0.3$, $\mu = 0.06$, and $r = 0.1$.

values. From (2.11) and (2.12) we can obtain that both R^* and X^* increase with the uncertainty parameter σ. This is in line with the real options result that states that in case of increased uncertainty the firm invests more but delays investment (see, e.g., Dangl (1999) and Huisman and Kort (2015)). Here it holds that completing the R&D project creates an option to become active in the product market and this option value increases with uncertainty. Therefore, the firm invests more in R&D, which, at the same time, makes the R&D investment more expensive, implying that the firm starts the R&D project at a higher threshold X^*. Also the standard real options result that the value of waiting increases with uncertainty plays a role here. On the other hand, as mentioned before, the fact that the value of the created option to start producing is increasing in σ, gives the firm an incentive to start the R&D project earlier. Here, however, the total effect is such that the firm invests later when uncertainty goes up. For sufficiently low levels of uncertainty such that $\beta > \frac{1}{1-\gamma}$ in Corollary 2.1, the value of the created option to start producing upon completion of the R&D project is too low for the firm to undertake the R&D project at all. Nevertheless, the created option need not always be of low value. For example, a large expected growth rate μ will increase the option value, which, in turn, may offset the loss in option value due to little uncertainty. Consequently, the firm can still undertake the R&D project if there is little to no uncertainty. This is the case in Figure 2.2a in which both R^* and X^* remain positive as σ approaches zero.

We obtain a similar result for R^* with respect to the expected growth rate μ. The firm invests more in R&D in a growing market but invests less in a declining market. In fact, for sufficiently low values of μ, the firm does not undertake the R&D investment at all, irrespective of the values of σ and r. The sign of the derivative of X^* with respect to μ can go both ways because there are two opposing forces at play. The incentive to launch a new product is stronger if μ is large. The firm has two complementary tools to its disposal to bring the expected launch of the product forward: investing more or investing earlier. By investing more in R&D, the firm lowers the expected innovation time. However, investing more in R&D implies that investments are more expensive, and the firm is more inclined to invest a lot in the project if X is large. In other words, the firm can refrain from investing earlier if it knows that the increase in investment size is adequate for bringing the expected timing of the breakthrough forward. In such cases the investment threshold is increasing with μ. If investing more is in itself not sufficient enough, the firm will additionally invest at a smaller threshold level. We observe this when the drive to innovate is particularly high, i.e., for sufficiently large μ together with either a sufficiently large σ, low r, or both. In these cases the investment threshold is decreasing with μ.

With respect to the discount rate r the signs of the derivatives of R^* and X^* can go both ways. A lower discount rate leads to a higher net present value, which, in turn, incentivizes the firm to develop the product. Therefore, the firm will invest more and start its R&D project sooner. As the discount rate increases and the future becomes relatively less important, the firm invests less and delays investment. Eventually the quantity effect, in the sense that the firm invests earlier because it invests less, making the investment less expensive, dominates. This implies that the firm will start its R&D project sooner up to the point that it does not undertake the R&D project at all.



(a) (b) (c)

Figure 2.2: The optimal investment size, R^*, and the optimal investment threshold, X^*, as a function of σ, μ and r. Parameter values are $\sigma = 0.3$, $\mu = 0.06$, $r = 0.1$, $\alpha = 0.2$, $c = 0.2$, and $\gamma = \frac{2}{3}$.

2.1 R&D from a social perspective

Although the firm wants to maximize its producer surplus, a social planner wants to maximize the total surplus. The total surplus, or otherwise referred to as the *total welfare*, comprises the producer and consumer surplus.

The expected total surplus at the moment of launching the product is, for a fixed X and capacity level K, given by[5]

$$TS(X,K) = PS(X,K) + CS(X,K) = \frac{XK(2-\alpha K)}{2(r-\mu)}. \tag{2.13}$$

Maximizing (2.13) with respect to K gives the optimal production capacity of the social planner, which is equal to $K_W^* = \frac{1}{\alpha}$. If we compare this with the optimal production capacity of the firm, $K^* = \frac{1}{2\alpha}$, we conclude that the social planner chooses a capacity level that is twice that of the firm.

Then, by substituting K_W^* in (2.13), we obtain the expected total surplus upon breakthrough:

$$TS(X, K_W^*) = \frac{X}{2\alpha(r-\mu)} = 2\Omega(X),$$

which is twice the producer surplus as given in (2.3). Consequently, the corresponding optimal $R_W^*(X)$ follows from (see (2.5) and (2.6))

$$\Omega(X)\frac{\lambda'(R_W^*(X))(r-\mu)}{(\lambda(R_W^*(X))+r-\mu)^2} = \frac{1}{2}. \tag{2.14}$$

From (2.6) and (2.14) it is clear that, for every X, we have $R_W^*(X) \geq R^*(X)$ because

$$\frac{\partial}{\partial R}\frac{\lambda'(R)}{(\lambda(R)+r-\mu)^2} = \frac{\lambda''(R)(\lambda(R)+r-\mu) - 2(\lambda'(R))^2}{(\lambda(R)+r-\mu)^3} < 0.$$

Hence, for fixed X, the social planner invests more in R&D than the firm. However, given R, the optimal investment trigger $X_W^*(R)$ of the social planner is equal to

$$X_W^*(R) = \left(\frac{\beta}{\beta-1}\right)\frac{2\alpha(r-\mu)R(\lambda(R)+r-\mu)}{\lambda(R)}, \tag{2.15}$$

which, for fixed R, is half the optimal investment trigger of the firm as given in (2.10). From (2.14) and (2.15) it follows that the optimal investment size $R_W^* \equiv R_W^*(X_W^*)$ is implicitly determined by

$$\beta = \frac{\lambda(R_W^*)+r-\mu}{\lambda(R_W^*) + (1-\frac{R_W^*\lambda'(R_W^*)}{\lambda(R_W^*)})(r-\mu)}. \tag{2.16}$$

[5] We refer to the welfare section of Section 2 of Huisman and Kort (2015) for a derivation of the total surplus. Recall that we impose $\delta = 0$, which implies that the social planner launches the product upon breakthrough.

By comparing (2.16) with (2.9) we find that $R^* = R^*_W$, meaning that the firm and the social planner invest the same amount in R&D at their respective investment triggers. In other words, the social planner has a twice as low optimal investment threshold as the firm, but invests the same amount in R&D. In fact, this result is essentially the opposite of Huisman and Kort (2015). They find that the social planner invests at the same time as the firm, but chooses a capacity level that is twice that of the firm. We summarize these results in the following proposition.

Proposition 2.2. *The optimal investment size and optimal investment threshold of the social planner are equal to* $R^*_W = R^*$ (2.9) *and* $X^*_W = \frac{1}{2}X^*$ (2.10), *respectively.*

The disruption to the total surplus as a consequence of the firm's investment decisions is twofold. The first disruption is one on the product market in the sense that the firm chooses an optimal production capacity that is half that of the social planner (Huisman & Kort, 2015). The second, additional, disruption is caused by the technological uncertainty relating to the R&D project. The firm starts its R&D process later than the social planner and therefore, in expectation, it takes longer for the product to become available.

For a proper evaluation, we assume that the firm and the social planner undertake the R&D investment at their respective optimal thresholds, that is, we assume that $X \leq X^*_W = \frac{1}{2}X^*$. The expected total surplus upon initiation of the R&D project, with respect to the policy of the social planner, is equal to

$$\mathbb{E}\left[e^{-rT}TS(X(T), K^*_W) \,|\, X = X^*_W\right] = \frac{X^*_W}{2\alpha(r-\mu)} \frac{\lambda(R^*)}{\lambda(R^*) + r - \mu}. \tag{2.17}$$

Subsequently, we discount (2.17) to the start of the planning period by using the stochastic discount factor.[6] By doing so we obtain the expected total surplus at the start of the planning period with respect to the policy of the social planner, which is given by

$$T_W(X) = \left(\frac{X}{X^*_W}\right)^{\beta} \frac{X^*_W}{2\alpha(r-\mu)} \frac{\lambda(R^*)}{\lambda(R^*) + r - \mu}. \tag{2.18}$$

Similarly, the expected total surplus with respect to the policy of the firm, evaluated at the

[6]The stochastic discount factor is given by $\mathbb{E}[e^{-rT^*}] = (X/X^*_W)^{\beta}$, in which T^* is the random first time the geometric Brownian motion hits X^*_W starting at $X = X(0)$. See, e.g., pages 315-316 of Dixit and Pindyck (1994) for a derivation of the stochastic discount factor.

start of the planning period, is equal to

$$T_F(X) = \left(\frac{X}{X^*}\right)^{\beta} \frac{3X^*}{8\alpha(r-\mu)} \frac{\lambda(R^*)}{\lambda(R^*)+r-\mu} = \left(\frac{1}{2}\right)^{\beta-1} \frac{3}{4} T_W(X). \tag{2.19}$$

The expected total surplus corresponding to the firm's policy can be decomposed as follows. First, apart from the R&D phase, it is equal to 75% of the expected total surplus corresponding to the policy of the social planner. This loss in welfare corresponds to the different choice of the optimal production capacity when the firm launches the product. Indeed, Huisman and Kort (2015) observe that, in the absence of an R&D project, the welfare loss equals 25%. The second loss in welfare is given by $\left(\frac{1}{2}\right)^{\beta-1}$ and corresponds to the different choice in R&D investment timing. In contrast to the first welfare disruption, the second one depends on the state of the market. If β is large, which happens when σ is low, μ is low, and r is large, investing happens relatively early, as Figure 2.2 exemplifies. Then discounting matters so that the firm investing twice as late results in a considerable welfare reduction.

3 Towards the socially-desirable outcome

The previous section learns that there are two disruptions to the total welfare, which are the direct result of installing a too small production capacity, and of initiating the R&D project too late. The government can provide subsidies to nudge the firm towards the socially-desirable investment decision. We assume that the government provides two forms of subsidies to combat both types of welfare disruptions. First, the government provides a subsidy that moves the production capacity of the firm towards that of the social planner, which we call a productive investment subsidy. Second, the government provides a subsidy in the form of a one-time capital injection, which we call an R&D investment subsidy. As we show later, the effect of this subsidy is that the firm will invest in R&D earlier.

The optimal production capacities of the firm and the social planner are $K^* = \frac{1}{2\alpha}$ and $K_W^* = \frac{1}{\alpha}$, respectively. Here, we impose that the firm produces with quantity $K_\theta^* = \frac{1+\theta}{2\alpha}$, in which $\theta \in [0, 1]$. Note that $\theta = 0$ corresponds to the optimal firm capacity size, and that $\theta = 1$ corresponds to the optimal social planner capacity size. The parameter θ reflects the imposed increase in production capacity of the firm, i.e., $\theta = \frac{1}{3}$ means that the firm installs 33% more capacity. At the same time this θ is a measure of the size of the productive investment subsidy. The more the firm has to deviate from its optimal capacity level, the more the subsidy has to make up for. If the firm produces with quantity K_θ^* upon breakthrough, then the expected total surplus becomes

$$TS(X,\theta) = TS(X, K_\theta^*) = \frac{(\theta+1)(3-\theta)X}{8\alpha(r-\mu)}. \tag{3.1}$$

Clearly, the expected total surplus increases if θ increases from zero to one, at which it admits its maximal value.

If the firm produces with quantity K_θ^* upon breakthrough, then the expected producer surplus becomes

$$\Omega(X,\theta) = PS(X, K_\theta^*) = \frac{(\theta+1)(1-\theta)X}{4\alpha(r-\mu)}, \tag{3.2}$$

admitting the maximal value for $\theta = 0$. Consider $\theta > 0$. Then the firm will produce with quantity K_θ^* only if the reduction in producer surplus is compensated by the government. Hence, the productive investment subsidy the government must provide is equal to

$$S_p(X,\theta) = \Omega(X,0) - \Omega(X,\theta) = \frac{\theta^2 X}{4\alpha(r-\mu)}. \tag{3.3}$$

Therefore, given θ, the firm receives a productive investment subsidy of $S_p(X(T),\theta)$ upon breakthrough at the stochastic innovation time T. If $\theta = 0$, the firm receives no subsidy. If $\theta = 1$, the firm's payoff would be zero without a subsidy. Therefore, as confirmed by (3.2) and (3.3), the firm receives its full expected producer surplus as subsidy if $\theta = 1$. The cost of the productive investment subsidy is strictly convex in θ, i.e., the government compensates $\theta^2 \times 100\%$ of the producer surplus if the firm's production capacity is increased by $\theta \times 100\%$. Furthermore, the productive investment subsidy is linearly increasing in the geometric Brownian motion, X. The larger the demand, the more a deviation in capacity has an impact on the firm's payoff, so the more the firm needs to be compensated for over-investing.

Without an R&D investment subsidy the firm starts R&D at X^* with corresponding R&D investment R^*. The government therefore only provides an R&D investment subsidy for $X < X^*$.

From Proposition 2.2 we know that the optimal R&D investment from a social perspective is equal to $R_W^* = R^*$. Thus, the government provides an R&D investment subsidy only on the condition that the firm invests R^*. The firm will not reject this offer because the corresponding project value exceeds the option value to invest without subsidies. Denote the R&D investment subsidy by $S_\ell \in (0, R^*)$, and let $X^*(S_\ell) < X^*$ denote the investment threshold of the firm when receiving an R&D investment subsidy of size S_ℓ. Consequently, the value of the firm is given by (cf. Proposition 2.1)

$$V(X, S_\ell) = \begin{cases} A(S_\ell)X^\beta & \text{if } X < X^*(S_\ell), \\ \dfrac{X}{4\alpha(r-\mu)} \dfrac{\lambda(R^*)}{\lambda(R^*) + r - \mu} - R^* + S_\ell & \text{if } X^*(S_\ell) \leq X < X^*, \\ \dfrac{X}{4\alpha(r-\mu)} \dfrac{\lambda(R^*(X))}{\lambda(R^*(X)) + r - \mu} - R^*(X) & \text{if } X \geq X^*, \end{cases} \tag{3.4}$$

with

$$X^*(S_\ell) = \left(\frac{\beta}{\beta - 1}\right) \frac{4\alpha(r-\mu)(R^* - S_\ell)(\lambda(R^*) + r - \mu)}{\lambda(R^*)}, \tag{3.5}$$

and

$$A(S_\ell) = \frac{(X^*(S_\ell))^{-\beta}(R^* - S_\ell)}{\beta - 1}, \tag{3.6}$$

and in which $R^*(X)$ is implicitly determined by (2.6).

