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Coordination of Autonomic Functionalities in Communications Networks

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Abstract—**Future communication networks are expected to feature autonomic (or self-organizing) mechanisms to ease deployment, tune parameters automatically and repair the network. Self-organizing mechanisms are designed as stand-alone entities, even though multiple mechanisms run in parallel in operational networks. An efficient coordination mechanism will be the major enabler for large scale deployment of self-organizing networks. We model self-organizing mechanisms as control loops, and study the conditions for stability when running control loops in parallel. Based on control theory, we propose a distributed coordination mechanism to stabilize the system. In certain cases, coordination can be achieved without any exchange of information between control loops. The mechanism remains valid in the presence of noise via stochastic approximation. Instability and coordination in the context of wireless networks are illustrated with two examples. We are essentially concerned with linear systems, and the applicability of our results for non-linear systems is discussed. ([1], [2])**

Index Terms—**Self-Organizing Networks;Self-Organization;Self-Optimization;Self-configuration;Stochastic approximation;Stability;Coordination;Dynamical Systems;**

I. INTRODUCTION

Deployment, optimization and maintenance of communication networks are complex tasks which occupy many network engineers everyday. In order to ease this burden and reduce costs of operations, researchers and industrials have proposed to include autonomic functionalities in future networks. Networks with autonomic entities are often called Self-organizing networks (SON). Self-organization should enable at least partial automation of the configuration of newly deployed network nodes (self-configuration), of parameter tuning (self-optimization) and of reparation of faulty network nodes (self-healing).

In wireless networks, SON functionalities such as Inter-Cell Interference Coordination (ICIC), load balancing, management of Base Station (BS) sleep mode, mobility management and drive test automation have been identified as important use cases ([1]). Future standards for wireless networks such as Long Term Evolution (LTE) feature SON functionalities. In previous contributions, SON functionalities have been designed as stand-alone entities ([2], [3], [4], [5], [6]). Such contributions did not take into account the interaction between different SON functionalities operating simultaneously. Since SON functionalities are numerous, a large number of SON entities might operate simultaneously in the network. This poses a fundamental stability issue, if we cannot guarantee stable interaction of multiple SON entities. The solution for robust coordination of SON functionalities is probably the most important problem in the domain of SON, and a key enabler for large scale deployment of this technology.

Our contribution: The main contribution of this paper is to propose a mathematical model for the interaction of multiple SON mechanisms running in parallel, along with a practically implementable coordination mechanism. We model SON mechanisms as control loops, so that the system can be described by an Ordinary Differential Equation (ODE). Stability is studied using the Lyapunov approach used by control theorists. In particular, stability conditions can be stated in terms of *linear matrix inequalities* ([7]). When stability does not hold naturally, we propose a *coordination* mechanism which forces stability. The design of a coordination mechanism corresponds, in the context of control theory, to the concepts of *controllability* and *state-feedback synthesis*. We propose a generic coordination mechanism which can be implemented in a distributed way. When the network is controlled using measurements corrupted by additive noise, the results remain valid using stochastic approximation theorems ([8]).

To the best of our knowledge, no previous contribution has studied the problem of SON coordination using this control theory/stochastic approximation-based framework and provided a generic coordination mechanism which is practically implementable (distributed or based on local information).

The rest of the paper is organized as follows: in Section II we state the proposed model for interaction of SON mechanisms running in parallel and the coordination problem to be solved. In Section III we examine the case where performance indicators are linear functions of the parameters, and propose a practically implementable coordination mechanism. In Section IV we study fully distributed coordination when no exchange of information between SONs is needed. In Section V we illustrate the application of our model to traffic management in wireless networks with two examples. Section VI concludes the paper. In appendices A and B we recall the basic notions of stability for ODEs and linear ODEs respectively.

II. SON COORDINATION AS PARALLEL CONTROL LOOPS

A. The model

A SON mechanism is an entity which monitors a given performance indicator and controls a scalar parameter. The current value of the performance indicator is observed, and the parameter is modified accordingly to attain some objective. We consider $I > 1$ SON mechanisms operating simultaneously. We define θ_i the parameter controlled by the i-th SON mechanism and $\theta = (\theta_1, \ldots, \theta_I)$ the vector of parameters. The i-th SON mechanism monitors a performance indicator $F_i(\theta)$ and updates its parameter θ_i proportionally to it. $F(\theta) = (F_1(\theta), \ldots, F_I(\theta))$ is the direction of update of θ.

We say that the i-th SON mechanism operates in *stand-alone* mode if all parameters but θ_i are kept constant. Namely all the other mechanisms are shut down. The i-th SON mechanism operating in stand-alone is described by the ODE:

$$\dot{\theta}_i = F_i(\theta) \ , \ \dot{\theta}_j = 0, j \neq i. \tag{1}$$

We say that the SON mechanisms operate in *parallel* mode if all parameters are modified simultaneously, which is described by the ODE:

$$\dot{\theta} = F(\theta). \tag{2}$$

We say that the i-th SON mechanism is stable in stand-alone mode if there exists $\theta_i^{*,i}$ which is *asymptotically stable* for (1). The definition of asymptotic stability is recalled in appendix A. It is noted that $\theta_i^{*,i}$ can depend on θ_j, $j \neq i$. We say that the SON mechanisms are stable in parallel mode if there exists θ^* which is asymptotically stable for (2). Typically, the SON mechanisms are designed and studied in a stand-alone manner, so that stand-alone stability is verified.

