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J Stuart Bolton
Purdue University, bolton@purdue.edu

Yong-Joe Kim

Sung-Yop Lee

Yeon-June Kang

IDENTIFICATION OF COMPLEX DISPERSION RELATIONS IN CYLINDRICAL, FOAM-LINED DUCTS

°Yong-Joe Kim[1], J. Stuart Bolton[2], Sung-Yop Lee[3] and Yeon June Kang[4]

ABSTRACT

Complex dispersion relations in a cylindrical, foam-lined duct were successfully identified by using an iterative Prony series method. Techniques for using the iterative procedure successfully are described in detail, particularly with regard to model order selection and the identification of parameter starting values. It is shown that modal wave speeds and spatial attenuations per wavelength can be derived from the complex dispersion relations obtained using the iterative procedure. In addition, a finite element simulation is shown to well represent corresponding experimental measurement in terms of modal wave speeds and spatial attenuations.

1. INTRODUCTION

It is of interest to determine the wave speeds and especially the spatial decay rates of modes in foam-lined ducts [1]. The latter quantity is commonly used as a performance measure when fibrous or foam linings are used to suppress sound propagation along ducts. Both quantities can be calculated by measuring the spatial distribution of sound pressure at discrete positions along a finite, lined duct: the wave speed by calculating the spatial gradient of the phase, and the spatial decay rate by fitting exponential functions to the envelope of the sound pressure. These measurement methods, however, may be made inaccurate by reflections from the duct termination at low frequencies and by the appearance of higher modes at high frequencies. Wave number transform techniques may also be used in this context, but in that case the spatial Fourier transform yields only the real part of the dispersion relation, which then allows only the wave speed to be inferred. In addition, spatial transform procedures may suffer from poor resolution in the wave number domain. These various concerns have been addressed in the present work in which complex modal wave numbers were identified by fitting a Prony series, i.e., a sum of complex exponentials, to the spatial data. By doing so, it is possible to obtain complex modal dispersion relations and then the corresponding wave speeds and the spatial decay rates by curve fitting. Here, the conventional Prony method [2-5] has been supplemented by the application of an iterative Prony method [6]: the conventional Prony series method has been used to provide starting values for the iterative procedure. In this way, optimal performance in terms of estimation error and convergence speed is guaranteed: the modal properties are identified precisely and quickly. In particular, “true” system modes can be easily distinguished from those associated with noise.

This improved method has been applied to pressure data obtained from finite element

[1] Ray W. Herrick Lab., School of Mechanical Engineering, Purdue University: Graduate Research Assistant
[2] Ray W. Herrick Lab., School of Mechanical Engineering, Purdue University: Professor
[3] School of Mechanical and Aerospace Engineering, Seoul National University: Graduate Research Assistant
[4] School of Mechanical and Aerospace Engineering, Seoul National University: Assistant Professor

simulations that were used to obtain forced solutions for the sound field in a lined duct. Displacement-based finite element models can be used to analyze foam-lined ducts and to solve for modal properties directly [7]. However, the latter procedure is sometime made difficult by the appearance of a relatively large number of spurious eigenvalues: it is sometimes difficult to discriminate between these “false” modes and the “true” modes that are associated with the wave propagation process. Thus, the present approach may offer an easier way of identifying modal properties than is currently available. Experimental results are also presented here, and it is shown that the measured dispersion relations agree well with the results of the finite element simulations. It is shown, in particular, that the present procedure allows individual modal parameters to be determined even when several modes propagate simultaneously.

2. EXPERIMENT AND FINITE ELEMENT SIMULATION

2.1. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. A B&K impedance tube was modified to hold the 1 m foam lining and the anechoic termination. The inner diameter of the tube was 10 cm and the tube wall was sufficiently hard to be assumed rigid. A foam lining with a 10 cm outer diameter and an 8 cm inner diameter was carefully inserted into the tube. The boundary between the inner wall of the tube and the foam lining was assumed to be “lubricated” as the lining was not axially constrained by the tube wall. The absorption coefficient of the anechoic termination was equal to or greater than 0.95 at all frequencies higher than 100 Hz. To measure the sound pressure at the center of the airway, a long rod equipped with a ¼ in microphone (B&K Type 4187) at its end, was used. The microphone at the center of the cross-section could be moved from the inlet (x=0) of the lining to its termination. White noise, generated by a HP 35607A analyzer, and having a cut-off frequency of 12800 Hz, was fed to the amplifier and then the loudspeaker. By using the HP 35607A frequency analyzer, transfer functions between the sound pressure (output) and the loudspeaker (input signal) were measured at ninety points along the airway at a frequency spacing of 32 Hz. The sampling interval was 0.01 m and the first measurement position was at the inlet to the airway.

