Dynamics and spike trains statistics in conductance-based Integrate-and-Fire with chemical and electric synapses

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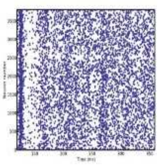
We consider conductance-based Integrate-and-Fire models coupled with gap junctions and chemical synapses, where conductances depend upon the spike-history of the network. We compute explicitly the time evolution operator and show that given the spike-history of the network and the membrane potentials at a given time, the further dynamical evolution can be written in a closed form. Moreover, spike train statistics is described by a Gibbs distribution whose potential can be approximated with an explicit formula, when the noise is weak.

Spike Trains



Spike state

$$\omega_k(n) \in \{0,1\}$$

Spike pattern

$$\omega(n) = (\omega_k(n))_{k=1}^{N}$$

Spike block

$$\omega_m^n = \{\omega(m)\,\omega(m+1)\ldots\omega(n)\}$$

Raster plot

$$\omega \stackrel{\text{def}}{=} \omega_0^T$$

The equation

We consider a network of N neurons, where dynamics depend in continuous and discrete dynamical variables. The sub-threshold variation of the membrane potential of neuron k at time t is given by:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t,\omega)(V_k - E_j) + \sum_j \overline{g_{kj}}\,(V_j - V_k) + I_k(t)$$

Where :

C_k is the capacitance of each neuron

$g_{L,k}$ is the leack conductance.

$\overline{g_{kj}}$ is the electrical conductance which is symmetric.

$I_k(t) = i_k^{(ext)}(t) + \sigma_B \xi_k(t)$ is the current term which is composed by:

$i_k^{(ext)}(t)$ the external current, and white noise term $\xi_k(t)$ whose magnitude is controlled by $\sigma_B > 0$

Conductance adaptation

$$g_{kj}(t) = g_{kj}(t_j^{(r)}(\omega)) + G_{kj}\alpha_{kj}(t - t_j^{(r)}(\omega)), \quad t > t_j^{(r)}(\omega)$$

$G_{kj} \geq 0$ is the maximal conductance

$\alpha_{kj}(t) = h(t)e^{-\frac{t}{\tau_{kj}}}H(t)$ is called "alpha function"

Matrix-Vector representation

$$C\frac{dV}{dt} + \left[\, G(t,\omega) - \overline{G} \,\right] V = I(t,\omega)$$

$$G(t,\omega) = (g_{L,k} + \sum_{j=1}^{N} g_{kj}(t,\omega))\delta_{kk}$$

$\overline{G}$ is the symmetric matrix of electric conductances, with entries $\overline{g_{kj}}$

$$I(t,\omega) = I^{cs}(t,\omega) + I^{ext}(t) + I^B(t)$$

The linear SDE

$$\frac{dV}{dt} = \underbrace{C^{-1}(\overline{G} - G(t,\omega))}_{\Phi(t,\omega)} V + \underbrace{C^{-1}I^{cs}(t,\omega) + C^{-1}I^{ext}(t)}_{f(t,\omega)} + C^{-1}I^B(t)$$

The flow and the solution

If we define recursively the following matrices:

$$M_0(t_0,t,\omega) = \mathcal{I}_N$$

$$M_k(t_0,t,\omega) = \mathcal{I}_N + \int_{t_0}^{t} \Phi(s,\omega)M_{k-1}(s,t)ds$$

Under very general conditions, this limits exist and is called "flow"

$$\Gamma(t_0,t,\omega) \stackrel{\text{def}}{=} \lim_{k\to\infty} M_k(t_0,t,\omega)$$

Considering both chemical and electric synapses, there exist a unique strong solution to the stochastic differential equation given by

$$V(t_0,t,\omega) = \Gamma(t_0,t,\omega)v + \int_{t_0}^{t}\Gamma(s,t,\omega)f(s,\omega)ds + \frac{\sigma_B}{c}\int_{t_0}^{t}\Gamma(s,t,\omega)dW(s)$$

This are examples when the flow takes an exponential form

(i) $\overline{G}$ is diagonal;

(ii) $\overline{G} = 0$;

(iii) $G(t,\omega) = G(t) = \kappa(t)\mathcal{I}_N$ where $\kappa(t)$ is a real function.

Spike Trains Statistics

The membrane potential can be decomposed in their deterministic and stochastic part:

$$V(t,\omega) = V^{(d)}(t,\omega) + V^{(noise)}(t,\omega)$$

The main result establishes that spike trains are distributed according to a Gibbs distribution whose potential can be obtained under Gaussian approximation of $V^{(noise)}(t,\omega)$

$$\widehat{\theta}_k(t,\omega) = \theta - V_k^{(d)}(t,\omega) \qquad \mathcal{J}_k(n,\omega) = \begin{cases}]-\infty, \widehat{\theta}_k(n-1,\omega)], & \text{if } \omega_k(n) = 0; \\ [\widehat{\theta}_k(n-1,\omega), +\infty[, & \text{if } \omega_k(n) = 1; \end{cases}$$

$$\mathcal{J}(n,\omega) = \prod_{k=1}^{N} \mathcal{J}_k(n,\omega) \qquad dv = \prod_{i=1}^{N} dv_i$$

$$\mathbb{P}\left[\,\omega(n) \,\middle|\, \omega_{-\infty}^{n-1}\,\right] = \int_{\mathcal{J}(n,\omega)} \frac{e^{\frac{-V^T\mathcal{Q}^{-1}(n-1,\omega)V}{2}}}{(2\pi)^{\frac{N}{2}}|\mathcal{Q}(n-1,\omega)|^{\frac{1}{2}}}dv$$

In the case without electric synapses:

$$\mathbb{P}\left[\,\omega(n) \,\middle|\, \omega_{-\infty}^{n-1}\,\right] = \prod_{k=1}^{N} \mathbb{P}\left[\,\omega_k(n) \,\middle|\, \omega_{-\infty}^{n-1}\,\right]$$

Gibbs distribution

The transition probabilities define a stochastic process in the set of rasters, where the probability of having a spike pattern depends on an infinite past. Such process are called "Chains with complete connections". Under suitable conditions define a unique probability distribution called Gibbs distribution.

$\phi(n,\omega) = \log \mathbb{P}\left[\,\omega(n) \,\middle|\, \omega_{-\infty}^{n-1}\,\right]$ is called Gibbs potential

$\mathbb{P}\left[\,\omega_m^n \,\middle|\, \omega_{-\infty}^{m-1}\,\right] = e^{\sum_{l=m}^{n}\phi(l,\omega)}$ is the conditional probability in terms of the Gibbs potential

Conclusions:

Including electric synapses spike statistics becomes indecomposable.

$$\mathbb{P}\left[\,\omega(n) \,\middle|\, \omega_{-\infty}^{n-1}\,\right] \neq \prod_{k=1}^{N} \mathbb{P}\left[\,\omega_k(n) \,\middle|\, \omega_{-\infty}^{n-1}\,\right]$$

Correlations are due to stimulus and dynamics.

Electric synapses have an effect in the memory of the system.

The Gibbs potential is largely more complex than the Ising Model used in retina spike train analysis.

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