However, stand-alone stability does not imply parallel stability. First consider a case where $F_i(\theta)$ does not depend on θ_j, for all pairs (i, j) , $j \neq i$. Then (2) is a set of I parallel independent ODEs, so that stand-alone stability implies parallel stability. On the other hand, if there exists $i \neq j$ such that $F_i(\theta)$ depends on θ_j, then the situation is not so clear-cut. We say that SON i and j *interact*. Namely, interaction potentially introduces *instablity*.

In the remainder of this article we will be concerned with conditions for parallel stability, and designing coordination mechanisms to force stability whenever possible.

B. Examples of parallel mechanisms

Two particular cases of parallel mechanisms will be of interest. The first case is what we will call *zero-finding* algorithms. Each SON mechanism monitors the value of a performance indicator and tries to achieve a fixed target value for this performance indicator. Namely:

$$F_i(\theta) = f_i(\theta) - \overline{f}_i, \tag{3}$$

where f_i is the performance indicator monitored by SON i and $\overline{f}_i$ - a target level for this performance indicator. The goal of the i-th SON mechanism is to find θ_i^* such that $f_i(\theta^*) = \overline{f}_i$.

If $\theta_i \mapsto f_i(\theta_1, \ldots, \theta_i, \ldots, \theta_I)$ is strictly decreasing $1 \leq i \leq I$ then stand-alone stability is assured.

Another case of interest is maximization algorithms. Each SON mechanism tries to maximize a given performance indicator. There exists a continuously differentiable function g_i such that:

$$F_i(\theta) = \nabla_{\theta_i} g_i(\theta). \tag{4}$$

In stand-alone mode, SON i indeed converges to a local maximum of g_i. If we restrict θ to a closed, convex and bounded set and if $\theta_i \mapsto g_i(\theta_1, \ldots, \theta_i, \ldots, \theta_I)$ is concave $1 \leq i \leq I$, we fall within the framework of *concave games* considered in [9]. An important result of [9] is that if we add an assumption called *diagonal strict convexity*, then parallel stability occurs. However, diagonal strict convexity is fairly restrictive, and without it there is no guarantee that parallel stability occurs, and coordination is needed.

C. Discrete time algorithms

Our model is based on ODEs, which are both *deterministic* and *continuous-time* objects. In a practical system, the parameters evolve in discrete time. Furthermore, the parameters evolve stochastically because they are updated based on noisy feedback, due to measurement noise. Denote by $\theta[t] = (\theta_1[t], \ldots, \theta_I[t])$ the value of parameters at time $t \in \mathbb{N}$. At each parameter update, performance indicators are estimated based on measurements so that SON i obtains the quantity $F_i(\theta[t]) + M_i[t]$, with $\mathbb{E}[M_i[t]] = 0$. The additive noise $\{M[t]\}_{t \in \mathbb{N}}$ is introduced by the measurements. The parameters are updated using the noisy feedback:

$$\theta[t+1] = \theta[t] + \epsilon(F(\theta[t]) + M[t]). \tag{5}$$

where $\epsilon > 0$ is a small constant. Stochastic approximation ([8], [10]) gives a strong link between the noisy, discrete-time (5) and the ODE studied in our model (2). In particular it can be shown that (under some technical conditions) $\{\theta[t]\}_{t \in \mathbb{N}}$ converges to *asymptotically stable* sets of the ODE when $\epsilon \to 0^+$. Studying ODEs is sufficient to study the stability of the discrete-time, noisy algorithms used in practical systems where ϵ does not vanish, but is a small constant.

III. COORDINATION FOR LINEAR SYSTEMS

A. Linear systems

In the remainder of this paper, we study the case where F is affine:

$$F(\theta) = A\theta + b, \tag{6}$$

with b a vector of size I and A a matrix of size $I \times I$. We assume that A is invertible and we define $\theta^* = -A^{-1}b$. The SON mechanisms running in parallel are described by the linear ODE:

$$\dot{\theta} = A\theta + b = A(\theta - \theta^*). \tag{7}$$

It is noted that in the linear case, we always fall within the scope of zero-finding algorithms described previously, by

defining:

$$f_i(\theta) = \sum_{1 \le j \le I} A_{i,j}\theta_j \ , \ \overline{f}_i = -b_i, \tag{8}$$

$$\dot{\theta}_i = f_i(\theta) - \overline{f}_i. \tag{9}$$

Assumptions 1. *A is invertible and* $A_{i,i} < 0$, $1 \le i \le I$.

Stand-alone stability occurs if and only if (iff) $A_{i,i} < 0$, $1 \le i \le I$, i.e all the diagonal terms of A are strictly negative. Basic results on linear ODEs are recalled in appendix B. Namely, parallel stability holds iff all the eigenvalues of A have a strictly negative real part. We say that A is a *Hurwitz matrix*. We use assumptions 1 for the rest of this article.

B. Coordination

1) Coordination mechanism: If A has at least one eigenvalue with positive or null real part, convergence to θ^* does not occur, and a coordination mechanism is needed. We consider a *linear* coordination mechanism, where A is replaced by CA with C a $I \times I$ real matrix. The ODE for the coordinated system is:

$$\dot{\theta} = CA(\theta - \theta^*). \tag{10}$$

The coordination mechanism can be interpreted as transforming the performance indicator monitored by SON i from f_i to a linear combination of the performance indicators monitored by all the SON mechanisms. For X, X' symmetric matrices, we write $X \prec X'$ if $X' - X$ is positive definite. As explained in appendix B, stability is achieved if there exists a symmetric matrix X such that:

$$(CA)^T X + XCA \prec 0 \ , \ 0 \prec X. \tag{11}$$

In particular,

$$V(\theta) = (\theta - \theta^*)^T X(\theta - \theta^*), \tag{12}$$

acts as a Lyapunov function.

