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Final Report

Steel Bridge Protection Policy
Volume IV of V
Life Cycle Cost Analysis and Maintenance Plan

Jon Fricker
Tarek Zayed

May 1999

Indiana
Department
of Transportation

Purdue
University

FINAL REPORT

STEEL BRIDGE PROTECTION POLICY

VOLUME IV

LIFE CYCLE COST ANALYSIS AND MAINTENANCE PLAN

FHWA/IN/JTRP-98/21

by

Jon D. Fricker
Principal Investigator

And

Tarek Zayed
Research Assistant

Purdue University
School of Civil Engineering

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16. Abstract

The study identifies various painting systems that are successfully used in Indiana's surrounding states and other industries. The identified systems are further screened and evaluated. After prudently comparing INDOT's inorganic zinc / vinyl system with the waterborne acrylic system, the moisture cure urethane coating system, and the 3-coat system of zinc-epoxy-urethane, the results show that the new 3-coat system fulfills INDOT's needs with the most benefits. Therefore, the 3-coat system is recommended to replace INDOT present inorganic zinc / vinyl system.

To deal with the problems facing the lead-based paint, a comparison between full-removal and over-coating alternatives is made. Results show that over-coating might provide a good protection for less than half the cost of full-removal; however it delays the lead full-removal process and does not completely solve the environmental problem.

The metalization of steel bridges is seemingly a potential protection policy. After reviewing standards and specifications on metalization, it is shown that metalization jobs require a higher degree of control. It suits on-shop practices, however, the initial cost is considerably high.

This study also describes a life cycle cost analysis that was done to determine an optimal painting system for INDOT. Herein, a deterministic method of economic analysis and a stochastic method of Markov chains process are used. The analysis not only reconfirms that the 3-coat system is the comparatively better painting system, but also generates an optimal painting maintenance plan for INDOT.

To assure the quality of paint material and workmanship after substantial completion of the painting contract, the development of legally binding and dependable warranty clauses is initiated in this study. The developed painting warranty clauses were primarily derived from the painting warranty clauses used by IDOT, MDOT, and INDOT's pavement warranty clauses. A comparative study was conducted on eleven essential categories. Among them, it was found that the warranty period, the definition of "defect", and the amount of the warranty bond all need further evaluation.

17. Key Words

coating, inspection, life cycle cost analysis, Markov Chains Process, metalization, painting, quality control, steel bridge, warranty Clauses.

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INTRODUCTION

Bridges of the United States are under constant attack from the environment. It may be sudden yet violent effects of floods or earthquakes or the long-term effects associated with sunlight, rain, deicing salts and freeze/thaw cycles.

The quality of the environment has always been of concern to the general populace. Concerns over the potential pollution of the environment, by lead and other heavy metals, during the removal of older paint systems from bridges has resulted in regulations severely limiting the options available for paint removal. Prior to 1985 open air blasting with grit was the standard method of paint removal. While inexpensive, it resulted in paint debris scattered over a wide area. Today, open air blasting is illegal and has been replaced with removal in containment. Containment means that the bridge or a portion of the bridge is enclosed, thereby containing the paint debris.

Additional regulations have been enacted limiting Volatile Organic Compounds (VOCs) for architectural and maintenance coatings. These regulations have resulted in the application, into bridges, of modified paint systems with limited available information associated with their durability. The use of these paints to replace or repair existing systems is a risk that the states must take in order to meet regulatory compliance.

The economic health of a nation is dependent on its ability to engage in commerce. This ability is directly related to the capability of its infrastructure to efficiently and safely respond to the demands placed upon it, not only by its users, but also by the environment. A recent survey indicates that of the nearly 600,000 bridges tabulated, just over 190,000 bridges were considered substandard. While the reasons for this classification are varied, a growing number are the result of the presence of lead-containing paints previously applied for corrosion protection. Both recently adopted and proposed future regulations have resulted from a growing awareness of the need to protect the environment from uncontrolled pollution, and to safeguard the health of workers engaged in renovation as well as that of the general populace. Under this research project, INDOT needs to evaluate the existing and new paint systems from cost point of view. Therefore, a life cycle cost analysis was done for the existing and some new paint systems to select the best paint type that is convenient to Indiana.

STUDY OBJECTIVES

The purpose of this project was to review aspects of the rehabilitation process in light of the above mentioned limitations, assess to the degree possible the state of the art, arrive at conclusions and make recommendations where applicable.

The objective of this study was to perform an economic study on INDOT steel bridge paint maintenance problems and life cycle cost analysis. A major goal of this study was to develop economic models that can be used to provide a rational framework for the evaluation of alternatives in the paint maintenance of steel bridges. To accomplish the objective, an extensive study of steel bridge maintenance practices was conducted. The purpose of this effort was to acquire cost data and detailed information on practices and performance experience, and gain a better understanding of the bridge maintenance problems as viewed from the owner's perspective. This study has included a literature search and a series of meetings and discussions with various groups and individuals within the bridge maintenance community, including various state highway department personnel, representatives of the paint industry, the different departments of transportation (DOTs) and bridge painting contractors. The data and experience from the bridge paint maintenance study were used to formulate the models and to provide input data for the completed models.

The sub-objectives of this study are:

1- Study the deterioration models for the existing paint systems using the deterministic and probabilistic methods. The deterministic method such as the regression analysis and the stochastic method such as the Markov chains process.

2- Compare the different existing paint systems according to life cycle cost analysis using economic traditional methods and the Markov decision process method.

3- Compare the best existing system with some new paint systems using the traditional economic methods. This step will identify the best solution that INDOT can use in the future.

4- Analyze the new system according to other states' experience for that type of paint. This analysis includes a deterioration model and life cycle cost analysis for different rehabilitation policies.

5- Establish an INDOT maintenance plan according to the analysis that is made in the previous step.

STUDY STEPS

The steps that are applied to this study are:

1- Data is collected from INDOT on the condition rating and paint age.

2- Data is classified into different existing paint types such as Lead based paint and Zinc/Vinyl.

3- Regression analysis is done in terms of paint types and categories such as Interstate roads and State roads.

4- Markov chains process is used to represent the deterioration models for data as a stochastic method.

5- Markov process as a stochastic method is carried out based on regression analysis, where regression concludes the best fitted models for data.

6- Paint type cost data is collected from INDOT for a life cycle cost analysis of the existing systems.

7- Economic traditional methods, such as present value (PV) and equivalent uniform annual cost (EUAC), are used as a deterministic method to compare different paint rehabilitation scenarios and different paint types.

8- The Markov decision process (MDP) is used as a stochastic method to compare different paint rehabilitation scenarios for each paint type.

9- A conclusion was drawn to select the best existing paint system according to the previous life cycle cost analysis.

10- Data is collected from different neighbor states about some other new paint systems that can be useful in Indiana. Cost data is also collected for these new types.

11- Comparison between the existing and the new systems is made to select the best paint system quantitatively or according to the economic analysis.

12- Different rehabilitation scenarios for the best paint system were analyzed to draw the maintenance plan for INDOT.

13- Based on the previous economic analysis, a maintenance plan for the new paint system is recommended for INDOT to be used in the future.

14- Conclusions and recommendations are made.

REPORT OVERVIEW

This report includes four chapters. The overview of these chapters is shown in Figure 1. This figure indicates that Chapter One discusses the derivation of deterioration models using the deterministic method, such as regression analysis, and the stochastic method, such as the Markov chains process. It also includes the comparison between the regression and Markov process results. Chapter Two discusses the life cycle cost analysis for the existing and new paint systems using the economic analysis methods and Markov decision process (MDP) method. The life cycle cost analysis for the best paint system is discussed in Chapter Three. This chapter indicates also the maintenance plan for the new system. Chapter Four shows the cited references that are used in this study. Appendix A shows the regression analysis results for the existing and new paint systems. The Markov transition probability matrix values are shown in Appendix B. Markov decision process (MDP) calculations and results for one paint system is indicated in Appendix C as an example. Appendix D includes the economic analysis results for the existing and new systems.

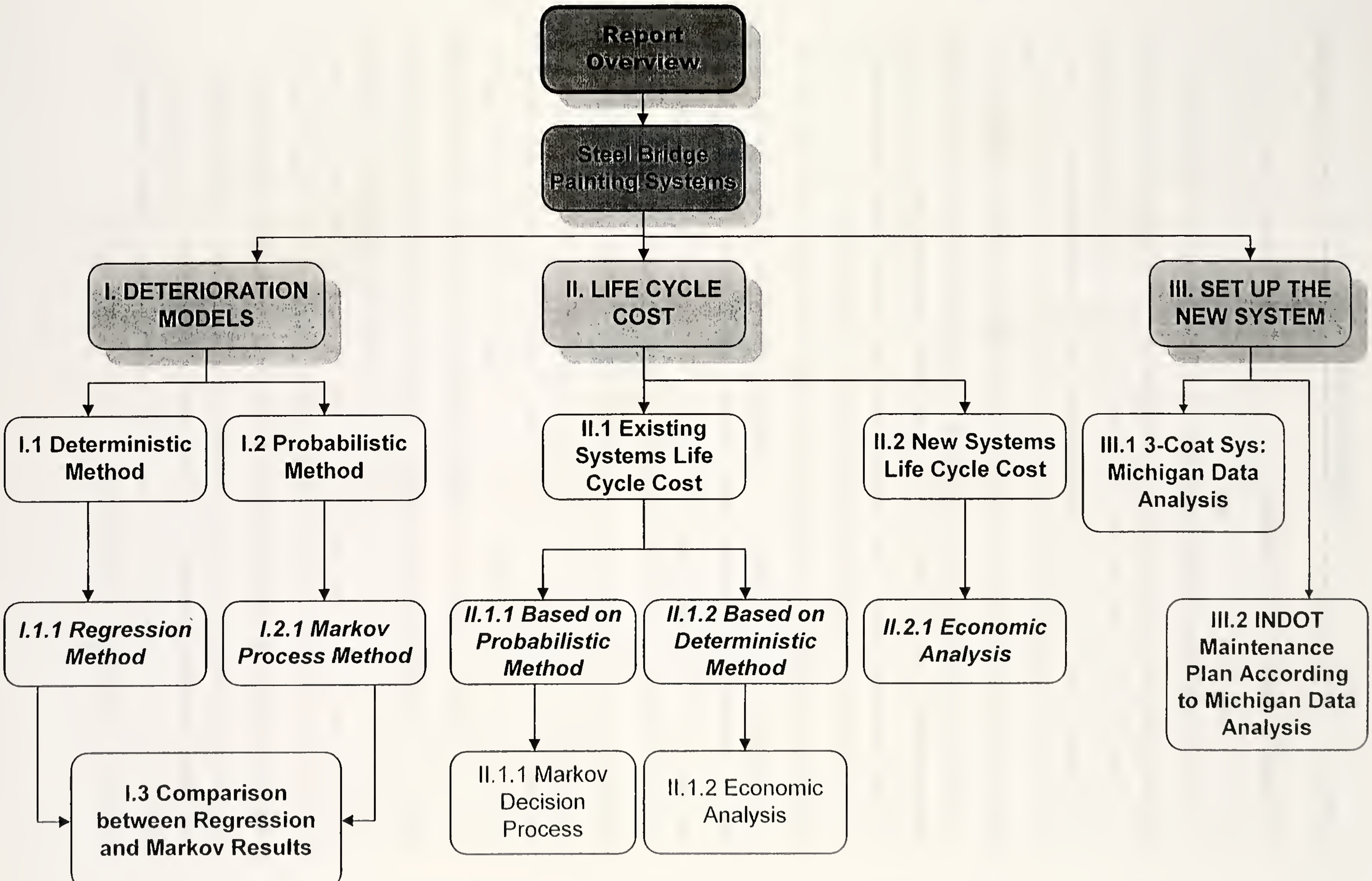


Figure 1: Steel Bridge Paint Life Cycle Cost Analysis: (Report Overview)

SUMMARY AND CONCLUSIONS

The application of life cycle cost analysis to the maintenance coating of steel bridges represents a major departure from the current practice of basing coating decisions on lowest initial cost. The economic analysis models and the Markov Decision Process (MDP) can be used in a variety of possibilities and leads to some interesting observations. A major problem becomes apparent upon a careful examination of the data that feeds the model. The cost and performance data is highly variable and considerable doubt exists about the validity of comparisons between different sources. The reasons for these problems are many. The costs are seldom available in a detailed breakout format. Typically, the cost data are simply a lump sum that may be transformed to a unit cost per area if the surface area is provided. INDOT still uses the practice of expressing coating costs as a cost per ton of steel. Given the wide range of geometry used in bridge construction, conversion of cost per ton to cost per square foot is difficult. Another major factor in the variability of the cost data is the presence of hidden costs. For example, we may consider two similar bridges with the same surface preparation and coating system that have different accessibility limitations. In one case, access for painting is unrestricted, while in the other case, painting can only be performed during periods of minimal traffic disruption. For the first bridge, scaffolding and containment can be erected and left in place until the job is complete, while the second bridge requires the scaffolding and containment to be assembled and disassembled before and after each daily painting period. If all other costs remain identical, these two jobs could easily differ by two or three times in cost per square foot. Another source of hidden costs results from typical contractor practices that stem from cash-flow problems. Contractors will typically shift labor costs to purchased materials so that they can front-load their invoicing. This practice introduces large variations in hardware and material costs. Performance data is also badly clouded. The judgment of when a coating system has failed can be very subjective, particularly if no uniform quantitative standards exist for judging failure. Additional consideration needs to be given to contracting practices. If life-cycle cost methods are to provide meaningful comparisons of alternatives, reliable cost data must be available. One approach that might aid the situation would be uniform coating contract standards that require detailed cost breakouts of the elements of the project. These cost elements need to hold up to audit standards so that hidden costs cease to exist. This process will not be popular with painting contractors, but if we are going to employ life-cycle costs as a criteria for judging coating alternatives, we must be sure that the data (cost and performance) are accurate and that we are indeed comparing alternatives on a rational basis.

Based on the previous discussion, the existing and new paint systems are analyzed and come up to the conclusion that the 3-coat system, Inorganic/Organic Zinc Epoxy Urethane, is the best and comparative paint system. INDOT just started implementing the 3-coat system. Therefore, there were inadequate data for life cycle cost analysis. However, based on MDOT data set, life cycle cost analysis seemingly indicated that the optimal policy for 3-coat system rehabilitation was doing spot painting every 15 years or when paint condition rating reaches 7, regardless of the age of the paint on the steel bridge. Accordingly, a detailed maintenance plan was developed based on the economical life cycle cost analysis results. Mainly, INDOT steel bridges could be painted whenever the paint age is 15 years and the paint condition rating reaches 7 because this plan will result in the lowest cycle cost for INDOT. However, this result was derived from MDOT data set. Condition rating 7 is defined in Table E.1, Appendix E, in MDOT description.

RECOMMENDATIONS AND FUTURE WORK

1- Multimedia could be used as a training tool for the inspectors to apply the inspection criteria to the steel bridges paint.

2- It is recommended to use Markov Decision Process for life cycle cost analysis of Steel Bridge paint in future. This method is based on the stochastic method, Markov chains, where it needs more data to analyze the life cycle cost. Consequently, a design for a program that uses these methods together to provide the best rehabilitation scenario is recommended to INDOT.

3- Maximizing the benefit of budget allocation for INDOT is very important issue. Consequently, INDOT steel bridge paint rehabilitation budget could be allocated every year to different bridges according to their importance. Dynamic Programming or Mixed Integer Programming techniques are recommended to be used based on the Markov chains results to accommodate or to maximize the benefit of INDOT budget allocation for steel bridge paint rehabilitation projects.

CHAPTER I

PAINT PERFORMANCE ANALYSIS: DETERIORATION MODELS

A performance function is the relationship between bridge paint condition rating and age that reflects the level of service of that paint. The performance functions for steel bridge paint in INDOT were developed using regression method. Bridge paint performance prediction models were also developed using Markov chains. The probabilistic model that was developed by Markov chains was done to reflect the stochastic nature of bridge paint conditions. This model can be used to predict the condition rating of a bridge paint at a given age.

This study used the techniques of regression, Markov chains, non-linear programming and a combination of those techniques to analyze bridge paint performance. The results exhibited the power of those techniques, especially of Markov chains approach in predicting or estimating future bridge paint conditions.

I.1. DEVELOPMENT OF PERFORMANCE FUNCTIONS USING DETERMINISTIC METHOD

I.1.1. Deterministic Method (Regression Analysis)

There are many factors that affect the performance of steel bridge paint. Some of these factors are climate, age and traffic conditions (interstate or state road). The data available is divided by road type: interstate steel bridges and state steel bridges. Regression analysis by using SAS 1996 is done to check if climate conditions have an effect on the performance function or not. Indiana State is divided into two different climate regions: north (INDOT Districts 1 and 3) and south (INDOT Districts 2, 4, 5, and 6). Since we have no data on weather conditions, the climate effect is analyzed by putting it in as a class variable with one indicator variable in the SAS analysis. This indicator variable is a qualitative variable that has value 1 if it is in the north and 0 if it is in the south. Different types of functions are used to express the performance functions to get the best fit. Data is checked for linear and polynomial (quadratic and cubic) functions and climate as a class variable is also checked for different paint types and road conditions. The F-test and t-test are performed to determine the best model and whether climate is significant. The assumptions for the regression models are checked to select the appropriate model to fit the data. SAS results are indicated in Appendix (A) that contains linear, quadratic and cubic models and their F-test and t-test results. A lack-of-fit test is performed for all the models because there are replications in the data. This test is done to check the adequacy of these models to fit the data.

Deterioration models information is indicated in Table I.1. It contains all the models that are built using SAS, along with some vital information about them to make comparison among them and select the best model that can represent the data or fit the data. (16)

For paint type (1), interstate roads have their performance function in linear, quadratic and cubic formulas. The r-squared value for each formula is calculated and indicated in the table, but the difference among them is very low. The lack-of-fit test is done for each case and it is not significant for any formula. This means that all the models are adequate to fit the data. The assumptions for the regression models are checked for the error constant variance, normality and independence. The results indicate that all the models are good according to these assumptions as indicated in Appendix A. The climate effect is measured when this factor is included in the regression analysis as a class variable with one indicator variable. The significance of that class variable or indicator variable is checked for all the models of that paint type and indicates that it is not significant enough to be included any model. The p-value of this test of significance of the climate factor is indicated in Table I.1 where all of the models indicate negative answers for the significance of that factor. In addition to this information, the significance of all models' parameters is checked to select the best model that can fit the data. According to these tests and arguments, the best model is selected for paint type (1) Interstate roads is a quadratic model: (16)

$$\textit{Paint Rating} = 9.06 - 0.0821 * \textit{Age} - 0.00178 * \textit{Age}^2.$$

Table I.1: Deterioration Models Information

Paint Description	Case	No. of Obs.	R-sq	lack of Fit Prob	Level of Signific 10%	5%	1%	Norma Assu	P-value	climate	Equation	Selection
Interstate1 Paint 1(**I1**)	*Linear*	*73*	*0.764*	*0.1415*	*O.K.*	*O.K.*	*O.K.*	*O.K.*	*0.347*	*N.O.K.*	*Paint Rate = 9.0 - 0.143 Age*	
	Quad	*73*	*0.775*	*0.2172*	*O.K.*	*O.K.*	*O.K.*	*O.K.*	*0.411*	*N.O.K.*	*Paint Rate = 9.06 - 0.0821Age- 0.00178 Age^2.*	*x*
	Cubic	*73*	*0.778*	*0.2158*	*O.K.*	*O.K.*	*O.K.*	*O.K.*	*0.35*	*N.O.K.*	*Paint Rate = 8.80 + 0.0063 Age- 0.00781Age^2+0.000116Age^3*	
Interstate2 Paint 2(**I2**)	*Linear*	*315*	*0.817*	*0.01*	*N.O.K.*	*.O.K*	*O.K.*	*O.K.*	*0.016*	*N.O.K.*	*Paint Rate = 9.0 - 0.124 Age.*	
	Cubic	*315*	*0.831*	*0.01*	*N.O.K.*	*.O.K*	*O.K.*	*O.K.*	*0.213*	*N.O.K.*	*Paint Rate = 9.06 - 0.201 Age+ 0.0103Age^2 - 0.000348Age^3*	*x*
State 1 Paint 1 (**S1**)	*Linear*	*60*	*0.852*	*0.0181*	*N.O.K.*	*.O.K*	*O.K.*	*O.K.*	*0.546*	*N.O.K.*	*Paint Rate = 9.0 - 0.142 Age*	
	Quad	*60*	*0.896*	*0.7404*	*O.K.*	*O.K.*	*O.K.*	*O.K.*	*0.044*	*N.O.K.*	*Paint Rate = 9.06 - 0.007 Age - 0.00517 Age^2.*	*x*
	Cubic	*60*	*0.898*	*0.7486*	*O.K.*	*O.K.*	*O.K.*	*O.K.*	*0.541*	*N.O.K.*	*Paint Rate = 8.9 - 0.0827Age - 0.126Age^2 + 0.000162Age^3.*	
State 2 Paint 2 (**S2**)	*Linear*	*277*	*0.853*	*0.01*	*N.O.K.*	*.O.K*	*O.K.*	*O.K.*	*0.215*	*N.O.K.*	*Paint Rate = 9.0 - 0.156 Age*	
	Cubic	*277*	*0.865*	*0.01*	*N.O.K.*	*.O.K*	*O.K.*	*O.K.*	*0.435*	*N.O.K.*	*Paint Rate = 9.03 - 0.0753 Age- 0.00489Age^2+0.000054Age^3*	*x*
Paint3 all Indiana(**P3**)	*Linear*	*13*	*0.919*	*0.9426*	*O.K.*	*O.K.*	*O.K.*	*O.K.*	*0.546*	*N.O.K.*	*Paint Rate = 8.88 - 0.123 Age*	*x*

For Paint type (2), interstate roads have their performance function in linear and cubic formulas. The r-squared value for each formula is calculated and indicated in the table, but the difference among them is very low. The lack-of-fit test is done for each case and it is significant for 10% and 5% levels of significance, but it is not for 1%. This indicates that all the models are adequate for fitting the data at only 1% level of significance. The assumptions of the regression models are checked for the error constant variance, normality and independence. The results indicate that all the models are good according to these assumptions. The climate effect is measured when this factor is included in the regression analysis as a class variable with one indicator variable. The significance of that class variable or indicator variable is checked for all the models of that paint type and indicates that it is not significant enough to be included any model. The p-value of this test of significance of the climate factor is indicated in Table I.1 where all of the models indicate negative answers for the significance of that factor. In addition to this information, the significance of all models' parameters is checked to select the best model that can fit the data. According to these tests and arguments, the best model is selected for paint type (2) Interstate roads is a cubic model: [16]

$$\textit{Paint Rating} = 9.06 - 0.201 * Age + 0.0103 * Age^{\wedge}2 - 0.000348 * Age^{\wedge}3.$$

For Paint type (1), state roads have their performance function in linear, quadratic and cubic formulas. The r-squared value for each formula is calculated and indicated in the table, but the difference among them is low. The lack-of-fit test is done for each case and it is not significant for quadratic and cubic formulas and it is only significance at 1% level for linear formula. This indicates that all the models are adequate to fit the data. The assumptions of the regression models are checked for the error constant variance, normality and independence. The results indicate that all the models are good according to these assumptions. The climate effect is measured where this factor is included in the regression analysis as a class variable with one indicator variable. The significance of that class variable or indicator variable is checked for all the models of that paint type and indicates that it is not significant to be included in any model. The p-value of this test of significance of the climate factor is indicated in Table I.1 where all of the models indicate negative answers for the significance of that factor. In addition to this information, the significance of all models' parameters is checked to select the best model that can fit the data. According to these tests and arguments, the best model is selected for paint type (1) State roads is a quadratic model: [16]

$$\textit{Paint Rating} = 9.06 - 0.007 * Age - 0.00517 * Age^{\wedge}2.$$

For Paint type (2), state roads have their performance function in linear and cubic formulas. The r-squared value for each formula is calculated and indicated in the table, but the difference among them is very low. The lack-of-fit test is done for each case and it is not significant for all formulas. This means that all the models are adequate to fit the data. The assumptions of the regression models are checked for the error constant variance, normality and independence. The results indicate that all the models are good according to these assumptions. The climate effect is measured where this factor is included in the regression analysis as a class variable with one indicator variable. The significance of that class variable or indicator variable is checked for all the models of that paint type and indicates that it is not significant to be included in any model. The p-value of this test of significance of the climate factor is indicated in Table I.1 where all of the models indicate negative answers for the significance of that factor. In addition to this information, the significance of all models' parameters is checked to select the best model that can fit the data. According to these tests and arguments, the best model is selected for paint type (2) State roads is a cubic model: [16]

$$\textit{Paint Rating} = 9.03 - 0.0753 * Age - 0.00489 * Age^{\wedge}2 + 0.000054 * Age^{\wedge}3.$$

For Paint type (3), interstate and state roads are combined in one model and have their performance function in linear formula only due to lack of data. The r-squared value is calculated and indicated in the table. The lack-of-fit test is done and it is not significant for this model. This means that the model is adequate to fit the data. The assumptions of the regression model are checked for the error constant variance, normality and independence. The results indicate that the model is good according to these assumptions. The climate effect is measured where this factor is included in the regression analysis as a class variable with one indicator variable. The significance of that class variable or indicator variable is checked for the model of that paint type and indicates that it is not significant to be included in any model. The p-value of this test of significance of the climate factor is indicated in Table I.1 where the model indicates negative answers for the significance of that factor. In addition to this information, the significance of model's parameters is checked to select the best model that can fit the data. According to these tests and arguments, the best model is selected for paint type (3) is a linear model: [16]

$$\textit{Paint Rating} = 8.88 - 0.123 * \textit{Age}.$$

I.2. STOCHASTIC METHOD (MARKOV CHAINS PROCESS)

Stochastic processes are processes that evolve over time in a probabilistic manner. A stochastic process is defined to be an indexed collection of random variables (S_t), where the index t runs through a given set of non-negative integers. One special type of stochastic process is called Markov chain. A stochastic process is a Markov chain if it has the Markovian property: the conditional probability of any future event, given any past event and the present state $S_t = i$, is independent of the past event and depends only upon the present state. This property can be written as: [22] [1]

$$P(S_{t+1} = i_{t+1} / S_t = i_t, S_{t-1} = i_{t-1}, \ldots\ldots, S_1 = i_1, S_0 = i_0) = P(S_{t+1} = i_{t+1} / S_t = i_t).$$

Many processes fit this description including this study of steel bridge paint conditions. In the model development, to reduce the complexity of the analysis, the future condition of bridge paint is assumed to depend only on the present state, and independent of the past conditions. It is further assumed that for all states i , j and all t, $P(S_{t+1} = i_{t+1} / S_t = i_t)$ is independent of t. The probability , P_{ij}, that bridge paint is in state i at time t and it will be in a state j at time t+1 does not change (remains stationary) over time. This stationary assumption is expressed by the following equation: $P(S_{t+1} = j / S_t = i) = P_{ij}$.

The term transition is used when the system moves from state i during one period to state j during the next period. Accordingly, the probabilities, P_{ij}'s , are referred to the transition probabilities. The transition probabilities are commonly displayed as n x n matrix called the transition probability matrix P. In this study there are five states associated with the five possible conditions of bridge paint ratings. State 1 corresponds to the best condition and state 5 corresponds to the worst condition. Then, the transition probability matrix can be written as:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} \end{bmatrix}$$

The above transition probability matrix is for one-step (one-period) transition. The n-step transition probability matrix, $P^{(n)}$, of the process that is in state i and will in state j after n periods is computed by Chapman-Kolmogorov equation: $P^{(n)} = P^n$. The n-step transition probability matrix is obtained by taking the n-th power of the one step transition matrix. [22] [1]

The Markov chain as applied to bridge paint performance prediction is based on the concept of defining states in terms of bridge paint condition ratings and obtaining the probabilities of paint condition changing from one state to another. These probabilities are represented in a matrix form that is called the transition probability matrix. Knowing the present state of bridge paint, or the initial state, the future conditions can be predicted through the multiplication of initial state vector and the transition probability matrix. [13],[12]

I.2.1. Development of Markov Prediction Model

Steel bridge paint begins its life in a near-perfect condition and is then subjected to a sequence of duty cycles that cause the paint condition to deteriorate. In this study the paint condition rating is defined by an inspector's judgement. The paint rating ranges from 0 to 9, where 9 is the perfect condition and 0 is the worst condition. A duty cycle for a paint type is defined as one year's duration of weather and traffic. A state vector indicates the probability of bridge paint being in each of the 10 conditions, 0-9, in any given year. Figure I.1 shows a stochastic representation of state, state vector and duty cycle for pavement conditions. [4] [5] The description of each of these conditions is indicated in Table E.1, Appendix E.

