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CREATING DIAGRAMS FOR PROBLEM-SOLVING IN MATHEMATICS: IS IT WORTH THE EFFORT?

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Abstract

Diagrams are ubiquitous in mathematics instruction. This investigation examines whether the mental effort – often referred to as cognitive load – that is required to construct and use a diagram in order to solve a problem is associated with success in arriving at accurate problem solutions. In this article, data from a series of experiments that were conducted during the past decade are re-analyzed to compare the self-rated effort of being trained to use a diagram with subsequent problem-solving performance, relative to interventions in which participants were trained to use only equations to solve the same word problems. The results demonstrate that the mental effort invested in diagram training is not uniformly beneficial across all types of mathematics problems. Specifically, diagram training is more efficacious for conditional-probability word problems than for joint- and total-probability word problems. Of particular note is the repeated finding that training in how to use Venn diagrams causes worse performance for undergraduates solving total-probability problems.

Keywords

Probability, Diagrams, Mental Effort, Cognitive Load, Problem-Solving

1. Introduction

Diagrams are frequently used in mathematics to concretize ideas that are abstract, as a tangible, visible tool to represent what may otherwise be unseen. A visit to any mathematics lesson, or a quick browse through any mathematics textbook, is sure to reveal some type of diagram employed to illuminate the material being explained.

Not only are diagrams commonly used, but they are also broadly recommended by professional groups in mathematics. For example, the Conference Board of the Mathematical Sciences (2012) and the Committee on the Undergraduate Program in Mathematics (2015) promoted — without reservation — the use of diagrams in mathematics instruction. The practice of using diagrams as an aid in mathematics work was also endorsed by the National Mathematics Advisory Panel (2008) and encouraged by the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The National Council for Teachers of Mathematics (2000) has also urged teachers to help students develop these skills.

Given the consistency with which professional bodies encourage diagram use in mathematics, one would expect the research literature to corroborate these endorsements. Yet the empirical picture investigating the effectiveness of diagrams is not consistently promising. Some studies have found a beneficial effect of introducing diagrams as an aid to solving mathematics problems (Clinton, Alibali, & Nathan, 2013; Cooper, Sidney, & Alibali, 2018), but even here the efficacy of diagrams intermingling with mathematics problem-solving is qualified. For example, Cooper et al. (2018) found that learners' ability level and attitude toward mathematics mediated the effectiveness of diagrams.

The focus of the present paper is a re-analysis of the data from a series of experiments conducted by the author during the past decade. These studies manipulated diagram training, using a variety of experimental designs, in the context of asking postsecondary students to solve probability word problems of three distinct types: conditional, joint, and total. These three types of probability are known to have varying levels of inherent difficulty for students (Sedlmeier & Gigerenzer, 2001). Specifically, undergraduate students solve joint- and total-probability problems more easily than conditional-probability problems. Thus, it is reasonable to wonder whether the effort required constructing a diagram in order to help solve a word problem is associated with superior problem solutions, compared with a situation in which no diagram was utilized.

2. General Methodology

The generic methodology for these studies included (a) a pretest; (b) instructional materials that included practice problems; (c) a self-rating of the mental effort required during the instructional portion just experienced; and (d) an immediate and, in some studies, a delayed posttest. The participants for all of these experiments were undergraduate students at an Eastern U.S. college.

The pretest was a nine-item instrument that was used as a measure of participants' knowledge of probability concepts prior to beginning the intervention; the total score from the pretest was used as a covariate to control for prior knowledge when analyzing the posttest data. The nine pretest items were word problems ranging from simple probability problems (e.g., "When a coin is flipped what is the probability that a 'head' will appear?") to more complex probability problems. Coefficient alpha (Cronbach, 1951) for this instrument varied across these experiments but reached .70 on at least one occasion.

The instructional materials provided a brief explication of basic probability as a background and then proceeded to introduce conditional/joint/total probability (depending on the specific experiment) without assuming any prior knowledge of those topics. Experimental manipulations contained integrated explanations of how to construct and use a diagram to solve problems of that particular probability type (conditional/joint/total); control groups received explanations on how to solve the problems using equations alone. Following the explanations was a series of worked examples (cf. Atkinson, 2000) that scaffolded the participants through the step-by-step solution of the particular type(s) of probability problems that had just been explained. These worked examples followed a fading approach (Renkl & Atkinson, 2003), with the first example fully worked out, the second example mostly worked out, etc., until the final example was left for the participants to solve independently with only the steps labeled.