2) Distributed implementation: It is noted that the choice for the coordination matrix C is not unique. For instance $C = -A^{-1}$ ensures stability. For the coordination mechanism to be scalable with respect to the number of SONs, C should be chosen to allow *distributed* implementation. We say that SON j is a neighbor of SON i if $\frac{\partial f_j}{\partial \theta_i} \ne 0$. We define $\mathcal{I}_i$ the set of neighbors of i. The coordination mechanism is distributed if each SON needs only to exchange information with its neighbors.

We give an example of a coordination mechanism which can always be distributed. The mechanism is based on a *separable* Lyapunov function. Define the weighted square error:

$$V(\theta) = \sum_{i=1}^{I} w_i(f_i(\theta) - \overline{f}_i)^2 = (\theta - \theta^*)^T A^T W A(\theta - \theta^*), \tag{13}$$

with $w = \{w_i\}_{1 \le i \le I}$ strictly positive weights and $W = \mathbf{diag}(w)$ i.e the diagonal matrix with diagonal elements $\{w_i\}_{1 \le i \le I}$. Coordination is achieved by following the gradient of $-V$ so that V is a Lyapunov function:

$$\dot{\theta} = -\nabla_\theta V(\theta) = -A^T W A(\theta - \theta^*). \tag{14}$$

Namely, we choose $C = -A^T W$. We can verify that the mechanism is distributed:

$$\dot{\theta}_i = \sum_{j=1}^{I} 2w_i \frac{\partial f_j}{\partial \theta_i}(f_j(\theta) - \overline{f}_j) = \sum_{j \in \mathcal{I}_i} 2w_j \frac{\partial f_j}{\partial \theta_i}(f_j(\theta) - \overline{f}_j). \tag{15}$$

Indeed, the update of θ_i only requires knowledge of $\frac{\partial f_j}{\partial \theta_i}$ and $f_j(\theta) - \overline{f}_j$, for $j \in \mathcal{I}_i$, and this information is available from the *neighbors* of i.

C. Applicability of the linear model and parameter estimation

For practical systems, performance indicators $F(\theta)$ are not linear functions of θ. However, as long as they are smooth, they can be approximated by linear functions using a Taylor expansion. Consider a point θ^* with $F(\theta^*) = 0$. If the values of θ are restricted to a small neighborhood of θ^*:

$$F(\theta) \approx JF(\theta^*)(\theta - \theta^*), \tag{16}$$

with $JF(\theta^*)$ the Jacobian of F evaluated at θ^*. The Hartman-Grossman theorem ([11]) states that on a neigbourhood of θ^*, stability of the system with linear approximation implies stability of the original, non-linear system. Hence implementing the proposed coordination mechanism where A is replaced by JF ensures stability if we constrain θ to a small neighborhood of θ^*.

The matrix A might be unknown, and we can only observe noisy values of $F(\theta)$ for different values of θ. The crudest approach is to estimate A through finite differences:

$$a_{i,j} \approx \frac{f_j(\theta + e_i\delta\theta_i) - f_j(\theta - e_i\delta\theta_i)}{2\delta\theta_i}, \tag{17}$$

with e_i the i-th unit vector. The results are averaged over several successive measurements and additive noise is omitted for notation clarity. In general, the measurements of F are obtained by calculating the time average of some output of the network during a relatively long time, so that a form of the central limit theorem applies and the additive noise is Gaussian. In this case, a better method is to employ *least-squares regression*. Least-squares regression is a well studied topic with very efficient numerical methods ([12]) even for large data sets so that estimation of A is not computationally difficult.

Finally, since practical systems do not remain stationary for an infinite amount of time, a database with values of A for each set of operating conditions must be maintained. In the context of wireless networks, the relationship between parameters and performance indicators changes when the traffic intensity changes because of daily traffic patterns. For instance, during the night traffic is very low, and traffic peaks are observed at the end of the day. If the network can be sampled for a few days, a database with estimated values of A at (for instance) each hour of the day can be constructed.

IV. FULLY DISTRIBUTED COORDINATION

In this section we study fully distributed coordination, where the coordination matrix C is *diagonal*. As said previously,

if $C_{i,j} \neq 0$, $i \neq j$ then SON i and j need to exchange information. In fully distributed coordination, no information is exchanged. We say that the system can be coordinated in a *fully distributed* way iff there exists $c \in \mathbb{R}^I$ such that $\mathbf{diag}(c)A$ is a Hurwitz matrix. The problem was introduced in [13], to solve linear systems iteratively.

For $I = 2$ fully distributed coordination can always be achieved. For $I = 3$ it is also possible iff A^{-1} has at least one non-zero diagonal element. These results are attractive from a practical point of view: if there are 3 or less SONs to coordinate, it suffices to modify their feedback coefficient, without any exchange of information or interface between them.

Consider $\mathcal{N}$ and $\mathcal{N}'$ subsets of $\{1, \cdots, I\}$ of size q, then the determinant of $(A_{i,j})_{i \in \mathcal{N}, j \in \mathcal{N}'}$ is called a minor of order q. It is a principal minor if $\mathcal{N} = \mathcal{N}'$, and it is a first minor if $q = I - 1$.

A sufficient and a necessary conditions were given for fully distributed coordination in terms of minors in [13].

Theorem 1 ([13]). *(i) If there exists a sequence of non-zero principal minors M_q, $q \in \{1, \cdots, I\}$ such that M_{q-1} is a first principal minor of M_q, there exists $c \in \mathbb{R}^I$ such that $\mathbf{diag}(c)A$ is a Hurwitz matrix.*

(ii) If there exists $q \in \{1, \cdots, I\}$ such that all principal minors of A of order q are zero then for all $c \in \mathbb{R}^I$ $\mathbf{diag}(c)A$ is not a Hurwitz matrix.

