Self Calibration of Gough Platform using leg mobility constraints

David Daney

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SELF CALIBRATION OF GOUGH PLATFORM USING LEG MOBILITY CONSTRAINTS

David Daney
David.Daney@sophia.inria.fr
INRIA Sophia-Antipolis - Projet SAGA
2004, Route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex France

Abstract

A new algorithm for the calibration of a gough platform is presented. It enables auto-calibration as it only uses internal measurements : the leg lengths. In order to obtain the necessary information to do so, without adding additional sensors, we introduce some constraints on two legs of the robot. First, the method is described. Second, it is validated by simulation.

Calibration, non-linear resolution, optimization, constraints.

1 Introduction

Due to errors such as manufacturing or assembly defects, it is well known that the geometry of robotic manipulators does not exactly match the design goals. A direct drawback lies in a reduced accuracy of the manipulator, as robot control uses the kinematic models. One way to tackle this problem consists in improving the knowledge of the kinematic model using kinematic calibrations. In most cases, internal and external information about the state of the robot are used, but it can be very interesting to do auto-calibration using only internal information.

More precisely, we perform auto-calibration by adding constraints on the robot's legs without the help of any redundant sensor. This restricts the mobility of the platform and also provides us some relation between the kinematic parameters.

Determining and solving these equations is the problem that we address in this contribution.

2 Calibration problem

According to the general paradigm of [Wampler et al. 95], a unified calibration formulation can be stated as follows. First, given the unknown kinematic parameters of the manipulator x and the measurements m provided by sensors, some loop equations $f(x, m) = 0$ have to be figured out. For Gough platform, the measures m can be of two types: either external measurements of the position and orientation of the robot's end-effector together with the legs lengths ; or only internal measurements of legs lengths and redundant sensors [Zhuang 97] — in the latter case, an additional advantage is that the forward Kinematic may be simpler. To obtain the loop equations, one can use Forward/Inverse Kinematic, closing loops, mobility constraints [Masory et al. 97, Everett 89, Nahvi et al. 94] on the legs or on the end effector. Then, these loop equations can be solved using some optimization [Masory et al. 97], linearization [Geng et al. 94] or resolution [Innocenti 95] machinery.

Some calibration method are also related to leg constraints. For example, Zhuang [Zhuang et al. 93] sets the length of one leg for each measurement configuration, to remove a kinematic parameter (offset on the leg length) and to lower the degrees of the equations. Murareci [Murareci 97] specifies the direction of a leg for a set of measurement to minimize the U-ball angles calculated from two measurement configurations. We shall use these ideas thereafter.

3 Problem description

We want to find the real kinematic parameters of a Gough platform. For that, we describe a new self calibration method which determines the real coordinates of the segment attachment points in a frame bound at a home configuration of the parallel manipulator. It is supposed that the offsets of the legs lengths are known and that the articulations are perfectly assembled — see [Masory et al. 97]. The only measurements available are the legs lengths .