Clearly, from (3.5) it follows that any positive R&D investment subsidy by the government brings the investment timing forward. The exact shift in investment timing depends on the size of the subsidy. For each $X < X^*$, the subsidy triggering immediate investment follows from $X^*(S_\ell) = X$, which, by (3.5), implies that

$$\bar{S}_\ell(X) = \left(\frac{X^* - X}{X^*}\right) R^*. \tag{3.7}$$

The subsidy $\bar{S}_\ell(X)$ is the maximal meaningful R&D investment subsidy the government provides, because it is just enough to trigger immediate investment. In other words, the government provides an R&D investment subsidy of $S_\ell = \bar{S}_\ell(X)$ to ensure that the firm invests at $X < X^*$.

For a proper evaluation, the expected total surplus upon breakthrough $TS(X, \theta)$, given by (3.1), as well as the productive investment subsidy $S_p(X, \theta)$, given by (3.3), will have to be discounted to the present. The expected total surplus at the start of the R&D project, i.e., at $X = X^*(S_\ell)$, is equal to

$$\mathbb{E}\left[e^{-rT} TS(X(T), \theta) | X = X^*(S_\ell)\right] = \frac{(\theta+1)(3-\theta)X^*(S_\ell)}{8\alpha(r-\mu)} \frac{\lambda(R^*)}{\lambda(R^*) + r - \mu}. \tag{3.8}$$

As obtained from (3.1), the first term of (3.8) denotes the expected total surplus upon breakthrough. The second term, being less than one, corrects for the fact that the breakthrough has not been achieved yet. Discounting (3.8) to the present using the stochastic discount factor gives us the expected present total surplus of the R&D project. For $X \leq X^*(S_\ell)$, it is given by

$$T(X, S_\ell, \theta) = \left(\frac{X}{X^*(S_\ell)}\right)^\beta \frac{(\theta+1)(3-\theta)X^*(S_\ell)}{8\alpha(r-\mu)} \frac{\lambda(R^*)}{\lambda(R^*) + r - \mu}. \tag{3.9}$$

Similarly, the expected present value of the productive investment subsidy can be determined. From (3.3) it follows that, for $X \leq X^*(S_\ell)$, it is given by

$$S_p(X, S_\ell, \theta) = \left(\frac{X}{X^*(S_\ell)}\right)^\beta \frac{\theta^2 X^*(S_\ell)}{4\alpha(r-\mu)} \frac{\lambda(R^*)}{\lambda(R^*) + r - \mu}. \tag{3.10}$$

3.1 Welfare-maximizing policy

We let the government implement a welfare-maximizing policy with the aim of attaining the first-best solution. In particular, the government maximizes total surplus on the condition that its own payoff when providing subsidies is never negative. The payoff of the government, being equal to the difference between the increase in total surplus due to the subsidies and the total subsidy expenses, is given by

$$P(X, S_\ell, \theta) = T(X, S_\ell, \theta) - T(X, 0, 0) - S_\ell - S_p(X, S_\ell, \theta). \tag{3.11}$$

The first two terms of (3.11) capture the expected present net gain in total welfare and the last two terms are the R&D investment and productive investment subsidies, respectively. The constraint that (3.11) is non-negative is also referred to as the *budget constraint*.

The government wants to bring the investment timing of the firm forward to some $X \in [X_W^*, X^*)$; to do so, the government provides the maximal R&D investment subsidy at X, which is given by $\bar{S}_\ell(X)$ (see (3.7)). Furthermore, note that for $X \in (0, X_W^*)$ it is not optimal to undertake the R&D investment from a social planner perspective. Therefore, the government refrains from providing the R&D investment subsidy in this case. Let $X = aX^*$ with $a \in [\frac{1}{2}, 1]$ be the value of X at which the government provides the maximal R&D investment subsidy such that the firm immediately invests at this X, in which a smaller a thus corresponds to a larger R&D investment subsidy, and let $\theta \in [0, 1]$. Then, the expected total surplus at $X \leq X_W^*$ is equal to

$$\left(\frac{X}{aX^*}\right)^{\beta} T(aX^*, \bar{S}_\ell(aX^*), \theta) = \left(\frac{1}{2a}\right)^{\beta-1} \frac{(\theta+1)(3-\theta)}{4} T_W(X), \tag{3.12}$$

in which $T_W(X)$ (see (2.18)) is the expected total surplus corresponding to the policy of the social planner. Clearly, one obtains the first-best solution for $a = \frac{1}{2}$ and $\theta = 1$; however, this might not be feasible with respect to the budget constraint. If $X = aX^*$, we obtain from (3.11) that the budget constraint, $P(aX^*, \bar{S}_\ell(aX^*), \theta) \geq 0$, is equivalent to

$$-3a\beta\theta^2 + 2a\beta\theta + \beta(5a - 3a^{\beta}) - 2(a + \beta - 1) \geq 0. \tag{3.13}$$

The government therefore requires to solve the following optimization problem:

$$\begin{aligned} &\max_{(a,\theta)\in[\frac{1}{2},1]\times[0,1]} \left(\frac{1}{2a}\right)^{\beta-1} \frac{(\theta+1)(3-\theta)}{4} \\ &\text{subject to } -3a\beta\theta^2 + 2a\beta\theta + \beta(5a - 3a^{\beta}) - 2(a + \beta - 1) \geq 0. \end{aligned} \tag{3.14}$$

We solve the optimization problem on an ad hoc basis by considering the budget constraint, i.e., the feasible region, in detail. If the government does not provide an R&D investment subsidy, i.e., $S_\ell = 0$ (or $a = 1$), then,

$$P(X, 0, \theta) = \frac{1}{3} T(X, 0, 0)(-3\theta^2 + 2\theta), \tag{3.15}$$

which is non-negative for $\theta \in [0, \frac{2}{3}]$, and strictly negative for $\theta \in (\frac{2}{3}, 1]$. We conclude that a large productive investment subsidy such that $\theta \in (\frac{2}{3}, 1]$ can only be feasible if it is accompanied by an

R&D investment subsidy. Correspondingly, we will consider $\theta \in [0, \frac{2}{3}]$ and $\theta \in [\frac{2}{3}, 1]$ separately.

The following proposition states that the government is always able to bring the investment timing of the firm towards that of the social planner if $\theta \in [0, \frac{2}{3}]$. More specifically, the payoff of providing an R&D investment subsidy is at least the payoff of providing no R&D investment subsidy, that is, at $X = X_W^*$ it holds that

$$P(X_W^*, \bar{S}_\ell(X_W^*), \theta) \geq P(X_W^*, 0, \theta) \geq 0.$$

In fact, for all $X \in [X_W^*, X^*]$ the government is willing to provide the maximal R&D investment subsidy, which means that the budget constraint given in (3.14) is satisfied for all $(a, \theta) \in [\frac{1}{2}, 1] \times [0, \frac{2}{3}]$.

Proposition 3.1. *Let $\theta \in [0, \frac{2}{3}]$. Then, the R&D investment subsidy the government is willing to provide is equal to*

$$S_\ell(X) = \begin{cases} 0 & \text{if } X < X_W^*, \\ \left(\dfrac{X^* - X}{X^*}\right) R^* & \text{if } X_W^* \leq X \leq X^*. \end{cases} \tag{3.16}$$

By plugging $S_\ell(X)$ from (3.16) into the value function (3.4) we obtain the corresponding value function of the firm:

$$V(X) = \begin{cases} \left(\dfrac{X}{X_W^*}\right)^\beta \dfrac{(1/2)R^*}{\beta - 1} & \text{if } X < X_W^*, \\ \dfrac{X}{4\alpha(r-\mu)} \dfrac{\lambda(R^*)}{\lambda(R^*) + r - \mu} - \left(\dfrac{X}{X^*}\right) R^* & \text{if } X_W^* \leq X < X^*, \\ \dfrac{X}{4\alpha(r-\mu)} \dfrac{\lambda(R^*(X))}{\lambda(R^*(X)) + r - \mu} - R^*(X) & \text{if } X \geq X^*, \end{cases}$$

in which $R^*(X)$ is implicitly determined by (2.6). The government brings the investment timing forward in such a way that the firm is indifferent between investing and not investing, thereby shifting the investment threshold of the firm from X^* to X_W^*. The R&D investment subsidy increases the project value in the sense that the firm only needs to incur a fraction of the investment costs. Figure 3.1 shows a typical pattern of the value function of the firm under the R&D investment subsidy according to (3.16). For $X < X_W^*$ the firm holds an option to invest, which is valued on the basis of the fact that the firm expects to receive an R&D investment subsidy of $\bar{S}_\ell(X_W^*)$ at X_W^*. This option value is larger than the option value to invest in absence of a subsidy. There is a kink at X_W^* because that is the threshold after which the firm receives an R&D investment subsidy which is just enough to induce immediate investment. At X^* there is another kink because for $X \geq X^*$ the firm already is enough incentivized by itself to invest in R&D immediately, which implies that no R&D investment subsidy is given.

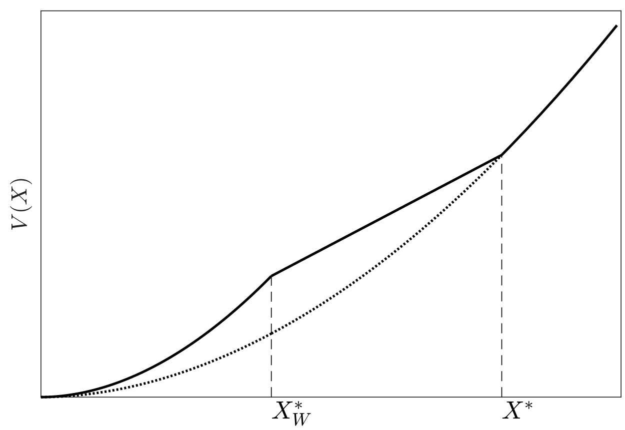


Figure 3.1: A typical pattern of the value function of the firm under the R&D investment subsidy according to (3.16). The dotted line shows the value without subsidy.

By providing the maximal R&D investment subsidy of $\bar{S}_\ell(X)$ at $X = X_W^*$ the government eliminates the loss in welfare as a result of investing too late. Let $X \leq X_W^*$ and let $\theta \in [0, \frac{2}{3}]$. Then, the expected total surplus, evaluated at the start of the planning period, is equal to

$$\frac{(\theta+1)(3-\theta)}{4} T_W(X), \tag{3.17}$$

in which $T_W(X)$ (see (2.18)) is the expected total surplus corresponding to the policy of the social planner. Recall that the government imposes a larger production capacity if θ is larger, which, in turn, results in an increase in total surplus. From (3.17) it follows that the total surplus is largest for $\theta = \frac{2}{3}$, given that $\theta \in [0, \frac{2}{3}]$.

To solve the optimization problem (3.14) we can consequently restrict ourselves to $(a, \theta) \in [\frac{1}{2}, 1] \times [\frac{2}{3}, 1]$. The government provides an R&D investment subsidy in combination with the productive investment subsidy if the government's payoff stays positive, that is, if condition (3.13) is satisfied. Given that the government provides an R&D investment subsidy of $\bar{S}_\ell(X)$ at $X = aX^*$, with $a \in [\frac{1}{2}, 1]$, the government chooses θ to approach one as much as possible subject to the budget constraint because it brings about the largest total surplus. Correspondingly, we have the following proposition.

Proposition 3.2. *Let $X = aX^*$ with $a \in [\frac{1}{2}, 1]$. Then, the corresponding optimal productive investment subsidy is equal to*

$$S_p(X) = \frac{(\theta^*_\beta(a))^2 X}{4\alpha(r-\mu)},$$

in which

$$\theta^*_\beta(a) = \min\left\{1, \frac{1}{3} + \sqrt{\frac{16}{9} - a^{\beta-1} - \frac{2(a+\beta-1)}{3a\beta}}\right\} \geq \frac{2}{3}. \tag{3.18}$$

Hence, the optimal solution should be of the form $(a^*_\beta, \theta^*_\beta) = (a^*_\beta, \theta^*_\beta(a^*_\beta))$. By substituting $\theta^*_\beta(a)$ (see (3.18)) into the value function of the optimization problem (3.14), we obtain

$$\left(\frac{1}{2a}\right)^{\beta-1} \frac{(\theta^*_\beta(a)+1)(3-\theta^*_\beta(a))}{4},$$

which has a strictly negative derivative for $a \in [\frac{1}{2}, 1]$. This implies that the optimal solution of the optimization problem (3.14) is given by $(a^*_\beta, \theta^*_\beta) = (\frac{1}{2}, \theta^*_\beta(\frac{1}{2}))$. In this way, the following proposition presents the optimal subsidy policy of the government under the welfare-maximizing policy.

Proposition 3.3. *Under the welfare-maximizing policy of the government, it is optimal to provide an R&D investment subsidy of $S^*_\ell = \frac{1}{2}R^*$ at $X = X^*_W$, and to provide a productive investment subsidy of*

$$S^*_p(X) = \frac{(\theta^*_\beta(\frac{1}{2}))^2 X}{4\alpha(r-\mu)}, \tag{3.19}$$

at the random innovation time T with $X = X(T)$, in which $\theta^*_\beta(\frac{1}{2})$ is given by (3.18).

It is thus optimal for the government to subsidize the firm such that the firm will undertake the R&D investment at the socially-optimal investment time. However, as Figure 3.2a exemplifies, providing a productive investment subsidy such that the firm acquires the socially-optimal production capacity is not always feasible in light of the constraint on the government's budget. Let $X \leq X_W^*$. Under the welfare-maximizing policy of the government, the expected total surplus, evaluated at the start of the planning period, is equal to

$$T^*(X) = \frac{(\theta_\beta^*(\frac{1}{2}) + 1)(3 - \theta_\beta^*(\frac{1}{2}))}{4} T_W(X), \tag{3.20}$$

in which $T_W(X)$ (see (2.18)) is the expected total surplus corresponding to the first-best policy.

Figure 3.2 shows that the first-best solution is only attainable for sufficiently large β. A large β corresponds to a situation with little uncertainty, a small drift rate, a high discount rate, or a combination of the three. In such a situation, the R&D project and the corresponding investment in production capacity have a relatively low net present value. Correspondingly, the government requires less funds to compensate the firm for the loss in payoff if it undertakes investments that are optimal from a social perspective.