[14] gives a constructive result showing that in certain cases, a suitable c can be found by choosing some of its components to be arbitrarily close to 0. This is equivalent to separating the time scales on which certain groups of components of the corresponding ODE evolve.

Theorem 2 ([14]). *Decompose A by blocks*

$$A = \begin{pmatrix} A^{1,1} & A^{1,2} \\ A^{2,1} & A^{2,2} \end{pmatrix} \tag{18}$$

with $A^{1,1}$ of size $q \times q$ and $c(\epsilon) = (c_1, \cdots, c_q, \epsilon c_{q+1}, \cdots, \epsilon c_I)$. Assume that $\mathbf{diag}(c_1, \cdots, c_q)A^{1,1}$ is a Hurwitz matrix.

Then there exists $(c_{q+1}, \cdots, c_I)$ such that $\mathbf{diag}(c(\epsilon))A$ is a Hurwitz matrix if there exists c' such that $\mathbf{diag}(c')(A^{2,2} - A^{2,1}(A^{1,1})^{-1}A^{1,2})$ is a Hurwitz matrix.

For $I = 2$ and $I = 3$, theorems 1 and 2 give a clear cut solution. We recall a consequence of the Routh-Hurwitz theorem ([15]).

Lemma 1. *(Routh-Hurwitz) Consider M a $I \times I$ matrix.*
(i) ($I = 2$) M is a Hurwitz matrix iff $\det(M)$ and $-\mathbf{tr}(M)$ are strictly positive,
(ii) ($I = 3$) M is a Hurwitz matrix iff $-\det(M)$ and $-\mathbf{tr}(M)$ and $\mathbf{tr}(M)\mathbf{tr}(M^{-1}) - 1$ are strictly positive.

Theorem 3. *With assumptions 1, for $I = 2$:*
i) the system can always be coordinated in a fully distributed way.
ii) If $(c_1, c_2) = (1, \mathbf{sign}\det(A)\epsilon)$, then $\mathbf{diag}(c)A$ is a Hurwitz matrix for small $\epsilon > 0$.

Proof. i) $\det(A) \neq 0$ and diagonal elements of A are non-zero. Hence all principal minors of A are non-zero, and theorem 1 proves the result.

ii) $\mathbf{tr}(\mathbf{diag}(c)A) = A_{1,1} + O(\epsilon)$ and $\mathbf{sign}(\det(\mathbf{diag}(c)A)) = 1$. Since $A_{1,1} < 0$, lemma 1 proves that $\mathbf{diag}(c)A$ is a Hurwitz matrix for small all $\epsilon > 0$. $\square$

Theorem 4. *With assumptions 1, for $I = 3$:*
i) If all diagonal entries of $B = A^{-1}$ are zero, then the system cannot be coordinated in a fully distributed way.
ii) Otherwise, the system can be coordinated in a fully distributed way. Assume that $B_{2,2} \neq 0$ without loss of generality. Define $C(\epsilon) = \mathbf{diag}(1, \epsilon c_2, \epsilon c_3)$ with:

$$\frac{B_{2,2}}{c_2} + \frac{B_{3,3}}{c_3} < 0 \ , \ c_2 c_3 \det(A) < 0. \tag{19}$$

A possible choice for (c_2, c_3) is $c_2 = -B_{2,2}$ and $c_3 = -2\mathbf{sign}(\det(A)c_2)|B_{3,3}|$ if $B_{3,3} \neq 0$ and $c_3 = -\mathbf{sign}(\det(A)c_2)$ otherwise. Then $C(\epsilon)A$ is a Hurwitz matrix for all small $\epsilon > 0$.

Proof. i) The diagonal entries of A^{-1} are proportional to the first principal minors of A by Cramer's rule. So by theorem 1 the system cannot be coordinated since all first principal minors are zero.

ii) $\mathbf{tr}(C(\epsilon)A) = A_{1,1} + O(\epsilon)$ and:

$$\mathbf{tr}(C(\epsilon)A)\mathbf{tr}((C(\epsilon)A)^{-1}) = \frac{A_{1,1}}{\epsilon}\left(\frac{B_{2,2}}{c_2} + \frac{B_{3,3}}{c_3}\right) + O(1).$$

Since $A_{1,1} < 0$, for small ϵ we have $\mathbf{tr}(C(\epsilon)A) < 0$ and $\mathbf{tr}(C(\epsilon)A)\mathbf{tr}((C(\epsilon)A)^{-1}) > 1$. Using lemma 1, $C(\epsilon)A$ is a Hurwitz matrix for small $\epsilon > 0$. $\square$

It is noted that for large I, theorems 1 and 2 are not sufficient since they require a non-polynomial number of operations to obtain a solution. Some search heuristics are given in [14]. However, to the best of our knowledge there exists no simple necessary and sufficient conditions on A for fully distributed coordination.

V. APPLICATION TO WIRELESS NETWORKS

In this section we illustrate instability and coordination in the context of wireless networks using two examples. We show that instability occurs even in simple models with as few as two SONs in parallel.