After the instructional materials and practice problems were completed, participants were presented with a series of questions and asked to rate their experience along several dimensions. These items are drawn from the NASA-TLX instrument (Hart & Staveland, 1988) that many researchers use for examining the demands of learning tasks on cognitive processing.

The mental-effort question is the first data point in focus in the present article; it asked, "How much mental and perceptual activity was required (e.g., thinking, deciding, calculating, remembering, looking, searching, etc.)?" Other questions inquired about temporal demand, perceived level of success, and frustration.

The posttests were constructed to test for participants' ability to solve the type(s) of problems for which they had received training (i.e., conditional/joint/total probability). These posttests were composed entirely of word problems and were completely unscaffolded (i.e., no hints for solutions were provided). Some problems closely resembled the worked examples from the instructional portion of the experiment, and other problems were constructed to require application of the probability principles learned in order to reach a solution; but all problems were able to be solved given only the instruction contained in the experiment. The posttests were scored based on conceptual accuracy, ignoring errors in rounding or fraction simplification. The posttest data provide the second focal data point of the present article.

Most experiments in this series were conducted in a paper-and-pencil format. A few of these studies were administered using computer-based software. The specific materials (content of the tutorial and problems used for worked examples and posttests) varied somewhat across these experiments, but all followed the above-described pattern.

3. Empirical Results

Although the experiments in this series had different experimental designs and other aspects not included here, all contained the mental-effort question at the same point in the experiment (i.e., after the training that included practice problems) and all had posttests with probability word problems that were scored in a similar fashion. We turn now to examine the relationship between the mental effort reported by participants and its association with problem-solving success, for situations with and without training in the use of diagrams as an aid to solving these problems.

Because the pattern of results varies by type of probability, the data from the studies examining conditional-probability will be presented first, followed by the data from the experiments investigating joint and total probability.

3.1 Conditional Probability

The visual pattern for conditional probability, shown in Figure 1, is striking. The two solid lines (in the middle of the figure) represent the diagram conditions, in which participants were trained to use a tree diagram to solve conditional-probability word problems. Notably, the dashed lines (representing the control conditions) are *outside* the solid lines, meaning that students trained to solve the same problems with only equations experienced higher levels of mental effort and a decreased level of performance, relative to their counterparts who were trained to use a diagram.

Table 1: *Correlations between Self-Rated Mental Efforts and Actual Performance for Conditional-Probability Problems*

	Dataset	*r*	*p*
1	Beitzel and Staley (2011)	-0.47	< *.01*
2	Beitzel and Staley (2015, Exp. 1)	-0.28	*.01*
3	Beitzel and Staley (2015, Exp. 2)	-0.30	*.02*
4	Beitzel, Gonyea, and Staley (2013)	-0.38	< *.01*
5	Beitzel, Gonyea, and Staley (2014)	-0.22	< *.01*

Note. The datasets in this table correspond to those in Figure 1.

As shown in Table 1, the correlations between mental demand and actual performance for these five experiments are all negative! Moreover, each of these correlations is statistically significant. The conclusion we can draw from these correlations is that the more mental demand participants experienced in these experiments, the worse they performed on the posttest (and vice versa). This is particularly intriguing when noticing that in Figure 1, the highest reported mental effort was always reported by the control group—who also achieved the lowest posttest performance.

We can also see in Figure 1 that the level of mental demand is quite high for these conditional-probability word problems. The average demand is 70% for the diagram conditions, and 79% for the control conditions. Together, these high values for mental effort and the significantly negative correlations between effort and performance help to explain why problem-solving performance in these experiments is relatively low (45% for the diagram conditions and 29% for the control conditions). Conditional probability tends to be a challenging topic even for postsecondary students, and these data corroborate other studies demonstrating the difficulty of this type of probability for this population. Diagrams do not appear to be a sufficiently powerful mechanism to increase the success of undergraduate students solving this type of problem.

