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1. Introduction. We report on our analysis of timing experiments performed on a parallel version of a Fast Fourier synthesis program.

Some other results, based on entirely different kinds of data, are given by K. L. Wang and D. C. Marinescu, "An Analysis of the Paging Activity of Parallel Programs," Technical Report CSD-TR-94-042, Computer Sciences Department, Purdue University, June 1994.

2. Outline of FFTsynth. Given a set of complex-valued structure factors (discrete Fourier coefficients)

$$F(h,k,l), \qquad |h| \le h_{max}, \quad |k| \le k_{max}, \quad 0 \le l \le l_{max},$$

FFTsynth does a 3-d discrete Fourier synthesis to compute values of electron density at grid points[1]:

$$\rho(x,y,z), \qquad x = 1,\ldots,N_x, \quad y = 1,\ldots,N_y, \quad z = 1,\ldots,N_z.$$

The number of complex valued structure factors is less than half the number of grid points:

$$2h_{max} + 1 \le N_x, \qquad 2k_{max} + 1 \le N_y, \qquad 2l_{max} + 1 \le N_z.$$

Because the electron density is real, only structure factors with nonnegative l are needed. The transformation is accomplished by carrying out three sets of 1-d FFT's: $k \to y$, $h \to x$, and $l \to z$.

[1]The grid point (x,y,z) corresponds to the point $([x-1]/N_x,\ [z-1]/N_z,\ [z-1]/N_z)$ in fractional coordinates.

The work is distributed as evenly as possible among P processors (nodes). First, each processor transforms k to y for all values of l and about[2] $\Delta h = (2h_{max} + 1)/P$ values of h. After inserting zeros for structure factors $F(h, k, l)$ with $k = k_{max} + 1, \ldots, N_y - k_{max}$, the program transforms an 'h-slab' having Δh planes of $(l_{max} + 1) \times N_y$ structure factors.

Next, the results are distributed ('exchanged') among different nodes so that each node gets a 'y-slab' (with zero fill where necessary). Then each node transforms h to x for all values of l and its allocated $\Delta y = (N_y)/P$ planes in its y-slab. Finally, l is transformed to z for all N_x values of x and Δy values of y.

If the amount of the collective local storage of the nodes is large enough so that all the data can be put into the nodes at one time, and if there is about an equal amount of buffer space, then the exchange of the results after the transformation $k \to y$ can be done by 'message passing' among nodes; we call this 'inter-nodal exchange'; otherwise, a scratch-file on disk must be used, which takes much more execution time than inter-nodal exchange.

The local memory of each node must hold the operating system and the program; if the amount of memory remaining is large enough to hold

$$[N_y \times (l_{max} + 1) \times \Delta h] + [2(l_{max} + 1) \times \Delta h \times \Delta y] + [N_x \times \Delta y \times N_z/2]$$

complex values, then inter-nodal exchange can be used. In the display above, the first term is the size of an 'h-slab' of structure factors. The third term is the size of a 'y-slab' of values resulting after the transformations $h \to x$ and $l \to z$. The second term is the size of a pair of buffers: one is used as an output buffer and the second is used as an input buffer; the space for these is small compared to the space needed for an h-slab or for a y-slab.

When inter-nodal exchange is possible, the action of Node q is:

1. Node q fills its h-slab with structure factors read from the Data-Input-File on disk.

2. Node q transforms its h-slab: $k \to y$.

[2]The slab width Δh is the same for each node only if $2h_{max} + 1$ is divisible by P.

3. For $p = 1, \ldots, P$, inter-nodal exchange:

   (a) Node q fills its output buffer with as many of its values as is needed by Node p.

   (b) Node q's output buffer is sent to node p as a message.

   (c) Node q fills its input buffer with a message from Node p.

   (d) Node q moves the data from its input buffer to the appropriate place in its y-slab.

4. When the exchange is complete, Node q has its y-slab filled; it carries out the two transformations $h \to x$ and $l \to z$.

5. Node q writes the electron density it has computed onto the Data-Output-File.

As can be seen from Step 3, Node q must have space for its h-slab, its y-slab, and two small buffers for input and output during the exchange.

To make this process work, it must be synchronized. All processors must complete step 2 before the exchange takes place. Then a pair of processors exchange data, while other pairs are doing similar exchanges. Because a processor does not send a message to itself, the exchange is done in $P - 1$ simultaneous pairwise exchanges on a hypercube. One expects such an exchange to take a little more time on a 2-d mesh than on a hypercube because the routing of a group of messages on a mesh is more complicated than on a hypercube.

If the memory is not large enough so that such an inter-nodal exchange can take place, then intermediate results are stored on a scratch-file on disk and steps 3 and 4 are replaced with

3. For $p = 1, \ldots, P$, disk scratch-file exchange:

   (a) Node q fills its output buffer with as many results as is needed to be transported to node p.

   (b) Node q's writes its output buffer onto a scratch-file on disk.

4. After all nodes have completed step 3, Node q reads data from disk and puts it into its y-slab and transforms $h \to x$ and $l \to z$.

As in the case of inter-nodal exchange, use of a scratch-file requires synchronization.

In this scratch mode of operation, space for an h-slab is needed during Steps 1 and 2; each node might have to transform several h-slabs. After all the disk-writes are complete, then space for an h-slab is no longer needed and that space can be used for a y-slab. Consequently, in this 'scratch exchange', a larger problem can be processed for a given number of nodes; the minimum amount of memory required is that for a slab containing a single plane and a small buffer.

Writing to and reading from disk takes considerably longer time than the inter-nodal exchange which involves only communication among nodes. Not only is the transfer-time between node and disk much longer than the transfer-time between node and node, but also there are only a few nodes which can communicate between nodes and disk, and all communications to and from disk must be routed through these special input/output (`I/O`) nodes.

3. Test cases. We collected data for two test cases. The pertinent parameters for our study are listed below.

Human Rhinovirus 16 ('HRV16'):

Parameters for structure factors: $h_{max} = 106$, $k_{max} = 102$, $l_{max} = 98$;
Number of complex structure factors: $(2h_{max}+1)(2k_{max}+1)(l_{max}+1) = 4,322,835$
Parameters for unit cell partition: $N_x = 360$, $N_y = 352$, $N_z = 336$;
Number of real electron density values: $N_x \times N_y \times N_z = 42,577,920$;
Size of plane for $k \to y$: $N_y \times (l_{max}+1) = 34{,}848$ (complex);
Size of plane for $h \to x$: $N_x \times (l_{max}+1) = 35{,}640$ (complex);
Size of plane for $l \to z$: $N_x \times N_z = 120{,}960$ (real).

Coxsackievirus B3 ('CVB3'):

Parameters for structure factors: $h_{max} = 143$, $k_{max} = 75$, $l_{max} = 130$;
Number of complex structure factors: $(2h_{max}+1)(2k_max+1)(l_{max}+1) = 11,310,957$;
Parameters for unit cell partition: $N_x = 480$, $N_y = 256$, $N_z = 432$;
Number of real electron density values: $N_x \times N_y \times N_z = 53,084,160$;
Size of plane for $k \to y$: $N_y \times (l_{max}+1) = 33{,}536$ (complex);
Size of plane for $h \to x$: $N_x \times (l_{max}+1) = 62{,}880$ (complex);

Size of plane for $l \to z$: $N_x \times N_z = 207{,}360$ (real).