A. Admission control and resource allocation

1) Model: We consider a BS in downlink, serving elastic traffic. Users enter the network according to a Poisson process with arrival rate λ, to download a file of exponential size σ, with $\mathbb{E}[\sigma] < +\infty$. The BS has x_{max} parallel resources available, and we write $x \in [0, x_{max}]$ the amount of resources used. We ignore the granularity of resources, either assuming that there are a large number of resources or using time sharing, using each resource a proportion $\frac{x}{x_{max}}$ of the time. Depending on the access technology, resources can be: codes in Code Division Multiple Access (CDMA), time slots in Time

Division Multiple Access (TDMA), time-frequency blocks in Orthogonal Frequency-Division Multiple Access (OFDMA) etc. When a user is alone in the system, his data rate is Rx. Users are served in a processor sharing manner (for instance through Round Robin scheduling): if there are n active users, each user has a throughput of $\frac{xR}{n}$. Admission control applies. We define $\beta \geq 0$ a blocking threshold and the probability of accepting a new user when n users are already present in the system is $\phi(n - \beta)$ with $\phi : \mathbb{R} \to [0, 1]$ a smooth, strictly decreasing function and $\phi(n) \underset{n \to +\infty}{\to} 0$. We choose ϕ as a logistic function for numerical calculations:

$$\phi(n) = \frac{1}{1 + e^n}. \qquad (20)$$

Define $n(t)$ the number of active users in the system at time t, then $n(t)$ is a continuous time Markov chain. $n(t)$ is ergodic because the probability of accepting a new user goes to 0 as $n \to \infty$. We define the load:

$$\rho(x) = \frac{\lambda \mathbb{E}[\sigma]}{xR}. \qquad (21)$$

We write π the stationary distribution of $n(t)$. $n(t)$ is reversible, and π can be derived from the detailed balance conditions:

$$\pi(n, x, \beta) = \frac{\rho(x)^n \prod_{k=0}^{n-1} \phi(k - \beta)}{\sum_{n \geq 0} \rho(x)^n \prod_{k=0}^{n-1} \phi(k - \beta)}. \qquad (22)$$

Using Little's law, the mean file transfer time is given by the average number of active users divided by the arrival rate:

$$T(x, \beta) = \frac{1}{\lambda} \sum_{n \geq 0} n \pi(n, x, \beta). \qquad (23)$$

Let R_{min} a minimal data rate required to ensure good Quality of Service (QoS). We say that there is an outage in a state of the system if users have a throughput lower than R_{min}. When there are n active users in the system, outage occurs iff:

$$n > \frac{xR}{R_{min}}. \qquad (24)$$

Denote by $\mathbf{1}(.)$ the indicator function, the outage probability is:

$$O(x, \beta) = \sum_{n \geq 0} \pi(n, x, \beta) \mathbf{1}_{(0,+\infty)} \left(n - \frac{xR}{R_{min}} \right). \qquad (25)$$

In this model, $x \to O(x, \beta)$ is not smooth, which is why we introduce the smoothed outage $\tilde{O}$:

$$\tilde{O}(x, \beta) = \sum_{n \geq 0} \pi(n, x, \beta) \psi \left(n - \frac{xR}{R_{min}} \right). \qquad (26)$$

with ψ a smooth function approximating $\mathbf{1}_{(0,+\infty)}$. We also choose ψ as a logistic function for numerical calculations.

This queuing system is controlled by two mechanisms, and that control occurs on a time scale much slower than the arrivals and departures of the users, so that the mean file transfer time and outage probability are relevant performance metrics, and can be estimated from (noisy) measurements. The mechanisms are:

- *Resource allocation:* a mechanism adjusts the amount of used resources to reach a target outage rate. Such mechanisms have been considered in green networking when a BS can switch off part of its resources in order to save energy.

  Another application is interference coordination: using more resources will create inter-cell interference in other BSs and degrade their QoS. Hence BSs should use as little resources as possible, as long as their target QoS is met.

- *Admission control:* another mechanism adjusts the admission control threshold to reach a target file transfer time. In particular, it is noted that without admission control, the mean file transfer time becomes infinite in overload.

It is noted that $x \to \tilde{O}(x, \beta)$ is strictly decreasing and $\beta \to T(x, \beta)$ is strictly increasing. Using the notations of Section II, we have $I = 2$ control loops, $\theta_1 \equiv x$, $\theta_2 \equiv \beta$, $f_1 \equiv \tilde{O}$, $f_2 \equiv -T$. Consider $\theta^* = (x^*, \beta^*)$ an operating point. The stability in the neighborhood of (x^*, β^*) can be calculated. The system will fail to converge to the desired operating point as long as the Jacobian matrix has a negative determinant, so that there are two eigenvalues of opposite sign:

$$-\frac{\partial \tilde{O}}{\partial x} \frac{\partial T}{\partial \beta}(x^*, \beta^*) + \frac{\partial \tilde{O}}{\partial \beta} \frac{\partial T}{\partial x}(x^*, \beta^*) < 0 \qquad (27)$$

2) Results: We now evaluate the stability region of the system numerically by checking condition (27) for various operating points. We choose the following parameter values: $\lambda = 0.5$ users/s, $\mathbb{E}[\sigma] = 10$ Mbits, $R = 15$ Mbits/s, $R_{min} = 2$ Mbits/s, $x_{max} = 1$. Figure 1 presents the results. In the white region the system is stable, and in the gray region it is unstable. Even in such a simple setting with 1 BS and 2 SON mechanisms, instability occurs for a large set of operating points.

B. Distributed interference control

1) Model: We turn to a case in which the SONs are not co-located, and distributed coordination is required. We study a model in which each BS can choose its transmitted power in order to control the QoS in the network. We consider a dense network, and the objective of each BS is to make sure that other BSs achieve a target coverage probability, by not transmitting at too high power. We define the coverage probability as the proportion of users whom achieve a given minimum data rate. This is relevant to current wireless networks, where BSs are linked with an interface (called X2 interface in the LTE standard), so that a BS experiencing low data rates or congestion can send an alarm to force other BSs to transmit at lower power to reduce inter-cell interference, or to offload some of its traffic to reduce congestion.