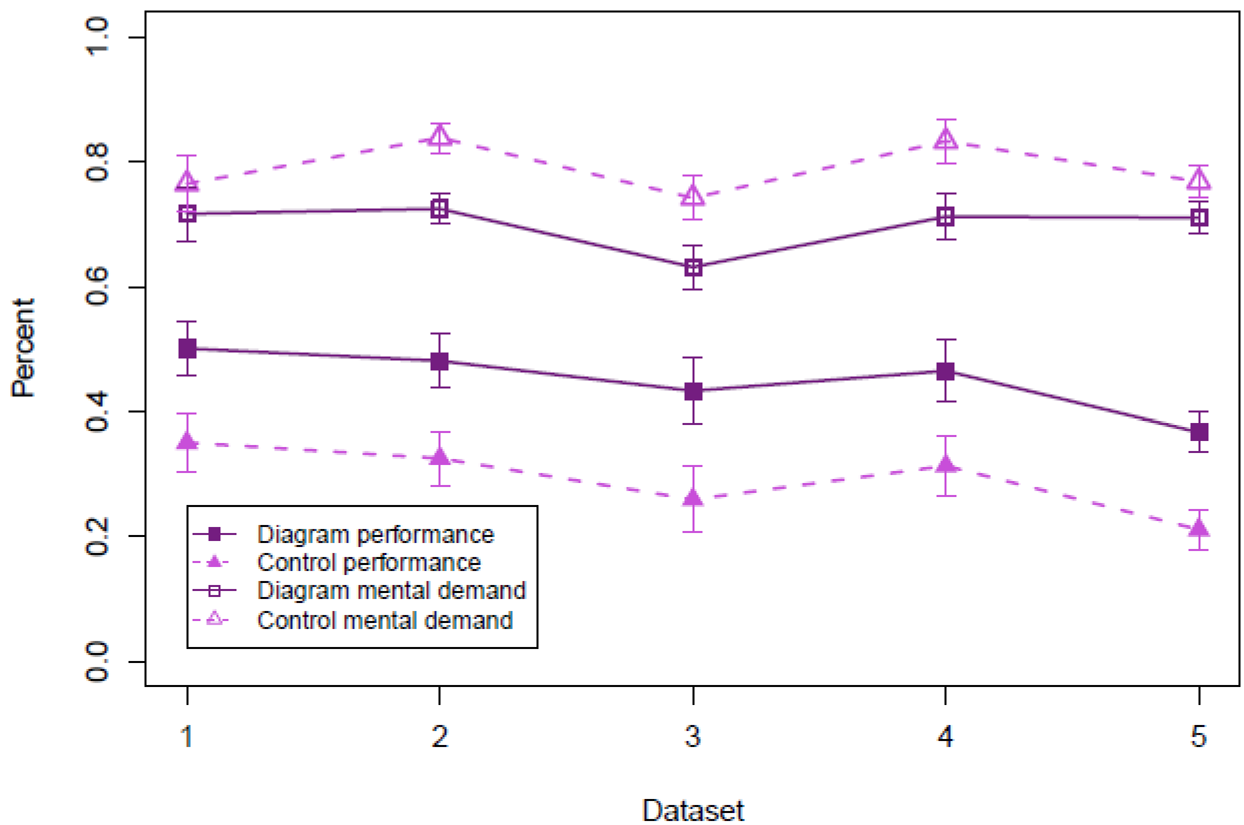

Figure 1: *Mental effort and problem-solving performance for conditional- Probability problems (adjusted for prior knowledge), with standard errors*
See Table 1.1 for the sources for each dataset

3.2 Joint and Total Probability

For joint and total probability, Figure 2 displays a distinctly different pattern. For these datasets, the solid lines (representing the diagram conditions) tend to be *outside* the dashed lines (representing the control conditions). This pattern suggests that participants using only equations to solve joint- and total-probability problems experience less mental demand and achieve higher performance than their counterparts using diagrams who experience greater mental demand and achieve lower performance. Yet this is not such a clear-cut picture; the lines are closer together than in Figure 1 and some lines even cross over each other, indicating that the pattern is not consistent

Table 2: *Correlations between Self-Rated Mental Efforts and Actual Performance for Joint- and Total-Probability Problems*

Dataset		Type	*r*	*p*
6	Beitzel, Staley, and DuBois (2011)	Total	-0.25	*.04*
7	Beitzel, Boss, and Gonyea (2015)	Joint	-0.31	*.00*
8	Beitzel et al. (2015, Exp. 2)	Total	-0.46	*.07*
9	Beitzel, Staley, Holmes, and Snow (2017)	Total	-0.16	*.17*
10	*(unpublished)*	Total	0.03	*.89*

Note. The datasets in this table correspond to those in Figure 2.