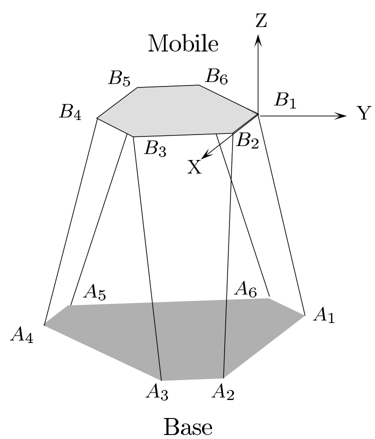
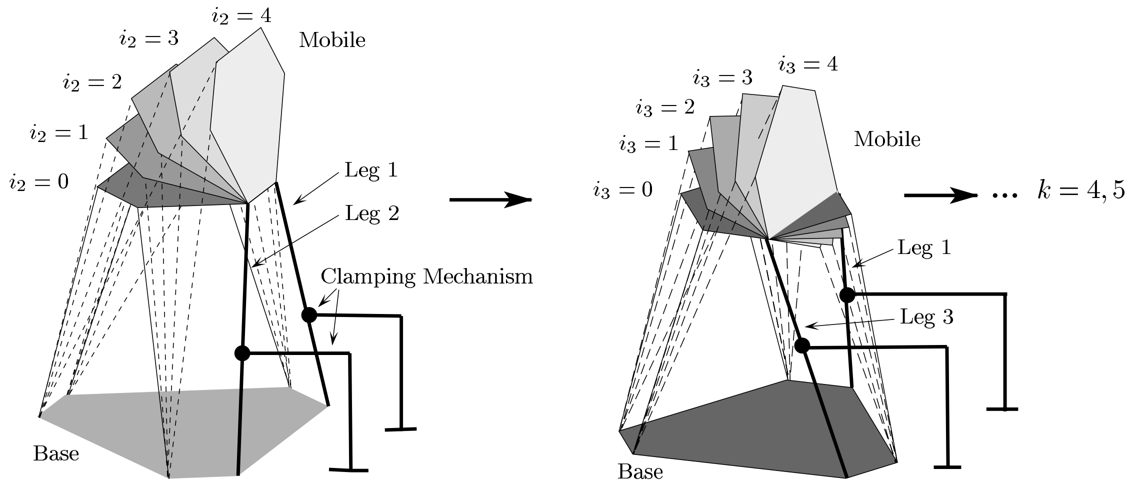


Figure 1: Home configuration

4 The idea

One contribution of this article is to show that the self-calibration is possible even without the addition of redundant sensors. In order to compensate this lack of information about the global configuration of the robot, we have to add external constraints on the mobility of the robot. One way to do this is to fix, with the help of a clamping mechanism, both the direction and the length of two legs of the robot — see Figure 2. Consequently, the attachment points of these legs on the mobile platform will remain exactly in the same position all along the different measures. Hence, the mobile platform can only move around the axis defined by these attachment points. This reduces the degree of freedom of the mobile platform to one, but we still have four pieces of information to use, namely the length measurements of the four free legs. So each new measurement configuration introduces only one new parameter — the rotation angle around the fixed axis, corresponding to the degree of freedom of the platform — and provides four equations on the legs lengths : these redundant informations are used to calibrate the robot.

However, fixing two particular legs is not enough to identify all kinematic parameters, so we fix consecutively four different pairs of legs.



(a) $k = 2$ (b) $k = 3$

Figure 2: The direction and the length of two legs (1 and k) are fixed

5 The algorithm

We denote by :

- - $k \in \{2, 3, 4, 5\}$ the index of the second fixed leg (the first one is always leg 1),
  - $i_k = 0...N_k$ the index of the N_k measurement configurations for each pair of fixed leg $(1, k)$ (with $i_k = 0$ for the home position),
  - $j \in \{2, 3, 4, 5, 6\}$ and $j \neq k$ index of unconstrained leg ,
- - $L^k_{i_k,j}$ the leg length of the segment j for the measurement configuration i_k when the second fixed leg is k, this data is provided by measurement,
  - θ_{i_k} the angle between the home position and the configuration i_k,
  - $R^k_{i_k}$ the 3×3 rotation matrix around the axis (b_1, b_k) of angle θ_{i_k} :

$$R^k_{i_k} = [I]\, cos\, \theta_{i_k} + \left(\frac{b_k}{||b_k||}\right)\left(\frac{b_k}{||b_k||}\right)^T (1 - cos\, \theta_{i_k}) + \left[\frac{b_k}{||b_k||}^{\times}\right] sin\, \theta_{i_k}$$

  where $[v^\times]$ is the 3×3 matrix of the cross product by v, $[I]$ the 3×3 identity matrix and M^T is the transposed of a matrix M.

First of all, we put the Gough platform into an arbitrary configuration ($\mathcal{X} = [\mathcal{P}, \mathcal{R}]$, a position and an orientation) called *home configuration* — see Figure 1. To set this configuration as a reference, we can put all the legs at their minimal length. In this home configuration, we note the coordinates of the attachment points of the six legs to the mobile and to the base as $(b_1, b_2, b_3, b_4, b_5, b_6)$ and $(a_1, a_2, a_3, a_4, a_5, a_6)$ respectively. We then define a reference frame such as $b_1 = (0, 0, 0)$, $b_2 = (x_2, 0, 0)$ and $b_3 = (x_3, y_3, 0)$.