We consider N_s BSs serving a bounded area $\mathbb{A}$. The zone served by BS i is written $\mathbb{A}_i$. Interference from other BSs is treated as Gaussian noise. We denote by $h_i(r)$ the signal
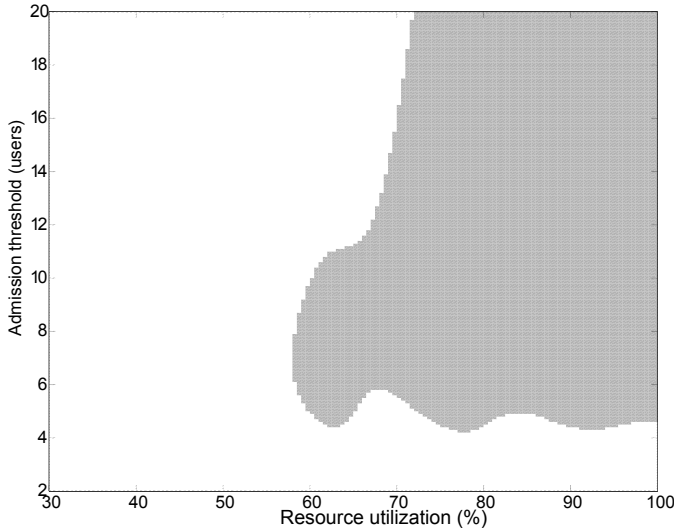
Fig. 1. Stability region of the system

attenuation between BS i and location r. $h_i(r)$ includes both path-loss and shadowing. For numerical calculations, we use the classical model where the signal attenuation (in dB) is a linear function of the distance, and shadowing is (in dB) a centered Gaussian random variable. Fast-fading is ignored. BS i transmits at power P_i. The Signal to Interference plus Noise Ratio (SINR) at location $r \in \mathbb{A}_s$ is calculated by:

$$S_i(r) = \frac{h_i(r)P_i}{N_0 + \sum_{j \neq i} h_j(r)P_j}, \qquad (28)$$

with N_0 the thermal noise. We denote by $R_i(r)$ the data rate at location r when no other users are being served by BS i. The data rate $R_i(r)$ is calculated using the Shannon formula:

$$R_i(r) = w \log_2(1 + S_i(r)), \qquad (29)$$

with w the system bandwidth. Consider a target data rate R_{min}. We define K_i the coverage probability for BS i, which is the probability that a user arrives in $\mathbb{A}_i$ at a location r such that $R_i(r) \geq R_{min}$. The coverage probability can be calculated by:

$$K_i = \frac{1}{|\mathbb{A}_i|} \int_{\mathbb{A}_i} \mathbf{1}_{\{R_i(r) \geq R_{min}\}}(r)dr. \qquad (30)$$

Consider users arriving in the network according to a space-time Poisson process of intensity λ, i.e the average number of users arriving during time dt in a region of size dr centered at location r is $\lambda dr dt$. Then the number of users arriving during time interval $[0, T]$ in $\mathbb{A}_i$ which have a data rate superior to R_{min}, divided by $|\mathbb{A}_i|\lambda T$ is an unbiased estimate of K_i. We define $\mathcal{B}_i$ the set of BSs exchanging information with i. The relation can either come from geographical proximity, or from propagation conditions, so that BSs exchanging information are BSs which strongly interfere each other.

We define G_i the coverage probability of the BSs exchanging information with i by:

$$G_i = \frac{\sum_{j \in \mathcal{B}_i} |\mathbb{A}_j| K_j}{\sum_{j \in \mathcal{B}_i} |\mathbb{A}_j|}. \qquad (31)$$

We study the case where each BS adjusts its transmitted power P_i to avoid degrading the network QoS and make sure that other BSs reach a target coverage probability $\overline{G}_i$. BS i transmitting at higher power decreases the data rate of users in $\mathbb{A}_j$, $j \in \mathcal{B}_i$, so that $P_i \mapsto G_i$ is strictly decreasing. It is noted that we work with P_i in dBm (not in linear scale). Using the notations of Section II, we have $I = N_s$ SONs, with $\theta_i \equiv P_i$, and $f_i \equiv G_i$.

2) Results: For numerical calculations, the signal attenuation (in dB) at distance d (in km) is $128 + 36.4 \log_{10}(d)$. The shadowing standard deviation is 6 dB. The thermal noise power spectral density is -174 dBm/Hz, the system bandwidth w is 20 MHz. The minimal data rate R_{min} is 20 Mbits/s, corresponding to a minimal SINR of 0 dB.

We first consider an hexagonal network with 12 BSs and inter-site distance of 500 m, using a wrap-around to avoid border effects and ensure that all BSs are symmetric. Namely the BSs are placed on a torus. We are interested in stability of the operating point P^* , $P_i^* = 46$ dBm , $1 \leq i \leq I$. The corresponding target value for G_i is $\overline{G}_i = 80\%$, $1 \leq i \leq I$. Without coordination, this operating point is unstable as shown by calculating the Jacobian of G at P^* using finite differences and calculating its eigenvalues. Another illustration of instability is given by plotting trajectories of the corresponding (non-linear) ODE starting in a neighbourhood of P^*. On figures 2 and 3 we represent the transmitted powers and coverage probabilities respectively as a function of time obtained by discretization of the ODE. Only BSs 1, 2 and 3 are represented for clarity, where BSs 2 and 3 exchange information with BS 1. Clearly, P^* is not stable and the plotted solution does not remain close to P^*. On figures 4 and 5 we show the same quantities when the coordination mechanism is applied with $C = -(JG(P^*))^T$ as explained in the previous section. Indeed, the solution stays close to P^* at all times.