Computers:

128-Node Intel iPSC/860 hypercube at NIH; 8 I/O nodes; 16 Mbytes memory per node;
56-Node Intel Paragon 2-d mesh at Cal Tech; 3 I/O nodes; 32 Mbytes memory per node;
140-Node Intel Paragon 2-d mesh at Purdue; 14 I/O nodes; 32 Mbytes memory per node;
512-Node Intel Paragon 2-d mesh at Cal Tech; 22 I/O nodes; 32 Mbytes memory per node.

Each node on an Intel iPSC/860 hypercube has 16 Mbytes of local memory; nodes on an Intel Paragon 2-d mesh have 32 Mbytes. Thus some problems which require use of scratch-disk on an iPSC/860 can use inter-nodal exchange on a Paragon.

Because of the amount of memory on the nodes, inter-nodal exchange could be used for the HRV16 example on 8 or more nodes of a Paragon, but it required at least 16 nodes on an iPSC/860. Similarly, CVB3 could be run on 16 or more nodes of a Paragon, but it required at least 32 nodes on an iPSC/860.

4. Timings. We recorded times on each node at several points during execution of the program. All times below are given in seconds.

A time was recorded on a node by a subroutine call. The subroutine incremented a counter, stored the time elapsed since the node started, stored the time elapsed since the previous call of the subroutine, and stored a message identifying the time. The last call to the subroutine caused the times and messages to be printed, one line for each of call of the subroutine.

The output-file contained the usual output from Node 0 and interspersed among these lines were the lines printed at the last call of the timing subroutine by each node. The timing statements were extracted from the output, sorted, and processed.

The times recorded are described in Table 1.

Table 1. Times recorded during experiments.

Startup(p):	The time between the start of Node p and its initial reading of data from the Data-Input-File.
Read(p):	The time to read all the structure factor data from the Data-Input-File.
First FFT(p):	time for $k \to y$ for Node p's h-slab.
Exchange(p):	The time required for the exchange of data ('sending message' plus 'receiving message' — or — 'writing to disk' plus 'reading from disk').
Second FFT(p):	$h \to y$, for Node p's y-slab.
Third FFT(p):	$l \to z$, for Node p's y-slab.
Write(p):	The time required to write the electron density values to the Data-Output-File.
Total(p):	The time between the beginning of execution of Node p and its termination

Minimum and maximum values (with respect to p for a given run) were determined for each item. To reduce the quantity of information, we formed averages. For example, the 'average' start-up time is

$$\langle Startup \rangle = \frac{1}{P} \sum_p Startup(p)$$

Similarly,

$$\langle Total \rangle = \frac{1}{P} \sum_p Total(p),$$

whereas

$$Total = \max_p \{ Total(p) \}$$

is the total execution time. We combined the FFT times:

$$\langle \mathrm{FFT} \rangle = \frac{1}{P} \sum_p [First\ \mathrm{FFT}(p) + Second\ \mathrm{FFT}(p) + Third\ \mathrm{FFT}(p)]$$

We also calculated the percent of time used by these different tasks, defined, for example, by

$$100 \times \langle Startup \rangle / \langle Total \rangle$$

Part 2. Conclusions from the experimental results.

5. Parallel processing reduces total execution time. The observed values of the percents, mentioned immediately above, show that the percent of time devoted to FFT decreases as the number of nodes increases. Moreover, they also show that the 'overhead' time required for start-up, input, exchange, output, etc., dominate the execution time — more time is spent doing this 'overhead' than in spent doing the actual FFT calculations. Nevertheless, the *total execution time* does decrease as the number of nodes increases (up to a certain point depending on the size of the problem). Thus, in spite of the amount of overhead time, one does obtain results of FFT calculations quicker by using several nodes. For a specific problem, its size, kind of computer, and the state of operation (e.g., the number of other users) determine the 'optimal number of nodes': the number of nodes which result in the smallest amount of execution time.

Our experimental results indicate that with inter-nodal exchange the optimal numbers of nodes are approximately those given in Table 2. For the CVB3 128-Node iPSC/860, the time was 46.5 seconds using 32 nodes as well as 64 nodes.

Table 2. Estimate of optimal number of nodes.

HRV16 128-node iPSC/860:	32 nodes in 27 seconds
HRV16 56-node Paragon:	16 nodes in 57 seconds
HRV16 140-node Paragon:	32 nodes in 28 seconds
HRV16 512-node Paragon:	16 nodes in 37 seconds
CVB3 128-node iPSC/860:	32 or 64 nodes in 47 seconds
CVB3 56-node Paragon:	32 nodes in 116 seconds
CVB3 140-node Paragon:	64 nodes in 37 seconds
CVB3 512-node Paragon:	16 nodes in 53 seconds

6. FFT times were consistent and scalable. The only times which showed *consistent variation* as the number of processors changed on a fixed machine were the FFT times.

We divided each of the three FFT times on a node by the number of planes it transformed – the number of planes can be different for different nodes. For example (see Table 3), for HRV16 with $2h_{max} + 1 = 213$ and 16 nodes, Nodes 0 through 4 transformed $\Delta y = 14$ planes and Nodes 5 through 15 transformed $\Delta y = 13$ planes. We divided the measured times for the first FFT times on Nodes 0–4 by 14 and the times on Nodes 5–15 by 13. We then averaged these 16 ratios (sum divided by 16). The minimum, the average, and the maximum ratios are listed in Table 3. For a given FFT ('first', 'second', or 'third') are almost identical, independent of the number of nodes.

For example, the times for the first FFT of HRV16 in Table 3, the smallest value, 0.41893, differs from the largest, 0.42137, by 0.00244 and the percent difference, $100 \times 0.00244/([0.41893 + 0.42137]/2)$, is 0.21%. The results listed in Table 3 are typical of those in all of our runs: the difference between minimum and maximum FFT time was negligible, independent of the number of nodes on a given machine.

The times per plane of the 'first', 'second', and 'third' is different from one another because the number of values being transformed is different for each of these. Also, the time to execute a 1-*d* FFT depends on the prime factorization of the number of items in the transformation. For our test cases, these are

	HRV16	CVB3
First FFT:	$N_y 352 = 2^4 \times 11$	$N_y = 256 = 2^8$
Second FFT:	$N_x = 360 = 2^3 \times 3^2 \times 5$	$N_x = 480 = 2^5 \times 3 \times 5$
Third FFT:	$N_z = 336 = 2^4 \times 3^2 \times 7$	$N_z = 432 = 2^4 \times 3^3$

Table 3. HRV16, 128 Node NIH hypercube FFT time per plane.

	Number of planes	Minimum	Average	Maximum
			First FFT	
16 Nodes	14 for Nodes 0–4; 13 for 5–15	0.41893	0.41935	0.41962
32 Nodes	7 for Nodes 0–20; 6 for 21–31	0.41957	0.42010	0.42050
64 Nodes	4 for Nodes 0–20; 3 for 21–63	0.42025	0.42102	0.42133
			Second FFT	
16 Nodes	6 for Nodes 0–8; 5 for 9–15	0.27217	0.27260	0.27283
32 Nodes	3 for Nodes 0–24; 2 for 25–31	0.27167	0.27251	0.27300
64 Nodes	2 for Nodes 0–24; 1 for 25–63	0.27200	0.27400	0.27500
			Third FFT	
16 Nodes	6 for Nodes 0–8; 5 for 9–15	0.80350	0.80438	0.80480
32 Nodes	3 for Nodes 0–24; 2 for 25–31	0.80533	0.80628	0.80750
64 Nodes	2 for Nodes 0–24; 1 for 25–63	0.79800	0.80277	0.80750

Furthermore, the FFT time is the *only* one of the times which is scalable. Table 4 lists ⟨FFT⟩ times and these times multiplied by P.