The case of a small β corresponds to a situation of highly uncertain demand, a large drift rate, a low discount rate, or a combination of the three. Then, the project's net present value is large, and, as a result, the government's productive investment subsidy needs to be large to compensate the firm for losing the large producer surplus when it invests according to the socially-optimal scheme. Consequently, the budget constraint, requiring that the welfare gain should exceed the corresponding subsidy expenses, will be violated in such a case and the first-best solution cannot be reached.

Figure 3.2b shows that in our numerical example the loss in welfare is relatively small despite the fact that the first-best solution ($\theta_\beta^*(\frac{1}{2}) = 1$) cannot be obtained. In the worst case, thus when $\theta_\beta^*(\frac{1}{2}) = \frac{2}{3}$, the total welfare is 97.22% of the first-best solution. Moreover, if the firm installs 90% more capacity instead of 100% more, as the first-best solution demands, the loss in total welfare is only 0.25%.

In Figure 3.2b we additionally observe that, as β increases, the relative total welfare under the policy of the firm in absence of subsidies decreases whereas the relative total welfare in the presence of subsidies increases. The reason that the relative total welfare under the policy of the unsubsidized firm decreases so rapidly is due to the fact that the firm starts R&D too late.

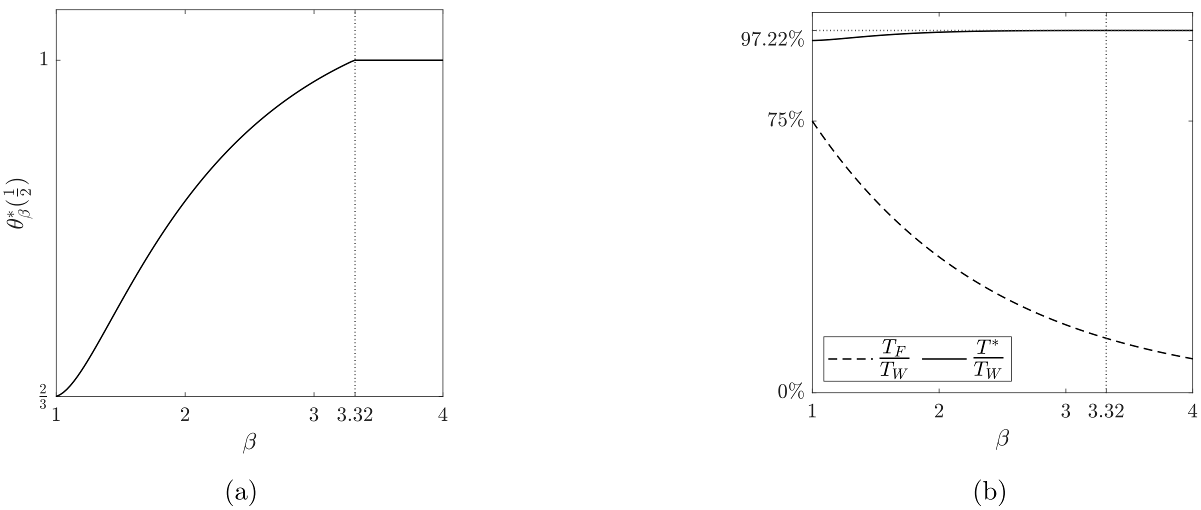


Figure 3.2: Panel (a) depicts the optimal increase in production capacity, $\theta^*_\beta(\frac{1}{2})$. Panel (b) visualizes the relative total welfare in absence of subsidies, T_F/T_W (see (2.19)), and the relative total welfare under the optimal subsidy policy, T^*/T_W (see (3.20)). At $\beta \geq 3.32$ the first-best solution is reached, meaning that $T^*/T_W \times 100\% = 100\%$.

4 R&D under an isoelastic demand function

After reading the previous section, one may wonder in how far our results depend on our demand function (2.2) being linear. To investigate, in this section we replace the inverse demand function (2.2) by

$$P(t) = X(t)K^{-\nu},$$

in which $\nu \in (0, 1)$ is the price elasticity of demand parameter such that this elasticity equals $-\frac{1}{\nu}$, meaning that, all else constant, a 1% increase in price results in a $\frac{1}{\nu}$% decrease in demand.[7] We assume that upon launching the product the firm incurs an investment cost equal to $\delta_0 + \delta_1 K$. However, we require $\delta_0 = 0$ to ensure that it is optimal for the firm to launch the product upon breakthrough (see Huisman and Kort (2015)). We additionally impose that $\nu\beta > 1$. If this condition is not satisfied, waiting indefinitely is preferred over undertaking the R&D project (see, e.g., p. 365 of Dixit and Pindyck (1994)). The derivations and proofs of the results in this section can be found in Appendix B.

[7]In our case with $\nu \in (0, 1)$ it implies that the demand for the new product is elastic.

The expected value of the firm at the moment it launches the product at X with capacity level K, i.e., the expected producer surplus, is equal to

$$PS(X,K) = \frac{XK^{1-\nu}}{r-\mu} - \delta_1 K; \tag{4.1}$$

the expected consumer surplus is equal to

$$CS(X,K) = \frac{\nu}{1-\nu}\frac{XK^{1-\nu}}{r-\mu}; \tag{4.2}$$

and the expected total surplus satisfies

$$TS(X,K) = \left(1 + \frac{\nu}{1-\nu}\right)\frac{XK^{1-\nu}}{r-\mu} - \delta_1 K. \tag{4.3}$$

From the above expressions we conclude that within the total surplus more weight is put on the producer surplus if $\nu \in (0, \frac{1}{2})$, and, conversely, more weight is put on the consumer surplus if $\nu \in (\frac{1}{2}, 1)$. Indeed, when ν is large, that is, when the price elasticity of demand is low, the decline in quantity is limited when the price increases. Therefore, firms intend to ask high prices for their products and consumers need to be protected against it. Correspondingly, the decisions of the social planner should be more similar to the decisions of the firm for $\nu \in (0, \frac{1}{2})$, whereas they should be more dissimilar for $\nu \in (\frac{1}{2}, 1)$.

Like in the case with linear demand the firm and social planner install different production capacities, which are in this case given by

$$K^*(X) = \left(\frac{(1-\nu)X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}}, \text{ and } K_W^*(X) = \left(\frac{X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}}, \tag{4.4}$$

respectively. The firm installs too little capacity from a welfare perspective because $K_W^*(X) = (1-\nu)^{-1/\nu}K^*(X) > K^*(X)$ for each X. This is especially true for large ν, confirming that in case the price elasticity of demand is low, thus when ν is large, the firm aims for high output prices, thereby acquiring a small production capacity.

Analogous to Proposition 2.2 in the linear demand case, the social planner starts R&D earlier than the firm, and the firm and the social planner invest the same amount in R&D at their respective investment triggers.

Proposition 4.1. *Let R^* and X^* be the optimal investment size and the optimal investment threshold of the firm, respectively. Then, the optimal investment size and optimal investment threshold of the social planner are equal to $R_W^* = R^*$ and $X_W^* = (1-\nu)X^*$, respectively.*

We observe two welfare disruptions. First, despite the fact that the firm and the social planner install exactly the same capacity at their respective investment triggers, i.e., $K^*(X^*) = K_W^*(X_W^*)$, the firm still installs too little capacity from a welfare perspective — after all, it is socially optimal to install more capacity at the level $X = X^*$, i.e., $K_W^*(X^*) > K^*(X^*)$. Second, the firm starts R&D too late.

Let $X \leq X_W^* = (1-\nu)X^*$. The expected total welfare at the start of the planning period with respect to the policy of the firm is given by

$$T_F(X) = \big[\underbrace{(1-\nu)^{\beta-1}(1-\nu+\nu\beta)}_{(1)} - \underbrace{(1-\nu)^{\beta-1}(\nu\beta-1)}_{(2)}\big]T_W(X), \tag{4.5}$$

in which $T_W(X)$ is the expected present total welfare under the policy of the social planner. The first term (1), being less than one, captures the welfare loss from installing too little capacity. Indeed, in the absence of an R&D project, Huisman and Kort (2015) observe the same welfare loss. The second term (2) captures the additional loss in welfare as a result of the different choice in R&D investment timing. Expression (4.5) additionally shows that the relative total welfare, $T_F(X)/T_W(X)$, decreases as ν increases. This is in line with our discussion before, namely that the decisions of the firm should be more dissimilar to the decisions of the social planner as ν increases, thereby resulting in an increase in the relative total welfare loss.

The government imposes that the firm installs a production capacity equal to

$$K_\theta^*(X) = \left(1 + \frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}} K^*(X),$$

in which $\theta \in [0,1]$ again is the indicator for the productive investment subsidy. If $\theta = 0$, the capacity corresponds to the optimal firm capacity size; if $\theta = 1$, the capacity corresponds to the optimal social planner capacity size.

Producing with $K_\theta^*(X)$ is sub-optimal for the firm if $\theta > 0$. The resulting loss in expected producer surplus must therefore be compensated by the government in the form of the productive investment subsidy, the amount of which is given by

$$S_p(X,\theta) = \Omega(X,0) - \Omega(X,\theta) = \Omega(X,0)\left[1 - (1-\theta)\left(1 + \frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}\right],$$

in which $\Omega(X,\theta)$ is the expected producer surplus if the firm installs a production capacity of $K_\theta^*(X)$. Like in the linear demand case, the firm's profit equals zero if $\theta = 1$. Therefore, the government needs to compensate the full expected producer surplus for $\theta = 1$.

The R&D investment subsidy triggering immediate investment is given by

$$\bar{S}_\ell(X) = \left(1 - \left(\frac{X}{X^*}\right)^{\frac{1}{\nu}}\right) R^*, \tag{4.6}$$

in which X^* is the optimal investment trigger in absence of an R&D investment subsidy. If the government provides the maximal R&D investment subsidy to induce immediate investment at X, the firm will invest $R_W^* = R^*$.

Recall that the payoff function of the government is given by (see (3.11))

$$P(X, S_\ell, \theta) = T(X, S_\ell, \theta) - T(X, 0, 0) - S_\ell - S_p(X, S_\ell, \theta).$$

Let $X = aX^*$ with $a \in [1-\nu, 1]$ be the value of X at which the government provides the maximal R&D investment subsidy given by (4.6), and let $\theta \in [0,1]$. Then, the expected total surplus at $X \leq X_W^*$ is equal to[8]

$$\left(\frac{1-\nu}{a}\right)^{\beta} a^{\frac{1}{\nu}} \left(\frac{1}{1-\nu} + (1-\theta)\right)\left(1 + \frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1} T_W(X). \tag{4.7}$$

The government maximizes the expression in (4.7) with respect to $(a,\theta) \in [1-\nu, 1] \times [0,1]$ subject to the budget constraint, $P(aX^*, \bar{S}_\ell(aX^*), \theta) \geq 0$.

Now, consider $\theta = 1$ at which point the cost of the productive investment subsidy equals the

[8]It is remarkable that for $\nu = \frac{1}{2}$, we obtain the $(1/2a)^{\beta-2}((\theta+1)(3-\theta)/4)T_W(X)$, in which $\beta > \frac{1}{\nu} = 2$. This expression is alike the expression in the linear demand case as given in (3.12). In the linear demand case the firm produces with quantity $K_\theta^*(X) = (1+\theta)K^*(X)$, whereas in the isoelastic demand case with $\nu = \frac{1}{2}$ the firm produces with quantity $K_\theta^*(X)^\nu = (1+\theta)K^*(X)^\nu$; furthermore, if $\nu = \frac{1}{2}$, then $X_W^* = \frac{1}{2}X^*$ and the productive investment subsidy is equal to $\Omega(X,0)\theta^2$, which are the same expressions as we found in the linear demand case, see Proposition 2.2 and (3.3), respectively.

expected producer surplus. If the government provides no R&D investment subsidy for $\nu \in (0, \frac{1}{2})$, the productive investment subsidy results in a negative payoff for the government, which is not the case for $\nu \in [\frac{1}{2}, 1)$. The reason being is that for $\nu \in [\frac{1}{2}, 1)$ more weight is put on the consumer surplus in the total surplus. Hence, the increase in expected consumer surplus as a result of producing more is sufficient to offset the incurred cost of the productive investment subsidy as well as the decrease in producer surplus. In particular, in addition to providing the maximal productive investment subsidy, the government can bring the investment timing of the firm forward such that the first-best solution is always obtained.

Proposition 4.2. *The first-best solution is always attainable if* $\nu \in [\frac{1}{2}, 1)$.

On the other hand, the government may not be able to attain the first-best solution for $\nu \in (0, \frac{1}{2})$ because its payoff is negative when providing the maximal productive investment subsidy in combination with no R&D investment subsidy. And, because less weight is put on the consumer surplus in the expression for the total surplus, providing an R&D investment subsidy may not always be enough to offset this negative payoff.

Let $\nu \in (0, \frac{1}{2})$. Then, like in Proposition 3.1, the government can always bring the investment timing forward as long as the welfare gain due to the productive investment subsidy exceeds the productive investment subsidy itself. Here, θ_ν denotes the value of θ for which the government breaks even on the productive investment subsidy — that is, if $\theta \in [0, \theta_\nu]$ the government can afford the productive investment subsidy, which is not the case if $\theta \in (\theta_\nu, 1]$. In other words, the following proposition states that the budget constraint, $P(aX^*, \bar{S}_\ell(aX^*), \theta) \geq 0$, is always satisfied for $(a, \theta) \in [1 - \nu, 1] \times [0, \theta_\nu]$.