It is assumed that all bridge paint conditions are in state 1 (condition 9) at an age of 0 year. Thus the state vector in Duty Cycle 0 (age=0) is given by (1,0,0,0,0) because it is known that all the bridges' paint must be in state 1 at an age of 0 year with a probability of 1.0. [4] [5] To model the way in which bridge paint deteriorates with time, it is helpful to establish a Markov probability transition matrix. In this research, the assumption is made that the bridge paint condition will not drop by more than one state in a single year. Thus, the bridge paint will either stay in its current state or degrade to the next lower state in 1 year. It should be desirable to examine historical data for each bridge, and determine how often a paint condition dropped by more than two units between biennial bridge inspections. However, only recent data on bridge paint condition have been preserved. For an alternative analytical method, Table I.2 is used to support the assumption that the bridge paint condition may not drop by more than one state in a single year. Based on INDOT data, the deterioration rate index (year(s) per unit change in condition rating) is calculated. This index is found by dividing the paint age by the difference between the highest condition rating, which is 9, and the current paint condition rating. After calculating this index for each bridge, an average of the indices for bridges that have the same current condition rating is calculated. Table I.2 shows these averages, and their corresponding standard deviations. For the 189 bridges in the INDOT data set that zinc-based paint at rating 6, for example, the mean index is 4.254 years per unit change in paint rating. Our assumption is that the index is never (or seldom) below

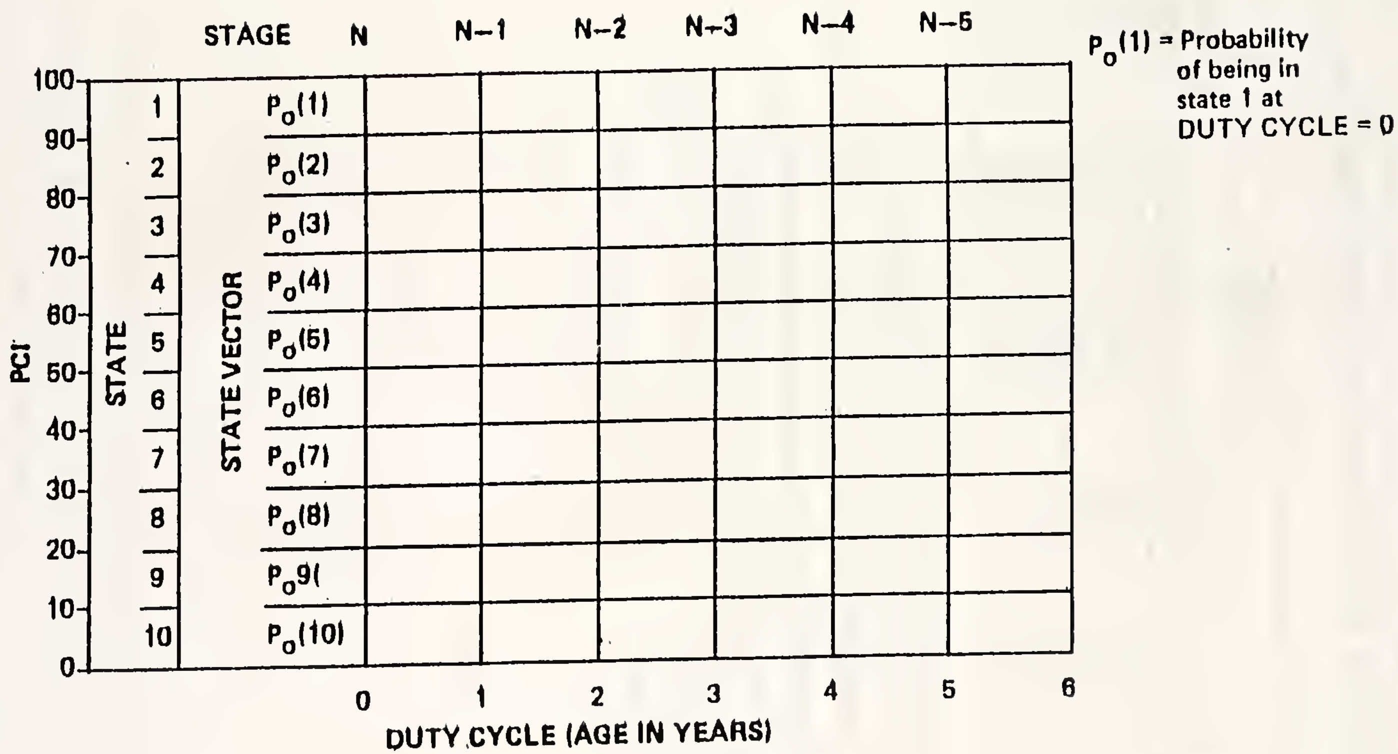


Figure I.1: Stochastic Representation of State, State Vector and Duty Cycle.

Table I.2: Markov chains Assumption Validation

Condition Rating	Paint type					
	Lead Based			Zinc Based		
	Ave. Rate*	STDEV	Under 1yr	Ave. Rate*	STDEV	Under 1yr
5	*5.43*	*1.065*	*None*	*4.208*	*0.681*	*None*
6	*7.644*	*0.676*	*None*	*4.254*	*1.719*	*None*
7	*10.66*	*1.962*	*None*	*4.031*	*2.294*	*3.90%*
8	*NA*	*NA*	*NA*	*4.443*	*3.149*	*None*

NA = Not Available.

* These rates are calculated by dividing the bridge age by the difference between the highest condition rating, which is 9, and the current condition rating for that bridge, as follows:

Deterioration Rate Index = Age/(9 - current condition rating)

After calculating the rates, an average is calculated for these rates for bridges having the same condition rating. All these values are based on INDOT data.

one. Looking at the individual indices that went into the mean value, we find that only 3.9% of the index values of condition rating 7 in zinc-based paints were less than 1.0. This justifies our assumption, which is invoked to reduce the computational effort associated with the Markov method. Consequently, the probability transition matrix has the following form: [4] [5]

$$P = \begin{bmatrix} p(1) & q(1) & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 \\ 0 & 0 & 0 & p(4) & q(4) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where p(j) is the probability of a bridge paint staying in state j during one duty cycle, and q(j) = 1 – p(j) is the probability of the bridge paint transiting down to next state (j+1) during one duty cycle. The entry of 1 in the last row of the transition matrix corresponding to state 5 indicates an “absorbing” state. The bridge paint can not transit from this state unless repair action is performed. The state vector for any duty cycle n is obtained by multiplying the initial state vector S(0) by the transition matrix P raised to the power of n. Then, [4] [5]

$S(1) = S(0) * P.$

$S(2) = S(1) * P = S(0) * P^2.$

....

....

$S(n) = S(n-1) * P = S(0) * P^n.$

With this procedure, if the transition probability matrix can be obtained, the future state of the bridge paint can be predicted at any duty cycle, n. It should be noticed that the lowest rating number before a bridge paint is repainted again is 5 (as indicated by FHWA and INDOT). Consequently, the corresponding transition probability p(5) is equal to 1.

This Markovian model provides a reliable mechanism for developing prediction models. The markov process imposes a rational structure on the deterioration model because it explains the deterioration as uncertain issue and it also ensures that the projections beyond the limits of data will continue to have worsening condition pattern with age. This model has been successfully used in other types of infrastructure deterioration modeling such as pavement and bridges. [4] [5] [7] [22] [1]

I.2.2. Transition Probability Matrix Determination

To estimate the transition probability matrix, a non-linear programming approach is used. The objective is to determine the five parameters, p(1) through p(5), that would minimize the absolute distance between the actual data points and the expected (predicted) bridge paint condition for the corresponding age generated by the Markov chain using these five parameters. Due to the large number of data and the complexity of applying this method for each data available, the regression models output values are used instead of this data to construct the transition probability matrix. This is because the regression models values are the output of least square method which is the best way to predict the data. Based on the fact that regression model is the best fit of data using the least square method, we are confident that these models are the best we can use in predicting the paint conditions. Consequently, if markov probability matrices are constructed according to these models, it will give the best fit for the data using the stochastic approach. Then, the objective function of the non-linear programming is to minimize the absolute difference between the average condition rating ate time t estimated by the regression function and the estimated value of condition rating using Markov chain at time t. [4] [5]

The objective function has the following form: [4] [5] [12] [13] [1] [22]

$$\textbf{Minimize} \sum_{t=1}^{N} |Y(t) - E(t,P)|$$

Subject to: $0 \leq p(i) \leq 1.0,$ $i = 1, 2, \ldots\ldots\ldots, I.$

Where:

N = 4, the number of years in one age range.

I = 4, the number of unknown probabilities.

P = [p(1), p(2),.........,p(I)], a vector of length I.

Y(t) = the average condition ratings at time t, estimated by regression function.

E(t,P) = the estimated value of condition rating by Markov chain at time t.

The solution to this function is obtained by using GAMS program for solving non-linear programming. This program is available on the Engineering Computer Network system (ECN) at Purdue University. The values of the corresponding regression function are taken as the average condition rating to solve the non-linear programming.

The bridge paint life is divided into age zones or ranges of 4 years. These 4-year ranges are selected to facilitate the calculation of probabilities and optimization procedure. It is assumed that each zone or range has a constant rate of deterioration and, hence, that a constant duty cycle has been assumed within each range. The rate of deterioration is assumed to vary from range or zone to another; therefore, different duty cycles have been assigned to different ranges or zones. [12] [13]

Because the duty cycle within a range is assumed to be constant, a homogeneous Markov chain has been used for each range and a separate transition matrix has been developed for each range. The duty cycle varies from range to another. Therefore, a non-homogeneous Markov chain has been used for transition from range to another. [12] [13]

The maximum rating of bridge paint condition is 9 and it represents a near-perfect condition of bridge paint. It is almost always true that a new bridge paint has condition rating of 9. In other words, abridge paint at age 0 year has condition rating of 9 with 1.0 probability. Therefore, the initial state vector S(0) for a new bridge paint is always [1,0,0,0,0], where the numbers are the probabilities of having condition rating of 9,8,7,6 and 5 at age 0 year respectively. That is the initial vector of the first range for developing the bridge performance curve or transition probability matrix. Second range takes the last state vector of first range as its starting state vector. Similarly, range or zone n takes the last state vector of range n-1 as its starting state vector. The rest of the work to obtain the overall bridge paint performance curve is nothing rather than conducting the following matrix multiplication: [12] [13]

$S(1) = S(0) * P.$

$S(2) = S(1) * P = S(0) * P^2.$

$S(3) = S(2) * P = S(0) * P^3$

....

....

$S(n-1) = S(n-2) * P = S(0) * P^{n-1}$

$S(n) = S(n-1) * P = S(0) * P^n.$

Where: S(n) represents the condition state vector at age n.

Let R be the column vector of condition ratings,

$$R = \begin{bmatrix} 9 \\ 8 \\ 7 \\ 6 \\ 5 \end{bmatrix}$$

Then, the estimated condition rating at age n by Markov chain is,

$$E(n,P) = S(n) * R$$

For illustration, let the performance function, using regression, for the steel bridge paint gives the values of Y(1), Y(2), Y(3) and Y(4) for the predicted condition ratings in the first four years of paint life. The corresponding values of prediction of condition ratings by Markov chain method can be expressed by the following equations: [12] [13]

$$E(1,P) = S(1) * R = S(0) * P * R$$

$$E(2,P) = S(1) * R = S(0) * P^2 * R$$

$$E(3,P) = S(1) * R = S(0) * P^3 * R$$

$$E(4,P) = S(1) * R = S(0) * P^4 * R$$

Since the E(n,P)s are functions of p(1), p(2), p(3) and p(4), the non-linear programming objective function should be solved to get the values of these probabilities. Based on these probabilities' values, the Markov transition probability matrices could be constructed using these values. (12) (13)

I.2.3 Steps of NLP Application

1- Paint age is divided into 4-year ranges with four unknown probabilities in each range.

2- Maple mathematics program is used to prepare the file of matrix multiplication as illustrated in the previous section. The output of this file is the equations that contain p(1), p(2), p(3) and p(4) as unknowns. Therefore, there are four equations with four unknowns that can be solved to get the values of p(1), p(2), p(3) and p(4).

3- The output of the previous step is substituted to the non-linear programming equation as E(t,P), which is subtracted from the part Y(t). This part, Y(t), is calculated using the regression formulas for the various models' types.

4- After constructing these equations, they are put into a GAMS program input file to solve the NLP problem based on the constraints that are illustrated in the previous section. This input file requests from GAMS to make minimization for the absolute value of Y(t)-E(t,P) for each four years of paint age. The output of this step are the values p(1), p(2), p(3) and p(4) that minimize the NLP objective function.

5- By knowing the values of these probabilities in each 4-year range, the last year state vector for each range can be calculated by using the Maple mathematics program. This state vector is calculated to be used as the initial state vector for the next range of four years.

6- Steps from 1 to 5 are repeated for each 4- year range of paint age.

7- Steps from 1 to 6 are repeated also for various paint categories such as: Interstate roads paint type (1) and (2), State roads paint type (1) and (2) and paint type (3).

8- Figures from I.2 to I.4 shows the input file used for Maple to get probability equations, the input file for GAMS, and the input file for Maple to get the last state vector. (12) (13) (23) (24) (25)

Figure I.2: Maple Input File to get Probabilities Equations.

```
maple
with(linalg):
P :=matrix(5,5,[p1,1-p1,0,0,0,0,p2,1-p2,0,0,0,0,p3,1-p3,0,0,0,0,p4,1-p4,0,0,0,0,1]);
S(0) :=matrix(1,5,[.8422907597 , .03693100731 , .03855028105 , .04024055068 ,
.0419874012]);
R :=matrix(5,1,[9,8,7,6,5]);
P2 :=multiply(P,P);
P3 :=multiply(P2,P);
P4 :=multiply(P3,P);
E(1,P) :=multiply(S(0),P,R);
E(2,P) :=multiply(S(0),P2,R);
E(3,P) :=multiply(S(0),P3,R);
E(4,P) :=multiply(S(0),P4,R);
```

Output:

```
E(1, P) :=
  .842290759 *p1 + 7.639284574 + .0369310073 *p2 + .0385502811 *p3
   + .0402405507* p4;

E(2, P) :=
    7.580616837 *p1**2  + 6.738326078 *p1 *(1 - p1) + 6.738326078 *(1 - p1)* p2
   + .2954480585* p2**2  + 5.896035318 *(1 - p1) *(1 - p2)
   + .2585170512 *p2 (1 - p2) + .2585170512 (1 - p2) p3 + .2698519674* p3**2
   + .2215860439 *(1 - p2) *(1 - p3) + .2313016863 *p3 *(1 - p3)
   + .2313016863 *(1 - p3) *p4 + .2414433041* p4**2 + .4111397594
   + .1927514053 *(1 - p3) *(1 - p4) + .2012027534 *p4 *(1 - p4)
   - .2012027534 *p4;

E(3, P) :=
    7.580616837 *p1**3  + 6.738326078 *(p1**2) * (1 - p1)
   + 6.738326078 *(p1* (1 - p1) + (1 - p1) *p2) *p2 + .2954480585 *p2**3
   + 5.896035318 *(p1 *(1 - p1) + (1 - p1) *p2) *(1 - p2)
   + 5.896035318 *(1 -* p1) *(1 - p2)* p3 + .2585170512* (p2**2)  *(1 - p2)
   + .2585170512 *(p2 (1 - p2) + (1 - p2)* p3)* p3 + .2698519674 *p3**3
   + 5.053744558 *(1 - p1) *(1 - p2)* (1 - p3)
   + .2215860439 *(p2 *(1 - p2) + (1 - p2)* p3) *(1 - p3)
   + .2215860439 *(1 - p2) *(1 - p3) *p4 + .2313016863 *(p3**2)  *(1 - p3)
   + .2313016863 *(p3 *(1 - p3) + (1 - p3) *p4)* p4 + .2414433041* p4**3
   + .4111397594 + .1846550366* (1 - p2)* (1 - p3)* (1 - p4)
   + .1927514053 *(p3 *(1 - p3) + (1 - p3) *p4)* (1 - p4)
   + .1927514053 *(1 - p3) *(1 - p4) + .2012027534* (p4**2) * (1 - p4)
   + .2012027534 *p4 *(1 - p4) - .2012027534 *p4;
```

```
E(4, P) :=
   .1927514053 *(1 - p3) *(1 - p4) - .2012027534 *p4 + .2012027534 *p4* (1 - p4)
   + .1846550366* (1 - p2)* (1 - p3)* (1 - p4)
   + .1927514053 *(p3* (1 - p3) + (1 - p3) *p4) *(1 - p4)
   + .2012027534 *(p4**2) * (1 - p4) + 6.738326078* (p1**3) * (1 - p1)
   + 6.738326078 *((p1**2) * (1 - p1) + (p1* (1 - p1) + (1 - p1) *p2) *p2)* p2
   + 7.580616837 *(p1**4)  + .2012027534* (p4**3) * (1 - p4) + 5.896035318*
   ((p1 *(1 - p1) + (1 - p1) *p2) *(1 - p2) + (1 - p1)* (1 - p2) *p3)* p3
   + 5.896035318 *((p1**2) * (1 - p1) + (p1* (1 - p1) + (1 - p1) *p2) *p2)* (1 - p2)
   + .2414433041 *p4**4  + 5.053744558*
   ((p1 *(1 - p1) + (1 - p1)* p2) *(1 - p2) + (1 - p1) *(1 - p2) *p3)* (1 - p3)
   + 5.053744558 *(1 - p1) *(1 - p2) *(1 - p3)* p4 + .2954480585 *p2**4
   + 4.211453799 *(1 - p1) *(1 - p2) *(1 - p3)* (1 - p4)
   + .2585170512 **(p2**3) * (1 - p2)
   + .2585170512 *((p2**2) * (1 - p2) + (p2 *(1 - p2) + (1 - p2)* p3) *p3)* p3
   + .2215860439 *((p2**2)  *(1 - p2) + (p2 *(1 - p2) + (1 - p2)* p3) *p3) *(1 - p3) +
   .2215860439*
   ((p2 *(1 - p2) + (1 - p2) *p3)* (1 - p3) + (1 - p2) *(1 - p3)* p4)* p4 +
   .1846550366*
   ((p2 *(1 - p2) + (1 - p2) *p3) *(1 - p3) + (1 - p2) *(1 - p3) *p4)* (1 - p4)
   + .2698519674 *p3**4  + .2313016863* (p3**3) * (1 - p3)
   + .2313016863 *((p3**2) *(1 - p3) + (p3 *(1 - p3) + (1 - p3) *p4) *p4)* p4
   + .1927514053 *((p3**2)  *(1 - p3) + (p3* (1 - p3) + (1 - p3)* p4) *p4) *(1 - p4)
   + .4111397594;
```

Figure I.3: **GAMS Input File.**

```
VARIABLE p1, p2, p3, p4, DIF1, DIF2, DIF3, DIF4, OBJ, M1, M2, M3, M4;
EQUATION E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, E12, E13, E14, E15, E16,
E17;
E1 .. M1 =E= P1 + 8;
E2 .. M2 =E= 9*p1**2 + 8*p1*(1 - p1) + 8*(1 - p1)*p2 + 7*(1 - p1)*(1 - p2);
E3 .. M3 =E= 9*p1**3 + 8*(p1**2)*(1 - p1) + 8*(p1*(1 - p1) + (1 - p1)*p2)*p2 +
7*(p1*(1 - p1) + (1 - p1)*p2)*(1 - p2) + 7*(1 - p1)*(1 - p2)*p3 + 6*(1 - p1)*(1 - p2)*(1 -
p3);
E4 .. M4 =E= 9*p1**4 + 8*(p1**3)*(1 - p1) + 8*((p1**2)*(1 - p1) + (p1*(1 - p1) + (1 -
p1)*p2)*p2)*p2 + 7*((p1**2)*(1 - p1) + (p1*(1 - p1) + (1 - p1)*p2)*p2)*(1 - p2) +
7*((p1*(1 - p1) + (1 - p1)*p2)*(1 - p2) + (1 - p1)*(1 - p2)*p3)*p3 +
6*((p1*(1 - p1) + (1 - p1)*p2)*(1 - p2) + (1 - p1)*(1 - p2)*p3)*(1 - p3) + 6*(1 - p1)*(1 -
p2)*(1 - p3)*p4 + 5*(1 - p1)*(1 - p2)*(1 - p3)*(1 - p4);
E5 .. DIF1 =E= 9.00 - M1;
E6 .. DIF2 =E= 8.896 - M2;
E7 .. DIF3 =E= 8.74 - M3;
E8 .. DIF4 =E= 8.60 - M4;
E9 .. OBJ =E= ABS(DIF1) + ABS(DIF2) + ABS(DIF3) + ABS(DIF4);
E10 .. P1 =L= 1.0;
E11 .. P1 =G= 0.0;
P1.LO = 0.0001;
E12 .. P2 =L= 1.0;
E13 .. P2 =G= 0.0;
P2.LO = 0.0001;
E14 .. P3 =L= 1.0;
E15 .. P3 =G= 0.0;
P3.LO = 0.0001;
E16 .. P4 =L= 1.0;
E17 .. P4 =G= 0.0;
P4.LO = 0.0001;
MODEL TAREK / ALL /;
option dnlp = minos5;
SOLVE TAREK USING DNLP MINIMIZING OBJ;
display p1.l, p2.l, p3.l, p4.l;
```

Figure I.4: **Input File For Maple to get Last State Vector**

```
maple
with(linalg):
P:=matrix(5,5,[0.958,0.042,0,0,0,0,0.0001,0.9999,0,0,0,0,0.0001,0.9999,0,0,0,0,0.0001,0.
9999,0,0,0,0,1]);
S(0) :=matrix(1,5,[1,0,0,0,0]);
P2 :=multiply(P,P);
P3 :=multiply(P2,P);
P4 :=multiply(P3,P);
S(4) :=multiply(S(0),P4);
S(4) :=[.8422907597 , .03693100731 , .03855028105 , .04024055068 , .04198740126]
```

I.2.4 Application of the Markov Chains Model

Once the transition probability matrix is obtained, the prediction of the future condition by Markov chains becomes a matter of matrix multiplication. Let us take the paint type (2) state road steel bridges performance curve. As mentioned earlier, the initial state vector of the first range of four years for new paint is always [1,0,0,0,0]. Therefore, the major problem is to obtain the transition probability matrix for paint type (2) State road. [13]

The values of Y(t) obtained from the regression function are used to solve the NLP equation. This solution provides the transition probability matrix for different paint age ranges of four years. The following matrix is the transition probability matrix for state road paint type (2) bridges in the first range of four years.

$$P := \begin{bmatrix} .935 & .065 & 0 & 0 & 0 \\ 0 & .766 & .234 & 0 & 0 \\ 0 & 0 & .301 & .699 & 0 \\ 0 & 0 & 0 & .0001 & .9999 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

p(1) = 0.935 for the first range that indicates that the probability of state road bridges of paint type (2) of paint age 4 years or less transition from state 1 (condition rating 9) to state 1 (remaining in state 1) in one year period is 0.935, and the probability of transition from state 1 to state 2 (condition rating 8) is q(1) = 0.065. Similarly, p(2) = 0.766 for the first range indicates that the probability of state road bridges of paint type (2) of paint age 4 years or less transition from state 2 (condition rating 8) to state 2 (remaining in state 2) in one year period is 0.766, and the probability of transition from state 2 to state 3 (condition rating 7) is q(2) = 0.234. [13]

To calculate the condition rating value by using a Markov chains, the value of E(t,P) can be calculated by using the equations described before.

S(0) = [1,0,0,0,0]

E(0,P) = S(0) * R = 9.0

where: R is the vector of condition rating 9,8,7,6 and 5.

E(1,P) :=multiply(S(0),P,R); E(1, P) = [8.935]

E(2,P) :=multiply(S(0),P2,R); E(2, P) = [8.859015]

E(3,P) :=multiply(S(0),P3,R); E(3, P) = [8.765686375]

E(4,P) :=multiply(S(0),P4,R); E(4, P) = [8.647524705] [13], [23]

Similarly, the condition rating for each year of paint age can be calculated for the same type of paint and road. Consequently, the deterioration curve for that type of paint can be drawn using the values of condition rating calculated by using the Markov chains. Figure I.5 indicates this curve. Table I.3 indicates the paint rating prediction for different ages. Table I.4 indicates the predicted ages that corresponding to different states.

These calculations could be made for all the paint categories: Interstate Paint types (1) & (2), State paint types (1) & (2) and Paint type (3). Figures from I.5 to I.9 indicate these curves respectively.

I.3. DIFFERENCE BETWEEN REGRESSION AND MARKOV PROCESS

Even though bridge paint performance curves have been developed by using regression, it is still necessary to use the Markov chains model to predict individual bridge paint conditions. As a matter of fact, both regression and Markov chains models play important roles in analyzing bridge paint systems. The regression model can be used to estimate the extent of condition improvement as a measure of effectiveness in response to alternate rehabilitation and repair strategies. However, when condition prediction is concerned, the Markov chains model provides more reasonable estimates of bridge paint conditions. This procedure is described in the following section. [12] [13]

The prediction models currently in use vary in complexity from simple straight-line extrapolation, regression models, to probability-based, Markov, models. Straight-line extrapolation is used to predict the condition of bridge paint. When sufficient data is available, it is found that the shape of the deterioration curves is generally curvilinear, rather than the straight line that results from straight-line extrapolation.

The probability-based Markov model was first developed for the Arizona PMS to describe pavement condition changes. Intuitively, the behavior of pavement is not deterministic but probabilistic. Similarly, the behavior of the steel bridge paint is also not deterministic but probabilistic. Consequently, the selection of an appropriate repair strategy for pavement or paint is also an uncertain procedure. Because of the probabilistic nature of steel bridge paint, it is decided to develop probability-based prediction models to predict its behavior. (5) (4)

The usage of a Markov probability decision process has the following advantages:

1- Future decisions on preservation actions are not fixed, but depend on how bridge paint actually perform.
2- Actions to be taken now can be identified. Also, actions likely to be taken in the next few years can be identified with a high degree of probability.
3- It is possible to compare the expected proportions in given condition states with the actual proportions observed in the field. In this way possible defects in construction, materials, quality and so on can be identified for bridge paint.
4- A dynamic decision model has the potential for significant cost savings by selecting less conservative rehabilitation actions that will still satisfy the prescribed performance standards. (5) (12) (13)

Regression extrapolation techniques are deterministic and do not attempt to explain the variability among the data points; they merely fit the best line to the data. Regression techniques are powerful tools, but in many cases the models are chosen for the best fit without regard to the suitability or intrinsic relevance to the variables selected. Polynomials of different degrees and mathematical functions can be manipulated to fit the data; but when these functions are projected beyond the bounds of the data results could be totally misleading. (5)

It is known that the rate of paint deterioration is uncertain. Therefore, the predictive model should reflect this, rather than using the erroneous assumption of deterministic behavior. The Markov process imposes a rational structure on the deterioration model. This form of predictive methodology has the further advantages of ensuring that projections beyond the limits of the data will continue to have the classic pattern of worsening condition with age, something that the regression models cannot guarantee. [(5)]

Another advantage of probability based models is the ease with which they can be integrated into optimization processes. The Markov process is a natural tool that is used in alliance with dynamic programming to produce the optimal solutions. It is believed that the application of Markov process in conjunction with dynamic programming will produce optimal maintenance and rehabilitation (M&R) strategies for selected steel bridges quickly and efficiently. [(5)]

Since a performance curve for steel bridges paint represents the average or mean condition rating at any given bridge paint age, it is obvious that both Markov chains method and regression method can be used to predict the average condition ratings for bridge paint. However, the Markov process has great advantages over regression in predicting individual bridge paint condition. [(13)]

Table I.3: Paint Rating Prediction Using Regression and Markov.