Table 4. 140-Node Paragon, ⟨FFT⟩ times

Nodes	16	32	48	64	96	128
HRV16	10.455	5.237	3.493	2.614	1.734	1.295
(time $\times P$)	167.28	167.58	167.66	167.30	166.46	165.76
CVB3	16.471	8.234	5.486	4.117	2.748	2.045
(time $\times P$)	263.54	263.49	263.33	263.49	263.81	261.76

These results show that, for this set of data, the following accurately model the experimental results:

$$\text{HRV16 } \langle \texttt{FFT} \rangle \text{ time} \approx 168./P, \qquad \text{CVB3 } \langle \texttt{FFT} \rangle \text{ time} \approx 263./P.$$

Extrapolating these approximations, we find that a single node would require 168 seconds and 263 seconds to do just the FFT's for these two test cases. But, in addition to the transformation times, there is a great deal of overhead: reading and writing the data and exchange of results among the nodes takes a large amount of time. Nevertheless, use of several processors can reduce the total execution time. For example, Tables 15 and 22 show that the entire calculation (FFT plus overhead) can be done in about 30 seconds using 48 nodes for HRV16 and about 37 seconds using 64 nodes for the CVB3 problem. Table 4 shows that the ⟨FFT⟩ times are only 3.493 seconds for HRV16 and 4.117 seconds for CVB3. That is, about 90% of the execution time is overhead, because

$$100 \times (30 - 3)/30 = 90\% \qquad \text{and} \qquad 100 \times (37 - 4)/37 = 89\%.$$

7. The inter-nodal exchange takes an order of magnitude less time than the use of a disk scratch-file. Table 5 lists ratios of scratch-file and inter-nodal exchange times. Not only are these ratios large, but their sizes increase as the number of nodes increases. That is, the larger the number of nodes, the greater is the savings in execution time when inter-nodal exchange is used instead of scratch-file exchange.

Table 5. Ratio (Scratch)/(Inter-nodal)

Nodes	NIH 128 hypercube		Cal Tech 512 Paragon	
8			58.573/5.064 = 11.6	
16	24.873/3.357 = 7.41		58.050/3.365 = 17.3	
32	44.174/2.407 = 18.35		103.692/3.716 = 27.9	
64	69.278/1.857 = 37.31		220.986/3.721 = 59.4	
	$100 \times \langle Exchange \rangle / \langle Total \rangle$			
Nodes	Scratch	Inter-nodal	Scratch	Inter-nodal
8			63.5%	13.2%
16	45.4%	10.5%	68.3%	12.1%
32	69.6%	11.2%	76.7%	12.0%
64	83.1%	10.5%	78.0%	6.6%

The table also lists the percent of $\langle Total \rangle$ used by the exchanges on the NIH 128 node hypercube. For the scratch-file, the amount of time is between 45% and 83% of the average total time and thus this scratch exchange *dominates* the execution time; the fraction of time spent in the exchange increases as the number of nodes increase. The inter-nodal exchange takes only 7% to 11%; in contrast to the scratch-file exchange, the larger the number of nodes, the smaller is the percent of time devoted to the exchange.

Results from other runs show that initially the inter-nodal exchange time decreases and then increases as the number of nodes increases; see, for example, HRV16 on the 140-node Paragon results in Table 15. Nevertheless, in all comparisons, it is clear that execution time is significantly reduced by using inter-nodal rather than scratch-file exchange.

8. Except for ⟨FFT⟩, times are irreproducible. When the program is run several times with the same input data, then, with the exception of ⟨`FFT`⟩, the measured times differ by large amounts from run to run. This is probably due to contention with other users for available recourses. Table 6 lists times for the same problem run several times. It also lists the percent variation: for example for $\langle Startup \rangle$ with 64 nodes:

$$100 \times (8.739 - 6.239)/([6.239 + 8.739 + 7.090]/3) = 33.99$$

Only for ⟨`FFT`⟩ is the variation negligible: less than 0.5%. The other times vary form 9% to 100%

from one run to another. In particular, for these experiments, the variation in the crucial total execution time, *Total*, is 53% and 42% for 64 and for 128 nodes, respectively.

Table 6. CVB3 inter-nodal on 140-Node Paragon; repeated runs

Nodes	64	64	64	%*	128	128	128	%*
$\langle Startup \rangle$	6.239	8.739	7.090	33.99	22.814	20.383	20.528	9.24
$\langle Read \rangle$	1.460	1.440	1.600	10.68	3.634	1.758	4.205	76.50
$\langle \texttt{FFT} \rangle$	4.117	4.119	4.120	0.06	2.045	2.052	2.053	0.39
$\langle Exchange \rangle$	2.890	2.985	3.188	9.86	6.838	4.077	4.707	53.02
$\langle Write \rangle$	8.735	11.067	20.022	85.03	11.536	8.839	23.416	99.86
$\langle Total \rangle$	23.603	28.522	36.309	43.10	46.999	37.258	55.171	38.54
$Total$	36.674	41.627	61.529	53.33	75.325	52.595	81.725	41.68
$100 \times \langle Startup \rangle / \langle Total \rangle$	26.435	30.638	19.528	43.51	48.543	54.708	37.209	37.38
$100 \times \langle Read \rangle / \langle Total \rangle$	6.186	5.050	4.406	34.14	7.732	4.717	7.622	45.07
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	17.443	14.441	11.347	42.30	4.351	5.507	3.721	39.46
$100 \times \langle Exchange \rangle / \langle Total \rangle$	12.243	10.466	8.780	32.99	14.550	10.943	8.532	53.06
$100 \times \langle Write \rangle / \langle Total \rangle$	37.006	38.802	55.145	41.55	24.546	23.725	42.443	61.90

* 100×(maximum − minimum)/(sum/3)

9. Least squares approximations. To model the relationship between exectution time, we assumed $\langle time \rangle(P) \approx A\,P^{\nu}$ and determined the two coefficients, A and ν by least squares. That is, values of A and ν were determined so that

$$R^2 = 100^2 \times \left\{ \sum_P [\langle time \rangle(P) - A\,P^{\nu}]^2 \right\} \bigg/ \left\{ \sum_P [\langle time \rangle(P)]^2 \right\}$$

is minimized. This reduces to a linear system when expressed in terms of logarithms:

$$r^2 = \sum_P [\log(\langle time(P) \rangle) - \log(A) - \nu \log(P)]^2$$

Results are listed for a three examples in Tables 7, 8, and 9. Listed are the experimentally observed times ("Obser."), the approximate values ("Fit"), the differences (Fit − Obser.), and the percent differences (% Diff.). Clearly, such an approximation is inappropriate for those times which do not vary monotonically. The only time for which the approximation gives a good fit (less than 0.5%) is ⟨FFT⟩.