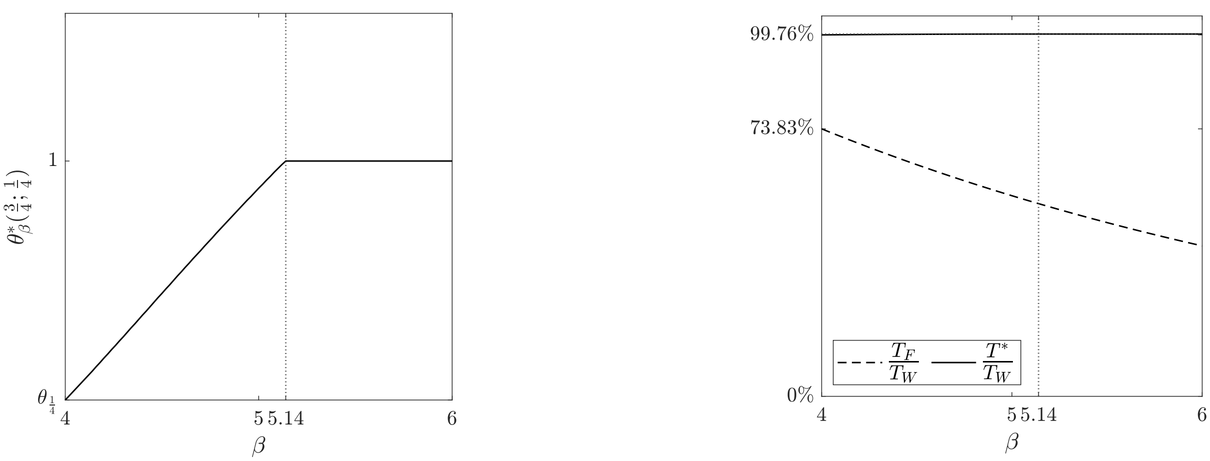
Proposition 4.3. *Let* $\nu \in (0, \frac{1}{2})$ *and let* $\theta \in [0, \theta_\nu]$. *Then, the R&D investment subsidy the government is willing to provide is equal to*

$$S_\ell(X) = \begin{cases} 0 & \text{if } X < X_W^*, \\ \left(1 - \left(\dfrac{X}{X^*}\right)^{\frac{1}{\nu}}\right) R^* & \text{if } X_W^* \leq X \leq X^*. \end{cases} \tag{4.8}$$

To solve the optimization problem for $\nu \in (0, \frac{1}{2})$ we can therefore restrict ourselves to $(a, \theta) \in [1 - \nu, 1] \times [\theta_\nu, 1]$. Given $a \in [1 - \nu, 1]$, it is optimal to choose $\theta \in [\theta_\nu, 1]$ as large as possible without violating the budget constraint. We denote this value of θ by $\theta^*_\beta(a;\nu)$. Just like the linear demand case, it is optimal for the government to provide the maximal R&D investment subsidy at $X = X^*_W$, and, in light of the budget constraint, to provide the productive investment accordingly. By doing so, the expected total surplus under the welfare-maximizing policy of the government, evaluated at $X \le X^*_W$, is equal to

$$T^*(X) = (1-\nu)^{\frac{1}{\nu}} \left(\frac{1}{1-\nu} + (1-\theta^*_\beta(1-\nu;\nu)) \right) \left(1 + \frac{\nu}{1-\nu} \theta^*_\beta(1-\nu;\nu) \right)^{\frac{1}{\nu}-1} T_W(X). \quad (4.9)$$

Figure 4.1 visualizes the situation in case $\nu = \frac{1}{4}$. The figure is qualitatively similar to Figure 3.2 in the linear demand case. Therefore, for its interpretation we refer to the corresponding text in Section 3.1.



(a) (b)

Figure 4.1: Panel (a) depicts the optimal increase in production capacity, $\theta^*_\beta(\frac{3}{4};\frac{1}{4})$. Panel (b) visualizes the relative total welfare in absence of subsidies, T_F/T_W (see (4.5)), and the relative total welfare under the optimal subsidy policy, T^*/T_W (see (4.9)). Here, $\theta_{\frac{1}{4}} = 0.9191$. At $\beta \ge 5.14$ the first-best solution is reached, meaning that $T^*/T_W \times 100\% = 100\%$.

5 Electric vehicle market

In Sections 3 and 4 we have seen that subsidies are an effective tool to increase total welfare in society. In fact, even if the first-best solution is not attainable, a substantial percentage of the welfare loss can be recovered. In this section we consider our model from a practical perspective.

In light of the goal of 100% zero emission cars and vans as outlined in the Introduction, we, for illustrative purposes, consider our model with respect to both the European and American electric vehicle market. We depart from the assumption that the demand for electric vehicles is isoelastic. Literature on the price elasticity of demand for vehicles indicates that demand for vehicles is price elastic, which corresponds to $\nu \in (0, 1)$ in our model (Fridstrøm and Østli (2021), Copeland, Dunn, and Hall (2011), Goldberg (1995), Wetzel and Hoffer (1982)). We put particular emphasis on results obtained by Fridstrøm and Østli (2021) as they compute price elasticities of demand for different types of electric vehicles. Fridstrøm and Østli (2021) show that price elasticities of demand for battery and plug-in hybrid electric vehicles are equal to -0.99 and -1.72, respectively. Because they additionally note that these estimates are likely to be underestimations, we deem that it is appropriate to assume that price elasticities of electric vehicles lie between -1 and -2. We therefore let $\nu \in [\frac{1}{2}, 1)$ and estimate the corresponding drift rate parameter, μ, and volatility parameter, σ, of the geometric Brownian motion. The estimation is based on monthly price and sales data.[9] Table 5.1 summarizes our findings for $\nu = \frac{1}{2}$ and $\nu = 1$. The estimations of μ and σ with respect to $\nu \in (\frac{1}{2}, 1)$ lie in between the values for $\hat{\mu}$ and $\hat{\sigma}$ provided in Table 5.1. We take 2010 as the starting period because that is the year Nissan introduced the world's first electric vehicle for the mass market, namely the Nissan LEAF (Nissan, 2020).

		$\nu = 0.5$		$\nu = 1$	
	Time period	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
European Union	2010-2017	-0.0003	0.0146	0.0005	0.0281
United States	2010-2019	-0.0019	0.0186	-0.0024	0.0370

Table 5.1: Estimation results of the trend and volatility parameters of the geometric Brownian motion given the two relevant boundary values of ν. See Appendix C for more details on the estimation procedure.

We require $r > \frac{1}{\nu}\left(\mu + \frac{1-\nu}{2\nu}\sigma^2\right)$ to guarantee that expected values are finite. Given this requirement, the results do not turn out to be sensitive to variations in the discount rate. Therefore, we set the discount rate, r, equal to some reasonable appropriate value, say 0.05.

In the previous section we concluded that, from a theoretical point of view, the relative loss in total welfare increases when the price elasticity of demand decreases, i.e., when ν increases. This is confirmed here, as Figure 5.1 exemplifies.

[9] See Appendix C for more details.

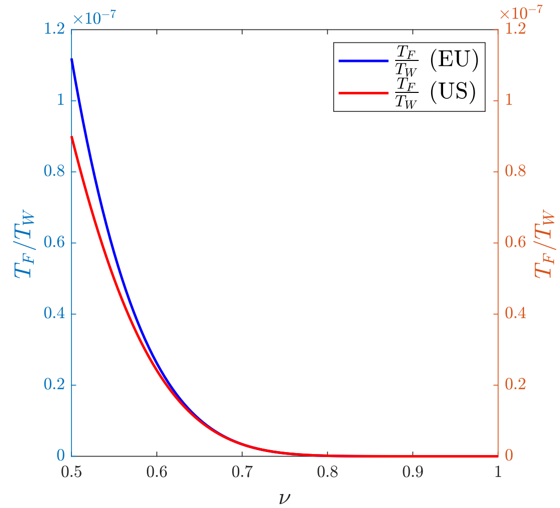


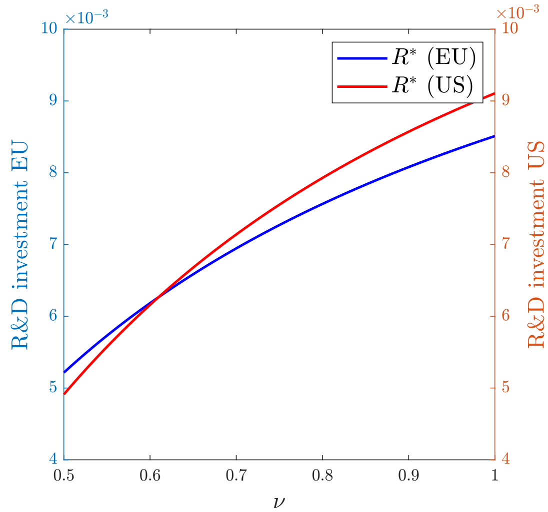
Figure 5.1: The total surplus under the policy of the firm, T_F, relative to the total surplus under the first-best policy, T_W, as a function of ν (see (4.5)).

Figure 5.1 additionally shows that, for each $\nu \in [\frac{1}{2}, 1)$, there is a considerable welfare loss. The reason is the negligible drift rate and the low volatility of the price, see Table 5.1, which give rise to a large value of β, and according to Figure 4.1b, then the welfare loss is large. Our model prescribes that in these instances subsidies are a particularly effective tool in recovering welfare. Indeed, as Proposition 4.2 states, the first-best solution is always attainable if $\nu \in [\frac{1}{2}, 1)$. In this sense our model corroborates the current way governments pave the way for electric vehicles. On the one hand, governments provide incentives on the consumer side in the form of tax benefits or purchase incentives (ACEA, 2021). By encouraging consumers to buy electric vehicles, and thus inducing car manufacturers to produce more, governments combat the welfare loss on the production side. On the other hand, governments provide incentives on the producer side by giving car manufacturers subsidies, tax breaks or loans. For example, in 2014 the U.S. State of Nevada provided electric car manufacturer Tesla 1.3 billion US dollars in tax breaks and other incentives for the construction of a lithium-ion battery factory, “Tesla Gigafactory 1” (Chereb, 2014). More recently, in 2021, twelve EU Member States agreed to free up 2.9 billion euros for research and development in batteries for electric vehicles among other things. A total of 42 companies, such as BMW, Fiat, Northvolt and Tesla, will participate in this European Battery Innovation project (European Commission, 2021). As the previous two instances demonstrate, governmental support on the producer side is aimed at accelerating investment in R&D projects, thereby reducing the welfare loss as a result of starting the R&D project too late.

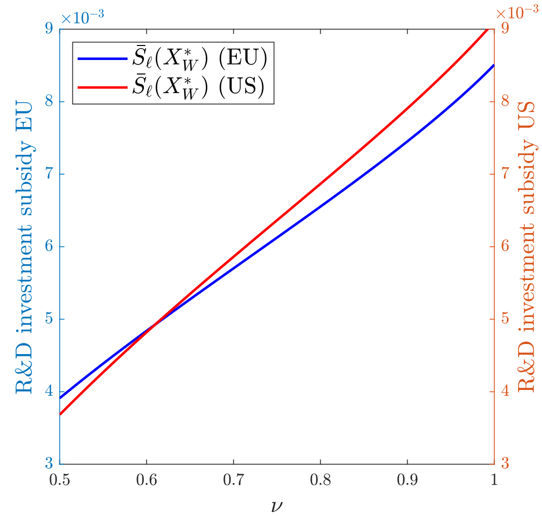
Now suppose that the arrival rate of the R&D breakthrough is given by $\lambda(R) = cR^{\gamma}$ with $c > 0$ and $\gamma \in (0, 1)$. Here, γ reflects the efficiency of the technological progress (see also Section 2), and is required to be sufficiently large in order for the firm and social planner to undertake the R&D project. We set $c = 0.20$ and $\gamma = 0.95$. The corresponding expressions for the optimal investment size, R^*, and the optimal investment threshold, X^*, are given in Appendix C.

Figure 5.2a shows the optimal R&D investment size, R^*, as a function of ν. As per Proposition 4.1, the optimal R&D investment size is the same for the firm and the social planner at their respective investment thresholds. Figure 5.2a shows that the optimal R&D investment size is increasing in ν. If the price elasticity of demand is low, i.e., when ν is large, one exerts more influence on the price of the new product in the sense that a change in the price has less effect on demand. Therefore, it is more attractive to invest more in R&D.

The first-best solution is always attainable for $\nu \in [\frac{1}{2}, 1)$, and therefore the government will provide an R&D investment subsidy such that the firm starts its R&D project at $X = X_W^*$. In particular, the R&D investment subsidy the government provides is given by $\bar{S}_\ell(X_W^*) = (1 - (1 - \nu)^{\frac{1}{\nu}})R^*$. Correspondingly, the investment cost of the firm is reduced to $(1 - \nu)^{\frac{1}{\nu}} R^*$. Figure 5.2b shows that the government is required to pay a larger R&D investment subsidy as ν increases. There are two reasons for this. First, under a higher value of ν there is a larger difference in welfare between the first-best and the firm solution. This welfare difference enlarges the upper bound of possible subsidy expenses, because condition (3.11) prescribes that these subsidy expenses should not exceed the resulting gain in welfare. Second, as ν increases, it is optimal for the firm and the social planner to invest more in R&D, as Figure 5.2a illustrates, and according to (4.6) this results in a larger R&D investment subsidy.



(a)



(b)

Figure 5.2: The optimal R&D investment size, R^*, and the optimal R&D investment subsidy provided by the government, $\bar{S}_\ell(X_W^*)$, as a function of ν. Parameter values are $r = 0.05$, and $\lambda(R) = cR^\gamma$ with $c = 0.20$ and $\gamma = 0.95$.

Figure 5.3 shows the optimal investment thresholds for the firm and the social planner. In general, for fixed ν, the investment thresholds are increasing in the cost of one unit of production capacity, δ_1. If it is large, producing is more expensive and therefore it becomes more attractive to delay R&D. Moreover, the installed production capacity is smaller if δ_1 is larger. The effect the elasticity parameter, ν, has on the investment threshold depends on the cost of one unit of capacity. We obtain a graph like the one in Figure 5.3a if the cost of one unit of production capacity, δ_1, is low, whereas we obtain a graph like the one in Figure 5.3b if the cost of one unit of production capacity is high.