Age	Paint Rating									
	Interstate1		State1		Interstate2		State2		Paint3	
	Reg	Markov	Reg	Markov	Reg	Markov	Reg	Markov	Reg	Markov
0	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00
1	8.98	8.94	9.05	9.00	9.07	8.94	8.95	8.94	8.76	8.79
2	8.89	8.88	9.03	8.99	8.90	8.88	8.86	8.86	8.63	8.63
3	8.80	8.80	8.99	8.98	8.74	8.76	8.76	8.77	8.51	8.50
4	8.70	8.71	8.95	8.95	8.60	8.60	8.65	8.65	8.39	8.39
5	8.61	8.61	8.90	8.90	8.47	8.47	8.54	8.47	8.27	8.26
6	8.50	8.51	8.83	8.83	8.35	8.35	8.41	8.35	8.14	8.14
7	8.40	8.40	8.76	8.76	8.24	8.23	8.28	8.24	8.02	8.02
8	8.29	8.29	8.67	8.67	8.13	8.12	8.14	8.15	7.90	7.90
9	8.18	8.18	8.58	8.58	8.03	8.03	8.00	7.99	7.77	7.77
10	8.06	8.07	8.47	8.47	7.93	7.94	7.84	7.84	7.65	7.65
11	7.94	7.98	8.36	8.36	7.83	7.83	7.68	7.68	7.53	7.53
12	7.82	7.88	8.23	8.23	7.73	7.74	7.52	7.52	7.40	7.40
13	7.69	7.69	8.10	8.10	7.62	7.63	7.34	7.34	7.28	7.28
14	7.56	7.55	7.95	7.95	7.51	7.51	7.17	7.18	7.16	7.16
15	7.43	7.42	7.79	7.80	7.39	7.39	6.98	6.98	7.04	7.04
16	7.29	7.30	7.62	7.63	7.26	7.26	6.79	6.81	6.91	6.92
17	7.15	7.15	7.45	7.44	7.11	7.10	6.60	6.59	6.79	6.79
18	7.01	7.00	7.26	7.25	6.95	6.93	6.41	6.39	6.67	6.66
19	6.86	6.86	7.06	7.06	6.77	6.77	6.20	6.20	6.54	6.54
20	6.71	6.70	6.85	6.88	6.58	6.61	6.00	6.04	6.42	6.42
21	6.55	6.54	6.63	6.65	6.36	6.33	5.79	5.63	6.30	6.29
22	6.39	6.38	6.40	6.38	6.12	6.07	5.58	5.58	6.17	6.17
23	6.23	6.22	6.16	6.16	5.85	5.85	5.37	5.38	6.05	6.05
24	6.06	6.10	5.91	5.97	5.56	5.67	5.15	5.15	5.93	5.94
25	5.90	5.90	5.65	5.54	5.24	5.24	4.94	5.00	5.81	5.80
26	5.72	5.70	5.38	5.26	4.88	5.00			5.68	5.67
27	5.55	5.55	5.10	5.10					5.56	5.56
28	5.37	5.37	4.81	5.03					5.44	5.44
29	5.18	5.18							5.31	5.30
30	5.00	5.00							5.19	5.18
31									5.07	5.05
32									4.94	5.00

Table I.4: The predicted Age Corresponding to Different States.

Paint Rating	Age				
	Interstate1	State1	Interstate2	State2	Paint3
9	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*
8	*10.50*	*13.66*	*9.32*	*9.00*	*7.15*
7	*18.04*	*19.30*	*17.70*	*14.90*	*15.28*
6	*24.38*	*23.66*	*22.45*	*20.00*	*23.41*
5	*29.97*	*27.35*	*25.67*	*24.70*	*31.54*

Figure I.5: Interstate Paint Type (1):
Regression Vs. Markov

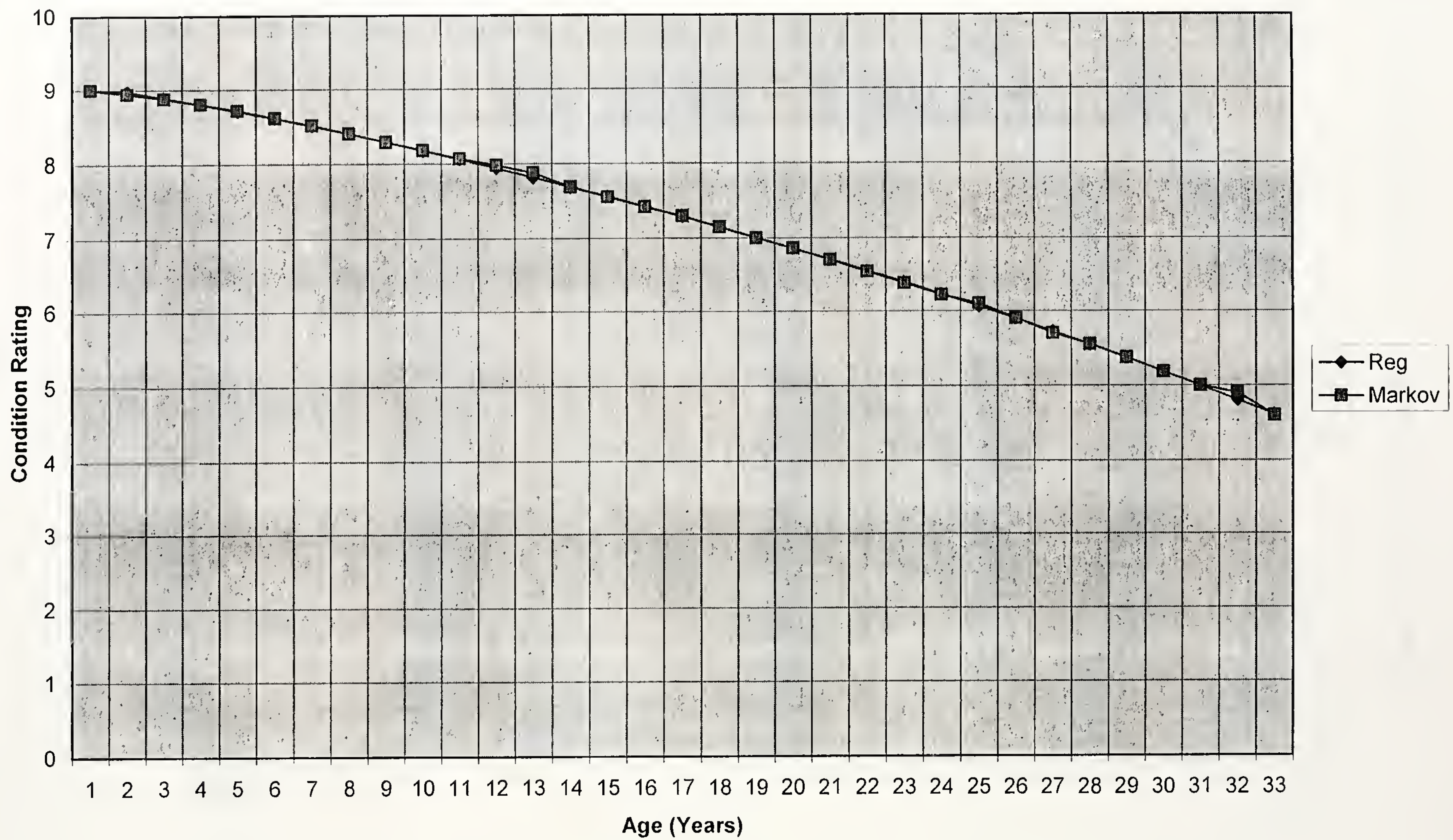


Figure I.6: Interstate Paint Type (2):
Regression Vs. Markov

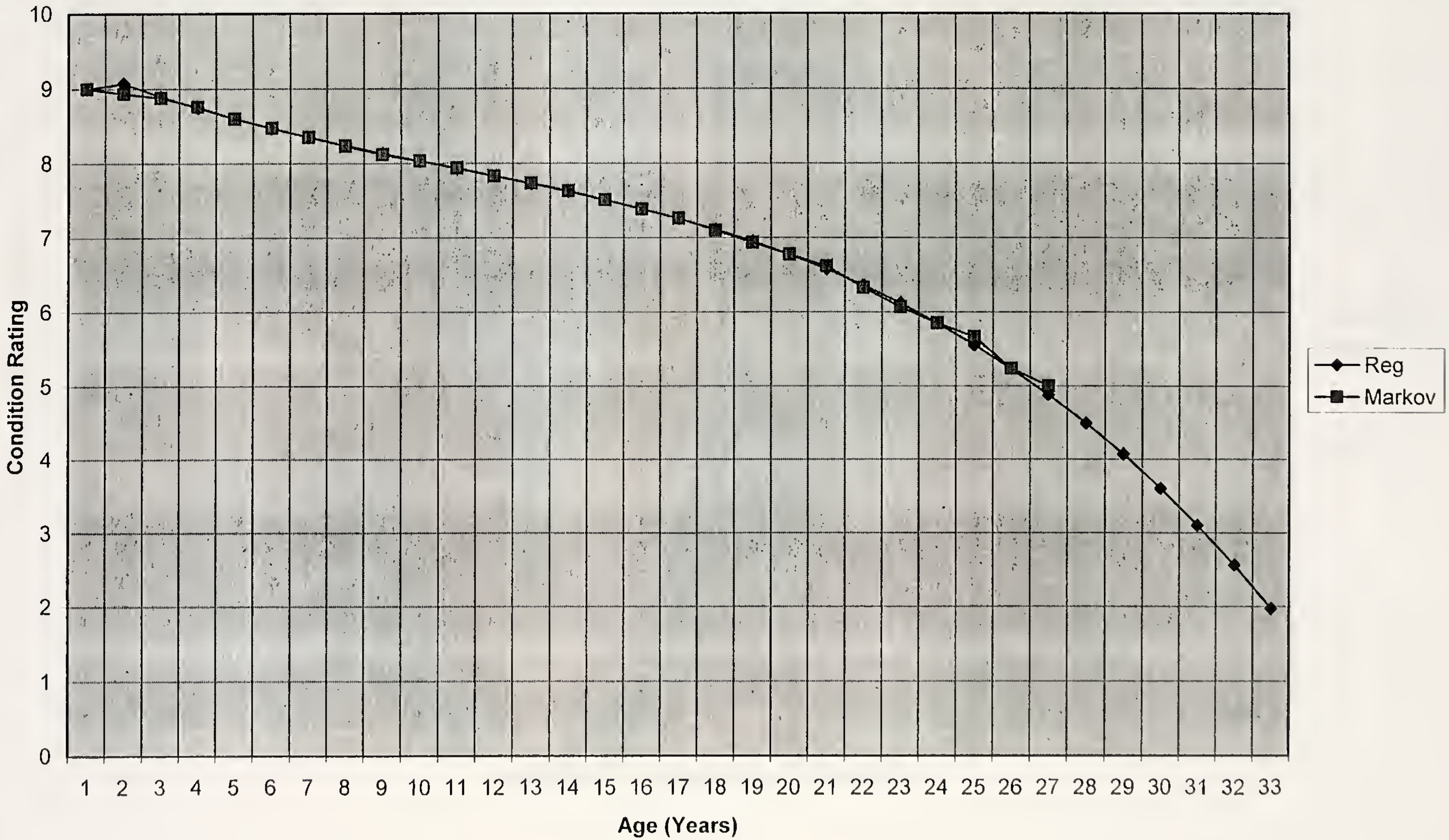


Figure I.7: State Paint Type (1):
Regression Vs. Markov

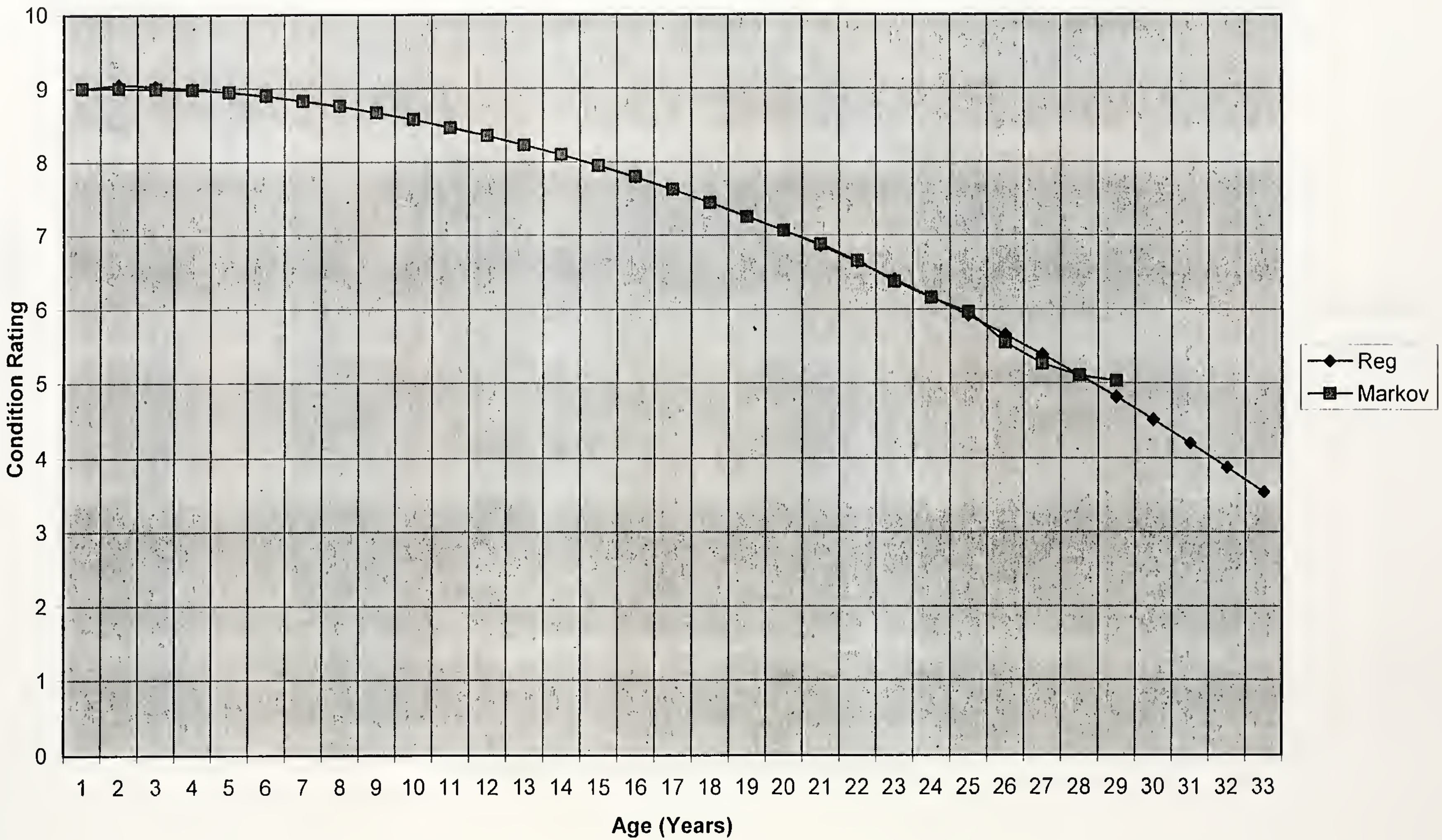


Figure I.8: State Paint Type (2): Regression Vs. Markov

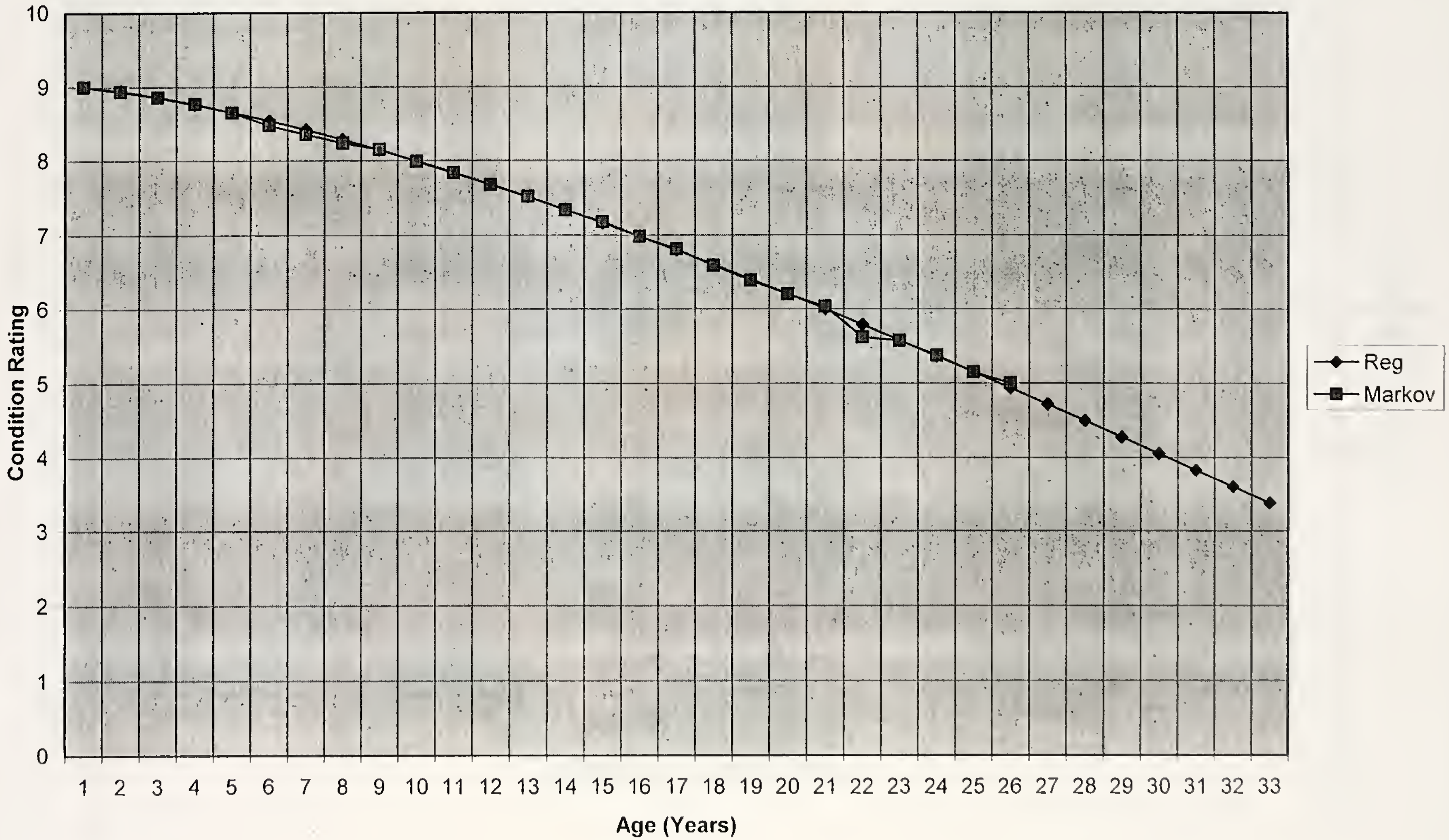
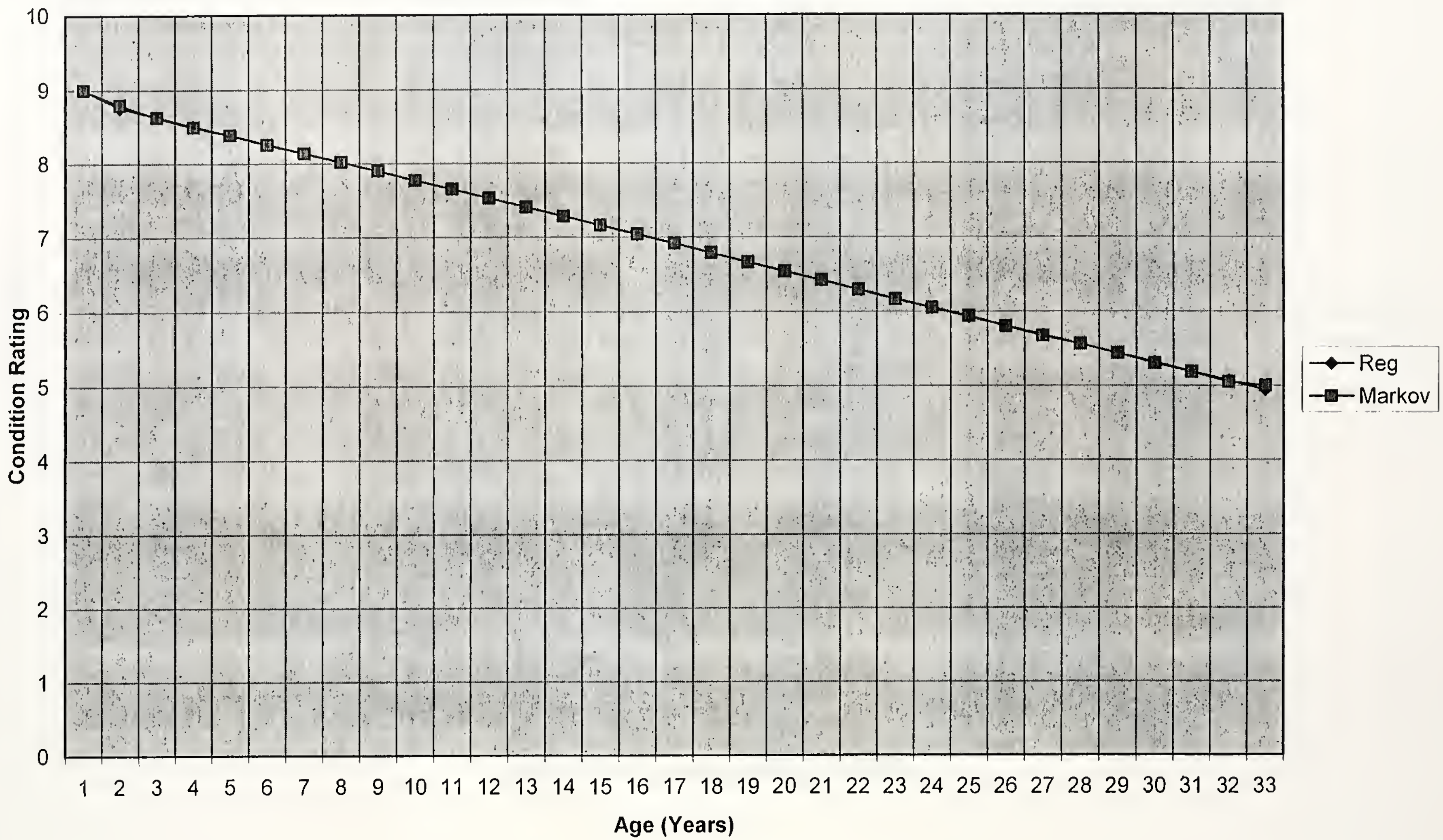


Figure I.9: Paint Type (3):
Regression Vs. Markov



CHAPTER II

STEEL BRIDGE EXISTING PAINT LIFE CYCLE COST ANALYSIS

II.1 EXISTING SYSTEMS LIFE CYCLE COST ANALYSIS

II.1.1 Markov Decision Process (MDP) (Based on Stochastic Method)

Managers need to make decisions regarding selecting the optimal rehabilitation action for each paint condition. The action that is chosen from several alternatives affects the transition probabilities in the Markov chains model, as well as their immediate and subsequent costs. The decision process for selecting the optimal actions for the respective states of paint when considering immediate and subsequent costs is referred to as the Markov Decision Process (MDP). [22]

Life cycle cost analysis for the steel bridges paint maintenance / Rehabilitation problem can be studied by using the Markov Decision Process (MDP). Life cycle cost analysis is performed on the basis of long-run behavior of the paint systems. As planning period n approaches infinity (long-run), there is a limiting probability (called the steady-state probability) that the system will be in state j after a large number of transitions, where this probability is independent of the initial state i. This long–run behavior holds under relatively general conditions and it holds for steel bridge paint rehabilitation problems as well.

Because the long-run behavior of a Markov chains exists for steel bridge paint, the rehabilitation problem can be solved using the assumption that the Markov Decision Process (MDP) will be operating indefinitely. The steps of that Markov Decision Process (MDP) can be summarized as follow: [22] [10]

1- There are five possible states observed after each transition (i = 1,2,3,4,5) associated with steel bridge paint condition rating, from 1 to 5, with 5 being the worst. The index i is used for the initial state, and the index j is used for the future state. Therefore, state 1 corresponds to condition rating 1 and state 2 corresponds to condition rating 2 and so on.

2- After each observation, a decision (action) k is chosen from a set of K possible decisions (k= 1,2,3,4). Some of the K decisions may not be relevant for some of the states. The following Table II.1 indicates the decisions and their relevant states.

Table II.1: Decisions and relevant states.

Decision k	Action Description	Relevant to States
1	*Do nothing*	*1,2,3,4*
2	*Spot Repair*	*2,3,4*
3	*Over-coating*	*3,4*
4	*Complete Repainting*	*5*

3- If decision $d_i = k$ is made in state i, an immediate cost is incurred that has an expected value C_{ik}. The costs in the following Table II.2 are the estimated unit cost ($/ton) for steel bridges.

Table II.2: Estimated Unit Cost of Paint Rehabilitation.

Decision	State	Description	Rehab. Cost Cr ($/ton) Paint Types (1)	(2)
1	*1,2,3,4*	*Do nothing*	*0.0*	*0.0*
2	*2*	*Spot Repair*	*25*	*20*
	3	~ ~ ~	*50*	*40*
	4	~ ~ ~	*90*	*75*
3	*3*	*Over-coating*	*110*	*100*
	4	~ ~ ~	*180*	*150*
4	*5*	*Complete Repainting*	*220*	*180*

Note: $/ton(English system) = 1.1023 $/ton(Metric system)

4- Decision $d_i = k$ in state i determines what the transition probabilities will be for the next transition from state i. These transition probabilities can be denoted by $P_{ij}(k)$ for $j = 1,2,3,4,5$. The parameter k in $P_{ij}(k)$ is used to indicate that the appropriate transition probability depends upon the decision k.

5- There are several policies that can be used to make rehabilitation for steel bridge paint according to its state. A policy is a set of decisions (actions) for each state. Table III.3 indicates the proposed policies or scenarios and their decisions at each state. (22) (10)

TableII.3 lists the relevant policies for steel bridge paint problems. For example, Policy Y1: $(d_1, d_2, d_3, d_4, d_5) = (1,1,1,1,4)$ describes a policy where decision 1 (do nothing) is made in states 1,2,3,4 and decision 4 (complete repainting) is made in state 5. The following transition probability matrix is assumed to be the matrix that corresponds to Policy Y1. This assumption is made for illustration purpose only.

$$P1 = \begin{bmatrix} 0.80 & 0.20 & 0 & 0 & 0 \\ 0 & 0.30 & 0.50 & 0.20 & 0 \\ 0 & 0 & 0.1 & 0.5 & 0.4 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Table II.3: Different Policies for Painting Rehabilitation.