Table 7. HRV16 inter-nodal exchange 140-Node Paragon. $Fit = A\,P^{\nu}$

⟨*Startup*⟩ $A = 0.0348$, $\nu = 1.3434$, $R = 13.63\%$ ‖ ⟨*Read*⟩ $A = 5.8183$, $\nu = -0.2675$, $R = 16.05\%$

Nodes	Obser.	Fit	Diff.	% Diff.	Obser.	Fit	Diff.	% Diff.
16	1.8550	1.4415	−0.4135	−22.2928	2.9390	2.7710	−0.1680	−5.7147
32	2.9900	3.6577	0.6677	22.3316	1.7830	2.3020	0.5190	29.1082
48	4.5940	6.3063	1.7123	37.2715	2.6930	2.0653	−0.6277	−23.3068
64	9.2920	9.2814	−0.0106	−0.1139	1.8030	1.9123	0.1093	6.0647
96	20.0500	16.0021	−4.0479	−20.1892	1.7910	1.7158	−0.0752	−4.2014
128	24.5000	23.5515	−0.9485	−3.8713	1.5070	1.5887	0.0817	5.4181

⟨FFT⟩ $A = 169.8650$, $\nu = -1.0043$, $R = 0.31\%$ ‖ ⟨*Exchange*⟩ $A = 1.1650$, $\nu = 0.2318$, $R = 17.87\%$

Nodes	Obser.	Fit	Diff.	% Diff.	Obser.	Fit	Diff.	% Diff.
16	10.4550	10.4908	0.0358	0.3421	2.6190	2.2151	−0.4039	−15.4232
32	5.2370	5.2298	−0.0072	−0.1379	2.4770	2.6011	0.1241	5.0099
48	3.4930	3.4804	−0.0126	−0.3594	2.4400	2.8574	0.4174	17.1059
64	2.6140	2.6071	−0.0069	−0.2637	2.6390	3.0544	0.4154	15.7407
96	1.7340	1.7350	0.0010	0.0602	3.0830	3.3554	0.2724	8.8340
128	1.2950	1.2997	0.0047	0.3610	4.6990	3.5867	−1.1123	−23.6710

⟨*Write*⟩ $A = 2.7457$, $\nu = 0.0915$, $R = 12,83\%$ ‖ ⟨*Total*⟩ $A = 7.3554$, $\nu = 0.2866$, $R = 21.16\%$

Nodes	Obser.	Fit	Diff.	% Diff.	Obser.	Fit	Diff.	% Diff.
16	4.0910	3.5391	−0.5519	−13.4902	22.2350	16.2832	−5.9518	−26.7676
32	3.5390	3.7710	0.2320	6.5549	16.2520	19.8620	3.6100	22.2128
48	3.5030	3.9136	0.4106	11.7210	16.9240	22.3098	5.3858	31.8235
64	3.2840	4.0180	0.7340	22.3516	19.7760	24.2274	4.4514	22.5091
96	4.7130	4.1700	−0.5430	−11.5218	31.4890	27.2132	−4.2758	−13.5788
128	4.7730	4.2813	−0.4917	−10.3025	36.9140	29.5522	−7.3618	−19.9431

Total $A = 9.0513$, $\nu = 0.3558$, $R = 19.89\%$

Nodes	Obser.	Fit	Diff.	% Diff.
16	30.9490	24.2721	−6.6769	−21.5740
32	28.1580	31.0603	2.9023	10.3071
48	29.0500	35.8801	6.8301	23.5117
64	30.1000	39.7470	9.6470	32.0497
96	51.0460	45.9148	−5.1312	−10.0521
128	64.5510	50.8631	−13.6879	−21.2048

Table 8. HRV16 inter-nodal exchange 512-Node Paragon. $Fit = A\,P^{\nu}$

$\langle Startup \rangle$: $A = 0.2944$, $\nu = 1.2139$, $R = 14.53\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	4.1330	3.6749	−0.4581	−11.0832
16	8.4290	8.5244	0.0954	1.1323
32	17.6800	19.7735	2.0935	11.8409
48	28.6860	32.3473	3.6613	12.7634
64	43.9670	45.8670	1.9000	4.3213
128	125.8730	106.3939	−19.4791	−15.4752

$\langle Read \rangle$: $A = 26.5834$, $\nu = -0.8345$, $R = 5.10\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	4.5500	4.6877	0.1377	3.0261
16	2.7990	2.6287	−0.1703	−6.0843
32	1.5150	1.4741	−0.0409	−2.7004
48	0.8900	1.0509	0.1609	18.0813
64	0.9240	0.8266	−0.0974	−10.5389
128	0.4610	0.4635	0.0025	0.5513

$\langle \mathrm{FFT} \rangle$: $A = 147.4722$, $\nu = -0.9977$, $R = 0.48\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	18.4240	18.5224	0.0984	0.5343
16	9.3050	9.2760	−0.0290	−0.3117
32	4.6620	4.6454	−0.0166	−0.3560
48	3.1070	3.0998	−0.0072	−0.2309
64	2.3300	2.3264	−0.0036	−0.1542
128	1.1590	1.1651	0.0061	0.5229

$\langle Exchange \rangle$: $A = 1.7823$, $\nu = 0.2870$, $R = 45.22\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	5.0640	3.2369	−1.8271	-36.0796
16	3.3650	3.9492	0.5842	17.3625
32	3.7160	4.8183	1.1023	29.6642
48	4.2780	5.4128	1.1348	26.5275
64	3.7210	5.8786	2.1576	57.9855
128	13.9460	7.1723	−6.7737	-48.5709

$\langle Write \rangle$: $A = 3.5875$, $\nu = 0.0614$, $R = 24.79\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	5.8300	4.0760	−1.7540	−30.0860
16	3.4930	4.2532	0.7602	21.7630
32	3.1520	4.4381	1.2861	40.8020
48	3.7020	4.5499	0.8479	22.9049
64	5.3020	4.6310	−0.6710	−12.6554
128	6.2180	4.8323	−1.3857	−22.2847

$\langle Total \rangle$: $A = 9.0908$, $\nu = 0.4673$, $R = 38.01\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	38.2540	24.0209	−14.2331	−37.2069
16	27.7650	33.2087	5.4437	19.6062
32	31.0650	45.9107	14.8457	47.7890
48	41.0250	55.4876	14.4626	35.2531
64	56.5390	63.4711	6.9321	12.2607
128	147.8840	87.7482	−60.1358	−40.6642

$Total$: $A = 12.8758$, $\nu = 0.4537$, $R = 32.74\%$

Nodes	Obser.	Fit	Diff.	% Diff.
8	50.8840	33.0758	-17.8082	-34.9976
16	37.0220	45.2991	8.2771	22.3573
32	44.9670	62.0395	17.0725	37.9668
48	54.6540	74.5696	19.9156	36.4394
64	82.1010	84.9664	2.8654	3.4901
128	180.3010	116.3659	-63.9351	-35.4602

Table 9. CVB3 INter-nodal exchange 140-Node Paragon. $Fit = A\,P^{\nu}$

	⟨*Startup*⟩ $A = 0.0401$, $\nu = 1.3209$, $R = 35.66\%$				⟨*Read*⟩ $A = 7.2567$, $\nu = -0.2641$, $R = 32.86\%$			
Nodes	Obser.	Fit	Diff.	% Diff.	Obser.	Fit	Diff.	% Diff.
16	1.8640	1.5608	−0.3032	−16.2686	4.4640	3.4890	−0.9750	−21.8411
32	4.0590	3.8991	−0.1599	−3.9404	3.6230	2.9053	−0.7177	−19.8091
48	4.9900	6.6613	1.6713	33.4920	1.6700	2.6103	0.9403	56.3030
64	6.2390	9.7406	3.5016	56.1242	1.4600	2.4193	0.9593	65.7036
96	29.7540	16.6411	−13.1129	−44.0711	1.9560	2.1736	0.2176	11.1236
128	22.8140	24.3339	1.5199	6.6619	3.6340	2.0145	−1.6195	−44.5641

	⟨FFT⟩ $A = 265.1912$, $\nu = -1.0019$, $R = 0.11\%$				⟨*Exchange*⟩ $A = 1.8256$, $\nu = 0.1773$, $R = 30.32\%$			
Nodes	Obser.	Fit	Diff.	% Diff.	Obser.	Fit	Diff.	% Diff.
16	16.4710	16.4884	0.0174	0.1058	3.9970	2.9843	−1.0127	−25.3370
32	8.2340	8.2335	−0.0005	−0.0061	3.1590	3.3744	0.2154	6.8193
48	5.4860	5.4848	−0.0012	−0.0215	2.6940	3.6259	0.9319	34.5907
64	4.1170	4.1114	−0.0056	−0.1361	2.8900	3.8156	0.9256	32.0265
96	2.7480	2.7388	−0.0092	−0.3331	3.6660	4.0999	0.4339	11.8355
128	2.0450	2.0530	0.0080	0.3925	6.8380	4.3144	−2.5236	−36.9058

