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Physically-based Preconditioner for the WCIP

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Abstract —A physically-based preconditioner for the 1d and 2d Wave Concept Iterative Procedure is introduced in this paper. Numerical results are provided to assess the efficiency of this technique.

Index Terms —Preconditioner, WCIP, modal method, multilayered planar circuit.

I. INTRODUCTION

The Wave Concept Iterative Procedure (WCIP) is a moment/modal method based on the wave concept [1]-[2] well suited for multilayered planar circuits. The initial scheme may be interpreted as a Richardson iteration for which the convergence is guaranteed but this convergence can be slow. In a previous work, several improvements were presented such as Krylov subspace acceleration [3]. In this paper, a physically-based right preconditioner is introduced. This preconditioner is first developed and analyzed for 1d TE and TM problems such as electromagnetic fields scattered by metallic surfaces printed over different dielectric substrate. This preconditioner contains two variable parameters optimized for the TE and TM case. Finally the same preconditioner is extended to the 2d case on an example.

II. 1D WCIP METHOD

The method is implemented on a 1d model circuit with one meshed surface separating two homogeneous media as represented on Fig. 1. In this particular case, a z-invariance is assumed. As a consequence, TE (E_y=0) and TM (H_y=0) modes are physically separated: if the source is polarized along z, only TE modes are generated, while if the source is polarized along x only TM modes are generated. The method is based on incident B_i and reflected A_i waves on the interface (where i stands for media numbered 1 or 2 in Fig. 1). These waves are defined by

$$\begin{cases} \mathbf{A_i} = \dfrac{1}{2\sqrt{\mathbf{Z_0}}}\left(\mathbf{E_{Ti}} + \mathbf{Z_0}\mathbf{H_{Ti}} \wedge \mathbf{n_i}\right), \\ \mathbf{B_i} = \dfrac{1}{2\sqrt{\mathbf{Z_0}}}\left(\mathbf{E_{Ti}} - \mathbf{Z_0}\mathbf{H_{Ti}} \wedge \mathbf{n_i}\right), \end{cases} \quad (1)$$

with Z_0 an arbitrary impedance, E_{Ti}, H_{Ti} and n_i the electric, magnetic tangential fields and unit normal outward vector to the surface.
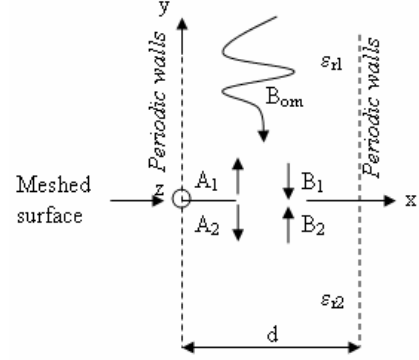

Fig. 1.Example of a 1d circuit.

A-Principles

Interface conditions (metallic or insulate) over the meshed surface are applied on each mesh through a spatial operator S:

$$\mathbf{A} = \mathbf{SB,} \text{ where } \mathbf{B} = \begin{pmatrix}\mathbf{B_1} & \mathbf{B_2}\end{pmatrix}^{\mathrm{T}} \text{ and } \mathbf{A} = \begin{pmatrix}\mathbf{A_1} & \mathbf{A_2}\end{pmatrix}^{\mathrm{T}}. \quad (2)$$

The homogeneous medium numbered i surrounding the surface is characterized through the modal operator Γ_i^α defined [1]-[2] (α=TE or TM) by

$$\mathbf{B} = \begin{pmatrix}\Gamma_1^\alpha & \mathbf{0} \\ \mathbf{0} & \Gamma_2^\alpha\end{pmatrix}\mathbf{A} = \Gamma^\alpha\mathbf{A}. \quad (3)$$

Conversion between the modal and spatial domains is performed through a Fast Modal Transform (FMT) and its inverse (FMT^{-1}). Modal sources are expressed through a known B_{om} vector added in equation (2). Consequently, the system to be solved is

$$\left(\mathbf{I} - \Gamma^\alpha\mathbf{VSV}^{-1}\right)\mathbf{B} = \mathbf{MB} = \mathbf{B_{om}} \quad (4)$$

where $\mathbf{V} = \begin{pmatrix}\mathbf{FMT} & \mathbf{0} \\ \mathbf{0} & \mathbf{FMT}\end{pmatrix}.$

B-Preconditioner

An ideal right preconditioner P^{-1} should be close to M^{-1}, since if MP^{-1} is close to the identity, the convergence of the iterative scheme is improved. The system to solve is then

$$MP^{-1}y = B_{om}.\qquad(5)$$

From the analytic solution in the perfectly conducting and insulating surface, a generic right preconditioner is determined and implemented. For a perfectly conducting case, S operator is equal to –I, which entails equation (4) becomes simply

$$\left(I + \Gamma^\alpha\right)B = B_{om}.\qquad(6)$$

Therefore $P = \left(I + \begin{pmatrix} \Gamma_1^\alpha & 0 \\ 0 & \Gamma_2^\alpha \end{pmatrix}\right)$.