Decision	Policy (Y1)	Policy (Y2)	Policy (Y3)	Policy (Y4)	Policy (Y5)
d_1	1	1	1	1	1
d_2	1	1	1	1	2
d_3	1	2	3	1	1
d_4	1	1	1	3	3
d_5	4	4	4	4	4

The long run behavior of P1 reached after seventeen transitions. It is noticed that the five rows have identical entries. This implies that the probability of being in state j after 17 periods appears to be independent of the initial paint condition.

$$(P1)^{17} = \begin{bmatrix} 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \end{bmatrix}$$

Because the long run behavior of Markov chains exists for all five relevant policies, paint rehabilitation problem can be solved using the assumption that the Markov decision process will be operating indefinitely.

6. The objective is to find an optimal Policy that minimizes the expected total discounted cost (immediate cost and discounted subsequent costs that result from the future processes). [(10),(22)]

II.1.2 Minimizing the Total Expected Discount Cost

Given a distribution $P\{X_0 = i\}$ over the initial states of the system and a Policy R, a system evolves over time according to the joint effect of the probabilistic laws of motion and sequence of decisions made (actions taken). In particular, when the system is in state i and decision $d_i\,(R) = k$ is made, then the probability that the system is in state j at the next observed time period is given by $p_{ij}(k)$. Furthermore, a known expected cost C_{ik} is incurred. Denoted by $V^n_i(R)$ the expected total discounted cost of a system starting in state i (at the first observed time period) and evolving for n time periods. Then $V^n_i(R)$ has two components: (1) C_{ik}, the cost incurred at the first observed time period as a result of the current state i and the decision $d_i\,(R) = k$ when operating under the Policy R, and (2) $\propto \sum_{j=0}^{M} p_{ij}(k)\, V^{n-1}_j(R)$, the expected total discounted cost of the process evolving over the remaining n-1 time periods. A discount factor $\propto < 1$ is specified, so that the present value of 1 unit of cost m periods in the future is $\propto^m$. $\propto$ can be interpreted as equal to 1/(i+1), where i is the current interest rate. Thus the recursive equation

$V^n_i(R) = C_{ik} + \propto \sum_{j=0}^{M} p_{ij}(k)\, V^{n-1}_j(R)$, for i= 0,1,2, …,M and $V^1_i(R)$- C_{ik} for all i is obtained.

This policy can be evaluated using the techniques associated with dynamic programming. It can be shown that as n approach infinity, this expression converges to:

$V_i(R) = C_{ik} + \propto \sum_{j=0}^{M} p_{ij}(k)\, V_j(R)$, for i= 0,1,2, …,M

where $V_i(R)$ can now be interpreted as the expected long run total discounted cost for a system starting in state i and continuing indefinitely. There are M+1 equations and M+1 unknowns, and hence $V_i(R)$ may be obtained by standard methods of solving simultaneous equations. [10],[22]

II.1.3 Policy Improvement Technique Algorithm

The steps for that algorithm are as follow:

(1) *Value determination*: For an arbitrary chosen policy R_1, use $p_{ij}(k_1)$ and C_{ik1} to solve the set of M+1 equations

$V_i(R_1) = C_{ik1} + \propto \sum_{j=0}^{M} p_{ij}(k_1)\, V_j(R_1)$, for i= 0,1,2, …,M

for all (M+1) unknown values of $V_i(R_1)$.

(2) *Policy Improvement*: Using the current values of $V_i(R_1)$, find the alternative policy R_2 such that, for each state i, $d_i(R_2) = k_2$ is the decision that makes

$C_{ik2} + \propto \sum_{j=0}^{M} p_{ij}(k_2)\, V_j(R_1)$, for i= 0,1,2, …,M

a minimum; that is, for each state i, find the appropriate value of k_2 that

$\underset{k_2=1,2,\ldots k}{Minimizes} \{C_{ik2} + \propto \sum_{j=0}^{M} p_{ij}(k_2)\, V_j(R_1)\}$, for i= 0,1,2, …,M

and then set $d_i(R_2)$ = minimizing the value of k_2. This procedure defines a new policy R_2. If R_2 does not equal to R_1, then return to step 1 by using R_2 instead of R_1, and solve for $V_i(R_2)$, i= 0,1,2, …,M. Using these values, go to step (2) and find R_3. Continue in this fashion until you find two successive R's to be equal. When you find them, the optimal policy is achieved, and the algorithm terminates. In fact, it can be shown that:

1- $V_i(R_{j+1}) \subseteq V_i(R_j)$ for i= 0,1,2, ...,M and j = 1,2,....
2- The algorithm terminates with the optimal solution in a finite number of iterations
3- The algorithm is valid without the assumption that Markov chain associated with every transition matrix is irreducible. [10],[22]

II.1.4 Application of Markov Decision Process (MDP) for Steel Bridge Paint

The previous procedure for MDP is applied to steel bridge paint according to the previous proposed policies and their costs. One transition probability matrix is estimated for each 4-year period or range along paint age. This transition probability matrix is changed or enhanced due to different decisions that can be taken at each paint state and according to different policies. Consequently, the procedure that is explained in the previous section should be repeated in each 4-year range or period, for the same paint type, to decide the optimum rehabilitation policy at each range of paint life.

Based on the transition probability matrix and the costs associated with the selected policy, five equations with five unknowns could be solved to get the values of the estimated costs at each state of the same range of life cycle. Therefore, for the selected policy, the procedure for checking this policy is applied. The results of the application indicate whether this policy is suitable for this range or not. If it is not, another policy is applied and checked and so on. If it is accepted, then this is the optimum policy for that range. This procedure is repeated for each range of the paint type life cycle. Consequently, maintenance plan based on the optimum policy can be planned for each paint type along its life cycle.

Based on the estimated costs that are indicated in the previous section, the optimum policy is calculated using MDP for each paint type. For Interstate Paint Types (1)&(2), State Paint Types (1)&(2) and Paint type (3), the policy Y1 is the optimum one for all the ranges along the life of each type. This policy proposes doing nothing for states

1,2,3 and 4 and does complete repainting for state 5. All the Markov decision process (MDP) calculations are shown in Appendix C.

II.1.5 Economic Analysis for Different Paint Types

In the past few years, the cost of maintenance and rehabilitation of existing coating systems on steel bridges has risen dramatically. This increase can be attributed to stricter environmental constraints and higher safety standards for workers. In addition, reduced government funding and sub-optimal maintenance scheduling have contributed to the increasing deterioration of these structures. Eventually, corrosive action can reduce the cross section of steel members and decrease load capacity. Due to these problems, a research project was jointly conducted between INDOT and Purdue University to investigate current rehabilitation methods to select the most economical method. [20]

One of the strategies used by bridge managers is to allow the coating system to deteriorate without implementing any maintenance activities. This option is certainly feasible under specific situations. For example, if the structure is near its expected design life, it is more economic to replace the structure. In most cases, this option will reduce the surface life of the structure. The reason for the decreased life span is attributed to the durability of the coating systems. Most coating systems have life spans that lie in the range between several years to approximately 30 years, depending on environmental conditions. However, as rough estimate on the basic service life of a bridge, Hiroshi (1988) found that the average life span of steel bridges is approximately 35 years. There may be some differences between Japanese, Canadian, and U.S. bridges, but the same guidelines are used by all of them. Therefore, most structures must require at least one rehabilitation activity during its service life. [20]

The service life of a steel bridge is limited by several factors such as fatigue, loading capacity, and corrosion. The first two factors, fatigue and loading capacity, depend on loading conditions and require either replacement of certain components or of the entire structure to maintain service. The third factor, corrosion, can be controlled by preventive maintenance. Application of rehabilitation activities can extend the service life of a structure. However, the cost of rehabilitation may sometimes be less economic than replacing the structure. The most economic choice will depend on the costs of these two options and their relative service lives. Therefore, some decision support techniques such as life-cycle cost analysis can be used to help bridge managers choose the best strategy for an existing or proposed steel bridge. [20]

Several researchers have begun to incorporate life cycle cost analysis (**Al-Subhi et al. 1990; Markow 1990; McNeil and Finn 1987**) or discounted cash flow methods (**Chen and John-ston 1990**) to minimize the cost of bridge maintenance. **Weyers et al**. (1988) has applied this approach to compare the equivalent uniform annual cost of various rehabilitation activities during the service life of a bridge. **Rajagopal and George** (1990) have applied this approach to road pavements in order to reduce their maintenance costs. In this research, a life-cycle cost analysis using equivalent annual cost is developed to compare the cost of three maintenance strategies: spot repair, over-coating and complete repainting. [20]

Life cycle cost analysis is a relatively simple approach for minimizing coating maintenance costs. The general objective of this approach is to determine all the costs associated with the corrosion protection of the structure throughout its remaining service life. The total cost for a combination of a particular maintenance strategy is compared to the total cost of another strategy. The strategy that yields the lowest cost is considered to be the optimal maintenance strategy for the specific structure. [20]

The basic components required in the life cycle cost analysis using equivalent annual cost are: [20]

1- Database: deterioration functions and maintenance costs.

2- Formula for equivalent annual costs.

3- Correlation between condition upgrades and rehabilitation activities.

4- Constraints: condition limits for each activity.

The database or deterioration curves are constructed for the INDOT steel bridges using the INDOT database. These functions are constructed for Interstate and State paint types (1) and (2) and paint type (3).

The basic formula used in the life cycle cost analysis can be obtained from any basic economics textbook (Riggs 1986). The equation for equivalent annual cost is:

$$A = F(A/F,i,N) * L = F \frac{i * L}{(1+i)^N - 1}$$

where: F = future cost; i = interest rate; A = annuity; L = inflation Factor = $(1+i)^N$ and N = maintenance period.

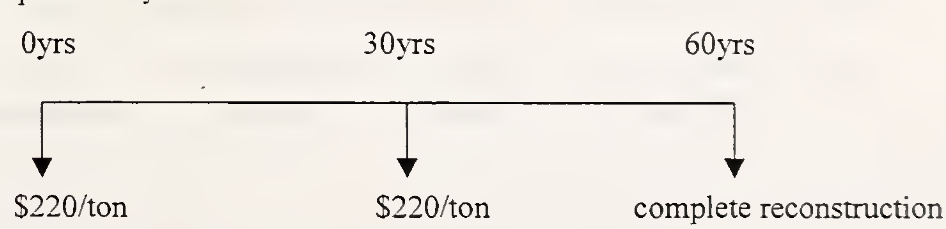
Also,

$$A = P(A/P,i,N) * L = P \frac{i(1+i)^N * L}{(1+i)^N - 1}$$

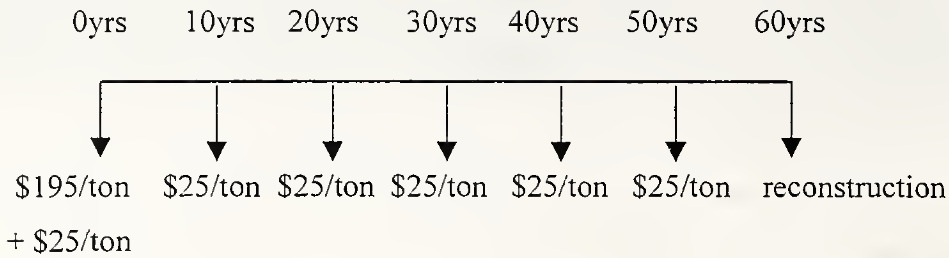
where: P = present cost; A = annual payment. [20]

These formulas are used to calculate the equivalent annual cost for five different categories of paint deterioration models according to five proposed rehabilitation and maintenance alternatives. The proposed alternatives of rehabilitation and maintenance are as follows: (***Note***: the $/ton numbers use the English system)

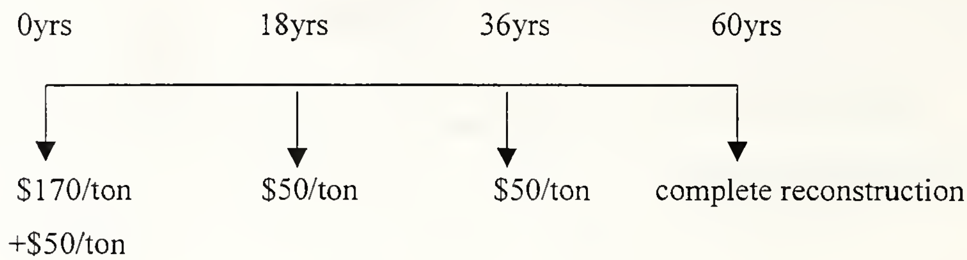
1- Do nothing and do complete repainting after reaching state 5 (approximately 30 years) Bridge life span is 60 years.



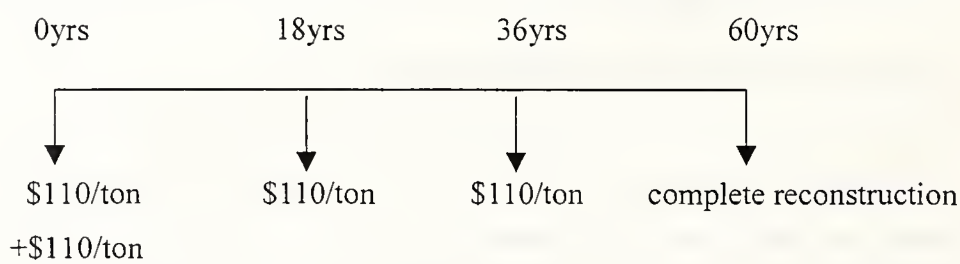
2- Spot repairs are made at state 2, occurs every 10 years until the end of the bridge life of 60 years.



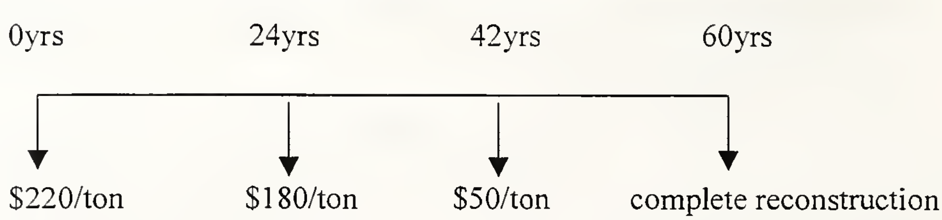
3- Spot repairs are made at state 3, which are repeated each 18 years until the end of the bridge life.



4- Over-coating is done at state 3, repeated each 18 years until the end of the bridge life.



5- Over-coating is done at state 4 after the first 24 years and spot repairs are done after 18 years, that is at 42nd year, until the end of the bridge life.



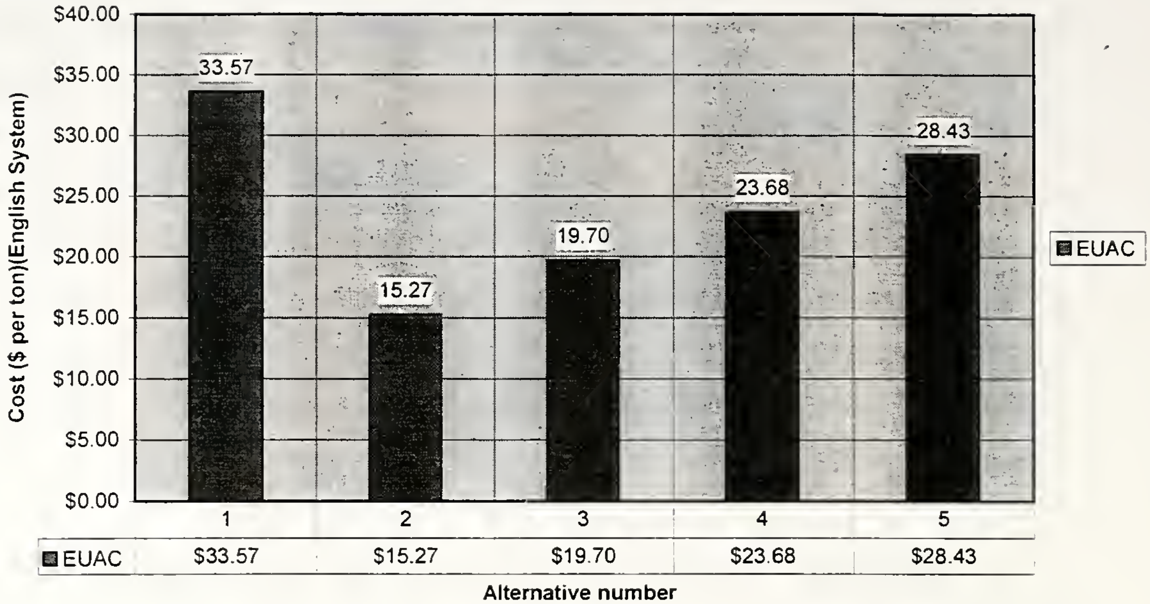
These options or alternatives are applied for the five categories of paint taking into consideration the difference in cost between different types of paint. The mentioned cost figures in the five policies are based on paint type (1) and costs for other paint types are indicated in Table II.2. The estimated number of years corresponding to bridge states is indicated in Table I.3. After calculating the equivalent annual cost for each alternative considering one paint category, the minimum cost alternative will be the optimum one. By analyzing the five categories, it is concluded that alternative number two (spot repairs at state 2 and repetition of spot repair along the life span of the bridge) is the optimum one. This happens for all the categories. The calculations for these alternatives are indicated in Figure II.1 to Figure II.5. The remaining calculations are indicated in Appendix D.

The bar chart in Figure II.1 indicates that the optimum alternative for Interstate 1 is alternative number 2, where spot repairs every 10 years should be done for the paint until the end of the bridge life. This optimum cost is $ 15.27 / ton ($16.83/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

The bar chart in Figure II.2 indicates that the optimum alternative for Interstate 2 is alternative number 2 where spot repairs every 9 years should be done until the end of the bridge life. This optimum cost is $ 12.32 / ton ($13.58/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

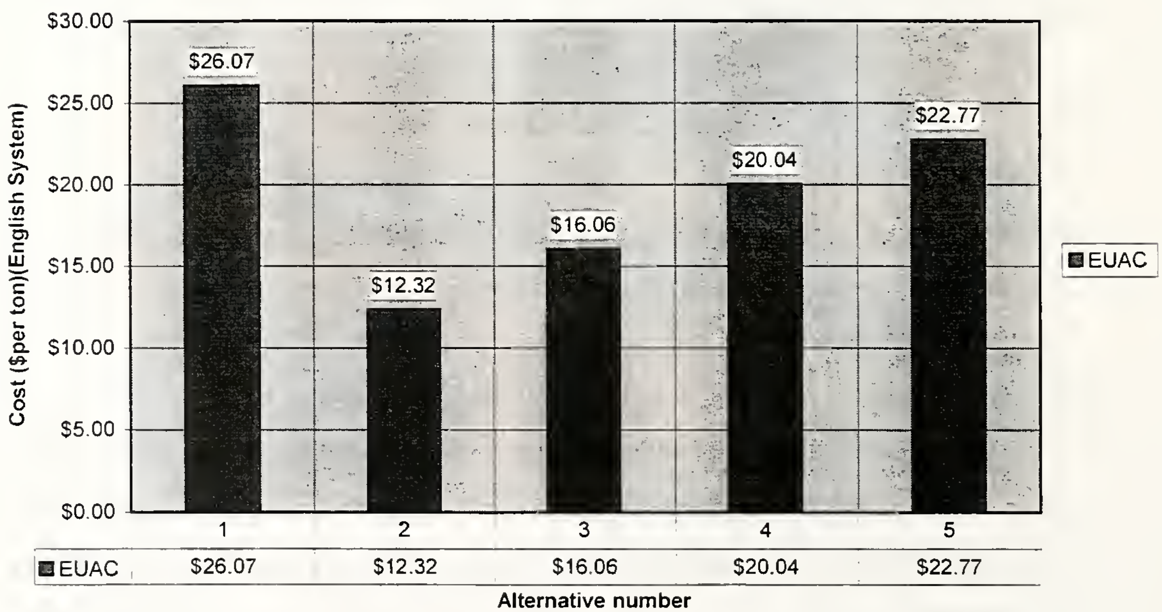
The bar chart in Figure II.3 indicates that the optimum alternative for State 1 is alternative number 2, where spot repairs every 14 years should be done until the end of the bridge life. This optimum cost is $ 16.36 / ton ($18.03/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

Figure (II.1): EUAC for Different alternatives for Interstate 1



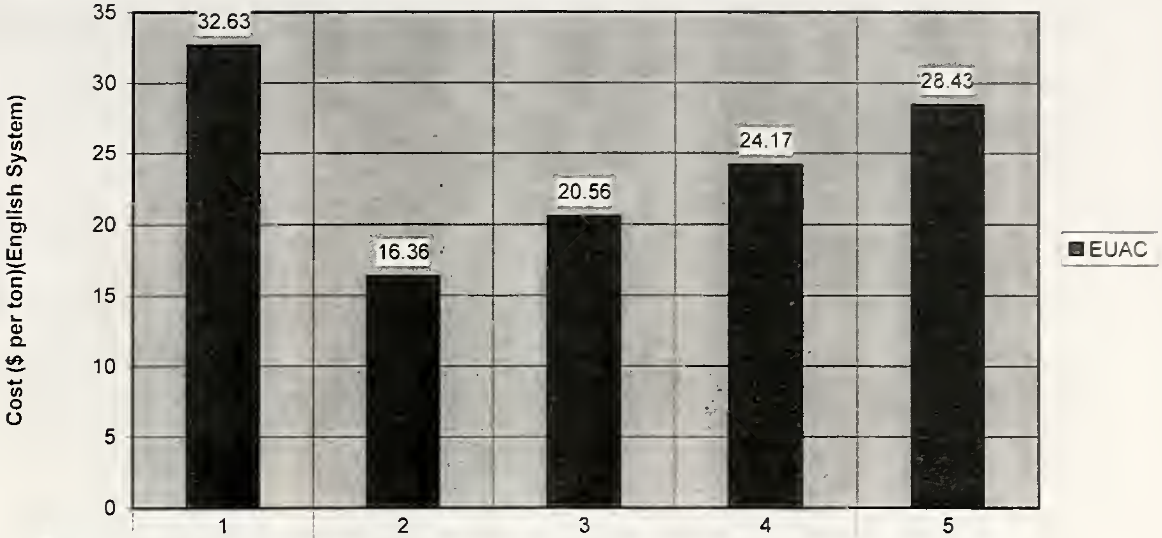
Note: \$/ton(English system) = 1.1023 \$/ton(Metric system)

Figure (II.2): EUAC for Different Alternatives for Interstate 2



Note: \$/ton(English system) = 1.1023 \$/ton(Metric system)

Figure (II.3): EUAC for Different Alternatives for State 1

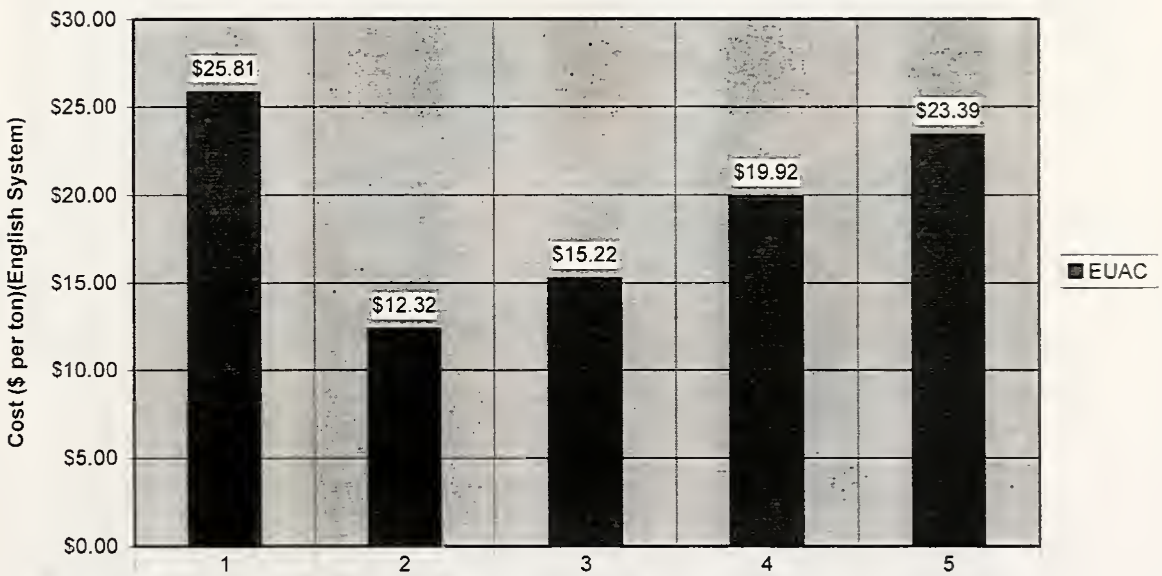


	1	2	3	4	5
EUAC	32.62790602	16.35941612	20.56072603	24.17487372	28.43286767

Alternative Number

Note: $/ton(English system) = 1.1023 $/ton(Metric system)

Figure (II.4): EUAC for Different Alternatives for State 2



	1	2	3	4	5
EUAC	$25.81	$12.32	$15.22	$19.92	$23.39

Alternative Number

Note: $/ton(English system) = 1.1023 $/ton(Metric system)

The bar chart in Figure II.4 indicates that the optimum alternative for State 2 is alternative number 2, where spot repairs every 9 years should be done until the end of the bridge life. This optimum cost is $ 12.32 / ton ($13.58/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

The bar chart in Figure II.5 indicates that the optimum alternative for Paint 3 is alternative number 2, where spot repairs every 7 years should be done until the end of the bridge life. This optimum cost is $ 15.08 / ton ($16.62/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

The bar chart in Figure II.6 indicates that the optimum alternative for paint Interstate 1 & 2 is alternative number 2 for Interstate 2, where spot repairs every 9 years should be done until the end of the bridge life. This optimum cost is $ 12.32 / ton ($13.58/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

The bar chart in Figure II.7 indicates that the optimum alternative for paint State 1 & 2 is alternative number 2 for State 2, where spot repairs every 7 years should be done until the end of the bridge life. This optimum cost is $ 12.32 / ton ($13.58/Metric ton). This alternative is less than half the cost of the do nothing alternative until complete repainting again (alternative number 1).

Figure (II.5): EUAC for Different alternatives for Paint Type (3)

Note: $/ton(English system) = 1.1023 $/ton(Metric system)

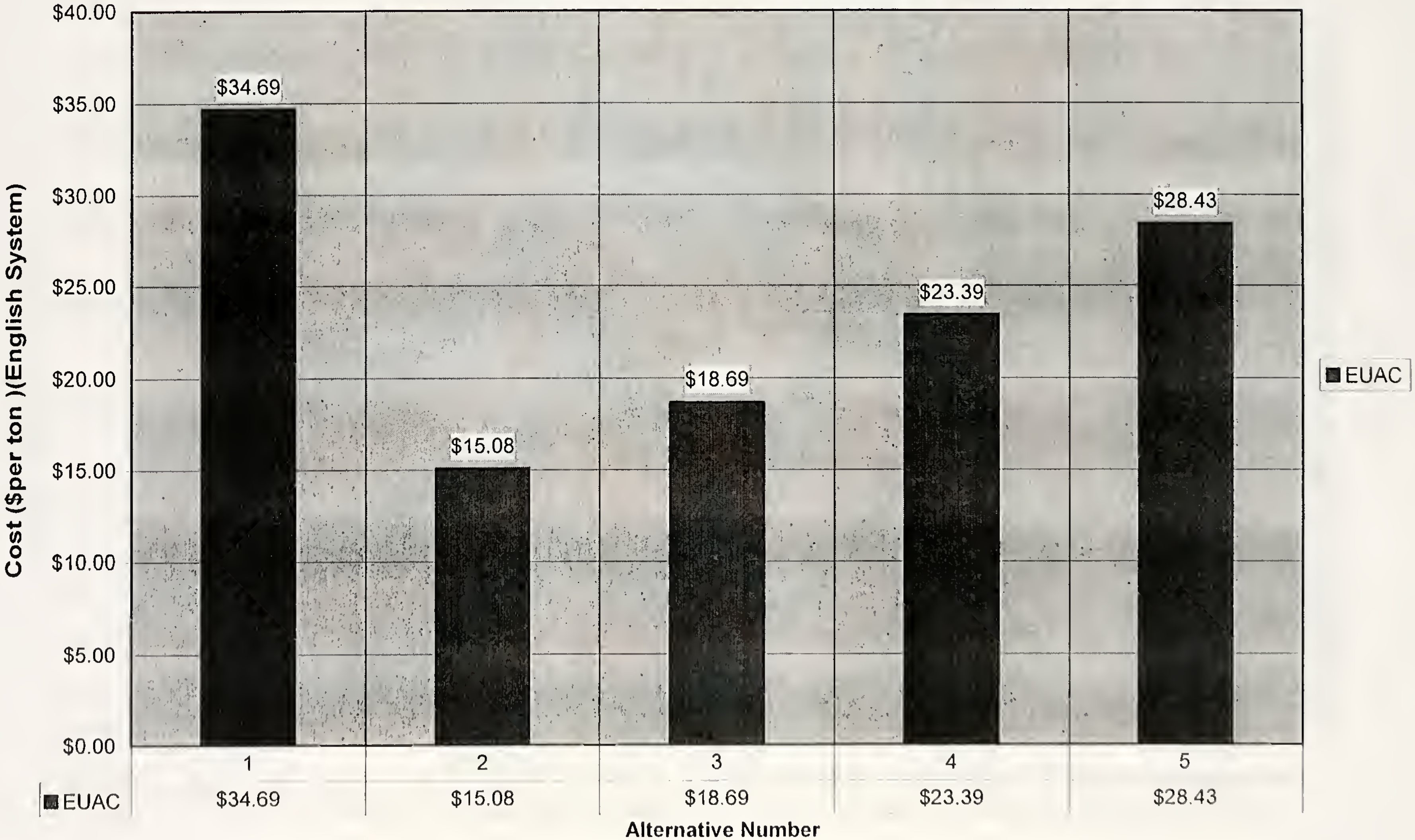


Figure (II.6): EUAC of Interstate I1 and Interstate I2 Comparison Using Economic Analysis.