	⟨*Write*⟩ $A = 11.0899$, $\nu = -0.0466$, $R = 17.59\%$				⟨*Total*⟩ $A = 19.5060$, $\nu = 0.1320$, $R = 26.56\%$			
Nodes	Obser.	Fit	Diff.	% Diff.	Obser.	Fit	Diff.	% Diff.
16	11.6980	9.7458	−1.9522	−16.6886	38.5980	28.1249	−10.4731	−27.1338
32	9.0770	9.4360	0.3590	3.9550	28.4910	30.8193	2.3283	8.1719
48	7.0400	9.2594	2.2194	31.5252	22.1240	32.5135	10.3895	46.9601
64	8.7350	9.1361	0.4011	4.5915	23.6030	33.7717	10.1687	43.0823
96	8.1900	8.9651	0.7751	9.4635	46.4960	35.6282	−10.8678	−23.3736
128	11.5360	8.8457	−2.6903	−23.3211	46.9990	37.0070	−9.9920	−21.2600

	Total $A = 20.7807$, $\nu = 0.2254$, $R = 22.43\%$			
Nodes	Obser.	Fit	Diff.	% Diff.
16	49.1370	38.8228	−10.3142	−20.9907
32	43.9280	45.3883	1.4603	3.3242
48	38.3250	49.7322	11.4072	29.7643
64	36.6740	53.0641	16.3901	44.6912
96	73.3990	58.1426	−15.2564	−20.7856
128	75.3250	62.0379	−13.2871	−17.6397

10. ⟨*Startup*⟩ times grow rapidly.

Ordering the exponents ν in Table 7, we have

$$-1.0043, \quad -0.2675, \quad 0.0915, \quad 0.2318, \quad 0.2866 \quad 0.3558, \quad 1.3434.$$

The first, -1.0043, for ⟨FFT⟩ together with the corresponding small percent differences shows that this time is inversely proportional to P for this set of data — this conclusion holds also for all of the sets of experimental data we have collected. The last value, 1.3434 indicates that ⟨*Startup*⟩ increases at a *faster* rate than P does. This model, $0.0348\,P^{1.34}$, for ⟨*Startup*⟩ is very much less accurate than the model $265\,P^{-1.00}$ for ⟨FFT⟩.

Nevertheless, as shown in Table 10, ⟨*Startup*⟩ does grow as P increases and in some cases the increase is very great, especially on the 512 node Paragon. Because of this rather unexpected experimental results, we describe in detail the nature of this measured time.

Table 10. ⟨*Startup*⟩ times: H = HRV16, C = CVB3, I = Inter-nodal, S = Scratch-file, Pa = Paragon, Hy = Hypercube

	Nodes	8	16	32	48	64	96	128
H I 56-node-Pa		2.085	1.814	3.957		6.950		
H S 56-node-Pa		2.166	2.745	4.465				
H I 128-node-Hy			3.733	4.557		3.869		
H S 128-node-Hy			3.566	4.269		4.376		
H I 140-node-Pa			1.855	2.990	4.594	9.292	20.050	24.500
H I 512-node-Pa		4.133	8.429	17.680	28.686	43.967		125.873
H S 512-node-Pa		5.070	11.657	22.673		57.372		
C I 56-node-Pa			3.203	4.392	13.710			
C S 56-node-Pa		2.287	2.199	6.110				
C I 128-node-Hy				3.687		4.979		4.688
C S 128-node-Hy			3.973	4.324		4.497		
C I 140-node-Pa			1.864	4.059	4.990	6.239	29.754	22.814
C I 512-node-Pa			6.892	18.868	27.093	55.979		522.844
C S 512-node-Pa		5.018	10.560	24.519		55.818		

When a job is sent to one of the parallel computers, a certain number of nodes is requested and a 'partition' of the machine with this many nodes is allocated to the job. The same program is loaded onto each of the nodes. The programs wait until all nodes are loaded before they are started. When the program starts, a 'clock' is turned on. Node 0 reads the 'Control-Input-file' from disk. This is a small amount of information, including file names for the Data-Input and the Data-Output files, and a dozen or so numerical values which control the operation of the program. Node 0 also opens and reads the 65536-byte header of the Data-Input file, which is on disk. After extracting some information from the header and doing some initialization, Node 0 sends ('broadcasts') about 3000 bytes of information to all the other nodes. After a node receives the information, it opens files and calls the subroutine which performs the first FFT; after some simple initializations, it begins to read data from the Data-Input file on disk. Immediately before initiating this read from disk, the timing subroutine is called to record the time; this is the $Startup(p)$ time for the Node p. If it happens that all of the nodes attempt to read information from the disk at the same time, then a particular node will have to wait until it has free access to the disk. Thus, it might be that this kind of contention causes blocking and, consequently, a large $\langle Startup \rangle$ time when there are a large number of nodes.

11. Additional tables. Tables 11–24 contain data from most of the runs we have made. These are followed (Tables 25–26) containing data which show samples of the extreme times and the average times for each of the calls to the timing subroutines for the 128-Node iPSC/860 hypercube at NIH.

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Table 11. HRV16 inter-nodal exchange on 128-Node hypercube

Nodes	16	32	64
$\langle Startup \rangle$	3.733	4.557	3.869
$\langle Read \rangle$	4.932	3.285	2.724
$\langle \texttt{FFT} \rangle$	11.573	5.796	2.900
$\langle Exchange \rangle$	3.357	2.407	1.857
$\langle Write \rangle$	7.279	4.952	5.257
$\langle Total \rangle$	32.005	21.581	17.646
$Total$	40.067	27.386	30.535
$100 \times \langle Startup \rangle / \langle Total \rangle$	11.665	21.117	21.927
$100 \times \langle Read \rangle / \langle Total \rangle$	15.410	15.220	15.439
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	36.160	26.859	16.432
$100 \times \langle Exchange \rangle / \langle Total \rangle$	10.488	11.152	10.525
$100 \times \langle Write \rangle / \langle Total \rangle$	22.743	22.948	29.793

Table 12. HRV16 scratch-file exchange on 128-Node hypercube

Node	16	32	64
$\langle Startup \rangle$	3.566	4.269	4.376
$\langle Read \rangle$	4.824	3.253	2.917
$\langle \texttt{FFT} \rangle$	11.573	5.796	2.900
$\langle Exchange \rangle$	24.873	44.174	69.278
$\langle Write \rangle$	8.535	4.857	2.987
$\langle Total \rangle$	54.823	63.461	83.347
$Total$	67.643	75.892	99.565
$100 \times \langle Startup \rangle / \langle Total \rangle$	6.504	6.727	5.251
$100 \times \langle Read \rangle / \langle Total \rangle$	8.799	5.126	3.500
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	21.110	9.133	3.479
$100 \times \langle Exchange \rangle / \langle Total \rangle$	45.369	69.608	83.119
$100 \times \langle Write \rangle / \langle Total \rangle$	15.568	7.653	3.584

