Adaptive Full-Multigrid algorithms based on Riemannian metrics

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Abstract. *We study the combination of successive refinements with Full Multigrid Method. For each FMG phase, we build the new meshes by adaptive anisotropic refinements, using Riemannian metrics. Then, we present test cases solved by this method.*

Keywords: refinement; anisotropic; multigrids; metrics.

1 INTRODUCTION

A method which is often employed to gain in efficiency in PDE's resolution is the method of successive refinements. In this study, we will use this principle in the case of the Full Multigrid Method. We describe the operation of the obtained algorithm and present some test cases.

2 METHOD

Firstly, we solve partially the equation with an iterative method on a given mesh, then we refine the mesh to solve on the new mesh, using an interpolation of the solution which was already obtained and so on. Using it with the Full Multigrid Method permits to consider resolutions of complexity $O(N)$, with N the mesh's number of nodes [2].

A very important condition for the good operation of these methods is the numerical convergence of the different phases. That means we must have, for each mesh $\mathcal{M}_k$ used in each phase, the following property:

$$|u - u_k| \approx K(N_k)^{\frac{-\alpha}{dim}}$$

with α the scheme's convergence order and N_k the number of points of the mesh $\mathcal{M}_k$.

In practice, this numerical convergence is not systematically obtained which harms to the efficiency and to the reliability of the performed calculations. The recent developments in adaptive meshes have shown that one of the keys of a good numerical convergence resides in the use of adaptive anisotropic meshes, especially those based on Riemannian metrics[3][4].

The offered contribution presents a method which associate:
-the generation of the meshes from the phases of successive refinements by anisotropic adaptation which is based on a metric
-the generation of the coarse levels from each phase by an anisotropic derefinement which is based on a metric too.

After each phase of the FMG algorithm, we have an approximate solution which we use to determine a metric, calculated thanks to the Hessians of the solution.
We obtain the following metric:

$$\mathcal{M}(x,y) = \mathcal{R}^t(x,y) \begin{pmatrix} \frac{1}{\Delta\xi^2(x,y)} & 0 \\ 0 & \frac{1}{\Delta\eta^2(x,y)} \end{pmatrix} \mathcal{R}(x,y)$$

with:
$\Delta\xi(x,y) =$ mesh size in the first characteristic dimension
$\Delta\eta(x,y) =$ mesh size in the second characteristic dimension
$\mathcal{R}(x,y) =$ eigenvectors matrix.

We want a refined mesh. To obtain it, we impose a new number of points and we change the mesh sizes. In the isotropic case, we divide both $\Delta\xi(x,y)$ and $\Delta\eta(x,y)$ by 2. In the anistropic case, we divide only the greatest mesh size. For exemple, if $\Delta\eta(x,y) < \Delta\xi(x,y)$, we divide $\Delta\xi(x,y)$ and we obtain a new metric:

$$\mathcal{M}(x,y) = \mathcal{R}^t(x,y) \begin{pmatrix} \frac{1}{(\frac{\Delta\xi}{2}(x,y))^2} & 0 \\ 0 & \frac{1}{\Delta\eta^2(x,y)} \end{pmatrix} \mathcal{R}(x,y) = \mathcal{R}^t(x,y) \begin{pmatrix} \frac{4}{\Delta\xi^2(x,y)} & 0 \\ 0 & \frac{1}{\Delta\eta^2(x,y)} \end{pmatrix} \mathcal{R}(x,y)$$

This metric gives us a finer mesh which is adapted to the solution and will be used as the finest mesh of the next phase. Then, we build the coarse meshes of the new phase. If the finest mesh is $mesh_0$, for each integer l going from 1 to the number of coarse meshes used in the phase, we have

$$\mathcal{M}_{l-1}(x,y) = \mathcal{R}^t(x,y) \begin{pmatrix} \frac{1}{\Delta\xi^2(x,y)} & 0 \\ 0 & \frac{1}{\Delta\eta^2(x,y)} \end{pmatrix} \mathcal{R}(x,y)$$

and we deduce, from it,

$$\mathcal{M}_l(x,y) = \mathcal{R}^t(x,y) \begin{pmatrix} \frac{1}{(2\Delta\xi(x,y))^2} & 0 \\ 0 & \frac{1}{\Delta\eta^2(x,y)} \end{pmatrix} \mathcal{R}(x,y) = \mathcal{R}^t(x,y) \begin{pmatrix} \frac{1}{4\Delta\xi^2(x,y)} & 0 \\ 0 & \frac{1}{\Delta\eta^2(x,y)} \end{pmatrix} \mathcal{R}(x,y)$$

in the case where $\Delta\xi(x,y) < \Delta\eta(x,y)$.
That permits us to deduce $mesh_l$ from $mesh_{l-1}$.
So, we have a series of adapted meshes and we use it to solve the problem with multigrids. We do that for each phase.

To make the calculation totally reliable, we use a stopping test which is inspired by the controlled FMG method, described in [1]. The new algorithm will be evaluated on two families of test cases of elliptic model which correspond to:
-calculations of interface between two fluids
-equilibrium calculation of confined plasma.

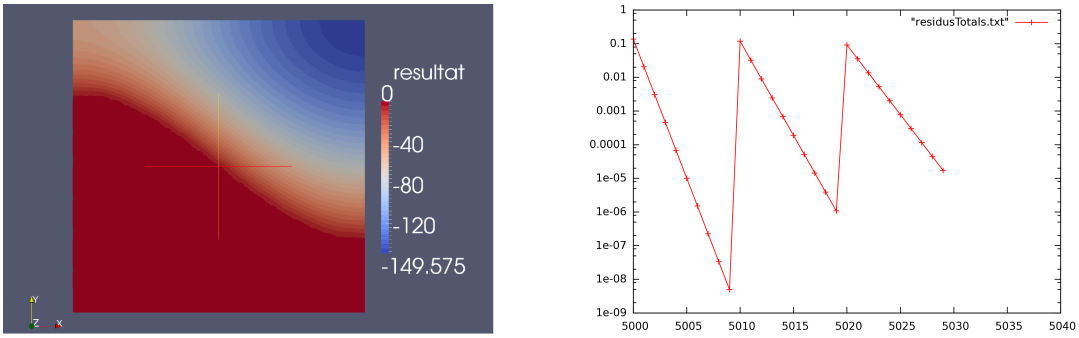

Figure 1: Pressure for a bi-fluidic interface and convergence curves for an FMG resolution.

3 CONCLUSIONS

We saw the combination of adaptive anisotropic refinements based on metrics and Full Multigrid Method gives good solutions for some anisotropic problems. We can now study how it works for more difficult problems like a bi-fluidic interface.

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