Note: $/ton(English system) = 1.1023 $/ton(Metric system)

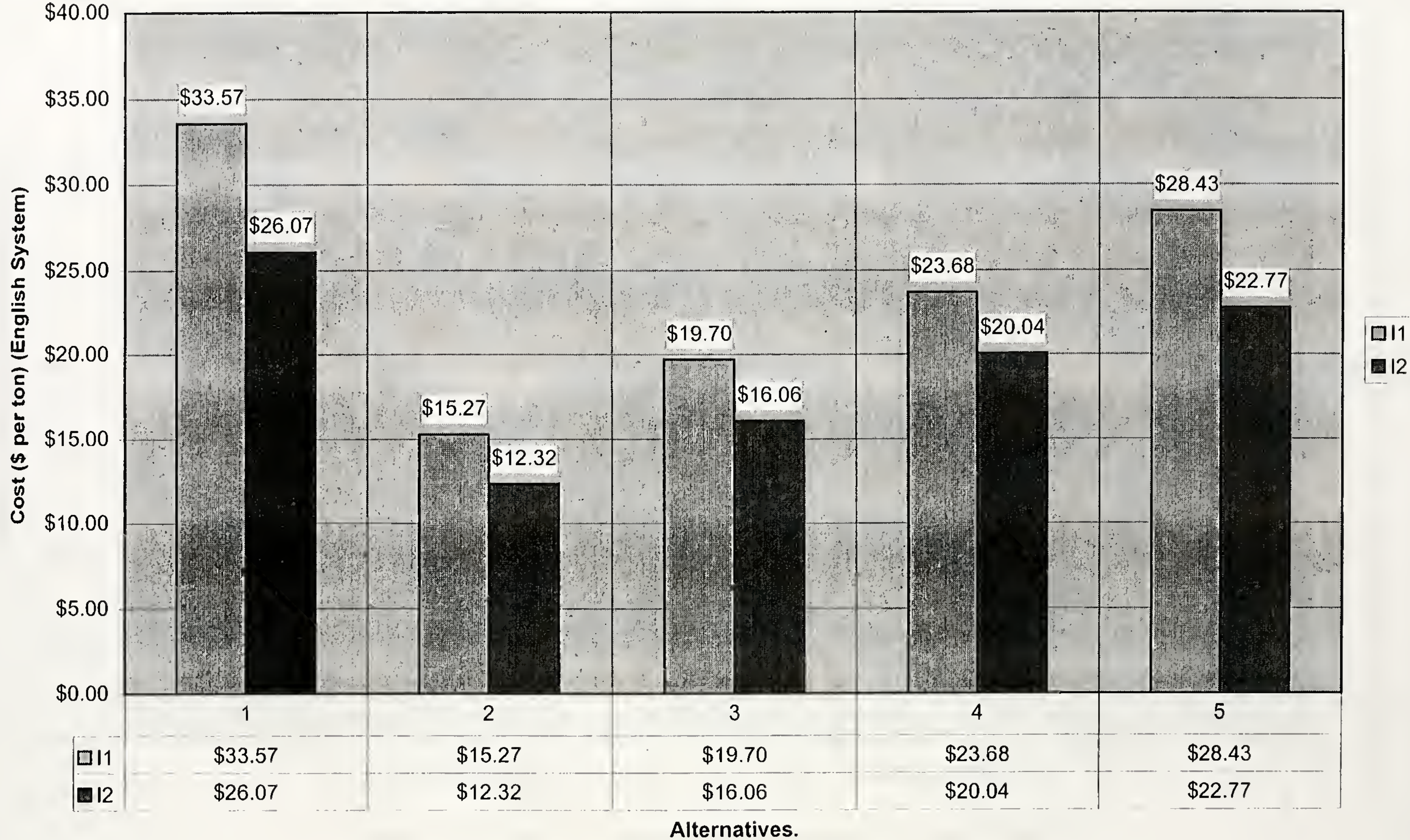
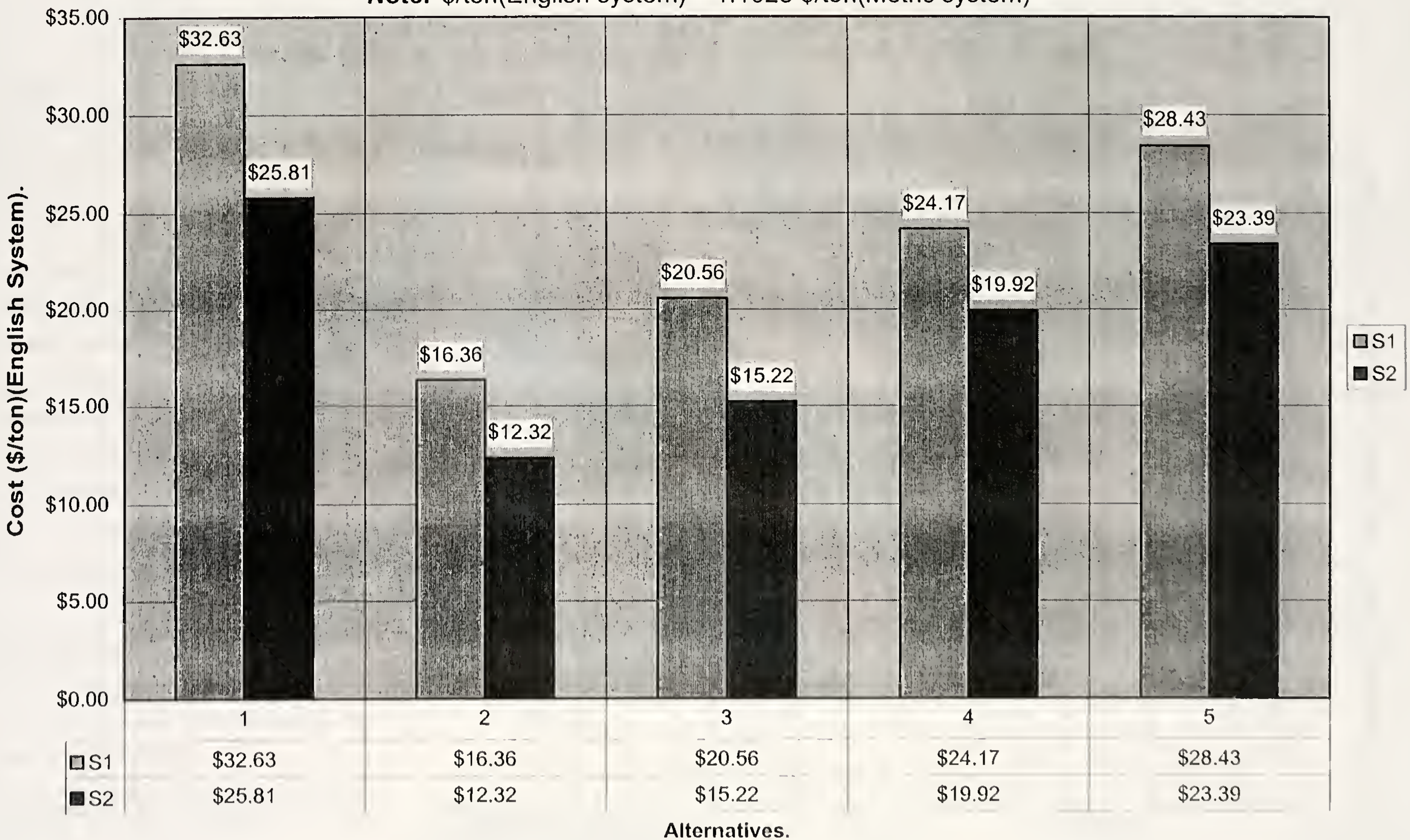


Figure (II.7): State S1 and State S2 Comparison Using EUAC.

Note: $/ton(English system) = 1.1023 $/ton(Metric system)



II.2 COMPARISON OF EXISTING WITH NEW SYSTEMS

According to life cycle cost analysis, Zinc/Vinyl is clearly the best existing paint system. Therefore, Zinc/Vinyl should be compared with some new systems to decide whether it is truly the best system for the future. This comparison is only from the cost basis. The available information from other states (Illinois, Michigan, Ohio, Kentucky and Connecticut) about new paint systems uses different units of measurement. Other states use $ per square foot as a cost reference, instead of using $ per ton. INDOT uses $ per ton as a cost reference; therefore a conversion factor is very important to compare these different cost references. Our study relies on the other states' experience to evaluate some new systems that did well in these states. Consequently, a conversion factor should be used to compare Zinc/Vinyl ($/ton) with the other new systems from other states ($\$/ft^2$). Table II.4 shows the calculation of the conversion factor from a sample of four bridges. This information is based on data from INDOT and some analysis from Mr. Maged Georgy's report. Mr. Ted Hoppwood[26] of KentuckyDOT said that the conversion factor is approximately $/ton = 125 $\$/ft^2$ ($/ton = 1344$\$/m^2$), based on his experience. Therefore, on average, a conversion factor of ***115*** can be used in our study until the calculation of the accurate number for INDOT is possible. The new systems that can be studied here and that are suitable to be used by INDOT are a 3-Coat System and Metalization. It is proposed to compare these new systems with the existing Zinc/Vinyl system.

Based on the information that is included in Table II.4, the Zinc/Vinyl system cost in $\$/ft^2$ can be calculated as the average of the two available bridges. This calculation concludes a cost of $\$2.5/ft^2$ ($\$26.88/m^2$) for this system. Based on data from INDOT for 3-coat system, Inorganic/Organic Zinc Epoxy Urethane (In/OZEU), the unit cost per square foot is calculated for that system. This unit cost is calculated taking into consideration two different old paint systems. The first application is done after the removal of Lead based paint as an old system. The second application is done after the removal of Zinc based paint as an old system. Table II.5 shows the calculation of unit

Table II.4 : Proposed Conversion Factor

Coating System	Vinyl			3 Coat System		
Sample #	1	2		3	4	
Contract Year	1997	1997		1998	1998	
Bridge Number	I-69-91-4783	I-69-113-4502		I-69-95-04768	I-69-110-02266NB	
Location	91 mile marker I-69 Huntington Co.	113 mile marker I-69 Allen Co.		95 mile marker I-69 Allen Co.	110 mile marker I-69 Allen Co.	
Cost/bridge (all items)	$20,000	$23,000		$27,000	$33,000	
Cost/bridge (*Only* clean and paint steel)	$14,000	$17,000		$19,000	$25,000	
Bridge Tonnage (MG)				79	71	
Bridge Tonnage (Ton)	65	84		87.10	78.28	
Number of beams	12	8	4	12	14	7
Length of beam (ft)	65	67.75	67.75	75.86	43	54
Type of beam	36 WF 135	36 WF 160	36 WF 170	36 WF 160	30 WF 108	30 WF 108
Width of flange (in)	11.95	12	12.03	12	10.745	10.475
Beam depth (in)	35.55	36.01	36.17	36.01	29.83	29.83
		5244.82	2632.36		4955.82	3084.37
Estimated surface area to be coated (ft^2)	7473.13	7877.19		8808.98	8040.19	
Cost in $/ton (all items)	$307.69	$273.81		$309.99	$421.56	
Cost in $/ton (*Only* clean and paint steel)	$215.38	$202.38		$218.14	$319.37	
Cost in $/$ft^2$ (all items)	$2.68	$2.92		$3.07	$4.10	
Cost in $/$ft^2$ (*Only* clean and paint steel)	$1.87	$2.16		$2.16	$3.11	
Conversion Factor(all items)	***114.97125***	*93.776015*		*101.1359763*	*102.710553*	
Conversion Factor(Only clean and paint steel)	***114.97125***	*93.776015*		*101.1359763*	*102.710553*	

Note: $/ton(English system) = 1.1023 $/ton(Metric system)

Table II.5: 3COAT Sytem over Lead Old paint

Original paint	New paint	Cost /MG	Cost/ton	Cost/ft^2
Lead	3COAT	207	187.7871	1.632931
Lead	3COAT	212	192.323	1.672374
Lead	3COAT	394	357.4305	3.108091
Lead	3COAT	143	129.7273	1.128064
Lead	3COAT	324	293.9276	2.555892
Lead	3COAT	702	636.8432	5.537767
Lead	3COAT	218	197.7661	1.719705
Lead	3COAT	556	504.3943	4.386037
Lead	3COAT	427	387.3676	3.368414
Lead	3COAT	421	381.9245	3.321082
Lead	3COAT	505	458.1279	3.983721
Lead	3COAT	562	509.8374	4.433369
Lead	3COAT	562	509.8374	4.433369
Lead	3COAT	585	530.7026	4.614806
Lead	3COAT	585	530.7026	4.614806
Lead	3COAT	563	510.7446	4.441257
Lead	3COAT	563	510.7446	4.441257
Lead	3COAT	556	504.3943	4.386037
Lead	3COAT	469	425.4693	3.699733
Lead	3COAT	480	435.4483	3.786507
Lead	3COAT	480	435.4483	3.786507
Average		***453.0476***	***410.9976***	***3.573892***
StDev		***152.0182***	***137.9085***	***1.199205***
Median		***480***	***435.4483***	***3.786507***
Confidence		***66.62361***	***60.43987***	***0.525564***
Average		***502***	***455.4064***	***3.960055***
StDev		***78.25258***	***70.98949***	***0.6173***
Median		***530.5***	***481.2611***	***4.184879***
Confidence		***39.60048***	***35.92493***	***0.312391***

Note: \$/ft^2 = 10.76 \$/m^2

Table II.6: 3COAT Sytem over ZINC Old paint

Original paint	New paint	Cost /MG	Cost/ton	Cost/ft^2
ZINC	3COAT	310	281.227	2.445453
ZINC	3COAT	338	306.6282	2.666332
ZINC	3COAT	338	306.6282	2.666332
ZINC	3COAT	342	310.2569	2.697886
ZINC	3COAT	268	243.1253	2.114133
ZINC	3COAT	465	421.8406	3.668179
ZINC	3COAT	465	421.8406	3.668179
ZINC	3COAT	419	380.1101	3.305305
ZINC	3COAT	337	305.721	2.658444
ZINC	3COAT	193	175.0865	1.522491
ZINC	3COAT	248	224.9816	1.956362
ZINC	3COAT	248	224.9816	1.956362
ZINC	3COAT	248	224.9816	1.956362
ZINC	3COAT	248	224.9816	1.956362
ZINC	3COAT	317	287.5773	2.500672
ZINC	3COAT	299	271.248	2.358678
ZINC	3COAT	317	287.5773	2.500672
ZINC	3COAT	312	283.0414	2.46123
ZINC	3COAT	287	260.3618	2.264016
ZINC	3COAT	459	416.3975	3.620847
ZINC	3COAT	479	434.5411	3.778619
ZINC	3COAT	479	434.5411	3.778619
ZINC	3COAT	422	382.8316	3.328971
ZINC	3COAT	531	481.7147	4.188824
ZINC	3COAT	531	481.7147	4.188824
ZINC	3COAT	413	374.667	3.257974
Average		***358.1923***	***324.9463***	***2.82562***
StDev		***96.87229***	***87.881***	***0.764183***
Median		***337.5***	***306.1746***	***2.662388***
Confidence		***37.97319***	***34.44867***	***0.299554***
Average		***304.75***	***276.4643***	***2.763739***
StDev		***46.6569***	***42.3264***	***0.643022***
Median		***311***	***282.1342***	***2.658444***
Confidence		***23.61118***	***21.41969***	***0.325408***

Note: \$/ft^2 = 10.76 \$/m^2

cost of that system application over an old Lead based paint. It is \$ 3.96/ft^2 (\$42.58/m^2) on average. Table II.6 shows the calculation of unit cost for that system application over an old Zinc based paint. It is \$2.80/ft^2 (\$30.11/m^2) on average.

Data was collected from different states for different paint types. Table II.7 shows the cost data that is collected and calculated for different paint systems from different departments of transportation. Zinc/Vinyl paint system cost for INDOT is calculated from Table II.4. The 3-coat system cost for INDOT is calculated as in Tables II.5 and II.6. Data was collected from other state DOTs, such as ODOT, MDOT and ILDOT, for that paint system. Metalization data was obtained from Maged Georgy's [(9)] report for INDOT. Other data was collected from ILDOT and CTDOT for the cost per square foot of metalization. Based on report no. FHWA-RD-96-058, metalization cost per square foot is taken to be included in this study.

Based on the collection of cost data for different paint systems, an economic analysis is done to compare these different systems. Table II.8 shows the present value (PV) calculation for the Zinc/Vinyl paint system. It is \$5.50 / ft^2. The EUAC calculated for the same paint type is \$0.39 / ft^2 . Table II.9 shows the calculation of PV for a 3-coat system where the old paint is Lead or Zinc. Two options are chosen for calculation based on 20 and 25 years of service life. The PV and EUAC of that paint type, based on these two different options, are indicated in Table II.9. Table II.10 shows the PV and EUAC calculation for metalization. This table shows that PV = \$18.91/ ft^2 and EUAC = \$1.347/ ft^2 for metalization. Figure II.8 shows the values of PV for different paint systems. It indicates that the minimum PV value is for 3-coat paint system, which is \$4.55/ ft^2. Figure II.9 shows that the EUAC for that paint system is the minimum, which is \$0.324/ ft^2. Therefore, a conclusion can be made that the best paint system is the 3-coat system according to the cost point of view.

Table II.7: Cost / ft^2 and Service Life for Different Paint Systems in Different States.

DOT's / Paint Sys.	Cost / ft^2 and Service Life for Different Paint Systems							
	INDOT	ODOT	MDOT	ILDOT	KDOT	CTDOT	FHWA	Service Life (Years)
Zinc-Vinyl	$2.50	x	x	x	x	x	x	15 - 25
3Coat/Lead	$3.96	$4.0-6.0	$9.29	$5	x	x	x	25 - 30
3Coat/Zinc	$2.80	$4.0-6.0	$9.29	$5	x	x	x	25 - 30
Metalization	$16.81	x	x	$6.0-9.0*	x	$12.0-15.0	$14.75	40 - 60

INDOT Numbers: calculated from data. ***Georgy and Chang*** *[9]*
ODOT Numbers: collected from Mr. ***Herald Schultz****[27] and Mr.* ***R. Bauer.*** *[28]*
MDOT Numbers: collected from Mr. ***Sonny Gduan****[29], Mr.* ***Brion Back****[30],*
Mr. ***Craig A. Russell****[31] and Mr.* ***Glenn Bukosky.*** *[32]*
ILDOT Numbers: collected from Mr. ***Gary Kowalski****. [33]*
CTDOT Numbers: collected from Mr. ***Eric Lohrey****. [34]*
FHWA Numbers: collected from report No. FHWA-RD-96-058. [14]
Service Life Numbers: are collected from:
Zinc-Vinyl from Mr. ***Ted Hopwood***[26]
3Coat-System from ILDOT and MDOT.
Metalization from ILDOT and CTDOT.

** This number is for new bridges only.*

Note: $\$/ft^2 = 10.76\ \$/m^2$

Table II.8: Zinc/Vinyl Economic Analysis

Age (yrs)	Cost initial/ft^2	Interest Rate ref no. (32)	
15	$2.50	7.00%	
Inflation Factor		0.035	
Item	n	Infl.Factor	Cost
Present Value for initial Cost:	0	1	$2.50
Present Value for Fv after 15 yrs:	15	1.675349	$1.52
Present Value for Fv after 30 yrs:	30	2.806794	$0.92
Present Value for Fv after 45 yrs:	45	4.702359	$0.56
Total Present Value TPV/ft^2			***$5.50***
EUAC			***$0.39***

Note: \$/ft^2 = 10.76 \$/m^2

Table II.9: 3-Coat Sytem (OZEU) Economic Analysis

Age (yrs)	Cost initial/ft^2	Interest Rate	Old Paint	Option
25	$4.00	7.00%	Lead	1
25	$2.80	7.00%	Zinc	1
20	$4.00	7.00%	Lead	2
20	$2.80	7.00%	Zinc	2

Option No. 1

Inflation Factor 0.035

Item	n	Infl.Factor	Lead Cost	Zinc Cost
Present Value for initial Cost	0	1	$4.00	2.8
Present Value for Fv after 25 yrs	25	2.363245	$1.74	1.219193
Present Value for Fv after 50 yrs	50	5.584927	$0.76	0.530868
Total Present Value TPV/ft^2			**$6.50**	**$4.55**
EUAC			**$0.46**	**$0.32**

Option No. 2

Inflation Factor 0.035

Item	n	Infl.Factor	CostLead	Cost Zinc
Present Value for initial Cost	0	1	$4.00	2.8
Present Value for Fv after 20 yrs	20	1.989789	$2.06	1.439758
Present Value for Fv after 40 yrs	40	3.95926	$1.06	0.740322
Total Present Value TPV/ft^2			**$7.11**	**$4.98**
EUAC			**$0.51**	**$0.35**

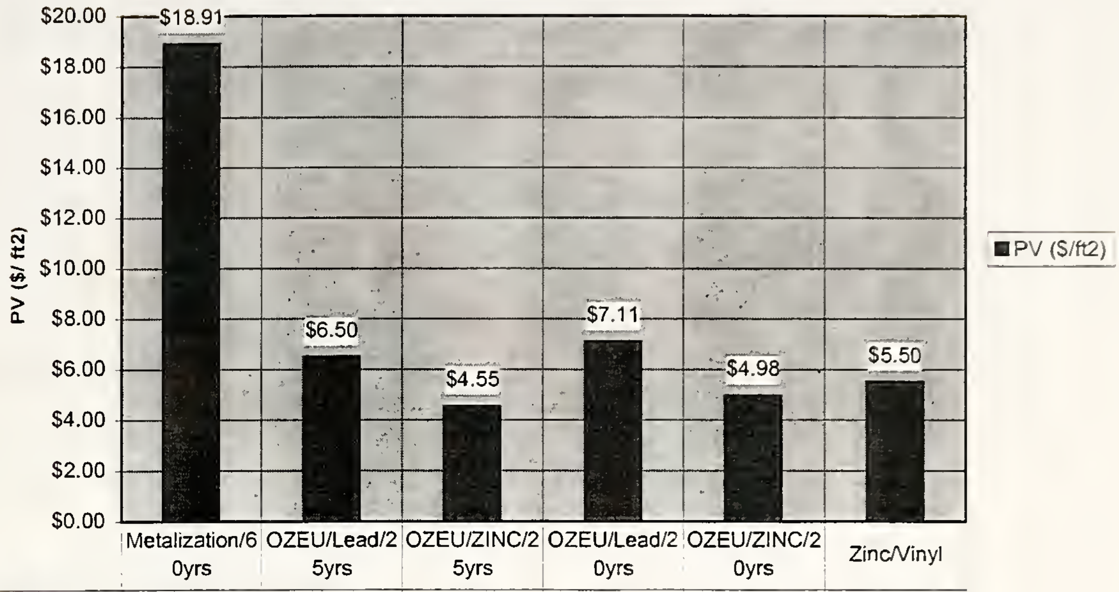
Note: \$/ft^2 = 10.76 \$/m^2

Table II.10: Metalization Economic Analysis

Met.Age (yrs)	Met.Cost initial	Coat Age (yrs)	Coat Cost initial	Coat Cost Maintenance	Coat Type	Interest Rate
60	*$14.75*	*10*	*$0.42*	*$1.82*	*Epoxy/Ur*	*7.00%*
Inflation Factor			*0.035*			
Item			**n**	**Infl.Factor**	**Cost**	
Present Value for initial Cost			*0*	*1*	*$15.17*	
Present Value for Fv after 10 yrs			*10*	*1.410598761*	*$1.31*	
Present Value for Fv after 20 yrs			*20*	*1.989788863*	*$0.94*	
Present Value for Fv after 30 yrs			*30*	*2.806793705*	*$0.67*	
Present Value for Fv after 40 yrs			*40*	*3.959259721*	*$0.48*	
Present Value for Fv after 50 yrs			*50*	*5.584926856*	*$0.35*	
Total Present Value TPV/ft^2					***$18.91***	
EUAC					***$1.35***	

Note: $/ft^2 = 10.76 $/m^2

Figure II.8: Present Value ($/ft2) of Different Paint Systems

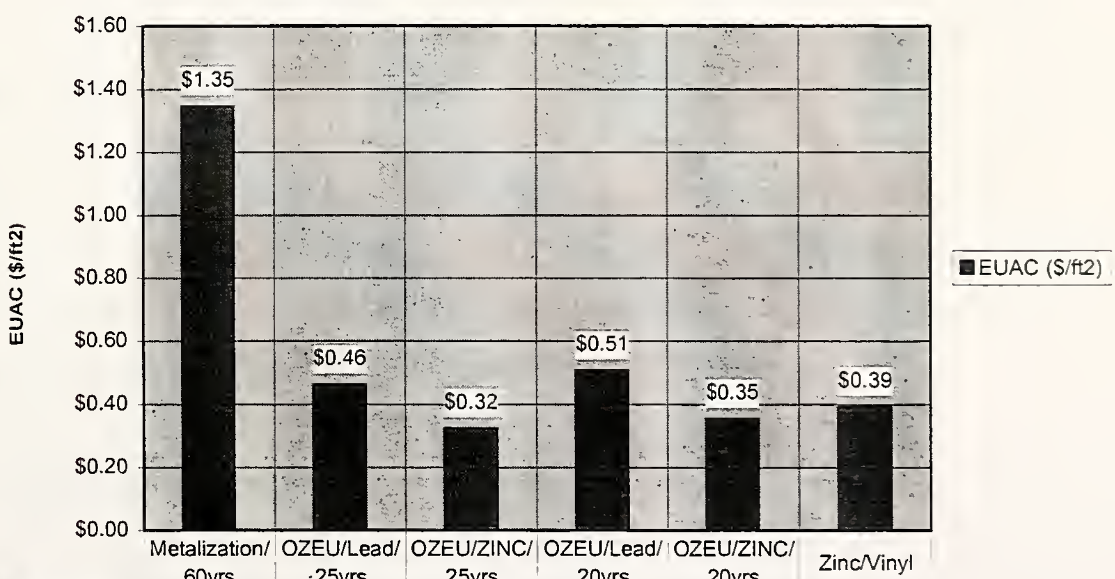


	Metalization/60yrs	OZEU/Lead/25yrs	OZEU/ZINC/25yrs	OZEU/Lead/20yrs	OZEU/ZINC/20yrs	Zinc/Vinyl
PV ($/ft2)	$18.91	$6.50	$4.55	$7.11	$4.98	$5.50

Different Paint Systems

Note: $\$/ft^2 = 10.76\ \$/m^2$

Figure II.9: EUAC ($/ft2) of Different Paint Systems



	Metalization/60yrs	OZEU/Lead/25yrs	OZEU/ZINC/25yrs	OZEU/Lead/20yrs	OZEU/ZINC/20yrs	Zinc/Vinyl
EUAC ($/ft2)	$1.35	$0.46	$0.32	$0.51	$0.35	$0.39

Different Paint Types

Note: $\$/ft^2 = 10.76\ \$/m^2$

CHAPTER III

STEEL BRIDGE NEW PAINT SYSTEM COST ANALYSIS

(3-Coat Paint System: In/OZEU)

III.1 3-COAT SYSTEM MICHIGAN DATA ANALYSIS

Based on Michigan's experience with 3-coat paint system, data was collected from MDOT to analyze that system. A regression function was fit to this data and the service life of that paint system was calculated. This analysis was done to support the economic analysis and service life estimate for the 3-coat system. SAS was used to fit the model to the data. Table III.1 shows the different models that can fit the data.