In the home configuration, we fix with a clamping mechanism the direction and the length of legs 1 and 2 ($k = 2$) — see Figure 2(a). We then change the length of a chosen leg, leaving the three other mobile legs unconstrained : we get four leg lengths for $j \in \{3, 4, 5, 6\}$ (a good way to do this would be to disengage the engines in these three legs) (**Note:** We can consider the constrained robot as a one DOF mechanism, the moving leg as internal sensor and the three unconstrained legs as redundant sensors). We repeat this four times ($i_2 = 1, 2, 3, 4$). So, we get a home position ($i_2 = 0$) and four new measurement poses ($i_2 = 1, 2, 3, 4$), each one defined by the angle θ_{i_2} of the rotation around the axis (b_1, b_2). We fix this angle to zero in the home position.

After saving the four legs lengths ($j \in \{3, 4, 5, 6\}$) of each of the four measurement poses ($i_2 = 1, 2, 3, 4$), we put the robot back into its home position ($i_2 = i_3 = 0$). Then, we free leg 2, clamp leg 3 ($k = 3$) — see Figure 2(b) and repeat the same operation (now, $j \in \{2, 4, 5, 6\}$ and $i_3 = 1, 2, 3, 4$).

We repeat this again with legs 1 and 4, and with legs 1 and 5. So that we have sequentially fixed legs (1,2) ($k = 2, j \in \{3, 4, 5, 6\}$), legs (1,3) ($k = 3, j \in \{2, 4, 5, 6\}$) , legs (1,4) ($k = 4, j \in \{2, 3, 5, 6\}$) and then legs (1,5) ($k = 5, j \in \{2, 3, 4, 5\}$). We can see that leg 1 always remains fixed in the home configuration. Therefore, the only information on the position of the point A_1 is the equation of the length of leg 1, hence we only know this point is in a sphere centered in B_1. We won't try to determine the coordinates of this point. The output is a set of 27 unknowns : the 3 coordinates of $a_2, a_3, a_4, a_5, a_6, b_4, b_5, b_6$ and x_2, x_3, y_3.

5.1 Equations

To simplify the resolution, we divide the problem for each pair of fixed leg $(1, k)$:

For $j \in \{2..6\}$ and $j \neq k$, we get the length of the unconstrained leg j as :

for the home configuration :

$$||a_j - b_j||^2 = a_j^T a_j + b_j^T b_j - 2b_j^T a_j = (L_{0,j})^2 \tag{1}$$

for $i_k = 1..N_k$:

$$||a_j - R^k_{i_k} b_j||^2 = a_j^T a_j + b_j^T b_j - 2b_j^T R^k_{i_k} a_j = (L^k_{i_k,j})^2 \tag{2}$$

To decrease the degree of each equation, we subtract the equation of the length of the segment j at the home position (equation 1) to every length equation of the unconstrained leg j at the measurement configuration i_k (equation 2).

For a chosen k, we get N_k equations for each j :

$$(cos\, \theta_{i_k} - 1)(X^k_j) - (sin\, \theta_{i_k})(Y^k_j) = \frac{(L^k_{i_k,j})^2 - (L_{0,j})^2}{2} \tag{3}$$

$$\text{with } \; X^k_j = b_j^T \, (\left[\frac{b_k}{||b_k||}^{\times}\right]^T \left[\frac{b_k}{||b_k||}^{\times}\right]) \; a_j \tag{4}$$

$$\text{and } \; Y^k_j = b_j^T \, (\left[\frac{b_k}{||b_k||}^{\times}\right]) \; a_j \tag{5}$$

5.2 Solving

5.2.1 Determination of X^k_j, Y^k_j

To solve this problem we need to find the values of the temporary unknowns $X^k_j, Y^k_j, j \in \{2..6\}, j \neq k$ and $\theta_{i_k}, i_k = 1..N_k$. This makes a total of $4 \times 2 + N_k$ unknowns and $4 \times N_k$ equations of type (equation 3) are available to calculate them. Therefore, for each k, we must have $3N_k \geqslant 8$ to solve this problem. This is done in 2 steps :