2.2. MEASURED DATA

Measured data at three frequencies are shown in Fig. 2. In each case, it is clear that the sound pressure attenuates exponentially as it propagates along the tube as a result of energy dissipation within the foam lining.

2.3. FINITE ELEMENT SIMULATION

An axisymmetric finite element formulation based on elastic porous material theory was derived to simulate the experimental results [1]. The macroscopic properties of the foam were considered to be: flow resistivity and bulk density (both measured), porosity, loss factor, and Poisson’s ratio (all assumed to have typical values), tortuosity and bulk Young’s modulus (both estimated from an optimization procedure in which the difference between the measured and predicted normal incidence sound absorption coefficient was minimized). The upstream boundary condition near the loudspeaker was specified in terms of a velocity: the particle velocity was assumed constant at all frequencies. It was also assumed that there was no axial constraint imposed between the foam and tube wall; a radial constraint was imposed, however.

3. PRONY METHOD

3.1. INTRODUCTION

When a complex wave number is used to represent a spatially attenuated wave such as the measured sound pressures shown in Fig. 2, its imaginary part accounts for the spatial attenuation. Thus, it is appropriate to identify the spatially attenuated wave by using a series of complex exponential functions whose exponents are themselves complex. This exponential series identification method, the Prony method, is very well known [2-5]. In the conventional implementation of the Prony method, a relatively large number of exponential terms (i.e., a large model order) is used so as to include roots that are necessary to model measurement noise. Because the bias error of the wave number estimate due to measurement noise decreases as the model order is increased, the model order must be as large as possible. To distinguish "true" roots from noise-related roots, Braun and Ram [4] have proposed a perturbation method that requires an in-depth understanding of the movement range associated with each type of root.

Here the iterative Prony method proposed by Therrien and Velasco [6] was used instead. This procedure can be used to find a Prony series representation of the spatial data that minimizes an estimation error norm. Therefore, without bias error, it can be used to identify Prony parameters with the same model order as the number of true roots. There are two concerns associated with this iterative scheme, however. The first concern is how to determine the model order. The second concern is that the error norm itself is not a quadratic function. All quadratic functions possess a unique optimal point where the value of the error norm is the local as well as the global minimum so that in that case the iterative optimization method would always yields the optimal solution regardless of the starting point. That is not the case in the present instance, however. To resolve the former concern, a singular value decomposition procedure was used: this procedure is well known [4-5] and can be applied in conjunction with the iterative method to estimate an appropriate model order. To resolve the second concern, it is necessary to select parameter starting values close to their optimal values to guarantee an optimal solution regardless of error norm definition. In the present case, parameter starting values were selected from amongst the solutions of the conventional Prony method in such a way that true roots were separated from those induced by noise. The convergence characteristics of the iterative procedure were improved in this way.

3.2. ITERATIVE PRONY METHOD

The discrete and finite sized data set measured at the N equally spaced points can be approximated by a complex exponential series written in matrix form: i.e.,

$$\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix} = \begin{bmatrix} r_1^0 & r_2^0 & \cdots & r_M^0 \\ r_1^1 & r_2^1 & \cdots & r_M^1 \\ \vdots & \vdots & \ddots & \vdots \\ r_1^N & r_2^N & \cdots & r_M^N \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix} - \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}$$

or, in more compact form,

$$\mathbf{f} = \mathbf{Rc} - \mathbf{e} \qquad (1)$$

where $r_m = \exp(ik_m\Delta x)$, M is the model order, Δx is the sampling interval, and the last vector term is the estimation error. The m-th complex wave number, k_m, has both a real part (β_m) that represents the propagation factor and an imaginary part (α_m) that represents the spatial attenuation factor. In the iterative procedure, the complex wave numbers and their complex amplitudes are found by an optimization scheme, essentially a steepest decent method, which has the effect of minimizing the error norm defined as

$$\mathrm{E} = \mathbf{e}^H \mathbf{e}\,. \qquad (2)$$

Once **R** is found separately from **c**, the optimal value of **c** can be calculated analytically from Eqs. (1) and (2). Thus, the unknowns **R** and **c** are separately updated under the assumption that one of them is constant in turn at each iteration step: i.e., **R** is updated using the steepest decent method assuming **c** is constant. Then **c** is calculated based on this new **R**. These steps are repeated until a convergence criterion is satisfied.