For a perfectly insulating surface, waves are directly transmitted to the other medium, implying equation (4) becomes

$$\left(I - \Gamma^\alpha V \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} V^{-1}\right)B = \left(I - \begin{pmatrix} 0 & \Gamma_1^\alpha \\ \Gamma_2^\alpha & 0 \end{pmatrix}\right)B = B_{om}.\qquad(7)$$

Therefore $P = \left(I - \begin{pmatrix} 0 & \Gamma_1^\alpha \\ \Gamma_2^\alpha & 0 \end{pmatrix}\right)$.

Finally, a right preconditioner is assumed with adjustable values (a_1, a_2) depending on the circuit shape:

$$P = \left(I + \begin{pmatrix} a_1\Gamma_1^\alpha & a_2\Gamma_1^\alpha \\ a_2\Gamma_2^\alpha & a_1\Gamma_2^\alpha \end{pmatrix}\right).\qquad(8)$$

For instance, in the perfectly conducting surface $(a_1, a_2)=(1,0)$ is chosen while $(0,-1)$ corresponds to the insulating surface.

III. NUMERICAL RESULTS

The surface is meshed into 400 cells, distance between periodic walls is 0.07m, source is composed of 20 modes. Results are presented as function of the mode type (TE or TM), the proportion of metal, the relative permittivity around the meshed surface. GMRes [4] tolerance is set to 10^{-9}, no restart is used and maximum iteration is set to 400.

A-Sensitivity to Metal/Insulate proportions
Iteration number (Niter) is given for several insulate/metal proportion at 300MHz in the TE case with and without the preconditioner in Table I for the best (a_1, a_2) within $[-1,1]^2$. Similar results are observed in the TM case. To compare preconditioner costs in computation time 10000 iterations are achieved in the case of Table I for 50% of metal. One iteration lasts 0.12ms without preconditioner and 0.15ms with the preconditioner (same results in the TM case). Consequently the decrease in the number of iterations

reported in the tables below translates in comparable computational time reduction.

Metal proportion		0%	25%	50%	75%	100%
No Precond	Niter	41	96	96	96	21
	CPU time (ms)	148	704	703	688	55
Precond	best (a_1,a_2)	(0,-1)	(0,-1)	(0,-1)	(0,-1)	(1,0)
	Niter	1	36	45	52	1
	CPU time (ms)	0.15	141	187	238	0.15
Reduction factor Niter		41	2.66	2.13	1.84	21
Reduction factor CPU		986	4.99	3.75	2.89	366

TABLE I: ITERATION NUMBER FOR SEVERAL METAL PROPORTIONS.

The number of iterations is at least divided at least by 1.8. Computation time is also reduced. Best couple (a_1, a_2) does not depend on the metal/insulate proportion but on the type of the excited modes (except for the perfectly insulate or metallic canonical cases). Many examples were tested and the best solutions for TE and TM problems are respectively (0,-1) and (1,0) whatever the frequency, the substrate permittivity and the metal/insulate proportion (except for the canonical cases: 100% metal or 100% insulate).

B-Sensitivity to dielectric permittivities
Let us consider different permittivities ε_{r2} for substrate 2, a ground plane at a distance 2mm for 50% metal surface, at a higher frequency (30.1GHz), so that 15 modes are propagating. Results are reported in Table II for the TM case. Iteration number is still divided at least by 2.6. Similar results are observed for the TE case.

ε_{r2}		-20	-10	1	10	20
No Precond	Niter	90	94	120	215	233
	CPU time (ms)	596	648	1099	4952	6195
Precond	Niter	26	30	45	57	62
	CPU time (ms)	89	104	191	270	308
Reduction factor Niter		3.46	3.13	2.66	3.77	3.75
Reduction factor CPU		6.69	6.23	5.75	18.3	20.1

TABLE II: ITERATION NUMBER FOR SEVERAL RELATIVE PERMITTIVITIES ε_{R2}.