Table 13. HRV16 inter-nodal exchange on 56-Node Paragon

Nodes	8	16	32	48
$\langle Startup \rangle$	2.085	1.814	3.957	6.950
$\langle Read \rangle$	5.807	3.801	3.210	2.335
$\langle \texttt{FFT} \rangle$	20.920	10.457	5.259	3.489
$\langle Exchange \rangle$	5.292	3.832	3.582	3.787
$\langle Write \rangle$	21.414	15.963	17.564	21.205
$\langle Total \rangle$	56.386	36.664	34.237	38.381
$Total$	83.915	57.122	64.022	76.281
$100 \times \langle Startup \rangle / \langle Total \rangle$	3.698	4.947	11.558	18.107
$100 \times \langle Read \rangle / \langle Total \rangle$	10.298	10.367	9.376	6.083
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	37.101	28.522	15.360	9.091
$100 \times \langle Exchange \rangle / \langle Total \rangle$	9.386	10.452	10.461	9.868
$100 \times \langle Write \rangle / \langle Total \rangle$	37.979	43.538	51.302	55.250

Table 14. HRV16 scratch-file exchange on 56-Node Paragon

Nodes	8	16	32
$\langle Startup \rangle$	2.166	2.745	4.465
$\langle Read \rangle$	6.029	3.974	3.748
$\langle \texttt{FFT} \rangle$	20.930	10.459	5.275
$\langle Exchange \rangle$	83.585	77.970	142.796
$\langle Write \rangle$	19.240	17.501	19.087
$\langle Total \rangle$	133.187	113.105	175.673
$Total$	164.125	142.214	203.654
$100 \times \langle Startup \rangle / \langle Total \rangle$	1.626	2.427	2.542
$100 \times \langle Read \rangle / \langle Total \rangle$	4.527	3.513	2.133
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	15.715	9.247	3.003
$100 \times \langle Exchange \rangle / \langle Total \rangle$	62.757	68.936	81.285
$100 \times \langle Write \rangle / \langle Total \rangle$	14.446	15.473	10.865

Table 15. HRV16 inter-nodal exchange on 140-Node Paragon

Nodes	16	32	48	64	96	128
$\langle Startup \rangle$	1.855	2.990	4.594	9.292	20.050	24.500
$\langle Read \rangle$	2.939	1.783	2.693	1.803	1.791	1.507
$\langle \texttt{FFT} \rangle$	10.455	5.237	3.493	2.614	1.734	1.295
$\langle Exchange \rangle$	2.619	2.477	2.440	2.639	3.083	4.699
$\langle Write \rangle$	4.091	3.539	3.503	3.284	4.713	4.773
$\langle Total \rangle$	22.235	16.252	16.924	19.776	31.489	36.914
$Total$	30.949	28.158	29.050	30.100	51.046	64.551
$100 \times \langle Startup \rangle / \langle Total \rangle$	8.343	18.397	27.148	46.986	63.674	66.370
$100 \times \langle Read \rangle / \langle Total \rangle$	13.217	10.974	15.915	9.116	5.687	4.083
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	47.021	32.223	20.641	13.216	5.506	3.509
$100 \times \langle Exchange \rangle / \langle Total \rangle$	11.780	15.244	14.420	13.346	9.790	12.729
$100 \times \langle Write \rangle / \langle Total \rangle$	18.400	21.776	20.697	16.606	14.968	12.930

Table 16. HRV16 inter-nodal exchange on 512-Node Paragon

Nodes	8	16	32	48	64	128
$\langle Startup \rangle$	4.133	8.429	17.680	28.686	43.967	125.873
$\langle Read \rangle$	4.550	2.799	1.515	0.890	0.924	0.461
$\langle \texttt{FFT} \rangle$	18.424	9.305	4.662	3.107	2.330	1.159
$\langle Exchange \rangle$	5.064	3.365	3.716	4.278	3.721	13.946
$\langle Write \rangle$	5.830	3.493	3.152	3.702	5.302	6.218
$\langle Total \rangle$	38.254	27.765	31.065	41.025	56.539	147.884
Total	50.884	37.022	44.967	54.654	82.101	180.301
$100 \times \langle Startup \rangle / \langle Total \rangle$	10.804	30.359	56.913	69.923	77.764	85.116
$100 \times \langle Read \rangle / \langle Total \rangle$	11.895	10.083	4.875	2.169	1.634	0.312
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	48.162	33.514	15.007	7.574	4.121	0.784
$100 \times \langle Exchange \rangle / \langle Total \rangle$	13.237	12.121	11.961	10.428	6.582	9.430
$100 \times \langle Write \rangle / \langle Total \rangle$	15.239	12.580	10.145	9.025	9.377	4.205

Table 17. HRV16 scratch-exchange on 512-Node Paragon

Nodes	8	16	32	64
$\langle Startup \rangle$	5.070	11.657	22.673	57.372
$\langle Read \rangle$	4.336	2.135	2.104	0.877
$\langle \texttt{FFT} \rangle$	18.426	9.314	4.694	2.336
$\langle Exchange \rangle$	58.573	58.050	103.692	220.986
$\langle Write \rangle$	5.504	3.391	1.655	1.464
$\langle Total \rangle$	92.300	84.996	135.145	283.367
Total	105.639	94.559	146.634	314.369
$100 \times \langle Startup \rangle / \langle Total \rangle$	5.493	13.714	16.777	20.246
$100 \times \langle Read \rangle / \langle Total \rangle$	4.697	2.512	1.557	0.310
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	19.963	10.958	3.473	0.824
$100 \times \langle Exchange \rangle / \langle Total \rangle$	63.459	68.298	76.727	77.986
$100 \times \langle Write \rangle / \langle Total \rangle$	5.964	3.990	1.225	0.517

Table 18. CVB3 inter-nodal exchange on 128-Node hypercube

Nodes	32	64	128
⟨*Startuplrangle*	3.687	4.979	4.688
⟨*Readlrangle*	3.433	2.992	2.316
⟨FFT*lrangle*	8.939	4.478	2.243
⟨*Exchangelrangle*	3.275	2.866	2.058
⟨*Writelrangle*	18.443	20.772	35.684
⟨*Totallrangle*	38.742	37.778	52.126
Total	46.543	46.515	74.181
100 × ⟨*Startup*⟩/⟨*Total*⟩	9.516	13.179	8.994
100 × ⟨*Read*⟩/⟨*Total*⟩	8.861	7.921	4.442
100 × ⟨FFT⟩/⟨*Total*⟩	23.074	11.853	4.302
100 × ⟨*Exchange*⟩/⟨*Total*⟩	8.452	7.586	3.948
100 × ⟨*Write*⟩/⟨*Total*⟩	47.606	54.984	68.456

Table 19. CVB3 scratch-file exchange on 128-Node hypercube

Nodes	16	32	64
⟨*Startup*⟩	3.973	4.324	4.497
⟨*Read*⟩	5.363	15.454	9.799
⟨FFT⟩	17.873	8.939	4.478
⟨*Exchange*⟩	66.079	77.625	134.207
⟨*Write*⟩	19.028	13.663	17.198
⟨*Total*⟩	112.962	109.558	164.382
Total	126.181	126.903	184.641
100 × ⟨*Startup*⟩/⟨*Total*⟩	3.517	3.947	2.735
100 × ⟨*Read*⟩/⟨*Total*⟩	4.748	14.106	5.961
100 × ⟨FFT⟩/⟨*Total*⟩	15.822	8.160	2.724
100 × ⟨*Exchange*⟩/⟨*Total*⟩	58.497	70.853	81.643
100 × ⟨*Write*⟩/⟨*Total*⟩	16.845	12.471	10.462