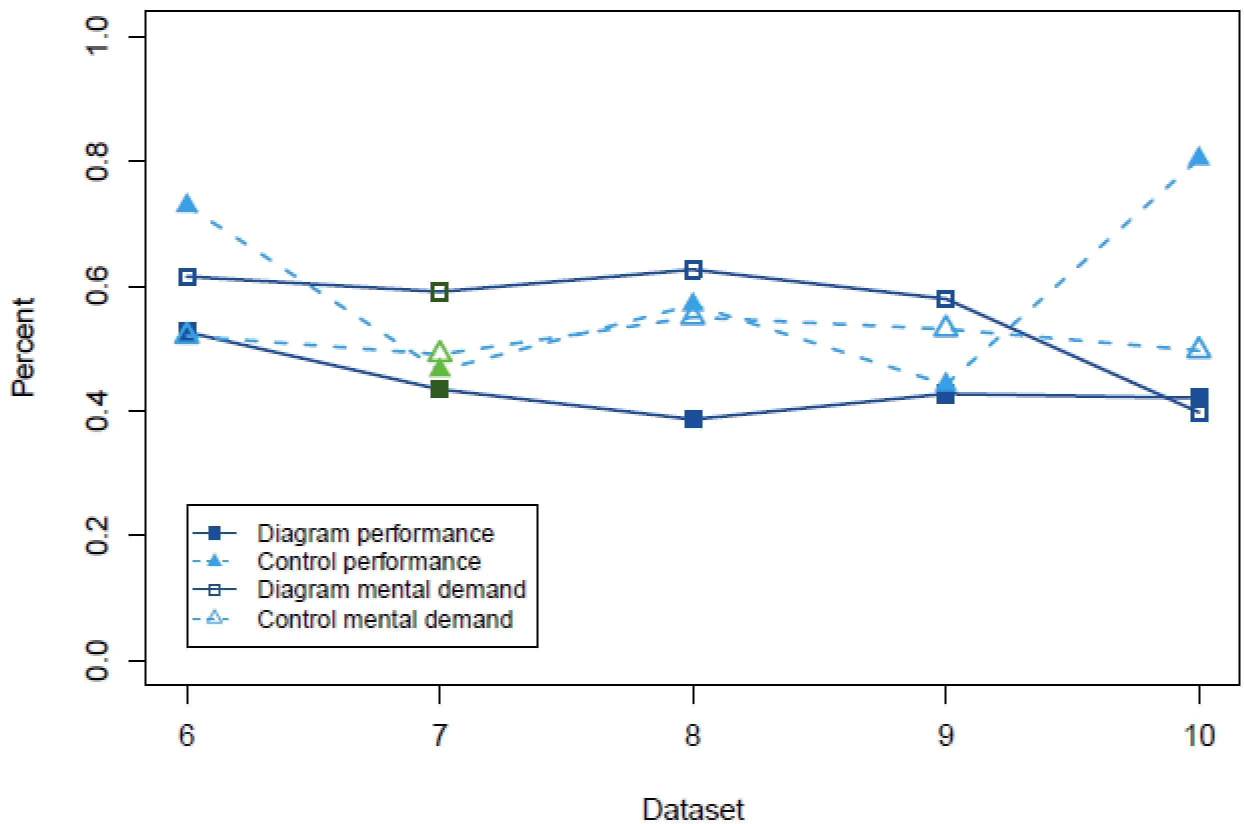

Figure 2: *Mental effort and problem-solving performance for joint- and Total-probability problems (adjusted for prior knowledge), with standard errors*
See Table 1.2 for the sources for each dataset

The data in Table 2 help explain the inconsistent pattern in Figure 2. Again, the correlations for these types of probability problems are mostly negative, but they are also mostly nonsignificant. Note how the data points for each dataset are closer together in Figure 2 than the data points in Figure 1, indicating that the investment of mental effort to produce a diagram reflected in Figure 2 is relatively less than the mental effort depicted in Figure 1. Additionally, one can also readily see that being trained to create a diagram to help solve these types of probability problems does not have much impact on posttest performance.

4. Conclusions

Although the patterns for joint- and total-probability studies (Figure 2) are not as consistent as the clear pattern for the conditional-probability studies (Figure 1), there is enough of a trend to suggest that these two types of probability word problems are associated with differing relative levels of mental effort. For conditional-probability problems, when participants use a diagram less mental effort is required for higher performance levels, compared with the situation in which participants are trained to use equations alone. But for joint- and total-probability problems, more effort is needed to process diagrams (relative to using only equations), and that appears to come at the cost of lower performance.

As with all studies, the interpretation of these results is constrained by a few limitations. First, the sample was obtained from students in a U.S. college that may not be representative of all undergraduates, especially international populations. Second, there are other types of mathematics problems not examined here that may exhibit differential patterns that do not mimic these results. Finally, these undergraduate samples should not be interpreted to generalize to other age-groups such as high-school students.

The takeaway message is that diagrams can impose additional demands on cognitive processing, and that extra effort does not always pay off. The type of problem being solved appears to play a role in determining the effectiveness of using a diagram. Therefore, educators and students alike should be aware that the work of creating a diagram to help solve a mathematics problem is not always worth the effort.

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