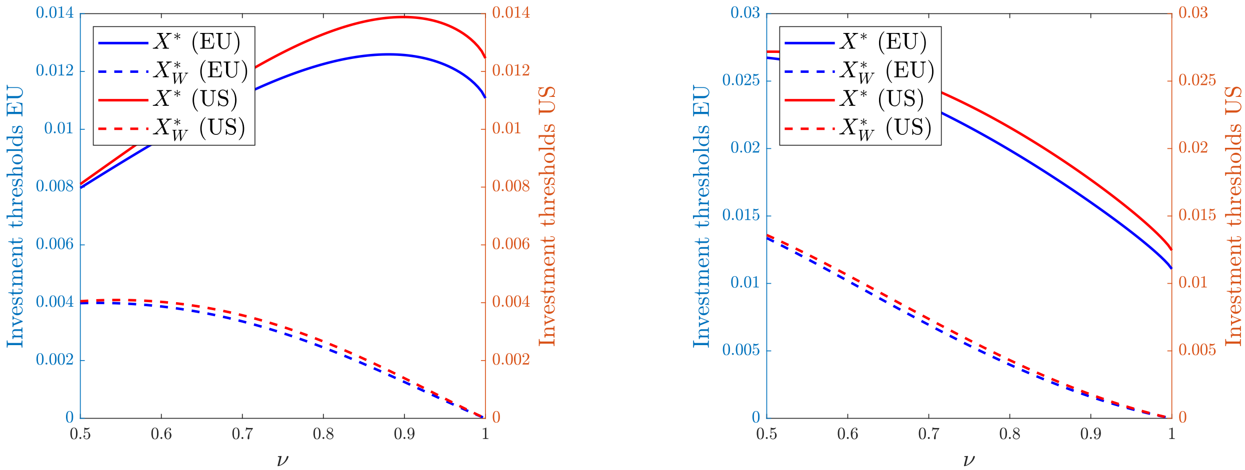
As far as the firm is concerned, there are three effects on the R&D investment threshold with respect to ν. First, as Figure 5.2a shows, the optimal investment size R^* increases as ν increases, which makes the investment more expensive so that one invests later and X^* goes up. Notice that this effect diminishes as ν increases because the increase in R^* slows down as ν increases. Second, as mentioned before, if the price elasticity of demand is low, one exerts more influence on the price of the new product, which makes R&D more attractive. This not only implies that the firm invests more in R&D, but also starts R&D earlier, i.e., the threshold is lower as ν increases. Third, when ν increases also σ increases, see Table 5.1, and the value of the option to produce increases as well.

Upon completion of the R&D project this option is created, which makes the R&D project more attractive and therefore the investment threshold decreases.

In case of the social planner these effects also hold, but in addition there is a fourth effect, coming from the equality $X_W^* = (1-\nu)X^*$. Hence, according to this effect the social planner threshold X_W^* is decreasing in ν. Figure 5.3 clearly shows that for this reason X_W^* is more inclined to decrease in ν than X^*.

Another conclusion we can draw from Figure 5.3 is that the investment thresholds for the firm and the social planner are decreasing to a larger extent with ν if productive investment costs are larger, i.e., if δ_1 is larger. Looking at the different effects, we conclude that only the first effect makes that the threshold is increasing in ν. However, this effect is not influenced by δ_1, because the optimal R&D investment size does not depend on δ_1. All the other negative effects are scaled up by the threshold size, which is obviously larger if productive investments are more expensive.

As we have argued before, the relative welfare loss increases as ν increases. Figure 5.3 seems to indicate the contrary because we observe that the absolute gap between the investment threshold of the firm and social planner need not necessarily increase as ν increases. What is missing, however, is the element of discounting. If it is optimal to invest relatively early from a social perspective, which happens when ν is large, discounting matters so that we observe a larger relative welfare loss. As a matter of fact, as ν approaches one, the social planner wants to start R&D immediately.



(a)

(b)

Figure 5.3: The optimal investment thresholds for the firm, X^*, and the social planner, X_W^*, as a function of ν. Parameter values are $r = 0.05$, $\lambda(R) = cR^\gamma$ with $c = 0.20$ and $\gamma = 0.95$, $\delta_1 = 0.04$ for panel (a) and $\delta_1 = 0.45$ for panel (b).

The US market is more volatile but the drift rates for both markets are roughly the same, see Table 5.1. By comparing the US investment policy with the one of the European Union, we can thus establish the effect of demand uncertainty. From Figures 5.2 and 5.3 we obtain that under higher demand uncertainty one invests more but delays R&D investment. This confirms what we already found in the linear investment scenario. However, note that for small ν the US investment policy prescribes to invest less in R&D, as seen in Figure 5.2. This is to be expected if one takes into account that, for small ν, the expected growth rate μ is smaller in the US, whereas the demand uncertainties are closer together, see Table 5.1. Again, this confirms what we have found in the linear case, namely that one invests more in R&D if μ is larger.

6 Concluding remarks

This article analyzes the interplay between a monopolist and a social planner in research and development under technological and market uncertainty. We employ a real options framework to answer the questions of (i) when to invest in R&D, (ii) how much to invest in R&D, and (iii) how much production capacity to install after the R&D breakthrough has been accomplished. A monopolist and a social planner have different incentives, meaning that their decisions do not align. The welfare loss resulting from the monopolist investing too late and installing a too small production capacity can be reduced by implementing an R&D investment subsidy and a productive investment subsidy, respectively. The productive investment subsidy is determined such that it exactly covers the loss in payoff the firm faces when investing as a social planner instead of applying its own profit-maximizing investment policy. We impose that the subsidy policy is restricted by taking into account that the total subsidy expenses cannot exceed the resulting net increase in total surplus. It depends on the market characteristics whether a subsidy policy that falls within the feasible region of this constraint, could enable the firm to invest just like a social planner would do. In fact, we find that a first-best solution can only be reached if the demand situation is relatively stable, i.e., when demand uncertainty as well as demand trend are limited, or when the price elasticity of demand is low. In such a situation the producer surplus is small so that expenses on the productive investment subsidy needed to cover the loss in the firm's payoff, are limited. However, when demand uncertainty is large, the market is expected to grow fast, and the discount rate is small, then the producer surplus is large and the productive investment subsidy needed to compensate for the loss in the firm's payoff is too large to satisfy the budget constraint.

We refrained from overcomplicating our model so as to keep our model analytically tractable. There are nonetheless natural ways of extending our model. A straightforward way is to include running costs that the firm pays for the duration of the development phase. Such a cost makes the R&D project less attractive and as a consequence the firm will both delay investment and invest less. It is therefore interesting to investigate how this affects welfare and subsidization by the government, especially if one takes into account that in the model without running costs the firm and social planner invest the same in R&D at their respective investment triggers.

Another natural extension of our work is to introduce competition in R&D. In competitive environments of R&D involving multiple firms, it may happen that firms invest too much in R&D on the aggregate level than what is socially desirable (Dasgupta & Stiglitz, 1980; Dixit, 1988; Loury, 1979). These articles also assume the same positive relationship between R&D investment and the arrival rate of the underlying Poisson process.

Furthermore, one could consider alternative stochastic processes such as a mean-reverting process, or a combination of a geometric Brownian motion and a Poisson process. Not just out of robustness sake, but also because it might better represent some markets such as the electric vehicle market.

Finally, in our model we made the assumption that the firm immediately launches the product when it is available. In the linear demand case this is optimal if the cost of one unit of capacity is zero; in the isoleastic demand case this is optimal if the fixed investment cost of launching the product is zero (Huisman & Kort, 2015). Dropping this assumption complicates the analysis considerably because it introduces an additional investment trigger, namely the product launch investment trigger. Consequently, as the duration of the R&D phase is random, there may or may not be a waiting period before launching the product. Despite this complication, one may still extract interesting results from a numerical analysis, which can subsequently be compared with the analytical results we obtain.

Appendix A

Proof of Proposition 2.1. Let X^* be the threshold whereupon the firm undertakes an irreversible investment in R&D. We distinguish between two regions on the basis of X^*, namely the waiting region and the R&D investment region. Specifically, in the waiting region it is optimal to wait, whereas in the R&D investment region it is optimal to invest.

Waiting region. Let $X < X^*$. The value of the option to invest is given by (see, e.g., Dixit and Pindyck (1994))

$$F(X) = AX^{\beta}, \tag{A.1}$$

in which A is some constant, which will be determined later on, and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{A.2}$$

is the positive root of the quadratic equation

$$Q(\beta) = \frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0.$$

It holds that $\beta > 1$ because $Q(\beta)$ is a strictly convex function with $Q(0) = -r < 0$ and $Q(1) = \mu - r < 0$.

R&D investment region. Let $X \geq X^*$. The value of the firm upon breakthrough at X is given by (see (2.3))

$$\Omega(X) = \frac{X}{4\alpha(r-\mu)}. \tag{A.3}$$

Let R denote an arbitrary, but non-negative, irreversible investment in R&D. Let $G(X, R)$ denote the expected value of the firm at the moment of investment. Then,

$$\begin{aligned}
G(X,R) &= \mathbb{E}\left[e^{-rT}\Omega(X(T)) - R \,|\, X(0)=X\right] \\
&= \frac{1}{4\alpha(r-\mu)}\mathbb{E}\left[\int_0^\infty \lambda(R)e^{-\lambda(R)t}e^{-rt}X(t)dt \,\middle|\, X(0)=X\right] - R \\
&= \frac{1}{4\alpha(r-\mu)}\int_0^\infty \lambda(R)e^{-\lambda(R)t}e^{-rt}\mathbb{E}\left[X(t)|X(0)=X\right]dt - R \\
&= \frac{1}{4\alpha(r-\mu)}\int_0^\infty \lambda(R)e^{-\lambda(R)t}e^{-rt}Xe^{\mu t}dt - R \\
&= \frac{X}{4\alpha(r-\mu)}\frac{\lambda(R)}{\lambda(R)+r-\mu} - R.
\end{aligned}$$

The second equality follows from $T \sim \text{EXP}(\lambda(R))$ and (A.3). Hence,

$$G(X,R) = \frac{X}{4\alpha(r-\mu)}\frac{\lambda(R)}{\lambda(R)+r-\mu} - R.$$

We will now determine the optimal investment size as well as the optimal investment timing. The second partial derivative of $G(X,R)$ with respect to R is given by

$$\frac{\partial^2 G(X,R)}{\partial R^2} = \frac{X}{4\alpha}\frac{\lambda''(R)(\lambda(R)+r-\mu) - 2(\lambda'(R))^2}{(\lambda(R)+r-\mu)^3} < 0. \tag{A.4}$$

The inequality follows from the fact that $\lambda''(R) < 0$. Therefore, given X, the optimal investment size, $R^*(X)$, provided that it exists, follows from the first-order condition and is implicitly determined by

$$\frac{X}{4\alpha(r-\mu)}\frac{\lambda'(R^*(X))(r-\mu)}{(\lambda(R^*(X))+r-\mu)^2} = 1. \tag{A.5}$$

If no positive solution to (A.5) exists, then the firm will not undertake the R&D investment, i.e., $R^*(X) = 0$.

The threshold $X^*(R)$, i.e., the indifference level, and the constant $A(R)$ are determined by the value matching and smooth pasting conditions, which are given by

$$F(X^*(R)) = G(X^*(R), R)$$
$$\text{and } \left.\frac{\partial F(X)}{\partial X}\right|_{X=X^*(R)} = \left.\frac{\partial G(X,R)}{\partial X}\right|_{X=X^*(R)},$$

respectively. Solving for $X^*(R)$ gives

$$X^*(R) = \left(\frac{\beta}{\beta-1}\right)\frac{4\alpha(r-\mu)R(\lambda(R)+r-\mu)}{\lambda(R)}. \tag{A.6}$$

Correspondingly, solving for $A(R)$ gives

$$A(R) = \frac{(X^*(R))^{-\beta}R}{\beta-1}.$$

Finally, the optimal investment size at the moment of investment, R^*, is determined by substituting (A.6) into (A.5), which gives

$$\beta = \frac{\lambda(R^*)+r-\mu}{\lambda(R^*)+(1-\frac{R^*\lambda'(R^*)}{\lambda(R^*)})(r-\mu)}.$$

Consequently, the investment threshold is equal to $X^* \equiv X^*(R^*)$, and the constant is equal to $A \equiv A(R^*)$. If no positive solution to R^* and X^* exists, then the firm does not invest in the R&D project. □

Proof of Proposition 3.1. We first show that the payoff of providing the maximal R&D investment subsidy at $X = X_W^*$ is at least the payoff of providing no subsidy, which, in turn, implies that the government is always willing to provide the maximal R&D investment subsidy. That is, it suffices to show that,

$$P(X_W^*, \bar{S}_\ell(X_W^*), \theta) \geq P(X_W^*, 0, \theta), \tag{A.7}$$

because the right-hand side is always non-negative (see (3.15)). The inequality (A.7) is equivalent to

$$\tilde{c}\left[X_W^*\frac{\beta(-3\theta^2+2\theta+5)-2}{2(\beta-1)}-\left(\frac{X_W^*}{X^*}\right)^\beta\left(X^*\frac{\beta(-3\theta^2+2\theta+3)}{2(\beta-1)}\right)-X^*\right]\geq 0,$$

in which $\tilde{c}=\frac{(\beta-1)\lambda(R^*)}{\beta 4\alpha(r-\mu)(\lambda(R^*)+r-\mu)}$ is a constant. Because $X_W^*=\frac{1}{2}X^*$, we are required to show that

$$\beta(-3\theta^2+2\theta+5)-2-(1/2)^{\beta-1}\beta(-3\theta^2+2\theta+3)-4(\beta-1)\geq 0,$$

which is equivalent to

$$\beta\left[(-3\theta^2+2\theta)(1-(1/2)^{\beta-1})+1-3(1/2)^{\beta-1}\right]+2\geq 0.$$

It holds that $1-(1/2)^{\beta-1}>0$; furthermore $-3\theta^2+2\theta\geq 0$ for $\theta\in[0,\frac{2}{3}]$. Hence, it suffices to show that, for all $\beta>1$,

$$f(\beta)=\beta\left[1-3(1/2)^{\beta-1}\right]+2\geq 0.$$

Note that $f(1)=0$. We proceed by showing that $f'(\beta)>0$ for all $\beta>1$. We have

$$f'(\beta)=3\,(1/2)^{\beta-1}\ln(2)\left[\beta-\ln(2)^{-1}\left(1-(1/3)2^{\beta-1}\right)\right].$$