III.1.1 Regression Analysis

For this paint type (3-coat system), the performance function is fitted to linear, quadratic and cubic formulas. The r-squared value for each formula is calculated and indicated in Table III.1, but the difference among them is very small. The lack-of-fit test was done for each case and it is not significant for any formula. This means that all the models are adequate to fit the data. The assumptions for the regression models are checked for the error constant variance, normality and independence. The results indicate that all the models are good according to these assumptions, as indicated in Appendix A. In addition to this information, the significance of all models' parameters is checked to select the best model that can fit the data. Based on these tests and arguments, the best model is selected for 3-coat system paint is a linear model: [16]

$$\textit{Paint Rating} = 8.08 - 0.116 * \textit{Age}.$$

III.1.2 Markov Chains Analysis

Markov chains process can be applied for the data available to draw the probabilistic deterioration curve for a 3-coat system. The same steps for applying Markov chains process that was used in the existing system analysis was used for the 3-coat system. The steps are:

1- The paint age is divided into 4-year ranges with four unknown probabilities in each range.
2- The Maple mathematics program is used to prepare the file of matrix multiplication as illustrated in the previous section. The output of this file are the equations that contain p(1), p(2), p(3) and p(4) as unknowns. Therefore, there are four equations with four unknowns that can be solved to get the values of p(1), p(2), p(3) and p(4).
3- The output of the previous step is substituted into the non-linear programming equation as E(t,P), which is subtracted from the part Y(t). This part, Y(t), is calculated using the regression formulas for the various models' types.
4- After constructing these equations, they are put into a GAMS program input file to solve the NLP problem based on the constraints that are illustrated in the previous section. This input file allows GAMS to minimize the absolute value of Y(t)-E(t,P) for each four years of paint age. The outputs of this step are the values p(1), p(2), p(3) and p(4) that minimize the NLP objective function.
5- By knowing the values of these probabilities in each 4-year range, the last year state vector for each range can be calculated by using the Maple mathematics program. This state vector is calculated to be used as the initial state vector for the next range of four years.
6- Steps 1 to 5 are repeated for each 4- year range of the 3-coat system paint age. [12] [13] [23] [24] [25]

After applying this procedure, transition probability matrices are constructed for each 4-year range of the 3-coat system paint age. The values of p(1), p(2), p(3) and p(4) for each transition probability matrix are shown in Appendix B.

III.1.3 Regression vs. Markov Process Results for 3-Coat System

The regression model provides the values of paint rating at various ages as shown in Table III.2. This table indicates the paint rating values for different ages using

Table III.2: Michigan Paint Rating Prediction
Using Regression and Markov.

Age	Paint Rating Mich. 3COAT	
	Reg	Markov
0	8.0819	8.0819
1	7.9657	8.0001
2	7.8496	7.856
3	7.7334	7.73
4	7.6173	7.618
5	7.5011	7.516
6	7.3849	7.425
7	7.2688	7.341
8	7.1526	7.264
9	7.0365	7.034
10	6.9203	6.918
11	6.8041	6.803
12	6.688	6.692
13	6.5718	6.57
14	6.4557	6.453
15	6.3395	6.34
16	6.2233	6.234
17	6.1072	6.105
18	5.991	5.985
19	5.8749	5.8751
20	5.7587	5.775
21	5.6425	5.632
22	5.5264	5.511
23	5.4102	5.41
24	5.2941	5.33
25	5.1779	5.15
26	5.0617	5.062
27	4.9456	5.0085
28	4.8294	
29	4.7133	
30	4.5971	

Table III.1: 3-COAT-System
from Michigan

Age	Paint Rating		
	1st order	2nd order	3rd order
5	7.5	7.68675	7.67153
10	6.92	6.77325	6.75053
15	6.34	5.29475	3.88953
20	5.76	3.25125	-2.30647
25	5.18	0.64275	-13.2325
30	4.6	-2.53075	-30.2835
26.5517	5		
R-square	0.438	0.461	0.465
Parameters t-tes	O.K.	N.O.K.	N.O.K.
Model F-test	O.K.	O.K.	O.K.
Normality Assum	O.K.	O.K.	O.K.
Lack of Fit test	O.K.	O.K.	O.K.
Selection	O.K.	No	No

regression and the Markov chains process. Consequently, the deterioration curve for that type of paint can be drawn using the values of condition rating calculated by using the Markov chains and regression. Figure III.1 shows these two curves. This figure indicates that the deviation between the two curves is very small and they look the same. As known, regression is based on Least Squared Method, which gives the best fit for the data. Therefore, the probabilistic model, Markov chains model, is good for fitting this data. Consequently, the deterministic and probabilistic fitted curves for this data are constructed and ready to be used in any applications. Table III.2 shows that at a paint condition rating of 5, the age of the 3-coat system will be between 26 and 27 years. This number supports the 25 years paint age estimate.

III.1.4 Markov Decision Process (MDP) Application for 3-Coat System

The steps in applying the Markov Decision Process (MDP) to the 3-coat system are the same as that of the existing system. These steps are:

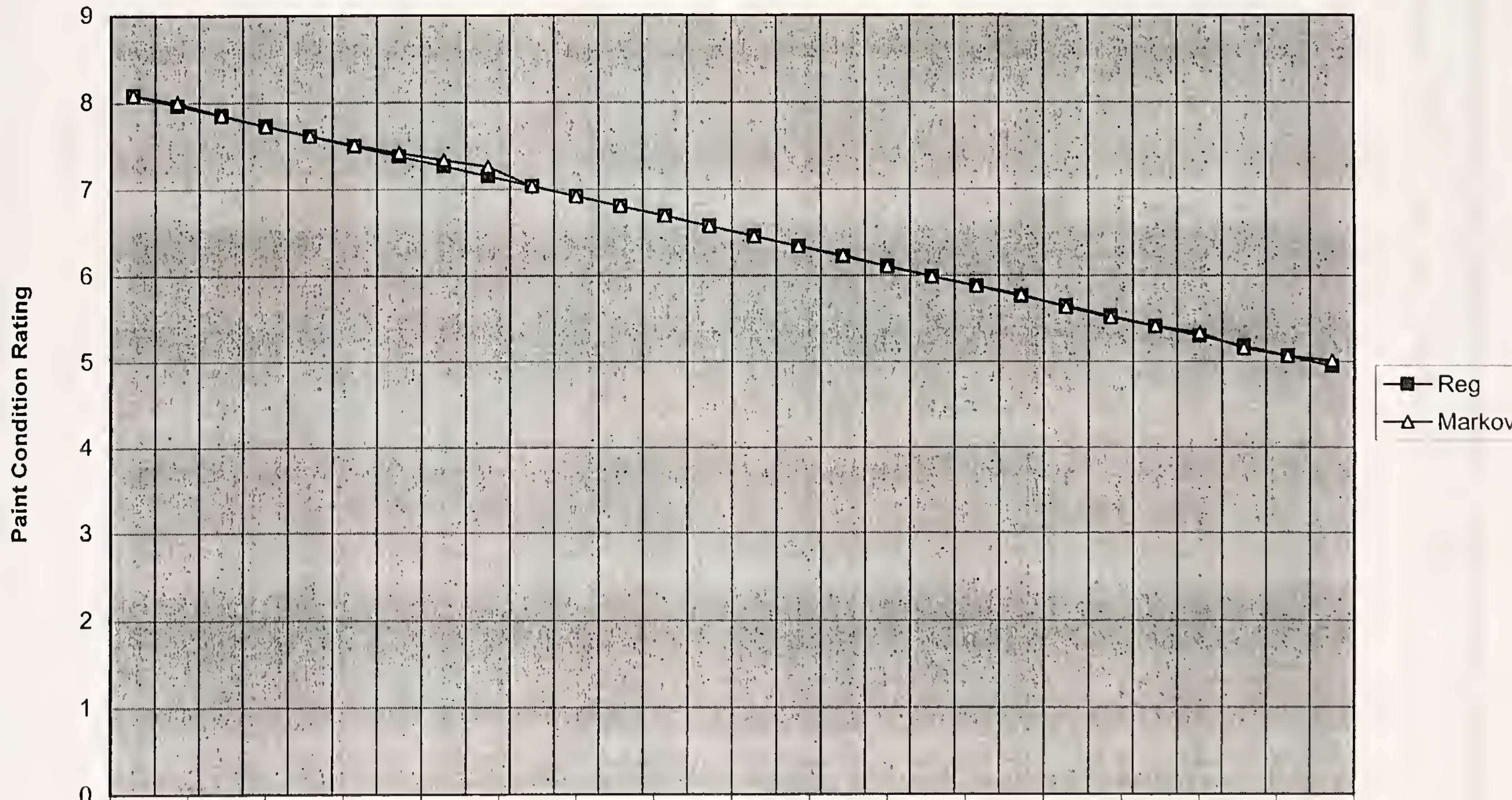
1- There are five possible states (i = 1,2,3,4,5) observed after each transition associated with steel bridge paint condition rating, from 1 to 5, with 5 being the worst. The index i is used for the initial state, and the index j is used for the future state. Therefore, state 1 corresponds to condition rating 1 and state 2 corresponds to condition rating 2 and so on.

2- After each observation, a decision (action) k is chosen from a set of K possible decisions (k= 1,2,3,4). Some of the K decisions may not be relevant for some of the states. Table III.3 indicates the decisions and their relevant states.

Table III.3 : Decisions and relevant states.

Decision k	Action Description	Relevant to States
1	*Do nothing*	*1,2,3,4*
2	*Spot Repair*	*3,4*
3	*Complete Repainting*	*5*

Figure III.1: Michigan Data Regression Results Vs. Markov Results



	1	3	5	6	8	10	12	13	15	17	19	20	22	24	26	27
Reg	8.082	7.85	7.617	7.501	7.269	7.036	6.804	6.688	6.456	6.223	5.991	5.875	5.643	5.41	5.178	5.062
Markov	8.082	7.856	7.618	7.516	7.341	7.034	6.803	6.692	6.453	6.234	5.985	5.875	5.632	5.41	5.15	5.062

3-COAT (OZEU) Paint Age (Years)

3- If decision $d_i = k$ is made in state i, an immediate cost is incurred that has an expected value C_{ik}. The costs in Table III.4 are the estimated unit cost ($/ton) for steel bridges.

Table III.4 : Estimated Unit Cost of 3-Coat Paint Rehabilitation.

Decision	State	Description	Rehab. Cost Cr ($/ft^2)		Disruption Cost (C_d)		Total Cost* C_{ik}($/ft^2)	
			Lead	Zinc	Lead	Zinc	Lead	Zinc
1	*1,2,3,4*	***Do nothing***	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
2	*3*	***Spot Repair***	*1.5*	*1.0*	*17.69*	*17.69*	*19.2*	*18.69*
	4	~ ~ ~	*2.5*	*2.0*	*26.53*	*26.53*	*29.03*	*28.53*
3	*5*	***Complete Repainting***	*4.0*	*2.8*	*35.37*	*35.37*	*39.37*	*38.17*

Note: $/ft^2 = 10.76 $/m^2

* Total Cost includes the rehabilitation cost and the disruption cost to travelers due to the rehabilitation work. Disruption Cost Calculations are included in Appendix D.

4- Decision $d_i = k$ in state i determines what the transition probabilities will be for the next transition from state i. These transition probabilities can be denoted by $P_{ij}(k)$ for j = 1,2,3,4,5. The parameter k in $P_{ij}(k)$ is used to indicate that the appropriate transition probability depends upon the decision k.

5- There are several policies that can be used to rehabilitate the steel bridge paint based on its status. A policy is a set of decisions (actions) for each state. Table III.3 indicates the proposed policies or scenarios and their decisions at each state. [22] [10]

Table III.14 lists the relevant policies for steel bridge paint problems. For example, Policy Y1: (d_1, d_2, d_3, d_4, d_5) = (1,1,1,1,4) describes a policy where decision 1 (do nothing) is made in states 1,2,3,4 and decision 4 (complete repainting) is made in state 5.

6. The objective is to find an optimal policy that minimizes the expected total discounted

cost (immediate cost and discounted subsequent costs that result from the future processes).[(10),(22)]

Table III.5 : Different Policies for Painting Rehabilitation.

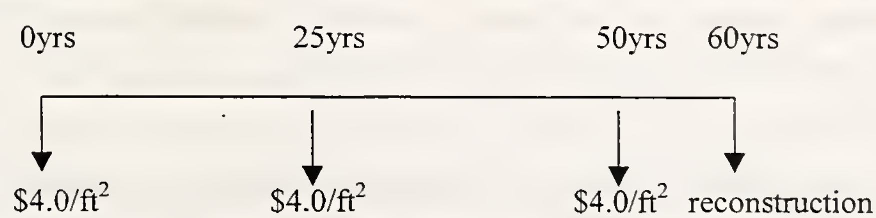
Decision	Policy (Y1)	Policy (Y2)	Policy (Y3)
d_1	*1*	*1*	*1*
d_2	*1*	*1*	*1*
d_3	*1*	2	*1*
d_4	*1*	*1*	2
d_5	*3*	*3*	*3*

After applying these steps to the 3-coat system, the calculations indicate that scenario or policy Y1 is the most economic one. This policy says do nothing until the bridge reaches state 5 and then do a complete repaint for the bridge. All the calculations are included in Appendix C.

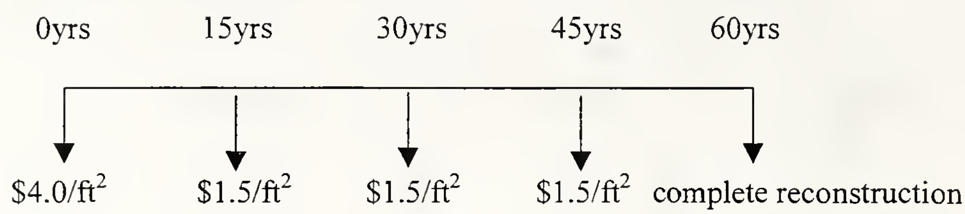
III.1.5 Economic Analysis for 3-Coat System

The economic formulas are used to calculate the present value (PV) and the equivalent uniform annual cost (EUAC) for the 3-coat system paint according to three proposed rehabilitation and maintenance alternatives. The proposed alternatives of rehabilitation and maintenance are as follows: (***Note***: $\$/ft^2 = 10.76\ \$/m^2$)

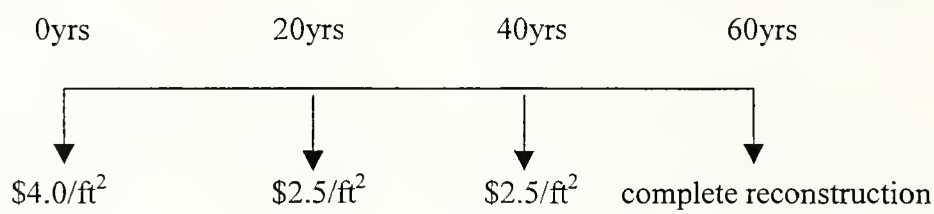
1- Do nothing and do complete repainting after reaching state 5 (approximately 25 years). Bridge life span is 60 years.



2- Spot repairs are made at state 3 (rating 7), occurs every 15 years until the end of the bridge life of 60 years.



3- Spot repairs are made at state 4 (rating 6), which occurs every 20 years until the end of the bridge life.



These options or alternatives are applied for the 3-coat system. The mentioned cost numbers in the three policies are based on the 3-coat system applied over Lead as a base for old paint and costs for that paint over Zinc old paint are indicated in Table III.4. The estimated number of years corresponding to bridge states are indicated in Table III.1. After calculating the PV and EUAC for each alternative considering one paint category, the minimum cost alternative will be the optimum one. By analyzing the five categories, it is concluded that alternative number two (spot repairs at state 3 and repetition of spot repairs over the life span of the bridge) is the optimum one. This happens for all the categories. The calculation for these alternatives PV and EUAC, for 3-coat over Lead as an old paint, are indicated in Figures III.2 and III.3. The calculations of PV and EUAC for the 3-coat over Zinc old paint are indicated in Figures III.4 and III.5.

The bar chart in Figure III.2 indicates that the optimum alternative for the 3-coat system is alternative number 2, where spot repairs every 15 years should be done. This optimum cost is PV = \$ 5.8/ft^2 (\$62.41/m^2). This alternative is approximately two third the cost of the do nothing alternative until complete repainting again (alternative number 1). Figure III.3 shows that the best policy is alternative no. 2 with EUAC = $0.413/ft^2$ ($4.44/m^2$).

The bar chart in Figure III.4 indicates that the optimum alternative for the 3-coat system is alternative number 2, where spot repairs every 15 years should be. This optimum cost is PV = $ 4.0/ft^2$($43.04/m^2$). This alternative is approximately two third the cost of the do nothing alternative until complete repainting again (alternative number 1). Figure III.5 shows that the best policy is alternative no. 2 with EUAC = $0.285/ft^2$ ($3.07/m^2$).

By adding the disruption cost to the paint cost, the best scenario does not change as indicated in Figures from III.7 to III.10 for 3-coat over old paint Lead based or Zinc based. The best scenario is making spot repairs every 15 years and/or the bridge reaches condition rating of 7. All disruption cost calculations are included in Appendix D.

III.1.6 SENSITIVITY ANALYSIS

Sensitivity analysis was carried out for the 3-coat system to check the best scenario to paint age and cost changes. Table III.6 shows the final results of trial and error method to get the minimum paint age where spot repairs remain the best scenario. The values of PV and EUAC for spot repairs at state 3 scenario look close to that of spot repairs at state 4 scenario. Consequently, conclusion can be made that spot repairs at state 3 scenario will remain the best scenario if the bridge reaches state 3 or condition rating 7 after 12 years old. Table III.6 shows this conclusion for the 3-coat system applied over Zinc as an old paint. Table III.7 shows this conclusion for the 3-coat system applied over Lead as an old paint. The initial cost and spot repairs cost were changed by trial and error to check the maximum cost value that maintain spot repairs at state 3 the best scenario. The results of this sensitivity analysis are shown in Table III.8 for 3-coat/Zinc and in Table III.9 for 3-coat/Lead. Table III.8 shows that the 3-coat spot repairs at state 3 will be the best scenario if the initial cost is less than $8.85/ft^2$ and the cost every 15 years is less than $3.16/ft^2 ($34.00/m^2) for 3-coat/Zinc. Table III.9 shows that the 3-coat spot repairs at state 3 will be the best scenario if the initial cost is less than $9.8/ft^2 ($105.45/m^2) and the cost every 15 years is less than $3.68/ft^2 ($39.60/m^2) for 3-coat/Lead.

III.1.7 Conclusion

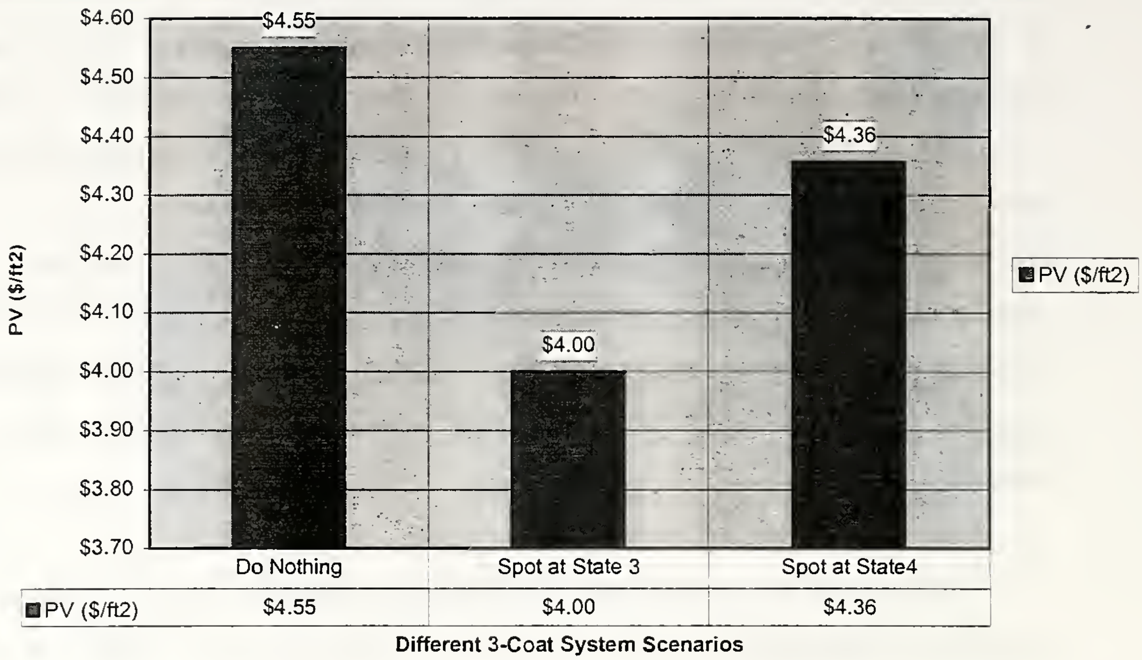
There is a difference between the results of the economic analysis and the results of Markov decision process (MDP). The economic analysis method uses deterministic values of cost and time where MDP uses probabilistic times. Therefore, research was done to investigate this conflict between the results of these two methods. The literature of probability theory applications and its relation with the deterministic theory applications shows that it is not necessarily getting the same results from both methods for the same application. For example, in Industrial Engineering, the TSP (**T**raveling **S**alesperson **P**roblem) can be solved using deterministic and probabilistic PTSP (**P**robabilistic **T**raveling **S**alesperson **P**roblem) values. The result of this application is shown in the following paragraph.

Bertsimas (1990) assigned the problem of how heuristic approaches to the deterministic problem TSP perform when applied to the corresponding probabilistic problem PTSP. This literature suggested that if the coverage probability p was large, the constant guarantee heuristics for the deterministic problem still behave well for the corresponding probabilistic problem. But if p tends to zero, the bound was not informative and indeed one can try with p tends to zero or infinity, the optimal deterministic solution was an arbitrarily bad approximation to the optimal a priori solution.

Jaillet (1988) presented an example showing that an optimal TSP tour was not necessarily a good solution to the corresponding PTSP problem. In this example, Figure III.6 contains 24 white nodes (and no black nodes) that are positioned at the vertices of two concentric 12-gons as shown in Figure III.6a. We assume $P = P_1$ (Bernouilli process with parameter p). In Figure III.6b, two tours have been designed through this set of nodes: tour *a* is an optimal TSP tour and tour *b* is an alternative tour (see Jaillet 1985 for the numerical derivations). One could then show that, for a probability of presence of 0.5, the expected length of tour *a* is 31% greater than the expected length of tour *b*. This numerical example raised the following question: in general, how well would a TSP tour do as a solution to a PTSP problem? Actually, PTSP introduced many features that were different from those of its famous special case, the TSP. The TSP is a special case of the PTSP in which all nodes are black. It is then natural to investigate the possible links between the two problems. At the end the author came up with a comment that in fact, one could show (using a generalization of the star-shaped example in Figure III.6) that the TSP could indeed be arbitrarily bad under the condition of Figure III.6 and his Theorem 2.

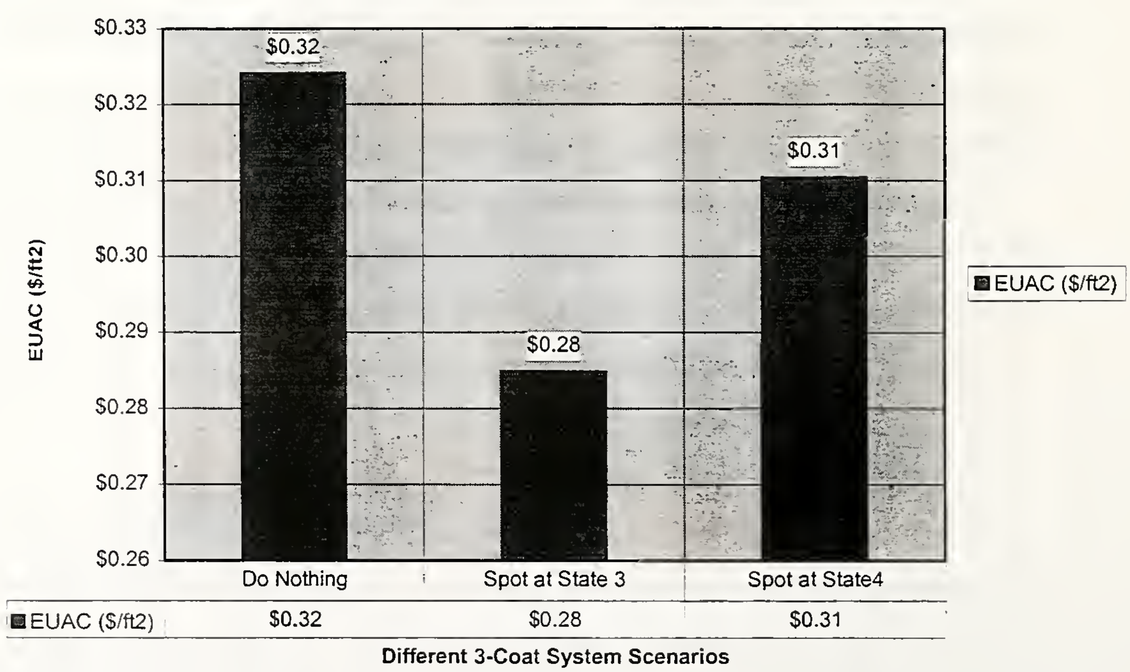
Based on the previous discussion, the deterministic method results are not necessarily the same as the probabilistic method results for the same problem. Consequently, there is a conflict between the economic analysis as a deterministic method and Markov decision process (MDP) as a probabilistic method. But the question that rises now is: which method should we go with? In fact, the economic analysis (the deterministic method) is simple, straightforward and easy to be applied by INDOT personnel. Therefore, the economic analysis results are considered in our study. Consequently, the conclusion here is that making spot repairs every 15 years and/or at condition rating of 7 is the best scenario that INDOT can apply for the 3-coat system in the future.