Practical networks do not exactly follow an hexagonal model due to non-uniformity of traffic and scarcity of sites with good propagation characteristics. A popular model to take into account this irregularity is to assume that BSs locations follow a Poisson point process on the plane. Based on measurements from operational networks, results in [16] suggest that from the point of view of coverage, the Poisson model is pessimistic while the hexagonal model is optimistic and reality lies somewhere in-between. We show that, like in the hexagonal case, instability occurs with a non-negligible probability. Hence instability is not an artifact of the hexagonal model.

We use the following procedure. For each snapshot we generate BS locations according to a Poisson process on a square area of 4 km^2 , and we find a point $P^* = \{P_i^*\}_{1 \leq i \leq I}$ at which all BSs have the same coverage probability. To be consistent with the hexagonal model, each BS exchanges

information with the 6 closest BSs (in terms of distance). We calculate the Jacobian matrix of G at P^* to assess its stability. We simulate 100 snapshots to estimate the probability of observing an unstable network. Figure 6 shows the probability of instability for different values of the number of BSs per unit of surface. There is a non-negligible probability of instability, and this probability rapidly goes to 1 when the network becomes denser. An intuitive explanation is that when the network gets denser, BSs become closer to each other, so that the coupling between the corresponding SON mechanisms becomes stronger and causes instability.
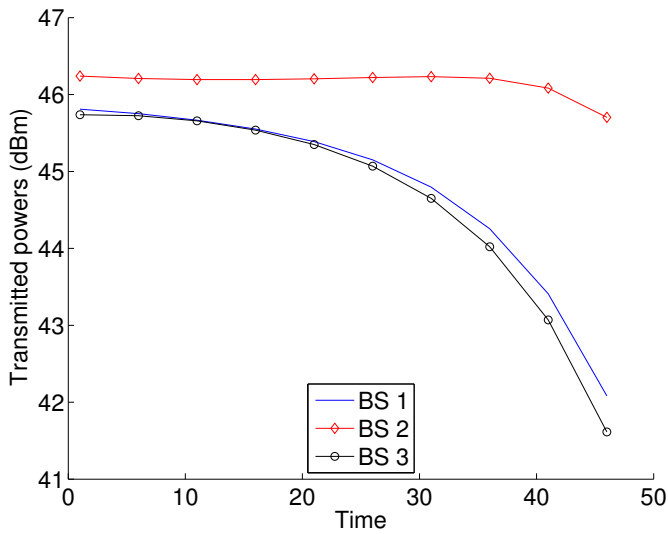

Fig. 2. Hexagonal network, no coordination, transmitted powers as a function of time
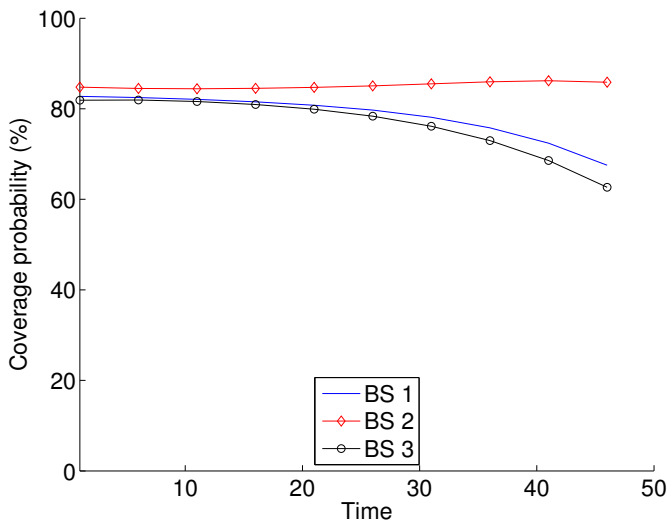

Fig. 3. Hexagonal network, no coordination, coverage probability as a function of time
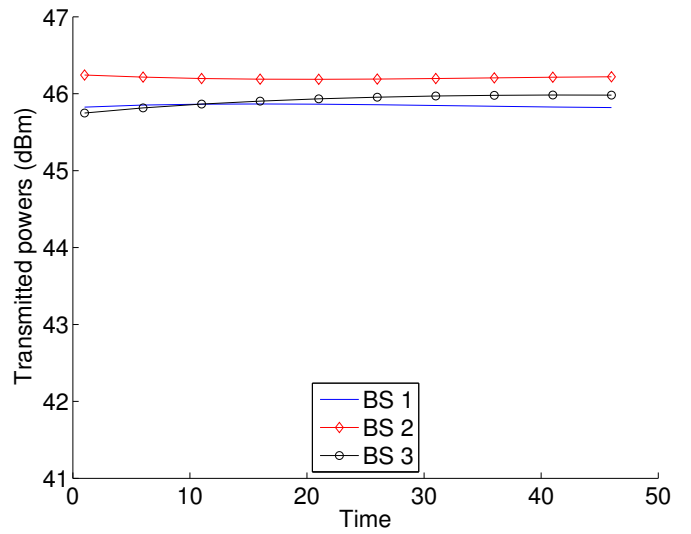