Let

$$g(\beta)=\beta-\ln(2)^{-1}\left(1-(1/3)2^{\beta-1}\right).$$

Then, $g(1)=1-\frac{2}{3}\ln(2)^{-1}>0$ and $g'(\beta)=1+\frac{1}{3}2^{\beta-1}>0$, which implies that $f'(\beta)>0$ for all $\beta>1$ so that also $f(\beta)>0$ for all $\beta>1$. Hence, we have shown that

$$P(X_W^*,\bar{S}_\ell(X_W^*),\theta)>P(X_W^*,0,\theta),\tag{A.8}$$

Finally, we need to show that, for all $X \in [X_W^*, X^*]$, we have

$$P(X, \bar{S}_\ell(X), \theta) \geq P(X, 0, \theta). \tag{A.9}$$

Let

$$\begin{aligned} G(X,\theta) &= P(X, \bar{S}_\ell(X), \theta) - P(X, 0, \theta) \\ &= \tilde{c}\left[X\frac{\beta(-3\theta^2+2\theta+5)-2}{2(\beta-1)} - \left(\frac{X}{X^*}\right)^\beta \left(X^*\frac{\beta(-3\theta^2+2\theta+3)}{2(\beta-1)}\right) - X^*\right], \end{aligned}$$

in which $\tilde{c} = \frac{(\beta-1)\lambda(R^*)}{\beta 4\alpha(r-\mu)(\lambda(R^*)+r-\mu)}$ is a constant. Then, as shown before, $G(X_W^*, \theta) > 0$ (see (A.8)); additionally it holds that $G(X^*, \theta) = 0$. Inequality (A.9) then follows from the fact that the function G is continuous and strictly concave in X on its relevant domain. □

Proof of Proposition 3.2. Define the budget constraint as a function of θ as follows:

$$f(\theta) = -3a\beta\theta^2 + 2a\beta\theta + \beta(5a - 3a^\beta) - 2(a + \beta - 1).$$

The function $f(\theta)$ is strictly concave and is maximized at $\theta = \frac{1}{3}$. Because $f(\frac{2}{3}) \geq 0$ for all $a \in [\frac{1}{2}, 1]$ and $\beta > 1$ (see Proposition 3.1), there always exists a positive root $\bar{\theta}_\beta(a) \geq \frac{2}{3}$ of $f(\theta)$. This positive root is given by

$$\bar{\theta}_\beta(a) = \frac{1}{3} + \sqrt{\frac{16}{9} - a^{\beta-1} - \frac{2(a+\beta-1)}{3a\beta}}.$$

If $\bar{\theta}_\beta(a) \leq 1$, then $\theta_\beta^*(a) = \bar{\theta}_\beta(a)$, otherwise $\theta_\beta^*(a) = 1$. □

Proof of Proposition 3.3. We require to show that $(a_\beta^*, \theta_\beta^*) = (\frac{1}{2}, \theta_\beta^*(\frac{1}{2}))$ is the optimal solution of the optimization problem (3.14), in which $\theta_\beta^*(\frac{1}{2})$ follows from

$$\theta_\beta^*(a) = \min\left\{1, \frac{1}{3} + \sqrt{\frac{16}{9} - a^{\beta-1} - \frac{2(a+\beta-1)}{3a\beta}}\right\} \geq \frac{2}{3}. \tag{A.10}$$

Correspondingly, the government provides the maximal R&D investment subsidy of $\bar{S}_\ell(\frac{1}{2}X^*) = \bar{S}_\ell(X_W^*) = \frac{1}{2}R^*$ at $X = \frac{1}{2}X^* = X_W^*$ (see (3.7)); the productive investment subsidy provided at the stochastic innovation time T with $X = X(T)$ is equal to $S_p(X, \theta^*_\beta(\frac{1}{2}))$ (see (3.3)).

By Proposition 3.2 it follows that the optimal solution should be of the form $(a^*_\beta, \theta^*_\beta) = (a^*_\beta, \theta^*_\beta(a^*_\beta))$. By substituting $\theta^*_\beta(a)$ (see (A.10)) into the value function of the optimization problem (3.14), we obtain

$$f(a) = \left(\frac{1}{2a}\right)^{\beta-1} \frac{(\theta^*_\beta(a) + 1)(3 - \theta^*_\beta(a))}{4}.$$

If $f'(a) < 0$ for all $a \in [\frac{1}{2}, 1]$, then the optimal solution is the boundary solution $a^*_\beta = \frac{1}{2}$. We have

$$\frac{\partial f(a)}{\partial a} = \frac{(2a)^{-\beta}}{2} \left[2a(1 - \theta^*_\beta(a))\frac{\partial \theta^*_\beta(a)}{\partial a} - (\beta - 1)(\theta^*_\beta(a) + 1)(3 - \theta^*_\beta(a))\right].$$

If $\theta^*_\beta(a) = 1$, then $\frac{\partial f(a)}{\partial a} < 0$. Otherwise, the derivative of $\theta^*_\beta(a)$ with respect to a is non-zero, and is given by

$$\frac{\partial \theta^*_\beta(a)}{\partial a} = \frac{\beta - 1}{2a^2\sqrt{\frac{16}{9} - a^{\beta-1} - \frac{2(a+\beta-1)}{3a\beta}}} \left[\frac{2}{3\beta} - a^\beta\right].$$

Consequently, $\frac{\partial f(a)}{\partial a} < 0$ is equivalent to

$$\frac{1}{a\sqrt{\frac{16}{9} - a^{\beta-1} - \frac{2(a+\beta-1)}{3a\beta}}} \left[\frac{2}{3\beta} - a^\beta\right] < \frac{(\theta^*_\beta(a) + 1)(3 - \theta^*_\beta(a))}{1 - \theta^*_\beta(a)}.$$

The right-hand side is at least equal to $11\frac{2}{3}$ because $\theta^*_\beta(a) \in [\frac{2}{3}, 1]$; the left-hand side is at most equal to 1 because $a\sqrt{\frac{16}{9} - a^{\beta-1} - \frac{2(a+\beta-1)}{3a\beta}} \geq \frac{1}{6}$ (see (A.10)) and $\beta > 1$. Therefore, the inequality always holds. □

Appendix B

The isoelastic demand function is given by

$$P(t) = X(t)K^{-\nu},$$

in which $\nu \in (0,1)$.

Optimal production capacity. The expected value of the firm at the moment it launches the product at X with capacity level K, i.e., the expected producer surplus, is equal to

$$PS(X,K) = \mathbb{E}\left[\int_0^\infty X(t)K^{-\nu}Ke^{-rt}dt - \delta_1 K \,\middle|\, X(0) = X\right] = \frac{XK^{1-\nu}}{r-\mu} - \delta_1 K. \tag{B.1}$$

Optimizing (B.1) with respect to K gives the optimal production capacity of the firm, $K^*(X)$, as given in (4.4) in the text. The consumer surplus is equal to

$$CS(X,K) = \mathbb{E}\left[\int_0^\infty \left(\int_{P(K)}^\infty D(P)dP\right) e^{-rt}dt \,\middle|\, X(0) = X\right] = \frac{\nu}{1-\nu}\frac{XK^{1-\nu}}{r-\mu}, \tag{B.2}$$

in which $\int_{P(K)}^\infty D(P)dP$ is the instantaneous consumer surplus. From (B.1) and (B.2) we obtain the total surplus, which is given by

$$TS(X,K) = \frac{1}{1-\nu}\frac{XK^{1-\nu}}{r-\mu} - \delta_1 K. \tag{B.3}$$

Optimizing (B.3) with respect to K gives the optimal production capacity of the social planner, $K_W^*(X)$, as given in (4.4) in the text.

Proof of Proposition 4.1. We distinguish between two regions on the basis of the investment triggers for the firm and social planner.

Waiting region. This region is characterized by values of X for which the firm and social planner are in the waiting region, which is the case for $X < X^*$ and $X < X_W^*$, respectively. Analogous to the proof of Proposition 2.1, the values of the option to invest for firm and social planner are given by

$$F(X) = AX^\beta \text{ and } F_W(X) = A_W X^\beta, \tag{B.4}$$

respectively, in which A and A_W are some constants, and in which $\beta > 1$ is given by (A.2).

R&D investment region. This region is characterized by values of X for which the firm and social planner are in the R&D investment region, which is the case for $X \geq X^*$ and $X \geq X_W^*$, respectively.

The value of the firm upon breakthrough is equal to

$$\Omega(X) = PS(X, K^*(X)) = \frac{\delta_1 \nu}{1-\nu}\left(\frac{(1-\nu)X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}}; \tag{B.5}$$

the value of the social planner upon breakthrough is equal to

$$\Omega_W(X) = TS(X, K_W^*(X)) = \frac{\delta_1 \nu}{1-\nu}\left(\frac{X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}}. \tag{B.6}$$

From (B.5) and (B.6) it follows that $\Omega_W(X) = (1-\nu)^{-1/\nu}\Omega(X)$.

Let R denote the size of the irreversible investment in R&D. Then, the expected value of the firm at the moment of investment is equal to

$$\begin{aligned} G(X,R) &= \mathbb{E}\left[e^{-rT}\Omega(X(T)) - R \,|\, X(0) = X\right] \\ &= \frac{\delta_1 \nu}{1-\nu}\left(\frac{(1-\nu)}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} \int_0^\infty \lambda(R) e^{-\lambda(R)t} e^{-rt} \mathbb{E}[X(t)^{\frac{1}{\nu}} | X(0) = X] dt - R \\ &= \frac{\delta_1 \nu}{1-\nu}\left(\frac{(1-\nu)X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} \frac{\lambda(R)}{\lambda(R) + r - c(\nu)} - R, \end{aligned}$$

in which $c(\nu) = \frac{1}{\nu}\left(\mu + \frac{(1-\nu)}{2\nu}\sigma^2\right)$. The third equality follows from the fact that

$$\mathbb{E}[X(t)^{\frac{1}{\nu}} | X(0) = X] = X^{\frac{1}{\nu}} e^{\frac{1}{\nu}(\mu + \frac{(1-\nu)}{2\nu}\sigma^2)t}.$$

We assume that $r > c(\nu)$ to guarantee that expected values are finite. Thus, it holds that

$$G(X,R) = \Omega(X)\frac{\lambda(R)}{\lambda(R) + r - c(\nu)} - R. \tag{B.7}$$

Moreover, the expected value of the social planner at the moment of investment is equal to

$$G_W(X,R) = \Omega_W(X)\frac{\lambda(R)}{\lambda(R)+r-c(\nu)} - R. \tag{B.8}$$

Optimal R&D investment triggers. The optimal R&D investment triggers, $X^*(R)$ and $X_W^*(R)$, follow in the usual way from the value matching and smooth pasting conditions with respect to the option values given in (B.4), the value of the firm given in (B.7), and the value of the social planner given in (B.8). The optimal investment trigger of the firm is equal to

$$X^*(R) = \frac{\delta_1(r-\mu)}{1-\nu}\left(\frac{R\beta(1-\nu)}{\delta_1(\nu\beta-1)}\frac{\lambda(R)+r-c(\nu)}{\lambda(R)}\right)^{\nu}; \tag{B.9}$$

the optimal investment trigger of the social planner is equal to

$$X_W^*(R) = \delta_1(r-\mu)\left(\frac{R\beta(1-\nu)}{\delta_1(\nu\beta-1)}\frac{\lambda(R)+r-c(\nu)}{\lambda(R)}\right)^{\nu}. \tag{B.10}$$

From (B.9) and (B.10) it follows that $X_W^* = (1-\nu)X^*$.

Optimal R&D investment sizes. The optimal R&D investment sizes for the firm and social planner, $R^*(X)$ and $R_W^*(X)$, respectively, follow from optimizing $G(X,R)$ and $G_W(X,R)$ (see (B.7) and (B.8)) with respect to R, respectively. By doing so, we obtain that the optimal R&D investment size of the firm, provided that it exists, follows from the first-order condition and is implicitly determined by

$$\Omega(X)\frac{\lambda'(R^*(X))(r-c(\nu))}{(\lambda(R^*(X))+r-c(\nu))^2} = 1; \tag{B.11}$$

in a similar fashion we obtain that the optimal R&D investment size of the social planner is implicitly determined by

$$\Omega_W(X)\frac{\lambda'(R_W^*(X))(r-c(\nu))}{(\lambda(R_W^*(X))+r-c(\nu))^2} = 1. \tag{B.12}$$

Because $X_W^* = (1-\nu)X^*$, it holds that

$$\Omega_W(X_W^*) = \frac{\delta_1 \nu}{1-\nu}\left(\frac{X_W^*}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} = \frac{\delta_1 \nu}{1-\nu}\left(\frac{(1-\nu)X^*}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} = \Omega(X^*);$$

hence, it must also hold that $R_W^* = R^*$ as both $R_W^* \equiv R_W^*(X_W^*)$ and $R^* \equiv R^*(X^*)$ satisfy (B.11) and (B.12). □

Welfare. Let $X \leq X_W^* = (1-\nu)X^*$. The expected total welfare at the start of the planning period with respect to the policy of the firm is equal to

$$T_F(X) = \left(\frac{X}{X^*}\right)^{\beta} \frac{\delta_1 \nu}{(1-\nu)}\left(\frac{(1-\nu)X^*}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} \left(1+\frac{1}{1-\nu}\right) \frac{\lambda(R^*)}{\lambda(R^*)+r-c(\nu)}. \tag{B.13}$$

The expected total welfare at the start of the planning period with respect to the policy of the social planner is equal to

$$T_W(X) = \left(\frac{X}{X_W^*}\right)^{\beta} \frac{\delta_1 \nu}{1-\nu}\left(\frac{X_W^*}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} \frac{\lambda(R_W^*)}{\lambda(R_W^*)+r-c(\nu)}. \tag{B.14}$$

Using $X_W^* = (1-\nu)X^*$ and $R_W^* = R^*$, we can write $T_F(X)$ (see (B.13)) in terms of $T_W(X)$ (see (B.14)) as follows

$$T_F(X) = (1-\nu)^{\beta-1}(2-\nu)T_W(X). \tag{B.15}$$

There is indeed a welfare loss because

$$(1-\nu)^{\beta-1}(2-\nu) < (1-\nu)^{\frac{1}{\nu}-1}(2-\nu) < e^{\nu-1}(2-\nu) < 1.$$

The first inequality follows from the fact that $\beta > \frac{1}{\nu}$; the second inequality follows from the inequality $e^{\nu} < (1-\nu)^{-1}$ that holds for $\nu < 1$; the last inequality follows from the fact that $g(\nu) = e^{\nu-1}(2-\nu)$ is an increasing function for $\nu < 1$ and the fact that $g(1) = 1$.

To relate (B.14) to the welfare loss observed in Huisman and Kort (2015), we write

$$T_F(X) = \left[(1-\nu)^{\beta-1}(1-\nu+\beta\nu) - (1-\nu)^{\beta-1}(\beta\nu-1)\right] T_W(X).$$

Productive investment subsidy. The government imposes the firm to produce with quantity

$$K_\theta^*(X) = \left(\frac{(1-\nu(1-\theta))X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} = \left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}} K^*(X),$$

in which $\theta \in [0,1]$.