3.3. DETERMINATION OF MODEL ORDER AND STARTING POINT

Consider first the conventional Prony method. The equation for the wave numbers can be separated from Eq. (1) by utilizing a characteristic equation defined as

$$r^M - a_{M-1}r^{M-1} - \cdots - a_1 r - a_0 = 0 \qquad (3)$$

of which the roots are $r = \exp(ik\Delta x)$. The coefficients of the characteristic equation, a_m can be calculated from

$$\begin{bmatrix} f_{M-1} & \cdots & f_0 \\ \vdots & \ddots & \vdots \\ f_{N-2M-2} & \cdots & f_{N-M-1} \end{bmatrix} \begin{bmatrix} a_{M-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} f_M \\ \vdots \\ f_{N-1} \end{bmatrix}. \qquad (4)$$

When the roots of Eq. (3) are known, the values of the wave amplitudes can be obtained from Eqs. (1) and (2) analytically, thus providing candidate starting values for the iterative procedure. It is first necessary, however, to estimate the model order, which in turn determines starting values required. The model order can be determined by examining the singular values of the coefficient matrix in Eq. (4): the model order is then assumed to equal the number of significant singular values (i.e., the rank of the coefficient matrix). The final parameter starting values are selected from amongst the solutions of the conventional method. When a dominant wave component is removed from the exponential series, the residual error should be large. Thus, the starting values are chosen to include only the wave components that make the error norm significantly large when one of them is removed from the Prony series.

4. RESULTS AND DISCUSSION

The iterative Prony method was applied to the spatial data obtained from both measurements and FE simulations. The rank inferred from the singular values of the coefficient matrix of Eq. (4) was equal to or less than three at all frequencies of interest. Thus, the model order was set to three. The results of the Prony method are shown in Fig. 3 where the real and imaginary wave numbers are plotted at each frequency. The brightness of the wave number at each frequency represents the magnitude of the complex amplitudes: thus the dominant mode at any frequency may be identified. Each of the four dispersion curves visible in Fig. 3 is associated with its own cross-sectional mode: the lowest curve is the (0,0) mode, the second the (1,0) mode, the third the (2,0) mode, and so on. Because of the symmetry of the present arrangement, only axisymmetric modes were significantly excited (although some contributions from non-axisymmetric modes may be visible in the experimental data, particularly near 5 kHz and 8 kHz). In general, there is good agreement between the FE results and the measurements.

The most interesting feature in Fig. 3 is the behavior near the cut-on frequency of the higher modes. For example, near the first cut-on frequency (approximately 2 kHz), the (0,0) mode begins to be significantly attenuated and the (1,0) mode “cuts on” when it has a large imaginary wave number: i.e., the (1,0) mode is initially nearly evanescent. As the frequency increases, the imaginary wave number of the (0,0) mode increases and that of the (1,0) mode decreases. Near 4 kHz, the dominant mode shifts from the (0,0) to the (1,0) mode: i.e., the latter becomes the least attenuated mode. Finally the (0,0) mode disappears for practical purposes. By fitting the dispersion curves of the (0,0) and (1,0) modes with polynomials, analytical expressions for the dispersion characteristics of these modes were obtained, even in the frequency range between 3 kHz and 4 kHz when both modes contributed significantly to the total field. By using these polynomial expressions, the wave speed and spatial attenuations per wavelength can be calculated: the former from the dispersion curves in the real wave number domain and the latter from those in imaginary domain. Both results are shown in Fig. 4. The results of both the experiment and FE simulation agree well with each other except for the phase speed of the (1,0) mode. The difference in this case results because the “cut-on” frequency of the (1,0) mode is underestimated in the FE simulation. Also, the attenuation per unit wavelength ($2\pi\beta/\alpha$) was found to be nearly constant at all frequencies except near the cut-on frequency.

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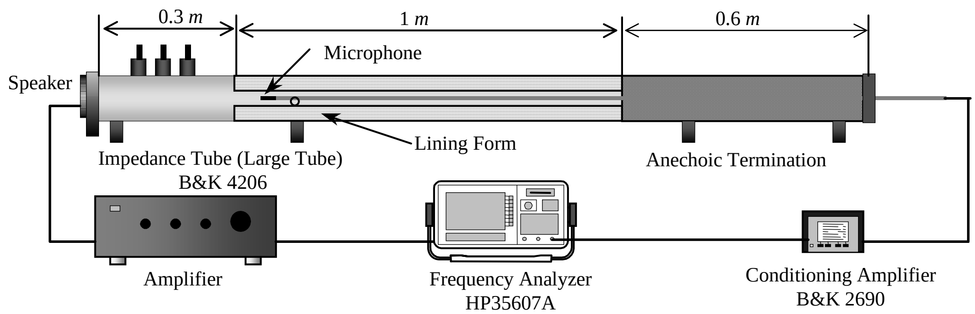