- As these equations are linear in X_j^k and Y_j^k, we use 8 equations (for $i_k = 1, 2$ and for $j \in \{2..6\}$ and $j \neq k$) to solve X_j^k and Y_j^k as functions of θ_{i_k} and $L_{i_k,j}^k$. There remains 4 equations in 3 unknowns $\theta_{i_k}, i_k = 1, 2, 3$,
- then, we solve the remaining equations by a Bezoutian method — see [Elkadi 98]. We get θ_{i_k} function of $L_{i_k,j}^k$, $i_k = 1, 2, 3$, so by substitution we get $X_j^k, Y_j^k, j \in \{2..6\}$ and $j \neq k$ function of $L_{i_k,j}^k$.

In practice we use $N_k = 4$ to get a robust estimation of the unknowns and alleviate problems due to a perfectly symmetrical mobile platform — some equations can be identical.

5.2.2 Determination of a_j, b_j

We have determined X_j^k and Y_j^k for $k \in \{2..5\}, j \in \{2..6\}$ and $j \neq k$: we get $2\times4\times4 = 32$ equations for 27 unknowns (for $a_2, a_3, a_4, a_5, a_6, b_2, b_3, b_4, b_5, b_6$).

Now, we want to solve equations (4) and (5). We put these equations in the following form :

$$X_j^k = v_j^T \left(\left[v_k{}^\times \right]^T \left[v_k{}^\times \right] \right) u_j \tag{6}$$

$$Y_j^k = v_j^T \left(\left[v_k{}^\times \right] \right) u_j \tag{7}$$

with v_j the direction of b_j, and $u_j = ||b_j|| a_j$ for $k \in \{2..5\}, j \in \{2..6\}$ and $j \neq k$.

These equations (6) and (7) are linear in u_j and we solve u_j in function of v_j. There remains 17 non-linear equations in 7 unknowns v_j. This highly redundant system can be solved by an optimization method with a estimate of v_j.

Warning : the equations of type (5) are not independent. Moreover, if the mobile platform is coplanar the equations of type (4) are no longer independent, and this system becomes under constrained, but with this hypothesis the system can be simplified and easily solved.

To finish up, we solve $||b_j||$ and $||a_j||$ by using the values of u_j, v_j and equations (1).

6 Results

We present a simulation using this method on the robot *Left-Hand* from INRIA. The real parameters of the robot are used to simulate the measurement poses and the associated leg lengths. To take into account the measurement noise, we add a random error uniformly distributed with an amplitude of Amp_{error} on the leg lengths simulation. The errors on the estimation of direction of the mobile attachment points are of about one millimeter.

After solving the temporary and kinematic parameters, all the equations are processed with an optimization method decreasing the noise influence and yielding better estimates.

To see how accurate this method is, we compute the magnitude of the difference between the real parameters (a_{kr} for the base, b_{kr} for the mobile) and the parameters provided by the method (a_{kc} for the base, b_{kc} for the mobile).

Error in (mm)	without noise	$Amp_{error} = 10^{-3} mm$
$\|\|a_{kr} - a_{kc}\|\|$	1.2×10^{-10}	0.191
$\|\|b_{kr} - b_{kc}\|\|$	1.8×10^{-11}	0.035

7 Conclusion

A new algorithm for the identification of the kinematic parameters of a Gough platform has been presented. It enables the determination of the relative coordinates of the 6 attachment points of the legs to the mobile platform and of 5 of the attachment points of the legs to the base platform. (The last one can easily be determined with another method once the coordinates of all the other points are known.) This algorithm needs an estimate of the directions of the mobile attachment points in the reference frame.

Due to the successive solving, this method is quite sensitive to measurement noise, but this can be improved by fixing the sixth leg (even though this requires four additional measurement configurations). In practice it can be difficult to implement on all types of platform as it is necessary to use the legs as simple length sensor (engines have to be disengageable).

However, this method can be very interesting as it only uses information on the length of the segments. It also shows the interest of constraining the platform and/or the legs.

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