C-Sensitivity to frequency
Let us consider different frequencies, a dielectric relative permittivity ε_{r2} of 4.4, a ground plane at a distance 2mm for 50% metal surface. Results are reported in Table III for the TE case. Iteration number is still divided at least by 2.1. The best couple value is not frequency dependent. Similar results are observed for the TM case.

Frequency (GHz)		0.3	3	30	300
No Precond	Niter	98	95	120	365
	CPU time (ms)	720	689	1107	22663
Precond	best (a_1,a_2)	(0,-1)	(0,-1)	(0,-1)	(0,-1)
	Niter	45	42	47	147
	CPU time (ms)	196	167	196	1837
Reduction factor Niter		2.17	2.26	2.55	2.46
Reduction factor CPU		3.67	4.12	5.64	12.33

TABLE III: ITERATION NUMBER FOR SEVERAL FREQUENCIES.

D-Operator Eigenvalue Analysis

To understand the convergence improvement, the eigenvalues of the operators described in equations (4) and (5) are compared at 300MHz and 30.1GHz for 50% metal proportion without ground plane in vacuum and represented respectively in Figures 2 and 3. Even if preconditioned eigenvalues are better clustered than eigenvalues of the WCIP without preconditioner, no rule has been established to find out the best couples (a_1, a_2) from the eigenvalue observation.
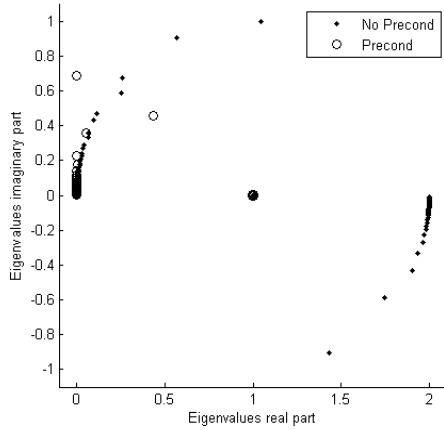

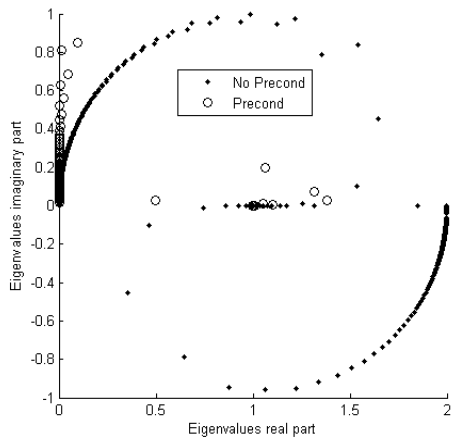Fig. 2.Eigenvalues representation at 300MHz.

Fig. 3.Eigenvalues representation at 30.1GHz.

IV. EXTENSION TO 2D PROBLEMS

Preconditioner (8) is applied in the 2d case with respect to (5) (TE and TM appear simultaneously). A square slot over an infinite dielectric (ε_{r2} of 4.4) is illuminated by a z-polarized electric field at 3GHz composed of 20 modes. The number of unknowns is 160000. Iteration number is observed for several metal proportions. For the sake of comparison, numerical experiments are reported in Table IV with and without preconditioner. Then its dual is studied: a square patch, results are reported in Table V. One can notice that the best couple does not depend on the metal proportion in both cases.

Metal proportion		19%	36%	50%	75%	90%
No Precond Niter		106	75	66	59	53
Precond	Niter	69	37	33	33	5
	best (a_1,a_2)	(0.9, -0.1)	(0.9, 0)	(0.9, 0)	(0.9, 0)	(1, 0)
Reduction factor Niter		1.53	2.02	2	1.78	10.6

TABLE IV: PRECONDITIONER IN THE SLOT CASE.

Metal proportion		19%	36%	50%	75%	90%
No Precond Niter		48	48	50	69	96
Precond	Niter	4	7	14	33	68
	best (a_1,a_2)	(0.4, -0.6)	(0.4, -0.6)	(0.4, -0.6)	(0.4, -0.6)	(0.4, -0.4)
Reduction factor Niter		12	6.85	3.57	2.09	1.4

TABLE V: PRECONDITIONER IN THE PATCH CASE.

V. CONCLUSION

A new physically-based right preconditioner is developed in this paper for the 1d and 2d WCIP. It allows at least to divide by 1.4 the iteration number compared to the initial Krylov subspace method without preconditioner for some examples. Several attempts to justify the adjustable parameter choice have been explored in the 1d case, but no rules emerge from it. Extension to the 2d case has been achieved on two examples and results are promising with a noticeable reduction of the number of iterations.

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