If the firm produces with quantity $K_\theta^*(X)$, the expected producer surplus is equal to

$$\Omega(X,\theta) = PS(X, K_\theta^*(X)) = \frac{\delta_1\nu(1-\theta)}{1-\nu(1-\theta)} K_\theta^*(X);$$

the expected consumer surplus is equal to

$$CS(X,\theta) = CS(X, K_\theta^*(X)) = \frac{\delta_1\nu}{1-\nu}\frac{1}{1-\nu(1-\theta)} K_\theta^*(X);$$

the expected total surplus is equal to

$$TS(X,\theta) = TS(X, K_\theta^*(X)) = \frac{1+(1-\nu)(1-\theta)}{1-\nu(1-\theta)}\frac{\delta_1\nu}{1-\nu} K_\theta^*(X).$$

The productive investment subsidy the government provides is equal to

$$S_p(X,\theta) = \Omega(X,0) - \Omega(X,\theta) = \Omega(X,0)\left[1-(1-\theta)\left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}\right].$$

R&D investment subsidy. The value of the firm with R&D investment subsidy $S_\ell \in (0, R^*)$ is given by

$$V(X,S_\ell) = \begin{cases} A(S_\ell)X^\beta & \text{if } X < X^*(S_\ell), \\ \frac{\delta_1\nu}{1-\nu}\left(\frac{(1-\nu)X}{\delta_1(r-\mu)}\right)^{\frac{1}{\nu}} \frac{\lambda(R^*)}{\lambda(R^*)+r-c(\nu)} - R^* + S_\ell & \text{if } X^*(S_\ell) \le X < X^*, \end{cases} \tag{B.16}$$

with investment trigger

$$X^*(S_\ell) = \frac{\delta_1(r-\mu)}{1-\nu}\left(\frac{(R^*-S_\ell)\beta(1-\nu)}{\delta_1(\nu\beta-1)}\frac{\lambda(R^*)+r-c(\nu)}{\lambda(R^*)}\right)^\nu, \tag{B.17}$$

and constant

$$A(S_\ell) = \frac{(X^*(S_\ell))^{-\beta}(R^* - S_\ell)}{\nu\beta - 1}.$$

The R&D investment subsidy triggering immediate investment follows from $X^*(S_\ell) = X$ (see (B.17)) and is given by

$$\bar{S}_\ell(X) = \left(1 - \left(\frac{X}{X^*}\right)^{\frac{1}{\nu}}\right) R^*.$$

Budget constraint. Let $X \leq X^*(S_\ell)$. The expected present total surplus of the R&D project is given by

$$T(X, S_\ell, \theta) = \left(\frac{X}{X^*(S_\ell)}\right)^\beta \Omega(X^*(S_\ell), 0) \left(\frac{1}{1-\nu} + (1-\theta)\right) \left(1 + \frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1} \frac{\lambda(R^*)}{\lambda(R^*) + r - c(\nu)}.$$

The expected present value of the productive investment subsidy is given by

$$S_p(X, S_\ell, \theta) = \left(\frac{X}{X^*(S_\ell)}\right)^\beta \Omega(X^*(S_\ell), 0) \left[1 - (1-\theta)\left(1 + \frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}\right] \frac{\lambda(R^*)}{\lambda(R^*) + r - c(\nu)}.$$

The payoff of the government is given by

$$P(X, S_\ell, \theta) = T(X, S_\ell, \theta) - T(X, 0, 0) - S_\ell - S_p(X, S_\ell, \theta). \tag{B.18}$$

Optimization problem. Let $X = aX^*$ with $a \in [1-\nu, 1]$ be the value of X at which the government provides the maximal R&D investment subsidy, $\bar{S}_\ell(X)$, and let $\theta \in [0,1]$. Then, the expected total surplus at $X \leq X_W^*$ is equal to

$$\left(\frac{X}{aX^*}\right)^\beta T(aX^*, \bar{S}_\ell(aX^*), \theta) = \left(\frac{1-\nu}{a}\right)^\beta a^{\frac{1}{\nu}} \left(\frac{1}{1-\nu} + (1-\theta)\right)\left(1 + \frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1} T_W(X), \tag{B.19}$$

in which $T_W(X)$ is the expected present total welfare under the policy of the social planner (see (B.14)). The expression in (B.19) is strictly decreasing in a and strictly increasing in θ, meaning that it is maximized at $a = (1-\nu)$ and $\theta = 1$. The budget constraint, $P(aX^*, \bar{S}_\ell(aX^*), \theta) \geq 0$, is equivalent to

$$a^{\frac{1}{\nu}}\left(\frac{1}{1-\nu}+2(1-\theta)\right)\left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}-a^{\beta}\left(\frac{1}{1-\nu}+1\right)-1+\frac{1}{\nu\beta}(1-a^{\frac{1}{\nu}})\geq 0.$$

Given $\nu\in(0,1)$, the optimization problem the government solves is given by

$$\max_{(a,\theta)\in[1-\nu,1]\times[0,1]}\left(\frac{1-\nu}{a}\right)^{\beta}a^{\frac{1}{\nu}}\left(\frac{1}{1-\nu}+(1-\theta)\right)\left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}$$

subject to

$$a^{\frac{1}{\nu}}\left(\frac{1}{1-\nu}+2(1-\theta)\right)\left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}-a^{\beta}\left(\frac{1}{1-\nu}+1\right)-1+\frac{1}{\nu\beta}(1-a^{\frac{1}{\nu}})\geq 0.$$

No R&D investment subsidy. If $S_\ell=0$, then

$$P(X,0,\theta)=\left(\frac{X}{X^*}\right)^{\beta}\frac{\Omega(X^*,0)\lambda(R^*)}{\lambda(R^*)+r-c(\nu)}\left[\left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}\left(\frac{1}{1-\nu}+2(1-\theta)\right)-\frac{1}{1-\nu}-2\right].$$

Define

$$g(\theta;\nu)=\left(1+\frac{\nu}{1-\nu}\theta\right)^{\frac{1}{\nu}-1}\left(\frac{1}{1-\nu}+2(1-\theta)\right)-\frac{1}{1-\nu}-2. \quad \text{(B.20)}$$

Then, g is a strictly concave function in θ, i.e., $g''(\theta;\nu)<0$, that is maximized at $\theta=\frac{1}{2}$, and for which it holds that $g(0;\nu)=0$. Consequently, if, for a given $\nu\in(0,1)$, it holds that $g(1;\nu)\geq 0$, then $g(\theta;\nu)\geq 0$ for all $\theta\in[0,1]$.

Lemma B.1. *It holds that $g(1;\nu)<0$ for $\nu\in(0,\frac{1}{2})$, and $g(1;\nu)\geq 0$ for $\nu\in[\frac{1}{2},1)$.*

Proof. We have

$$g(1;\nu)=\frac{1}{1-\nu}\left(\left(\frac{1}{1-\nu}\right)^{\frac{1}{\nu}-1}-(1+2(1-\nu))\right).$$

Now, define

$$f(\nu)=\left(\frac{1}{1-\nu}\right)^{\frac{1}{\nu}-1}-(1+2(1-\nu).$$

Then, $f(\frac{1}{2}) = 0$, $\lim_{\nu \uparrow 1} f(\nu) = 0$,

$$f'(\nu) = \frac{\nu + \ln(1-\nu)}{\nu^2} \left(\frac{1}{1-\nu} \right)^{\frac{1}{\nu}-1} + 2,$$

and

$$f''(\nu) = \frac{(1-\nu)\ln^2(1-\nu) - \nu^2}{\nu^4(1-\nu)} \left(\frac{1}{1-\nu} \right)^{\frac{1}{\nu}-1}.$$

It suffices to show that $f''(\nu) < 0$ because then we have $f(\nu) < 0$ for $\nu \in (0, \frac{1}{2})$ and $f(\nu) \geq 0$ for $\nu \in [\frac{1}{2}, 1)$. To this end, define

$$h(\nu) = (1-\nu)\ln^2(1-\nu) - \nu^2.$$

Then, $h(0) = 0$, $h'(0) = 0$, and

$$h''(\nu) = \frac{2}{1-\nu}(\nu + \ln(1-\nu)) < 0$$

because $e^\nu < (1-\nu)^{-1}$ for $\nu < 1$. Therefore, $\nu = 0$ is the global maximizer and $h(\nu) < 0$ for all $\nu \in (0, 1)$, which imply that $f''(\nu) < 0$ for all $\nu \in (0, 1)$. □

Solving the optimization problem. We consider the cases $\nu \in (0, \frac{1}{2})$ and $\nu \in [\frac{1}{2}, 1)$ separately, starting with $\nu \in [\frac{1}{2}, 1)$.

Let $\nu \in [\frac{1}{2}, 1)$. The first-best solution can be obtained if the budget constraint is satisfied at $X = X_W^*$ (or $a = (1-\nu)$) and $\theta = 1$, i.e., if

$$P(X_W^*, \bar{S}_\ell(X_W^*), 1) \geq 0.$$

In the following proof of Proposition 4.2 on the result that the first-best solution is always attainable if $\nu \in [\frac{1}{2}, 1)$, we show a stronger result, namely that, for every $X \in [X_W^*, X^*]$, i.e., $a \in [1-\nu, 1]$, the government's payoff when providing the maximal R&D investment subsidy is at least the payoff when providing no R&D investment subsidy. That is, for every $X \in [X_W^*, X^*]$, it holds that

$$P(X, \bar{S}_\ell(X), 1) \geq P(X, 0, 1), \tag{B.21}$$

in which the right-hand side of (B.21) is at least zero by Lemma B.1.

Proof of Proposition 4.2. Let $\nu \in [\frac{1}{2}, 1)$. It suffices to show that, for all $X \in [X_W^*, X^*]$,

$$P(X, \bar{S}_\ell(X), 1) \geq P(X, 0, 1), \tag{B.22}$$

because the right-hand side of (B.22) is at least zero by Lemma B.1.

Let $X = aX^*$ with $a \in [1-\nu, 1]$. Then, define

$$\begin{aligned}
g(a) &= P(aX^*, \bar{S}_\ell(aX^*), 1) - P(aX^*, 0, 1) \\
&= \tilde{c}_1 \left[a^\beta \left(1 - \left(\frac{1}{1-\nu} \right)^{\frac{1}{\nu}} \right) + \frac{1}{\nu\beta}(1 - a^{\frac{1}{\nu}}) + \left(\frac{a}{1-\nu} \right)^{\frac{1}{\nu}} - 1 \right] \\
&= \tilde{c}_2 \left[\nu\beta a^{\beta - \frac{1}{\nu} + 1}((1-\nu)^{\frac{1}{\nu}} - 1) + a(1-\nu)^{\frac{1}{\nu}}(a^{-\frac{1}{\nu}} - 1) + a\nu\beta - \nu\beta(1-\nu)^{\frac{1}{\nu}} a^{1-\frac{1}{\nu}} \right] \\
&= \tilde{c}_2 \left[\nu\beta a^{\beta - \frac{1}{\nu} + 1}((1-\nu)^{\frac{1}{\nu}} - 1) - a^{1-\frac{1}{\nu}}(1-\nu)^{\frac{1}{\nu}}(\nu\beta - 1) + a(\nu\beta - (1-\nu)^{\frac{1}{\nu}}) \right],
\end{aligned}$$

in which $\tilde{c}_1 = \frac{\delta_1 \nu}{1-\nu} \left(\frac{(1-\nu)X^*}{\delta_1(r-\mu)} \right)^{\frac{1}{\nu}} \frac{\lambda(R^*)}{\lambda(R^*)+r-c(\nu)}$ and $\tilde{c}_2 = \tilde{c}_1 \frac{1}{\nu\beta} \left(\frac{1}{1-\nu} \right)^{\frac{1}{\nu}} a^{\frac{1}{\nu}-1}$ are some positive constants. Now, define

$$h(a) = \nu\beta a^{\beta - \frac{1}{\nu} + 1}((1-\nu)^{\frac{1}{\nu}} - 1) - a^{1-\frac{1}{\nu}}(1-\nu)^{\frac{1}{\nu}}(\nu\beta - 1) + a(\nu\beta - (1-\nu)^{\frac{1}{\nu}}).$$

We will proceed by showing that $h(a) \geq 0$ for all $a \in [1-\nu, 1]$ because then also $g(a) \geq 0$ for all $a \in [1-\nu, 1]$. First, if $a = (1-\nu)$, then

$$\begin{aligned}
h(1-\nu) &= (1-\nu) \left[1 - (1-\nu)^{\frac{1}{\nu}} \right] \left[1 - \nu\beta(1-\nu)^{\beta - \frac{1}{\nu}} \right] \\
&> (1-\nu) \left[1 - (1-\nu)^{\frac{1}{\nu}} \right] \left[1 - \nu\beta e^{1-\nu\beta} \right] \\
&> 0.
\end{aligned}$$

The first inequality follows from the fact that $e^{\nu} < (1-\nu)^{-1}$ for $\nu < 1$ and the fact that $\beta > \frac{1}{\nu}$; the second inequality follows from the fact that the function $f(\beta) = 1 - \nu\beta e^{1-\nu\beta}$ is an increasing function in β for $\beta > \frac{1}{\nu}$ and the fact that $f(\frac{1}{\nu}) = 0$. Second, if $a = 1$, then $h(1) = 0$. Finally, the function h is strictly concave because

$$h''(a) = \frac{1}{a}\left[\nu\beta(\beta - \frac{1}{\nu} + 1)(\beta - \frac{1}{\nu})a^{\beta - \frac{1}{\nu}}((1-\nu)^{\frac{1}{\nu}} - 1) - \frac{1}{\nu}(\frac{1}{\nu} - 1)a^{-\frac{1}{\nu}}(1-\nu)^{\frac{1}{\nu}}(\nu\beta - 1)\right] < 0,$$

which implies that $h(a) \geq 0$ for all $a \in [1-\nu, 1]$. □

Second, let $\nu \in (0, \frac{1}{2})$. The government provides an R&D investment subsidy of $\bar{S}_\ell(X)$ at $X \leq X^*$ if

$$P(X, \bar{S}_\ell(X), \theta) \geq 0.$$

Let $\theta_\nu \in [0,1)$ be such that $P(X, 0, \theta_\nu) = 0$, that is, the government breaks even on its productive investment subsidy if $S_\ell = 0$. Because the function $g(\theta;\nu)$ (see (B.20)) is strictly concave in θ on its relevant domain that is maximized at $\theta = \frac{1}{2}$ and for which it holds that $g(0;\nu) = 0$, we have $P(X, 0, \theta) \geq 0$ for $\theta \in [0, \theta_\nu]$ and $P(X, 0, \theta) < 0$ for $\theta \in (\theta_\nu, 1]$. Moreover, $\theta_\nu \in (\frac{1}{2}, 1)$ because $g(\frac{1}{2};\nu) > 0$, and $g(1;\nu) < 0$ as per Lemma B.1. It is not possible to obtain an analytical expression for θ_ν, so we solve for it numerically. However, given $\theta \in [0,1]$, the function $g(\theta;\nu)$ is strictly increasing in ν, which means that $\theta_\nu > \lim_{\nu\downarrow 0}\theta_\nu$. In particular, we have

$$\lim_{\nu\downarrow 0} g(\theta;\nu) = e^{\theta}(3 - 2\theta) - 3,$$

so that $\lim_{\nu\downarrow 0}\theta_\nu \approx 0.8742$. In our analysis, we consider the cases $\theta \in [0, \theta_\nu]$ and $\theta \in (\theta_\nu, 1]$ separately.