Table 20. CVB3 inter-nodal exchange on 56-Node Paragon

Nodes	16	32	48
$\langle Startup \rangle$	3.203	4.392	13.710
$\langle Read \rangle$	3.914	3.439	4.914
$\langle \texttt{FFT} \rangle$	16.490	8.281	5.472
$\langle Exchange \rangle$	4.915	3.658	6.682
$\langle Write \rangle$	48.614	43.762	47.210
$\langle Total \rangle$	78.708	64.631	79.088
$Total$	126.662	116.359	143.114
$100 \times \langle Startup \rangle / \langle Total \rangle$	4.069	6.796	17.335
$100 \times \langle Read \rangle / \langle Total \rangle$	4.973	5.321	6.214
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	20.951	12.813	6.919
$100 \times \langle Exchange \rangle / \langle Total \rangle$	6.245	5.660	8.448
$100 \times \langle Write \rangle / \langle Total \rangle$	61.765	67.711	59.693

Table 21. CVB3 scratch-file exchange on 56-Node Paragon

Nodes	8	16	32
$\langle Startup \rangle$	2.287	2.199	6.110
$\langle Read \rangle$	5.698	3.997	4.656
$\langle \texttt{FFT} \rangle$	32.965	16.501	8.344
$\langle Exchange \rangle$	161.820	153.852	248.030
$\langle Write \rangle$	55.346	39.626	33.296
$\langle Total \rangle$	258.797	216.535	300.840
Total time	283.494	268.109	352.482
$100 \times \langle Startup \rangle / \langle Total \rangle$	0.884	1.015	2.031
$100 \times \langle Read \rangle / \langle Total \rangle$	2.202	1.846	1.548
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	12.738	7.620	2.774
$100 \times \langle Exchange \rangle / \langle Total \rangle$	62.528	71.052	82.446
$100 \times \langle Write \rangle / \langle Total \rangle$	21.386	18.300	11.068

Table 22. CVB3 inter-nodal on 140-Node Paragon

Nodes	16	32	48	64	96	128
$\langle Startup \rangle$	1.864	4.059	4.990	6.239	29.754	22.814
$\langle Read \rangle$	4.464	3.623	1.670	1.460	1.956	3.634
$\langle \texttt{FFT} \rangle$	16.471	8.234	5.486	4.117	2.748	2.045
$\langle Exchange \rangle$	3.997	3.159	2.694	2.890	3.666	6.838
$\langle Write \rangle$	11.698	9.077	7.040	8.735	8.190	11.536
$\langle Total \rangle$	38.598	28.491	22.124	23.603	46.496	46.999
$Total$	49.137	43.928	38.325	36.674	73.399	75.325
$100 \times \langle Startup \rangle / \langle Total \rangle$	4.829	14.248	22.553	26.435	63.992	48.543
$100 \times \langle Read \rangle / \langle Total \rangle$	11.566	12.716	7.549	6.186	4.207	7.732
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	42.672	28.902	24.797	17.443	5.910	4.351
$100 \times \langle Exchange \rangle / \langle Total \rangle$	10.355	11.089	12.175	12.243	7.884	14.550
$100 \times \langle Write \rangle / \langle Total \rangle$	30.308	31.859	31.822	37.006	17.613	24.546

Table 23. CVB3 inter-nodal exchange on 512-Node Paragon

Nodes	16	32	48	64	128
$\langle Startup \rangle$	6.892	18.868	27.093	55.979	522.844
$\langle Read \rangle$	3.681	2.680	1.310	2.068	0.520
$\langle \texttt{FFT} \rangle$	15.437	7.735	5.146	3.864	1.933
$\langle Exchange \rangle$	4.884	5.458	4.396	8.311	56.393
$\langle Write \rangle$	10.599	7.660	5.526	8.041	11.558
$\langle Total \rangle$	42.195	42.644	43.814	78.532	593.600
Total time	53.181	56.100	57.346	88.364	646.388
$100 \times \langle Startup \rangle / \langle Total \rangle$	16.334	44.246	61.837	71.282	88.080
$100 \times \langle Read \rangle / \langle Total \rangle$	8.723	6.284	2.990	2.634	0.088
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	36.586	18.138	11.745	4.921	0.326
$100 \times \langle Exchange \rangle / \langle Total \rangle$	11.574	12.798	10.033	10.583	9.500
$100 \times \langle Write \rangle / \langle Total \rangle$	25.120	17.962	12.611	10.238	1.947

Table 24. CVB3 scratch-file exchange on 512-Node Paragon

Nodes	8	16	32	64
$\langle Startup \rangle$	5.018	10.560	24.519	55.818
$\langle Read \rangle$	6.505	3.455	1.890	1.185
$\langle \texttt{FFT} \rangle$	30.514	15.465	7.756	3.870
$\langle Exchange \rangle$	102.666	107.716	176.101	313.970
$\langle Write \rangle$	12.810	5.916	3.838	4.063
$\langle Total \rangle$	158.007	143.620	214.444	379.246
$Total$	173.620	157.715	227.618	394.977
$100 \times \langle Startup \rangle / \langle Total \rangle$	3.176	7.363	11.434	14.718
$100 \times \langle Read \rangle / \langle Total \rangle$	4.117	2.406	0.881	0.313
$100 \times \langle \texttt{FFT} \rangle / \langle Total \rangle$	19.312	10.768	3.617	1.021
$100 \times \langle Exchange \rangle / \langle Total \rangle$	64.976	75.001	82.120	82.788
$100 \times \langle Write \rangle / \langle Total \rangle$	8.107	4.119	1.790	1.071

Table 25. Inter-nodal Exchange on 128-Node iPSC/860 at NIH

```
                 Inter-nodal Exchange on 128-Node iPSC/860 at NIH
Timing Groups:
   1   Begin execution
   2   Begin read input planes
   3   End read, begin y FFT
   4   End y FFT, begin exchang
   5   End exchange
   6   Begin x FFT
   7   End x FFT, begin z FFT
   8   End z FFT, begin write
   9   End write
  10   End of execution

HRV16 Number of Nodes =    16   on 128-Node iPSC/860 at NIH
For each group:                                 For FFT per plane
        Min time Aver time  Max time        Min time Aver time  Max time
   2      3.585     3.733     3.789
   3      0.787     4.932     7.234
   4      5.447     5.582     5.874         0.41893   0.41935   0.41962
   5      1.120     3.344     7.347
   6      0.002     0.013     0.029
   7      1.363     1.516     1.637         0.27217   0.27260   0.27283
   8      4.022     4.474     4.828         0.80350   0.80438   0.80480
   9      5.481     7.279     8.282
  10      0.016     1.130     8.010

HRV16 Number of Nodes =    32   on 128-Node iPSC/860 at NIH
For each group:                                 For FFT per plane
        Min time Aver time  Max time        Min time Aver time  Max time
   2      4.219     4.557     4.673
   3      0.329     3.285     4.960
   4      2.518     2.796     2.943         0.41957   0.42010   0.42050
   5      0.613     2.394     5.542
   6      0.003     0.012     0.033
   7      0.544     0.758     0.818         0.27167   0.27251   0.27300
   8      1.612     2.242     2.420         0.80533   0.80628   0.80750
   9      1.526     4.952     7.023
  10      0.002     0.583     4.321

HRV16 Number of Nodes =    64   on 128-Node iPSC/860 at NIH
For each group:                                 For FFT per plane
        Min time Aver time  Max time        Min time Aver time  Max time
   2      3.086     3.869     4.244
   3      0.126     2.724     4.033
   4      1.261     1.401     1.685         0.42025   0.42102   0.42133
   5      0.447     1.836     4.939
```