Fig. 4. Hexagonal network with coordination, transmitted powers as a function of time
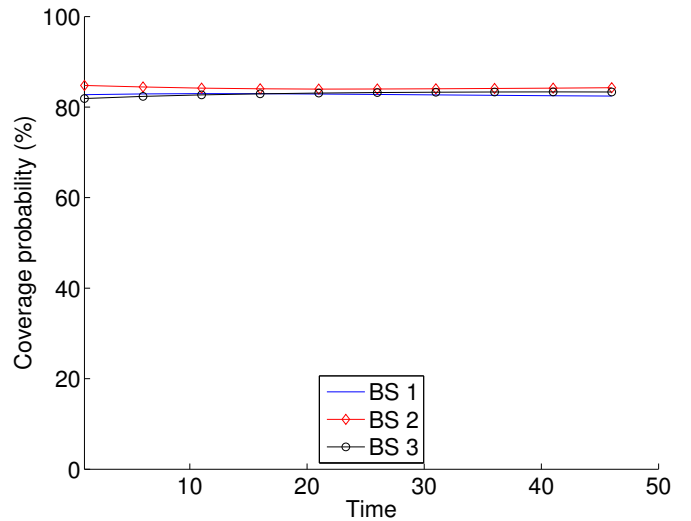

Fig. 5. Hexagonal network with coordination, coverage probability as a function of time

VI. CONCLUSION

In this paper we have studied the problem of coordinating multiple SON entities operating in parallel. Using tools from control theory and Lyapunov stability, we have proposed a co-ordination mechanism to stabilize the system. The mechanism can be implemented in a distributed fashion so it is scalable with respect to the number of SONs. We have shown that the mechanism remains valid in the presence of measurement noise, using stochastic approximation. Instability and coordination in the context of wireless networks have been illustrated with two examples. We have shown that even for two control loops, instability can occur, and the influence of network geometry has been investigated. An interesting continuation
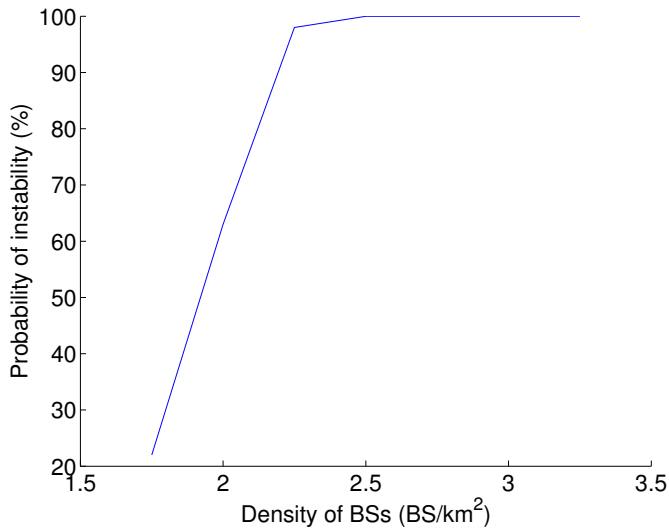
Fig. 6. Probability of observing an unstable network as a function of the density of BSs

of this work would be to investigate coordination without linearity/linearization. Coordination for non-linear systems is more challenging because we cannot rely on local analysis and Lyapunov functions which are quadratic forms.

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APPENDIX A
ASYMPTOTIC BEHAVIOR OF ODEs

Consider the ODE $\dot{\theta} = F(\theta)$, which we assume to have a unique solution for each initial condition defined on $\mathbb{R}^+$. We write $\Phi(t, \theta(0))$ the value at t of the solution for initial condition $\theta(0)$. We denote by $d_{\mathcal{U}}(\theta) = \inf_{u \in \mathcal{U}} \|\theta - u\|$ the distance to set $\mathcal{U}$. We say that $\mathcal{U}$ is *invariant* if $\theta(0) \in \mathcal{U}$ implies $\Phi(t, \theta(0)) \in \mathcal{U}$, $t \in \mathbb{R}^+$. We say that $\mathcal{U}$ is *Lyapunov stable* if for all $\delta_1 > 0$ there exists $\delta_2 > 0$ such that $d_{\mathcal{U}}(\theta(0)) \leq \delta_2$ implies $d_{\mathcal{U}}(\Phi(t, \theta(0))) \leq \delta_1$, $t \in \mathbb{R}^+$. A compact invariant set $\mathcal{U}$ is an *attractor* if there is an open invariant set $\mathcal{A}$ such that $\theta(0) \in \mathcal{A}$ implies $d_{\mathcal{U}}(\Phi(t, \theta(0))) \underset{t \to +\infty}{\to} 0$. $\mathcal{A}$ is called the *basin of attraction*. A compact invariant set $\mathcal{U}$ is *locally asymptotically stable* if it is both Lyapunov stable and an attractor. If its basin of attraction $\mathcal{A}$ is equal to the whole space then $\mathcal{U}$ is *globally asymptotically stable*. Asymptotic stability is often characterized using *Lyapunov functions*. A positive, differentiable function $V : \mathbb{R}^I \to \mathbb{R}$, is said to be a Lyapunov function if $t \mapsto V(\Phi(t, \theta(0)))$ is decreasing, and strictly decreasing whenever $V(\Phi(t, \theta(0))) > 0$. Then the set of zeros of V is locally asymptotically stable. If we add the condition $V(\theta) \underset{\|\theta\| \to +\infty}{\to} +\infty$, then we have global asymptotic stability.

APPENDIX B
LINEAR ODEs

Consider the ODE: $\dot{\theta} = A(\theta - \theta^*)$. Its solution has the form:

$$\theta(t) = \theta^* + e^{tA}(\theta(0) - \theta^*). \tag{32}$$

θ^* is asymptotically stable iff all the eigenvalues of A have a strictly negative real part. Alternatively, asymptotic stability applies iff there exists $0 \prec X$ such that $A^T X + XA \prec 0$. In this case, $V(\theta) = (\theta - \theta^*)^T X(\theta - \theta^*)$ is a Lyapunov function for the ODE. The reader can refer to [7] for the linear matrix inequality approach to stability.