Figure III.4: PV ($/ft2) Comparison of 3 COAT System/ZINC



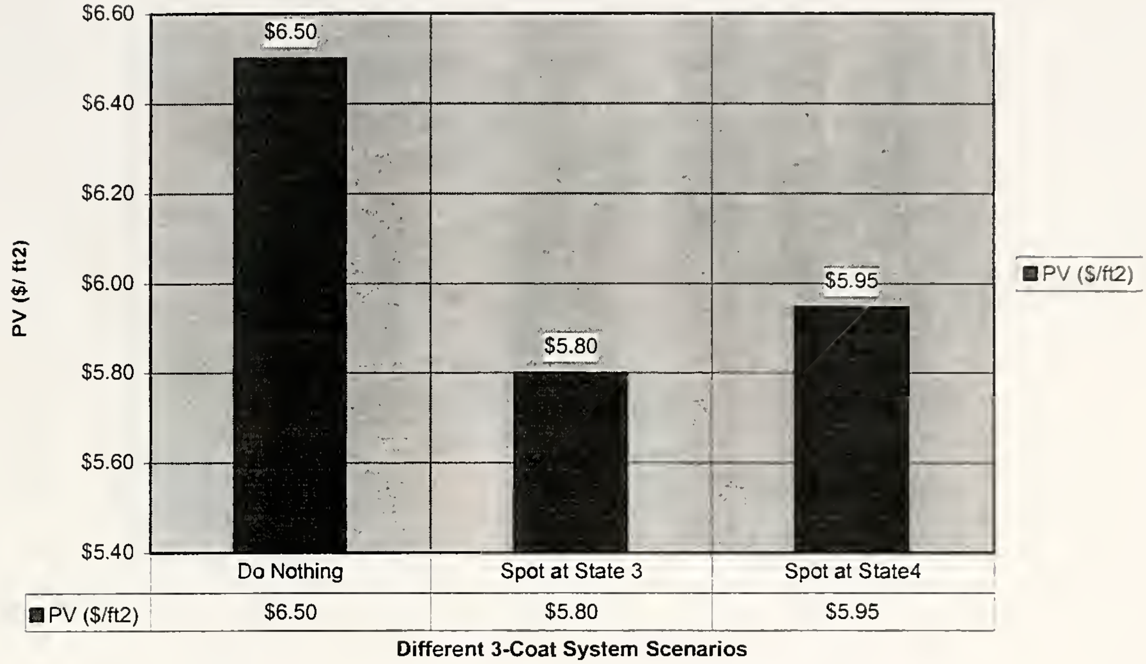
Note: $/ft^2 = 10.76 $/m^2

Figure III.5: EUAC ($/ft2) for 3-COAT System/ ZINC



Note: $/ft^2 = 10.76 $/m^2

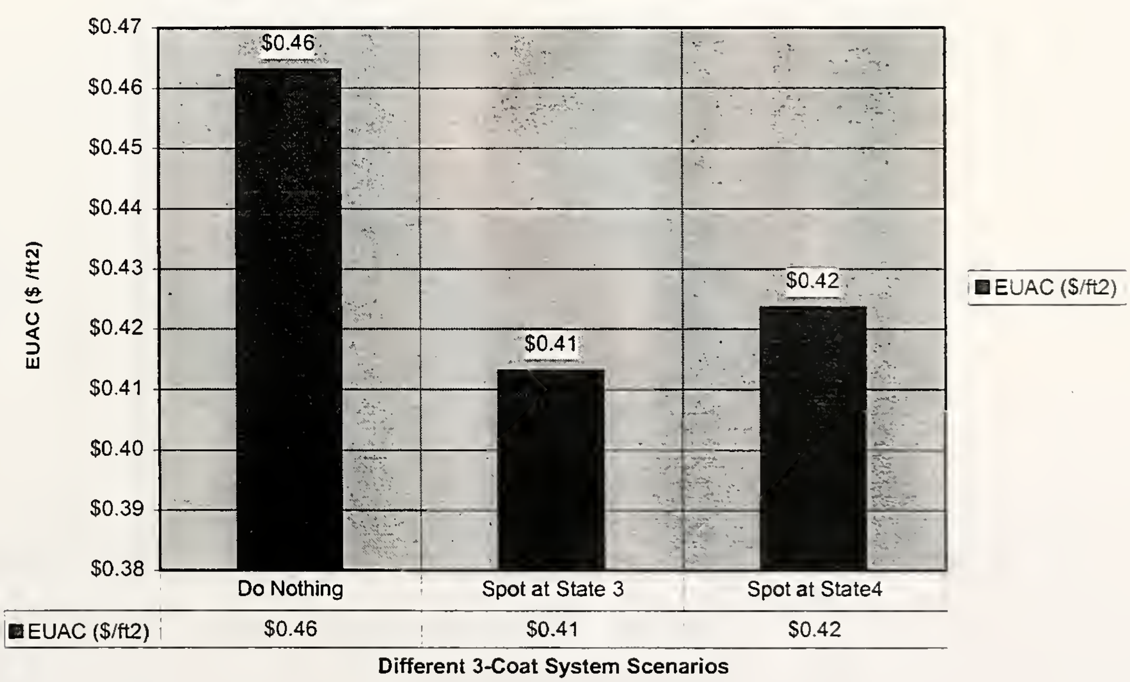
Figure III.2: PV Comparison of 3-COAT System /Lead for Different Scenarios



	Do Nothing	Spot at State 3	Spot at State4
PV ($/ft2)	$6.50	$5.80	$5.95

Note: $\$/ft^2 = 10.76\ \$/m^2$

Figure III.3: EUAC ($ /ft2) for 3-COAT System / Lead for Different Scenarios



	Do Nothing	Spot at State 3	Spot at State4
EUAC ($/ft2)	$0.46	$0.41	$0.42

Note: $\$/ft^2 = 10.76\ \$/m^2$

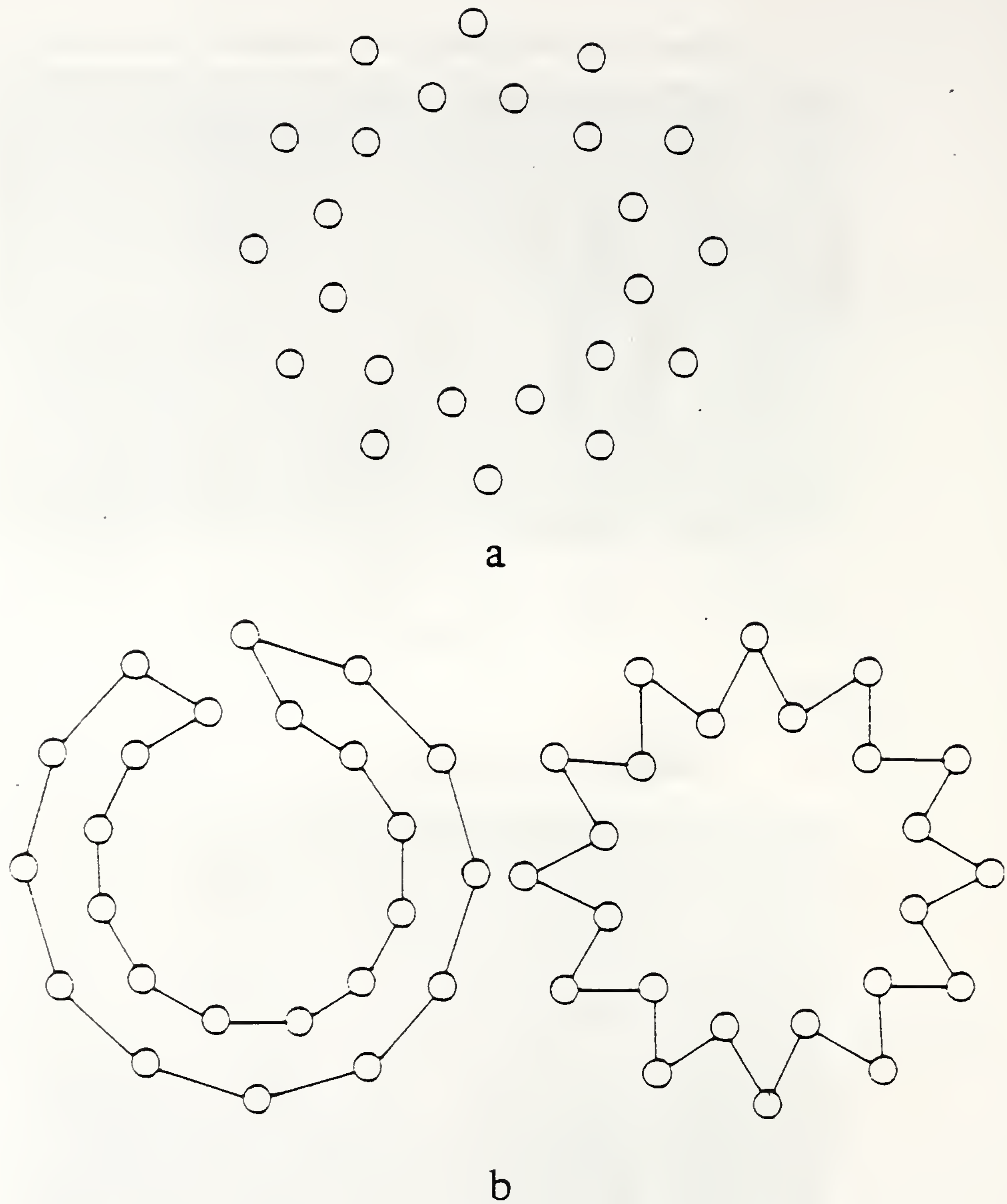
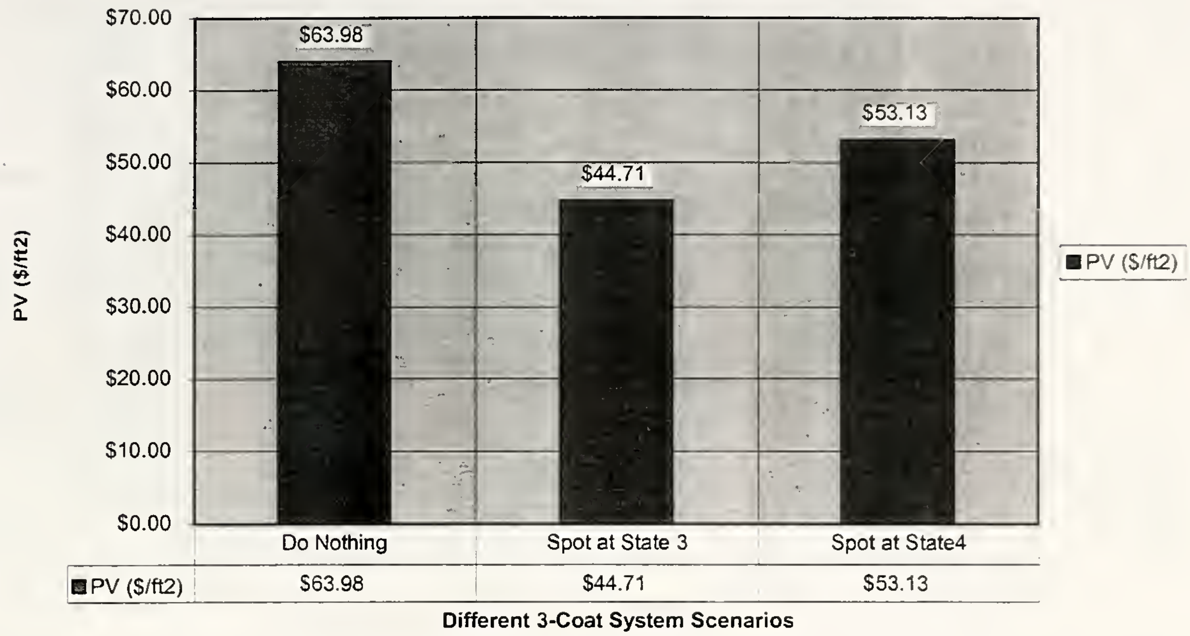


Figure III.6: Graph and tours for the numerical example. (a) A 24-node graph. (b) Two tours of the 24-node graph.

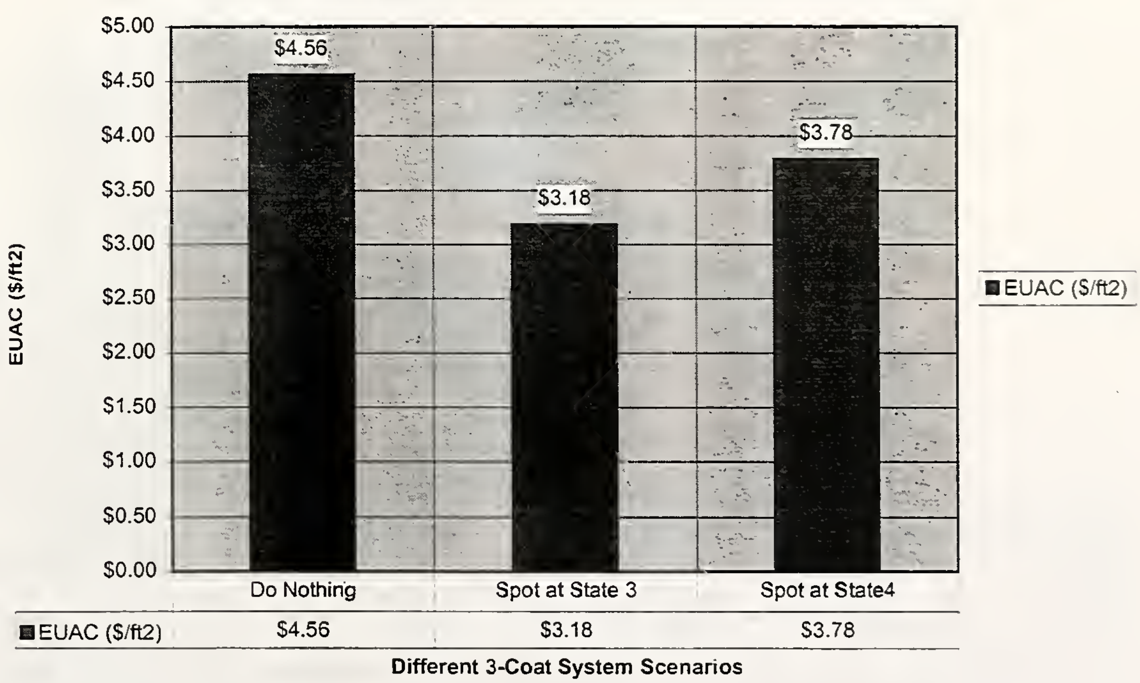
Figure III.7: PV ($/ft2) Comparison of 3-COAT System /Lead Including Disruption Cost



	Do Nothing	Spot at State 3	Spot at State4
PV ($/ft2)	$63.98	$44.71	$53.13

Note: $/ft² = 10.76 $/m²

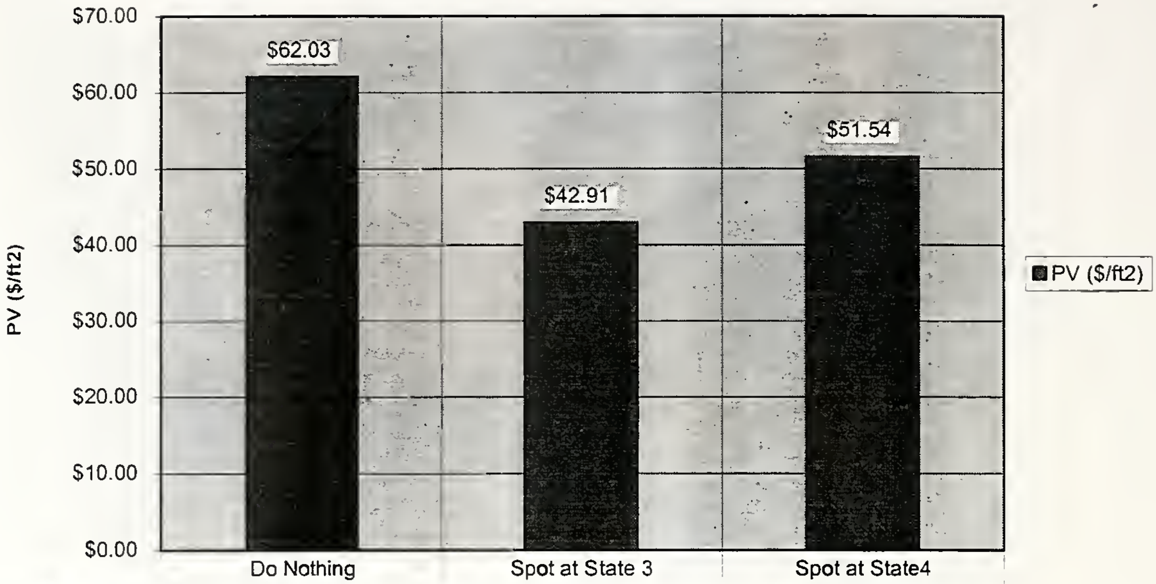
Figure III.8: EUAC ($/ft2) for 3-COAT System / Lead Including Disruption Cost



	Do Nothing	Spot at State 3	Spot at State4
EUAC ($/ft2)	$4.56	$3.18	$3.78

Note: $/ft² = 10.76 $/m²

Figure III.9: PV ($/ft2) of 3-COAT System/ZINC Including Disruption Cost

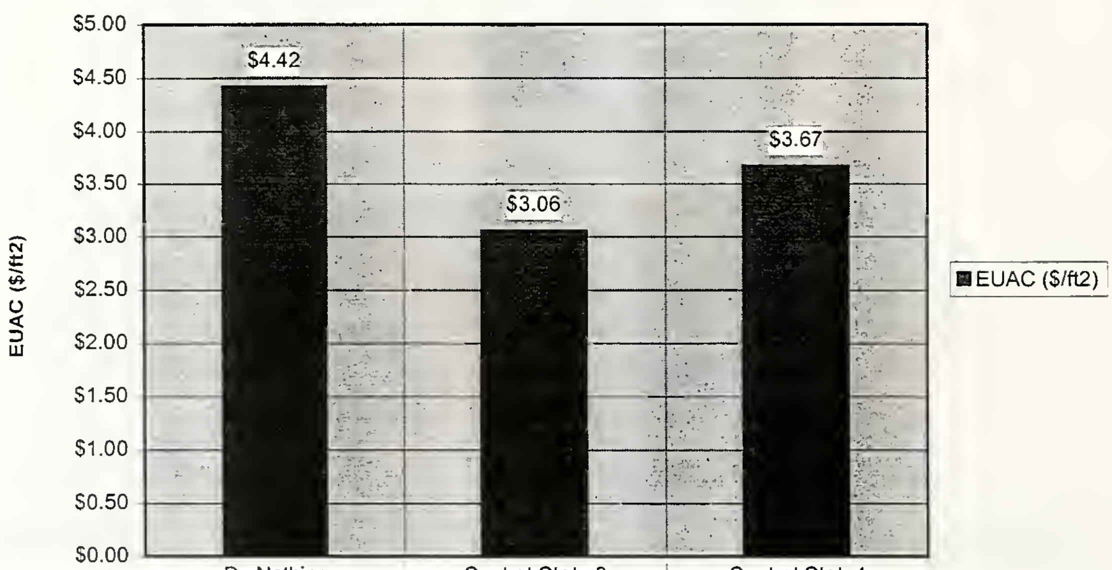


	Do Nothing	Spot at State 3	Spot at State4
PV ($/ft2)	$62.03	$42.91	$51.54

Different 3-Coat System Scenarios

Note: $\$/ft^2 = 10.76\ \$/m^2$

Figure III.10: EUAC ($/ft2) 3-COAT System/ ZINC Including Disruption Cost



	Do Nothing	Spot at State 3	Spot at State4
EUAC ($/ft2)	$4.42	$3.06	$3.67

Different 3-Coat System Scenarios

Note: $\$/ft^2 = 10.76\ \$/m^2$

Table III.6: Paint Age Sensitivity Analysis
3-Coat/Zinc
Spot Repairs at State 3

Interest Rate 7.00%					
Inflation Factor	0.035				
Item	Age (n)	Infl.Factor	Cost	Total	
Present Value for initial Cost	0	1	$2.80	$20.49	
Present Value for Fv after 15 yrs	12	1.511069	$1.00	$12.54	
Present Value for Fv after 30 yrs	24	2.283328	$1.00	$8.41	
Present Value for Fv after 30 yrs	36	3.450266	$1.00	$5.64	
Present Value for Fv after 45 yrs	48	5.213589	$1.00	$3.79	
					State 4 Spot
Total Present Value TPV/ft^2				**$50.86**	**$51.54**
EUAC				**$3.62**	**$3.67**

Note: $/ft^2 = 10.76 $/m^2

The values of PV and EUAC of state 3 including the disruption cost are close to that of state 4, therefore, this scenario is the best one if the bridge reaches state 3 or condition rating 7 after 12 years

Table III.8: Initial cost Sensitivity Analysis
3-Coat/Zinc
Spot Repairs at State 3

Interest Rate 7.00%					
Inflation Factor	0.035				
Item	Age (n)	Infl.Factor	Initi.Cost	Total(disrup.)	
Present Value for initial Cost	0	1	$8.85	$26.53	
Present Value for Fv after 15 yrs	15	1.675349	$3.16	$12.66	
Present Value for Fv after 30 yrs	30	2.806794	$3.16	$7.69	
Present Value for Fv after 45 yrs	45	4.702359	$3.16	$4.67	
					State 4 Spot
Total Present Value TPV/ft^2				**$51.54**	**$51.54**
EUAC				**$3.67**	**$3.67**

Note: $/ft^2 = 10.76 $/m^2

The values of PV and EUAC of state 3 including the disruption cost are the same as that of state 4, therefore, this scenario is the best one if the bridge is painted by 3-Coat system where the initial cost is less than $8.85/ft2 and the rehabilitation cost every 15 years is $3.16/ft2.

Table III.7: Paint Age Sensitivity Analysis
3-Coat/Lead
Spot Repairs at State 3

Interest Rate					
7.00%					
Inflation Factor	0.035				
Item	Age (n)	Infl.Factor	Init.Cost	Total(disrup.)	
Present Value for initial Cost	0	1	$4.00	$21.69	
Present Value for Fv after 12 yrs	12	1.511069	$1.50	$12.87	
Present Value for Fv after 24 yrs	24	2.283328	$1.50	$8.64	
Present Value for Fv after 36 yrs	36	3.450266	$1.50	$5.79	
Present Value for Fv after 48 yrs	48	5.213589	$1.50	$3.89	
					State 4 Spot
Total Present Value TPV/ft^2				**$52.88**	**$53.13**
EUAC				**$3.77**	**$3.78**

Note: \$/ft^2 = 10.76 \$/m^2

The values of PV and EUAC of state 3 including the disruption cost are close to that of state 4, therefore, this scenario is the best one if the bridge reaches state 3 or condition rating 7 after 12 years old.

Table III.9: Initial cost Sensitivity Analysis
3-Coat/Lead
Spot Repairs at State 3

Interest Rate					
7.00%					
Inflation Factor	0.035				
Item	Age (n)	Infl.Factor	Init.Cost	Total(disrup.)	
Present Value for initial Cost	0	1	$9.80	$27.49	
Present Value for Fv after 15 yrs	15	1.675349	$3.68	$12.97	
Present Value for Fv after 30 yrs	30	2.806794	$3.68	$7.88	
Present Value for Fv after 45 yrs	45	4.702359	$3.68	$4.78	
					State 4 Spot
Total Present Value TPV/ft^2				**$53.12**	**$53.13**
EUAC				**$3.78**	**$3.78**

Note: \$/ft^2 = 10.76 \$/m^2

The values of PV and EUAC of state 3 including the disruption cost are close to that of state 4, therefore, this scenario is the best one if the bridge is painted by 3-Coat system where the initial cost is less than \$9.8/ft^2 and the rehabilitation cost every 15 years is \$3.68/ft2.

III.2 INDOT MAINTENANCE PLAN

A maintenance plan is a very important objective. A life cycle cost analysis was done for different paint systems to determine the best scenario of rehabilitation for the maintenance plan. A maintenance plan is a set of rehabilitation procedure steps that satisfy the minimum rehabilitation cost.

III.2.1 Proposed Maintenance Plan Procedure

As well as the implementation described in the last section and based on the life cycle cost analysis for determination of the best rehabilitation scenario, a proposed maintenance plan and its procedural steps is summarized in Figure III.11.

The steel bridge paint should be inspected, as usual, every two years by INDOT inspectors. During the inspection, the paint condition will be rated. If the rate of paint condition reaches 7 or below, spot painting must be done on the bridge, regardless of whether or not the paint is new. In case of new paint, the question to the inspector is, Is the paint life over 15 years? If the answer is Yes, the spot painting will be automatically performed, even if the rating of the paint condition is away above 7. If the bridge has been painted within 15 years, the bridge still goes through routine biannually inspection. In other words, nothing is done to the paint job until the rating reaches 7 or the paint life is over 15 years.

When an old paint condition rating is above 7, the inspector should ask, "Was the bridge painted in the last 15 years?" If the bridge has not been painted in the last 15 years, the spot painting should be automatically done. Based on the recommendation of life cycle cost analysis, spot painting every 15 years is the most economic policy, despite the condition rating not reaching 7.

In case a bridge was painted in the last 15 years, the inspector should examine the factor of bridge life. When the bridge life is reaching 60 years, there is no point in repainting the bridge, since the bridge will be reconstructed or demolished.

After being spot-painted, any bridge can be categorized as an "old bridge". Thereafter, the maintenance procedures should be repeated until the bridge is reconstructed.

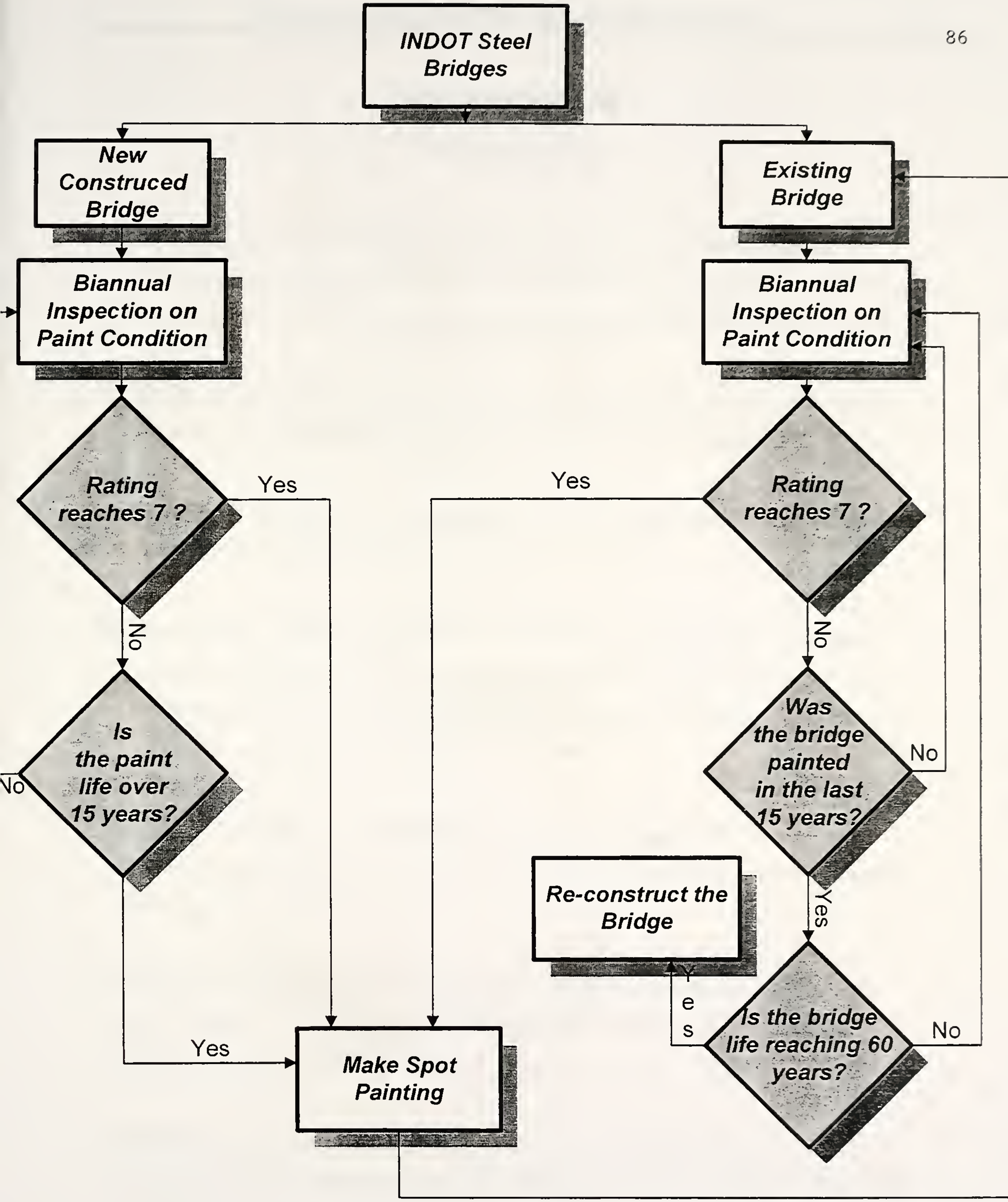


Figure III.11: *Maintenance Plan Procedure Flowchart*

CHAPTER IV

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(26) **Mr. Ted Hopwood,** Kentucky DOT, (606)257-4513.