6	0.004	0.022	0.050			
7	0.274	0.381	0.546	0.27200	0.27400	0.27500
8	0.798	1.118	1.615	0.79800	0.80277	0.80750
9	0.848	5.257	8.132			
10	0.085	1.038	10.934			

CVB3 Number of Nodes = 32 on 128-Node iPSC/860 at NIH

For each group:				For FFT per plane		
	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.272	3.687	3.742			
3	0.250	3.433	5.179			
4	1.617	1.814	1.821	0.20189	0.20222	0.20233
5	1.502	3.220	6.600			
6	0.008	0.054	0.101			
7	1.798	1.816	2.252	0.44950	0.45037	0.45075
8	5.263	5.310	6.581	1.31575	1.31721	1.31775
9	8.925	18.443	21.720			
10	0.038	0.965	6.917			

CVB3 Number of Nodes = 64 on 128-Node iPSC/860 at NIH

For each group:				For FFT per plane		
	Min time	Aver time	Max time	Min time	Aver time	Max time
2	4.099	4.979	5.040			
3	0.081	2.992	4.730			
4	0.797	0.909	1.015	0.19925	0.20260	0.20300
5	0.903	2.818	6.029			
6	0.008	0.048	0.076			
7	0.899	0.909	1.351	0.44950	0.45076	0.45100
8	2.637	2.661	3.953	1.31767	1.32006	1.32100
9	2.794	20.772	28.844			
10	0.170	1.691	8.554			

CVB3 Number of Nodes = 128 on 128-Node iPSC/860 at NIH

For each group:				For FFT per plane		
	Min time	Aver time	Max time	Min time	Aver time	Max time
2	2.803	4.688	4.881			
3	0.041	2.316	3.573			
4	0.397	0.454	0.612	0.19850	0.20260	0.20400
5	0.713	1.987	5.929			
6	0.006	0.071	0.130			
7	0.452	0.456	0.901	0.45050	0.45282	0.45300
8	1.319	1.332	2.643	1.31900	1.32154	1.32300
9	1.627	35.684	42.217			
10	0.873	5.138	21.371			

Table 26. Scratch-file Exchange on 128-Node iPSC/860 at NIH

```
Timing Groups:
   1   Begin execution
   2   Begin read input planes
   3   End read, begin y FFT
   4   End y FFT, begin exchang
   5   End exchange
   6   Begin read scratch
   7   End read scratch
   8   Begin x FFT
   9   End x FFT, begin z FFT
  10   End z FFT, begin write
  11   End write
  12   End of execution
```

HRV16 Number of Nodes = 16 on 128-Node iPSC/860 at NIH

For each group:				For FFT per plane		
	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.371	3.566	3.650			
3	0.784	4.824	7.098			
4	5.446	5.582	5.873	0.41886	0.41934	0.41962
5	3.652	18.225	21.223			
6	0.002	4.607	23.138			
7	1.753	2.041	2.221			
8	0.000	0.000	0.000			
9	1.363	1.516	1.637	0.27233	0.27262	0.27283
10	4.022	4.474	4.827	0.80350	0.80438	0.80480
11	6.622	8.535	9.859			
12	0.061	1.453	13.493			

HRV16 Number of Nodes = 32 on 128-Node iPSC/860 at NIH

For each group:				For FFT per plane		
	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.640	4.269	4.379			
3	0.342	3.253	5.051			
4	2.519	2.796	2.942	0.41957	0.42006	0.42033
5	4.740	38.696	41.393			
6	0.003	3.888	41.257			
7	1.060	1.589	1.783			
8	0.000	0.000	0.000			
9	0.544	0.758	0.818	0.27200	0.27254	0.27300
10	1.612	2.242	2.419	0.80533	0.80623	0.80700
11	1.574	4.857	6.548			
12	0.004	1.112	12.330			

HRV16 Number of Nodes = 64 on 128-Node iPSC/860 at NIH

For each group: For FFT per plane

	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.253	4.376	5.122			
3	0.255	2.917	4.496			
4	1.262	1.401	1.684	0.42025	0.42095	0.42133
5	51.448	62.682	67.880			
6	0.008	5.342	17.191			
7	0.537	1.253	1.692			
8	0.000	0.000	0.000			
9	0.274	0.381	0.546	0.27250	0.27403	0.27500
10	0.798	1.118	1.618	0.79800	0.80284	0.80900
11	0.708	2.987	6.035			
12	0.099	0.890	14.794			

Timing Groups:

1 Begin execution
2 Begin read input planes
3 End read, begin y FFT
4 End y FFT, begin exchang
5 End exchange
6 Begin read input planes
7 End read, begin y FFT
8 End y FFT, begin exchang
9 End exchange
10 Begin read scratch
11 End read scratch
12 Begin x FFT
13 End x FFT, begin z FFT
14 End z FFT, begin write
15 End write
16 Begin read scratch
17 End read scratch
18 Begin x FFT
19 End x FFT, begin z FFT
20 End z FFT, begin write
21 End write
22 End of execution

CVB3 Number of Nodes = 16 on 128-Node iPSC/860 at NIH

For each group:				For FFT per plane		
	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.755	3.973	4.033			
3	0.581	4.191	6.260			
4	2.825	2.828	2.830	0.20179	0.20200	0.20214
5	12.940	35.340	39.140			
6	0.000	0.000	0.000			
7	0.061	1.172	1.713			
8	0.608	0.798	0.811	0.20250	0.20270	0.20275
9	0.501	2.246	12.816			

10	0.002	4.089	25.921			
11	16.245	17.110	18.361			
12	0.000	0.000	0.000			
13	3.149	3.154	3.156	0.44986	0.45058	0.45086
14	9.198	9.208	9.212	1.31400	1.31544	1.31600
15	13.414	17.844	19.985			
16	0.000	0.000	0.000			
17	4.411	7.294	11.566			
18	0.000	0.000	0.000			
19	0.452	0.481	0.901	0.45050	0.45272	0.45300
20	1.320	1.404	2.642	1.32000	1.32106	1.32200
21	1.052	1.184	2.148			
22	0.005	0.645	8.813			

Timing Groups:

1 Begin execution
2 Begin read input planes
3 End read, begin y FFT
4 End y FFT, begin exchang
5 End exchange
6 Begin read scratch
7 End read scratch
8 Begin x FFT
9 End x FFT, begin z FFT
10 End z FFT, begin write
11 End write
12 End of execution

CVB3 Number of Nodes = 32 on 128-Node iPSC/860 at NIH

For each group: For FFT per plane

	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.800	4.324	4.461			
3	0.316	3.664	5.903			
4	1.617	1.814	1.821	0.20189	0.20221	0.20233
5	45.623	65.836	68.421			
6	0.004	3.883	27.532			
7	5.285	7.906	9.768			
8	0.000	0.000	0.000			
9	1.798	1.816	2.253	0.44950	0.45046	0.45075
10	5.264	5.310	6.582	1.31600	1.31722	1.31775
11	4.391	13.663	17.193			
12	0.014	1.341	14.556			

CVB3 Number of Nodes = 64 on 128-Node iPSC/860 at NIH

For each group: For FFT per plane

	Min time	Aver time	Max time	Min time	Aver time	Max time
2	3.438	4.497	4.755			
3	0.081	2.511	4.307			

4	0.797	0.908	1.014	0.19925	0.20258	0.20280
5	99.221	126.919	128.298			
6	0.004	2.645	33.564			
7	4.054	4.642	5.330			
8	0.000	0.000	0.000			
9	0.899	0.909	1.351	0.44950	0.45075	0.45100
10	2.637	2.661	3.954	1.31800	1.32005	1.32100
11	2.451	17.198	21.880			
12	0.387	1.491	15.289			