Figure 1. Experimental setup

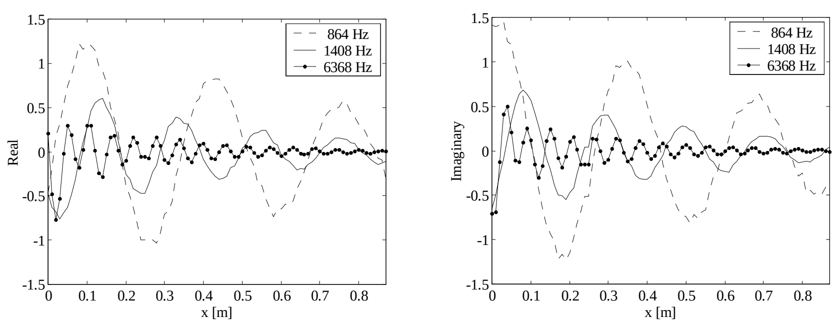

Figure 2. Measured sound pressures at 864 Hz, 1408 Hz, and 6368 Hz.

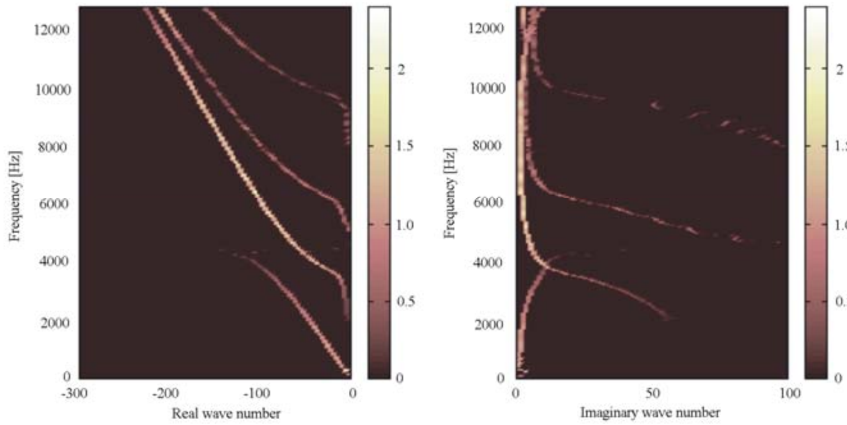

Figure 3(a). Complex dispersion relations obtained from FE simulation data.

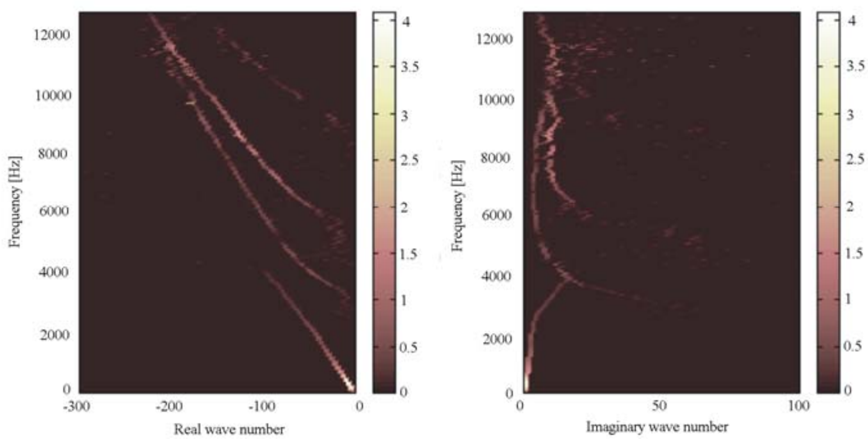

Figure 3(b). Complex dispersion relations obtained from experimental data.

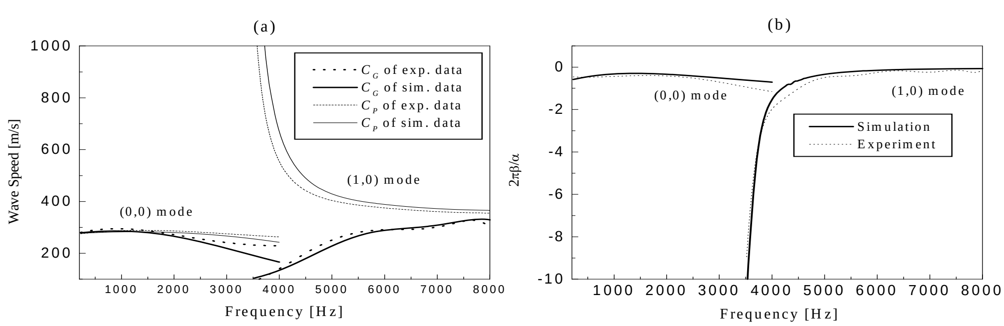

Figure 4. (a) Wave speeds. C_G represents the group speed and C_P the phase speed. (b) Spatial attenuation per wavelength.