First, let $\theta \in [0, \theta_\nu]$. Then, the government is always able to bring the investment timing forward as stated in Proposition 4.3.

Proof of Proposition 4.3. Let $\theta \in [0, \theta_\nu]$. It suffices to show that, for all $X \in [X_W^*, X^*]$,

$$P(X, \bar{S}_\ell(X), \theta) \geq P(X, 0, \theta)$$

because $P(X, 0, \theta) \geq 0$ for $\theta \in [0, \theta_\nu]$. Let $X = aX^*$ with $a \in [1-\nu, 1]$. Then, define

$$\begin{aligned} f(a, \theta) &= P(aX^*, \bar{S}_\ell(aX^*), \theta) - P(aX^*, 0, \theta) \\ &= \tilde{c}_1 \left[(a^{\frac{1}{\nu}} - a^\beta) \left(\frac{1}{1-\nu} + 2(1-\theta) \right) \left(1 + \frac{\nu}{1-\nu}\theta \right)^{\frac{1}{\nu}-1} - 1 + a^\beta + \frac{1}{\nu\beta}(1 - a^{\frac{1}{\nu}}) \right], \end{aligned}$$

in which $\tilde{c}_1 = \frac{\delta_1 \nu}{1-\nu} \left(\frac{(1-\nu)X^*}{\delta_1(r-\mu)} \right)^{\frac{1}{\nu}} \frac{\lambda(R^*)}{\lambda(R^*)+r-c(\nu)}$ is some positive constant. It suffices to show that $f(a, \theta) \geq f(a, 1)$ because the fact that $f(a, 1) \geq 0$ has been shown in the proof of Proposition 4.2, irrespective of the value $\nu \in (0, 1)$. The function $f(a, \theta)$ is increasing in θ for $\theta \in [0, \frac{1}{2})$ and decreasing in θ for $\theta \in (\frac{1}{2}, 1]$, so it is maximized at $\theta = \frac{1}{2}$. Therefore, to show $f(a, \theta) \geq f(a, 1)$, it suffices to show that $f(a, 0) \geq f(a, 1)$. From Lemma B.1 it follows that

$$\begin{aligned} f(a, 0) - f(a, 1) &= -\tilde{c}_1 (a^{\frac{1}{\nu}} - a^\beta) \left(\left(\frac{1}{1-\nu} \right)^{\frac{1}{\nu}} - \frac{1}{1-\nu} - 2 \right) \\ &= -\tilde{c}_1 (a^{\frac{1}{\nu}} - a^\beta) g(1; \nu) > 0. \end{aligned}$$

□

To solve the optimization problem for $\nu \in (0, \frac{1}{2})$ we can therefore restrict ourselves to $(a, \theta) \in [1-\nu, 1] \times [\theta_\nu, 1]$. Given $a \in [1-\nu, 1]$, it is optimal to choose $\theta \in [\theta_\nu, 1]$ as large as possible without violating the budget constraint. The budget constraint as a function of θ is given by

$$h(\theta) = a^{\frac{1}{\nu}} \left(\frac{1}{1-\nu} + 2(1-\theta) \right) \left(1 + \frac{\nu}{1-\nu}\theta \right)^{\frac{1}{\nu}-1} - a^\beta \left(\frac{1}{1-\nu} + 1 \right) - 1 + \frac{1}{\nu\beta}(1 - a^{\frac{1}{\nu}}) \geq 0,$$

which is increasing for $\theta \in [0, \frac{1}{2})$ and decreasing for $\theta > \frac{1}{2}$, so that it is maximized at $\theta = \frac{1}{2}$. Hence, because $\theta_\nu > \frac{1}{2}$ and the fact that $h(\theta_\nu) \geq 0$ (see Proposition 4.3), there exists a $\bar{\theta}_\beta(a; \nu) \geq \theta_\nu$ such that

$$h(\bar{\theta}_\beta(a; \nu)) = P(aX^*, \bar{S}_\ell(aX^*), \bar{\theta}_\beta(a; \nu)) = 0.$$

Consequently, the value of θ that, given $a \in [1-\nu, 1]$, optimizes the total welfare and satisfies the budget constraint is given by $\theta^*_\beta(a; \nu) = \min\{1, \bar{\theta}_\beta(a; \nu)\}$. We numerically solve for $\bar{\theta}_\beta(a; \nu)$.

Therefore, the optimal solution should be of the form $(a^*_\beta(\nu), \theta^*_\beta(\nu)) = (a^*_\beta(\nu), \theta^*_\beta(a^*_\beta(\nu);\nu))$. Substituting $\theta^*_\beta(a;\nu)$ into the objective function gives

$$f(a) = \left(\frac{1-\nu}{a}\right)^\beta a^{\frac{1}{\nu}} \left(\frac{1}{1-\nu} + (1-\theta^*_\beta(a;\nu))\right)\left(1+\frac{\nu}{1-\nu}\theta^*_\beta(a;\nu)\right)^{\frac{1}{\nu}-1}.$$

We proceed by showing that $f'(a) < 0$ for $a \in [1-\nu, 1]$, which implies that $a^*_\beta(\nu) = (1-\nu)$ and $\theta^*_\beta(\nu) = \theta^*_\beta(1-\nu;\nu)$. For notational convenience, let $\theta(a) = \theta^*_\beta(a;\nu)$. The derivative of $f(a)$ with respect to a is equal to

$$\frac{\partial}{\partial a} f(a) = \tilde{c}_1 \left[\frac{a\nu(1-\theta(a))}{1-\nu}\frac{\partial\theta(a)}{\partial a} - (\nu\beta - 1)\left(\frac{1}{1-\nu} + (1-\theta(a))\right)\left(1+\frac{\nu}{1-\nu}\theta(a)\right)\right],$$

in which $\tilde{c}_1 = \left(\frac{1-\nu}{a}\right)^\beta \frac{a^{\frac{1}{\nu}}}{a\nu}\left(1+\frac{\nu}{1-\nu}\theta(a)\right)^{\frac{1}{\nu}-2}$ is some positive constant. If $\theta(a) = 1$, then $f'(a) < 0$. Otherwise, the derivative of $\theta(a)$, which follows from $\frac{\partial h(\bar{\theta}_\beta(a;\nu))}{\partial a} = 0$, is non-zero and is equal to

$$\frac{\partial\theta(a)}{\partial a} = \frac{(\nu\beta-1)(1-a^\beta(\frac{1}{1-\nu}+1)\nu\beta)}{\nu\beta\frac{\nu}{1-\nu}a^{\frac{1}{\nu}+1}\left(1+\frac{\nu}{1-\nu}\theta(a)\right)^{\frac{1}{\nu}-2}(2\theta(a)-1)}. \tag{B.23}$$

Subsequently, we substitute (B.23) into the expression for $f'(a)$, and rewrite the condition $f'(a) < 0$ to

$$-\nu\beta a^{\frac{1}{\nu}}\left(\frac{1}{1-\nu} + (1-\theta(a))\right)\left(1+\frac{\nu}{1-\nu}\theta(a)\right)^{\frac{1}{\nu}-1} + \frac{1-\theta(a)}{2\theta(a)-1}\left[1-a^\beta\left(\frac{1}{1-\nu}+1\right)\nu\beta\right] < 0.$$

Because $\nu\beta > 1$, $a^{\frac{1}{\nu}} \geq (1-\nu)^{\frac{1}{\nu}}$, $\theta(a) > \frac{1}{2}$, and $a^\beta\left(\frac{1}{1-\nu}+1\right)\nu\beta > 0$, the above inequality also holds if

$$-\left(1+(1-\nu)(1-\theta(a))\right)\left(1-\nu(1-\theta(a))\right)^{\frac{1}{\nu}-1} + \frac{1-\theta(a)}{2\theta(a)-1} < 0.$$

Next, because $(1+(1-\nu)(1-\theta(a))) > 1$ and $(1-\nu(1-\theta(a)))^{\frac{1}{\nu}-1} > e^{\theta(a)-1}$, the above inequality also holds if

$$-e^{\theta(a)-1} + \frac{1-\theta(a)}{2\theta(a)-1} < 0.$$

Finally, because $\theta(a) \geq \theta_\nu > \lim_{\nu \downarrow 0} \theta_\nu = \theta_0 \approx 0.8742$, and the fact that the left-hand side in the above expression is decreasing in $\theta(a)$, the above inequality holds because $-e^{\theta_0 - 1} + \frac{1-\theta_0}{2\theta_0 - 1} \approx -0.7137 < 0$. Thus, we have established that $f'(a) < 0$ for $a \in [1-\nu, 1]$.

Let $X \leq X_W^*$. The expected total surplus under the welfare-maximizing policy of the government is equal to

$$T^*(X) = (1-\nu)^{\frac{1}{\nu}} \left(\frac{1}{1-\nu} + (1 - \theta_\beta^*(1-\nu;\nu)) \right) \left(1 + \frac{\nu}{1-\nu} \theta_\beta^*(1-\nu;\nu) \right)^{\frac{1}{\nu}-1} T_W(X),$$

in which $T_W(X)$ is the expected present total welfare under the policy of the social planner (see (B.14)).

Appendix C

The isoelastic demand function is given by

$$P(t) = X(t)K(t)^{-\nu},$$

in which $\nu \in (0,1)$. For our purposes, we consider $\nu \in (\frac{1}{2}, 1)$. By Itô's Lemma,[10] the rate of change in the natural logarithm of the price is given by

$$d \ln P(t) = \frac{-\nu dK(t)}{K(t)} + \left(\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma dZ(t).$$

Suppose that observations take place at increasing time points $t_0, t_1, \ldots, t_n$. Let $i \in \{1, \ldots, n\}$. Then,

$$Y_i = \ln \frac{P(t_i)}{P(t_{i-1})} + \nu \ln \frac{K(t_i)}{K(t_{i-1})} = \left(\mu - \frac{1}{2}\sigma^2 \right) \Delta t_i + \sigma \Delta Z_i,$$

in which $\Delta t_i = t_i - t_{i-1}$ and $\Delta Z_i = Z(t_i) - Z(t_{i-1})$. In our case, the time intervals are of equal length, namely one month, which implies that $\Delta t_i = \Delta t = 1$. Because the increments of a Wiener process are standard normally distributed, we have that $Y_i \sim N((\mu - \frac{1}{2}\sigma^2)\Delta t, \sigma^2 \Delta t)$. Moreover, $Y_1, \ldots, Y_n$ are independent as the increments of a Wiener process over any two different time

[10]See, e.g., page 79 in Dixit and Pindyck (1994).

intervals are independent. Consequently, we estimate μ and σ by maximum likelihood estimation. The maximum likelihood estimators of μ and σ are given by

$$\hat{\mu} = \frac{\bar{Y}}{\Delta t} + \frac{1}{2}\hat{\sigma}^2, \text{ and } \hat{\sigma} = \sqrt{\frac{1}{n\Delta t}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2},$$

respectively, in which $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. We consider both the European Union (EU) and United States (US) vehicle market. For the European Union our estimation is based on monthly price and sales data from January 2010 up and to including December 2017, whereas for the United States it is based on data from January 2010 up and to including December 2019.[11] The vehicle price is based on the consumer price index for motor cars and new vehicles in the EU and US, respectively, which is then converted to the real price using the consumer price index for all items.[12] Demand is based on seasonally adjusted retail sales of passenger cars.[13]

The optimal investment size at the moment of investment, $R^* \equiv R^*(X^*)$, is implicitly determined by

$$\nu\beta = \frac{\lambda(R^*) + r - c(\nu)}{\lambda(R^*) + (1 - \frac{R^*\lambda'(R^*)}{\lambda(R^*)})(r - c(\nu))}, \tag{C.1}$$

in which $c(\nu) = \frac{1}{\nu}\left(\mu + \frac{(1-\nu)}{2\nu}\sigma^2\right)$. Let $\lambda(R) = cR^{\gamma}$ with $\gamma \in (0,1)$. If $\nu\beta \leq \frac{1}{1-\gamma}$, then

$$R^* = \left(\frac{(r - c(\nu))(1 - \nu\beta(1-\gamma))}{c(\nu\beta - 1)}\right)^{\frac{1}{\gamma}},$$

and

$$X^* = \left(\frac{\delta_1}{1-\nu}\right)^{1-\nu}(r-\mu)\left(\frac{\nu\beta^2\gamma(r-c(\nu))^{\frac{1}{\gamma}}(1-\nu\beta(1-\gamma))^{\frac{1}{\gamma}-1}}{(\nu\beta-1)^{1+\frac{1}{\gamma}}c^{\frac{1}{\gamma}}}\right)^{\nu}.$$

[11] There are missing sales data points after 2017 for the EU, which is why the estimation period stops at 2017.

[12] See https://ec.europa.eu/eurostat/web/hicp/data/database for EU price data; we select code EU in geography (GEO) and codes CP00711 (motor cars) and CP00 (all items) in the classification of individual consumption by purpose (COICOP). For the US price data we take the consumer price index for all urban consumers, available under codes “CUSR0000SETA01” (new vehicles) and “CPIAUCSL” (all items) from https://fred.stlouisfed.org/.

[13] Available under codes “EU28SLRTCR03IXOBSAM” (EU) and “USASLRTCR03MLSAM” (US) from https://fred.stlouisfed.org/.

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