(27) **Mr. Herald Schultz**, Ohio DOT, (614)644-6628.

(28) **Mr. R. Bauer**, Ohio DOT, (614)644-6784.

(29) **Mr. Sonny Gduan**, Michigan DOT, (517)322-3316.

(30) **Mr. Brion Back**, Michigan DOT, (517)322-5722.

(31) **Mr. Craig A. Russell**, Michigan DOT, RUSSELLC@state.mi.us.

(32) **Mr. Glenn Bukosky**, Michigan DOT, (517)241-3062 and e.mail: bukoskig@ state.mi.us.

(33) **Mr. Gary Kowalski**, Illinois DOT, (217)785-2914.

(34) **Mr. Eric Lohrey**, Connecticut DOT, (860)258-0303.

(35) **Mr. Mike Long**, INDOT, (317)232-5506.

Appendix A:

Regression Output

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	2.5150380	2.5150380	7.01	0.0100
Error	71	25.4849620	0.3589431		
Corrected Total	72	28.0000000			

R-Square	C.V.	Root MSE	RATE Mean
0.089823	9.985310	0.5991	6.0000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	2.5150380	2.5150380	7.01	0.0100

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	2.5150380	2.5150380	7.01	0.0100

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	7.177012484	15.94	0.0001	0.45014850
AGE	-0.051327307	-2.65	0.0100	0.01939050

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	3.5663205	1.1887735	3.36	0.0237
Error	69	24.4336795	0.3541113		
Corrected Total	72	28.0000000			

R-Square	C.V.	Root MSE	RATE Mean
0.127369	9.917875	0.5951	6.0000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	2.5150380	2.5150380	7.10	0.0096
AGE2	1	0.0092365	0.0092365	0.03	0.8722
AGE3	1	1.0420460	1.0420460	2.94	0.0908

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	1.1818535	1.1818535	3.34	0.0720
AGE2	1	1.0511788	1.0511788	2.97	0.0894
AGE3	1	1.0420460	1.0420460	2.94	0.0908

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	11.21397561	4.55	0.0001	2.46417003
AGE	-0.79338953	-1.83	0.0720	0.43428471
AGE2	0.03972385	1.72	0.0894	0.02305595
AGE3	-0.00065516	-1.72	0.0908	0.00038192

General Linear Models Procedure

)ependent Variable: RATE

;ource	DF	Sum of Squares	Mean Square	F Value	Pr > F
1odel	2	2.8372088	1.4186044	3.95	0.0238
:rror	70	25.1627912	0.3594684		
:orrected Total	72	28.0000000			

R-Square	C.V.	Root MSE	RATE Mean
0.101329	9.992615	0.5996	6.0000

;ource	DF	Type I SS	Mean Square	F Value	Pr > F
.GE	1	2.5150380	2.5150380	7.00	0.0101
:1	1	0.3221708	0.3221708	0.90	0.3470

;ource	DF	Type II SS	Mean Square	F Value	Pr > F
.GE	1	2.7213358	2.7213358	7.57	0.0075
:1	1	0.3221708	0.3221708	0.90	0.3470

'arameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
NTERCEPT	7.404864946	14.50	0.0001	0.51074197
.GE	-0.053920039	-2.75	0.0075	0.01959700
1	-0.195126915	-0.95	0.3470	0.20611257

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	6.0044073	0.8577725	2.53	0.0227
Error	65	21.9955927	0.3383937		
Corrected Total	72	28.0000000			

R-Square	C.V.	Root MSE	RATE Mean
0.214443	9.695270	0.5817	6.0000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	2.5150380	2.5150380	7.43	0.0082
AGE2	1	0.0092365	0.0092365	0.03	0.8693
AGE3	1	1.0420460	1.0420460	3.08	0.0840
Z1	1	0.3058123	0.3058123	0.90	0.3453
AGE*Z1	1	0.0001544	0.0001544	0.00	0.9830
AGE2*Z1	1	2.0521323	2.0521323	6.06	0.0165
AGE3*Z1	1	0.0799878	0.0799878	0.24	0.6285

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	0.2455279	0.2455279	0.73	0.3975
AGE2	1	0.1849023	0.1849023	0.55	0.4624
AGE3	1	0.1368142	0.1368142	0.40	0.5271
Z1	1	0.2317617	0.2317617	0.68	0.4109
AGE*Z1	1	0.1719455	0.1719455	0.51	0.4785
AGE2*Z1	1	0.1205558	0.1205558	0.36	0.5527
AGE3*Z1	1	0.0799878	0.0799878	0.24	0.6285

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	38.81026093	1.15	0.2561	33.87278278
AGE	-4.59619975	-0.85	0.3975	5.39584736
AGE2	0.20218756	0.74	0.4624	0.27352339
AGE3	-0.00283768	-0.64	0.5271	0.00446282
Z1	-28.11616802	-0.83	0.4109	33.97398020
AGE*Z1	3.86121075	0.71	0.4785	5.41675567
AGE2*Z1	-0.16396185	-0.60	0.5527	0.27470082
AGE3*Z1	0.00217928	0.49	0.6285	0.00448241

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	3.8721328	0.9680332	2.73	0.0362
Error	68	24.1278672	0.3548216		
Corrected Total	72	28.0000000			

R-Square	C.V.	Root MSE	RATE Mean
0.138290	9.927817	0.5957	6.0000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	2.5150380	2.5150380	7.09	0.0097
AGE2	1	0.0092365	0.0092365	0.03	0.8723
AGE3	1	1.0420460	1.0420460	2.94	0.0911
Z1	1	0.3058123	0.3058123	0.86	0.3565

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	1.1235090	1.1235090	3.17	0.0796
AGE2	1	1.0170542	1.0170542	2.87	0.0950
AGE3	1	1.0342484	1.0342484	2.91	0.0923
Z1	1	0.3058123	0.3058123	0.86	0.3565

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	11.25654570	4.56	0.0001	2.46706630
AGE	-0.77441275	-1.78	0.0796	0.43520034
AGE2	0.03909080	1.69	0.0950	0.02308913
AGE3	-0.00065272	-1.71	0.0923	0.00038231
Z1	-0.19462403	-0.93	0.3565	0.20963993

I 1

Polynomial Regression Paint (1):

Y = 8.79682 + 6.30E-03X - 7.81E-03X**2 + 1.16E-04X**3
R-Sq = 0.778

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	3	29.6770	9.89234	82.0007	0
Error	70	8.4446	0.12064		
Total	73	38.1216			

SOURCE	DF	Seq SS	F	P
Linear	1	29.1387	233.554	0
Quadratic	1	0.4013	3.31996	7.27E-02
Cubic	1	0.1370	1.13573	0.290218

Macro is running ... please wait

Regression

The regression equation is
y = 9.49 - 0.143 x

Predictor	Coef	StDev	T	P
Constant	9.4888	0.2154	44.04	0.000
x	-0.142790	0.009343	-15.28	0.000

S = 0.3532   R-Sq = 76.4%   R-Sq(adj) = 76.1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	29.139	29.139	233.55	0.000
Error	72	8.983	0.125		
Total	73	38.122			

Macro is running ... please wait

Polynomial Regression

Y = 9.06329 - 8.21E-02X - 1.78E-03X**2
R-Sq = 0.775

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	29.5400	14.7700	122.200	0
Error	71	8.5816	0.1209		
Total	73	38.1216			

SOURCE	DF	Seq SS	F	P
Linear	1	29.1387	233.554	0
Quadratic	1	0.4013	3.31996	7.27E-02

Regression Analysis-Inter-Paint(1):

The regression equation is
Paint Rate = 9.49 - 0.143 Age

Predictor	Coef	StDev	T	P
Constant	9.4888	0.2154	44.04	0.000
Age	-0.142790	0.009343	-15.28	0.000

S = 0.3532 R-Sq = 76.4% R-Sq(adj) = 76.1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	29.139	29.139	233.55	0.000
Error	72	8.983	0.125		
Total	73	38.122			

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
14	26.0	5.0000	5.7763	0.0517	-0.7763	-2.22R
53	23.0	7.0000	6.2047	0.0412	0.7953	2.27R
54	10.0	8.0000	8.0609	0.1250	-0.0609	-0.18 X
58	23.0	7.0000	6.2047	0.0412	0.7953	2.27R
65	7.0	8.0000	8.4893	0.1517	-0.4893	-1.53 X
74	1.0	9.0000	9.3460	0.2063	-0.3460	-1.21 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 1.50 P = 0.1415 DF(pure error) = 58
9 rows with no replicates

Regression Analysis ✓ Z: $F^* = -0.9$, $p = 0.347$

The regression equation is
Paint Rate = 9.06 - 0.0821 Age - 0.00178 Agesq

Predictor	Coef	StDev	T	P
Constant	9.0633	0.3154	28.73	0.000
Age	-0.08210	0.03455	-2.38	0.020
Agesq	-0.0017832	0.0009787	-1.82	0.073

o.k.

S = 0.3477 R-Sq = 77.5% R-Sq(adj) = 76.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	29.540	14.770	122.20	0.000
Error	71	8.582	0.121		
Total	73	38.122			

Source	DF	Seq SS
Age	1	29.139
Agesq	1	0.401

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
14	26.0	5.0000	5.7231	0.0587	-0.7231	-2.11R

42	32.0	5.0000	4.6099	0.1948	0.3901	1.35 X
53	23.0	7.0000	6.2315	0.0432	0.7685	2.23R
54	10.0	8.0000	8.0639	0.1230	-0.0639	-0.20 X
58	23.0	7.0000	6.2315	0.0432	0.7685	2.23R
65	7.0	8.0000	8.4012	0.1570	-0.4012	-1.29 X
74	1.0	9.0000	8.9794	0.2858	0.0206	0.10 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 1.34 P = 0.2172 DF(pure error) = 58
9 rows with no replicates

Regression Analysis

The regression equation is
Paint Rate = 8.80 + 0.0063 Age - 0.00781 Agesq +0.000116 Agecubic

Predictor	Coef	StDev	T	P
Constant	8.7968	0.4023	21.87	0.000
Age	0.00630	0.08985	0.07	0.944
Agesq	-0.007809	0.005738	-1.36	0.178
Agecubic	0.0001158	0.0001086	1.07	0.290

S = 0.3473 R-Sq = 77.8% R-Sq(adj) = 76.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	29.6770	9.8923	82.00	0.000
Error	70	8.4446	0.1206		
Total	73	38.1216			

Source	DF	Seq SS
Age	1	29.1387
Agesq	1	0.4013
Agecubic	1	0.1370

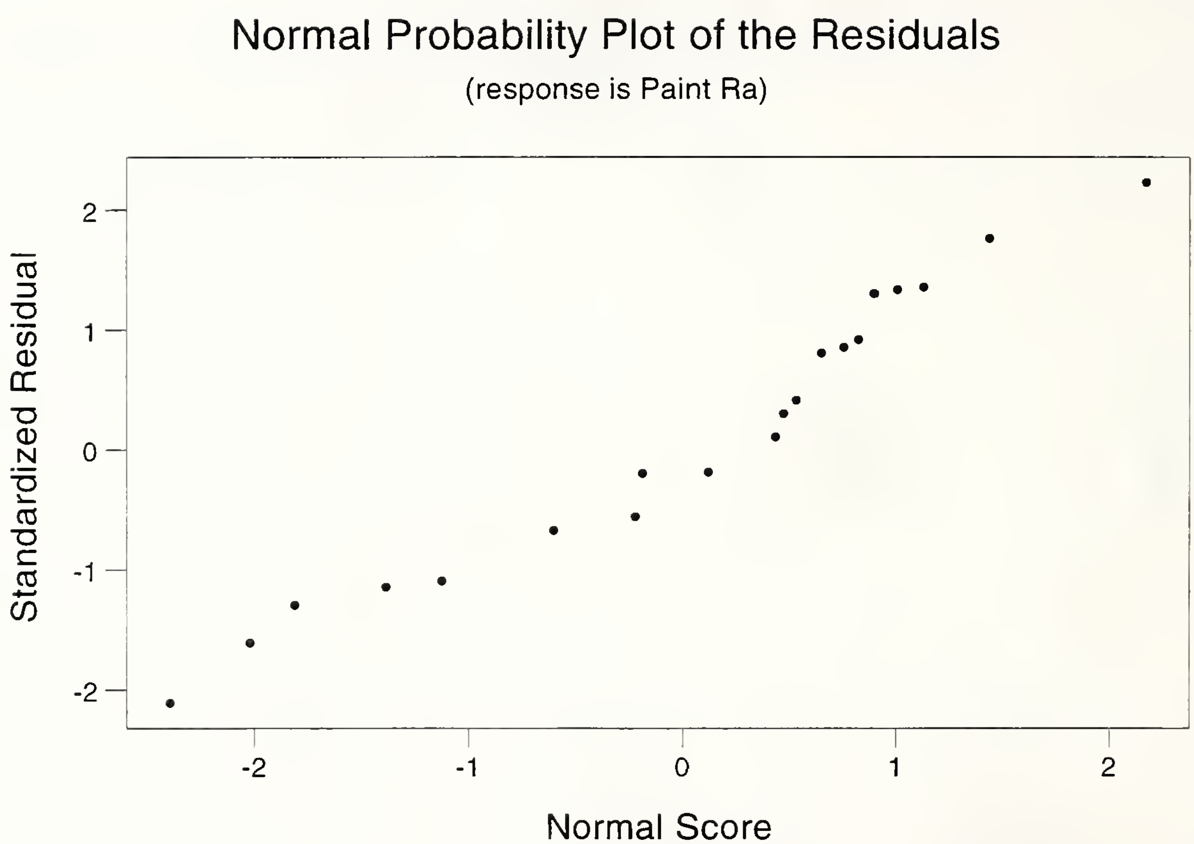
Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
3	13.0	8.0000	7.8133	0.1518	0.1867	0.60 X
14	26.0	5.0000	5.7163	0.0590	-0.7163	-2.09R
42	32.0	5.0000	4.7952	0.2609	0.2048	0.89 X
53	23.0	7.0000	6.2191	0.0447	0.7809	2.27R
54	10.0	8.0000	8.1946	0.1736	-0.1946	-0.65 X
58	23.0	7.0000	6.2191	0.0447	0.7809	2.27R
65	7.0	8.0000	8.4980	0.1812	-0.4980	-1.68 X
73	13.0	8.0000	7.8133	0.1518	0.1867	0.60 X
74	1.0	9.0000	8.7954	0.3337	0.2046	2.12RX

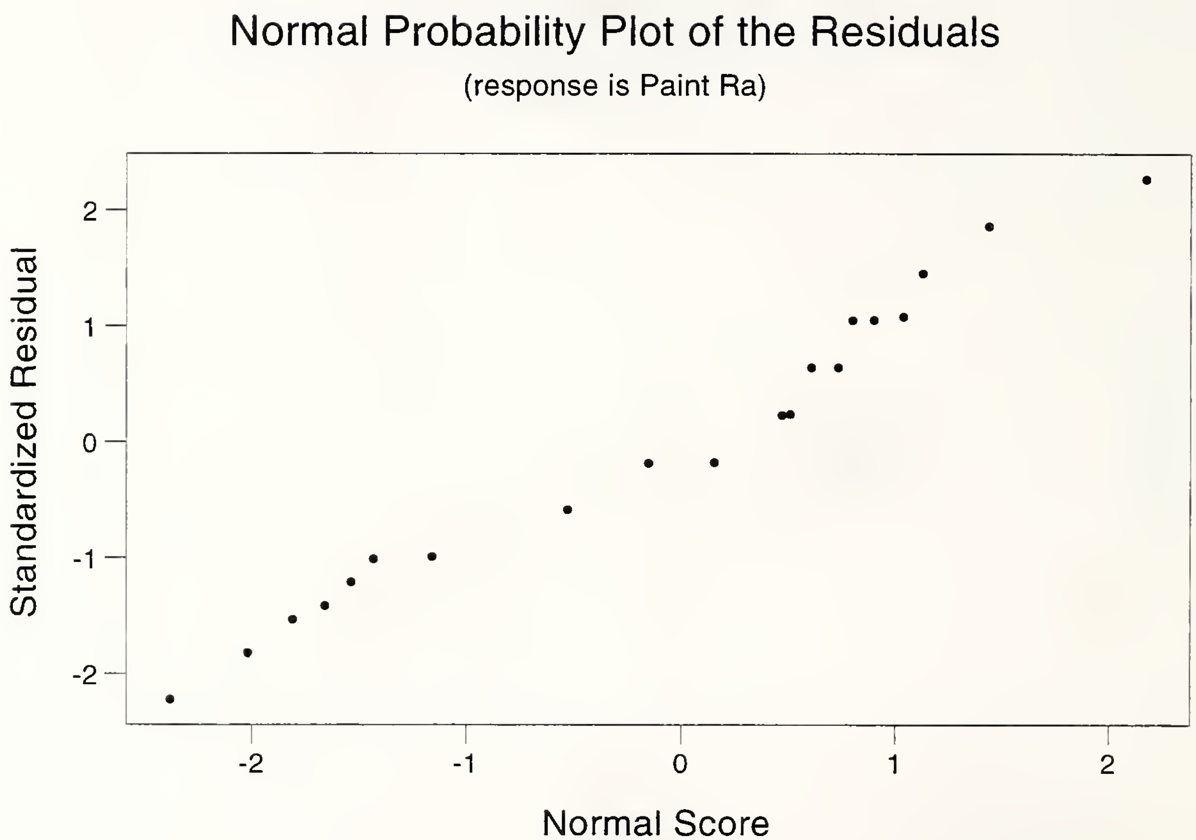
R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 1.35 P = 0.2158 DF(pure error) = 58
9 rows with no replicates


Regression Plot
Y = 9.06329 - 8.21E-02X - 1.78E-03X**2
R-Sq = 0.775
Paint Ra
9
8
7
6
5
0
10
20
30
Age

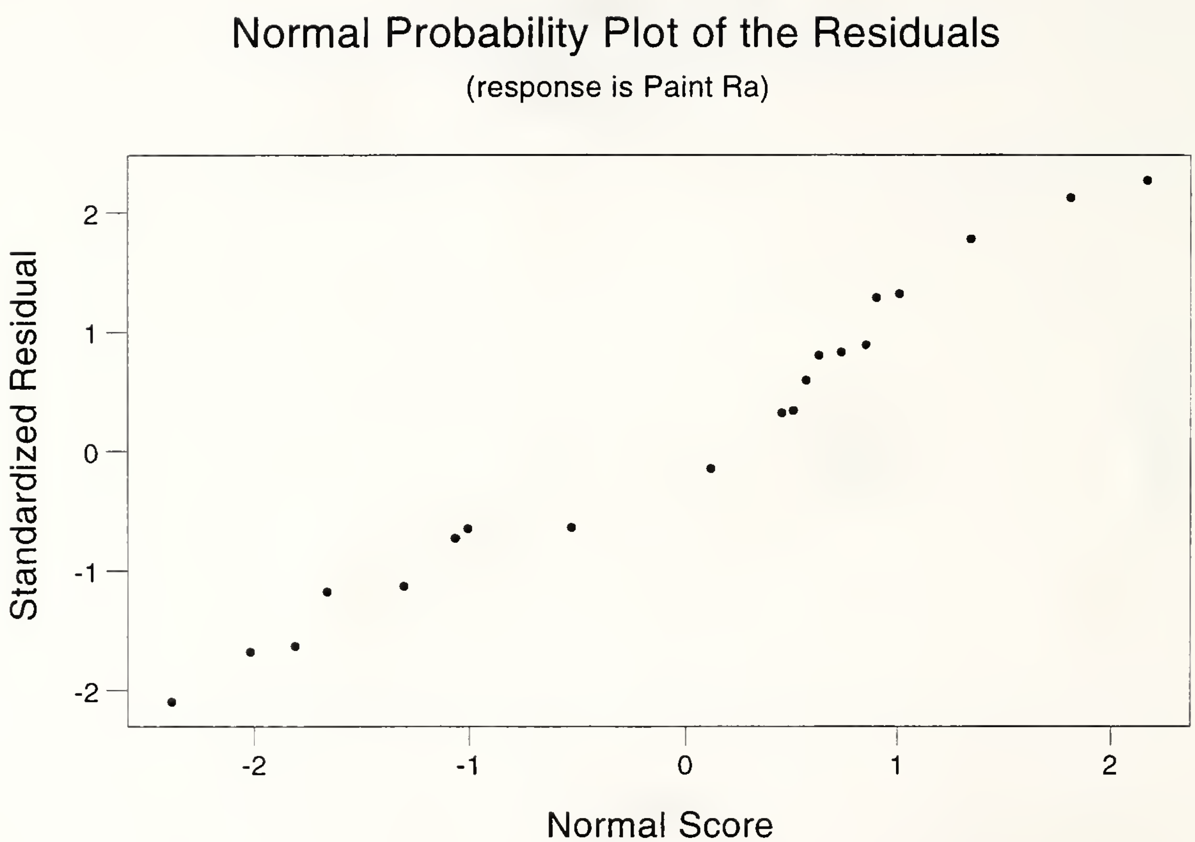

Normal Probability Plot of the Residuals
(response is Paint Ra)
Standardized Residual
2
1
0
-1
-2
-2
-1
0
1
2
Normal Score


Regression Plot
Y = 9.48882 - 0.142790X
R-Sq = 0.764
Paint Ra
9
8
7
6
5
0
10
20
30
Age



Normal Probability Plot of the Residuals
(response is Paint Ra)
Standardized Residual
2
1
0
-1
-2
-2
-1
0
1
2
Normal Score


Regression Plot
Y = 8.79682 + 6.30E-03X - 7.81E-03X**2 + 1.16E-04X**3
R-Sq = 0.778
Paint Ra
9
8
7
6
5
0
10
20
30
Age



Normal Probability Plot of the Residuals
(response is Paint Ra)
Standardized Residual
2
1
0
-1
-2
-2
-1
0
1
2
Normal Score

Interstate paint(2)

General Linear Models Procedure

ependent Variable: RATE

ource	DF	Sum of Squares	Mean Square	F Value	Pr > F
odel	1	173.24946	173.24946	1401.23	0.0001
rror	313	38.69975	0.12364		
orrected Total	314	211.94921			

R-Square	C.V.	Root MSE	RATE Mean
0.817410	4.388369	0.3516	8.0127

ource	DF	Type I SS	Mean Square	F Value	Pr > F
GE	1	173.24946	173.24946	1401.23	0.0001

ource	DF	Type II SS	Mean Square	F Value	Pr > F
GE	1	173.24946	173.24946	1401.23	0.0001

arameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
NTERCEPT	9.154824493	251.65	0.0001	0.03637923
GE	-0.124487791	-37.43	0.0001	0.00332562

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	176.08035	58.69345	508.90	0.0001
Error	311	35.86886	0.11533		
Corrected Total	314	211.94921			

R-Square	C.V.	Root MSE	RATE Mean
0.830767	4.238379	0.3396	8.0127

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	173.24946	173.24946	1502.15	0.0001
AGE2	1	0.51146	0.51146	4.43	0.0360
AGE3	1	2.31942	2.31942	20.11	0.0001

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	7.4068635	7.4068635	64.22	0.0001
AGE2	1	1.8075775	1.8075775	15.67	0.0001
AGE3	1	2.3194247	2.3194247	20.11	0.0001

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.258946262	140.49	0.0001	0.06590502
AGE	-0.200983652	-8.01	0.0001	0.02507969
AGE2	0.010336822	3.96	0.0001	0.00261106
AGE3	-0.000348433	-4.48	0.0001	0.00007770

General Linear Models Procedure

ependent Variable: RATE

ource	DF	Sum of Squares	Mean Square	F Value	Pr > F
odel	4	176.74860	44.18715	389.14	0.0001
rror	310	35.20061	0.11355		
orrected Total	314	211.94921			

R-Square	C.V.	Root MSE	RATE Mean
0.833920	4.205479	0.3370	8.0127

ource	DF	Type I SS	Mean Square	F Value	Pr > F
GE	1	173.24946	173.24946	1525.75	0.0001
GE2	1	0.51146	0.51146	4.50	0.0346
GE3	1	2.31942	2.31942	20.43	0.0001
1	1	0.66825	0.66825	5.89	0.0158

ource	DF	Type II SS	Mean Square	F Value	Pr > F
GE	1	7.2336428	7.2336428	63.70	0.0001
GE2	1	1.6279323	1.6279323	14.34	0.0002
GE3	1	2.0429357	2.0429357	17.99	0.0001
1	1	0.6682507	0.6682507	5.89	0.0158

arameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
NTERCEPT	9.236278094	139.82	0.0001	0.06605767
GE	-0.198754909	-7.98	0.0001	0.02490196
GE2	0.009840288	3.79	0.0002	0.00259887
GE3	-0.000328803	-4.24	0.0001	0.00007752
1	0.110566949	2.43	0.0158	0.04557743

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	179.22781	25.60397	240.22	0.0001
Error	307	32.72140	0.10658		
Corrected Total	314	211.94921			

R-Square	C.V.	Root MSE	RATE Mean
0.845617	4.074440	0.3265	8.0127

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	173.24946	173.24946	1625.47	0.0001
AGE2	1	0.51146	0.51146	4.80	0.0292
AGE3	1	2.31942	2.31942	21.76	0.0001
Z1	1	0.66825	0.66825	6.27	0.0128
AGE*Z1	1	1.50311	1.50311	14.10	0.0002
AGE2*Z1	1	0.08818	0.08818	0.83	0.3638
AGE3*Z1	1	0.88792	0.88792	8.33	0.0042

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	4.0387631	4.0387631	37.89	0.0001
AGE2	1	0.3851595	0.3851595	3.61	0.0582
AGE3	1	0.5603813	0.5603813	5.26	0.0225
Z1	1	0.1662791	0.1662791	1.56	0.2126
AGE*Z1	1	0.6915095	0.6915095	6.49	0.0113
AGE2*Z1	1	0.9631843	0.9631843	9.04	0.0029
AGE3*Z1	1	0.8879200	0.8879200	8.33	0.0042

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.220953652	130.29	0.0001	0.07076992
AGE	-0.166557032	-6.16	0.0001	0.02705737
AGE2	0.005338528	1.90	0.0582	0.00280833
AGE3	-0.000189415	-2.29	0.0225	0.00008261
Z1	0.208567849	1.25	0.2126	0.16698416
AGE*Z1	-0.182641478	-2.55	0.0113	0.07170461
AGE2*Z1	0.026217927	3.01	0.0029	0.00872148
AGE3*Z1	-0.000879608	-2.89	0.0042	0.00030475

Regression Analysis(I2):

```
The regression equation is
Paint Rate = 9.15 - 0.124 Age
               0

Predictor        Coef        StDev          T         P
Constant      9.15482      0.03638     251.65     0.000
Age         -0.124488     0.003326     -37.43     0.000

S = 0.3516      R-Sq = 81.7%     R-Sq(adj) = 81.7%

Analysis of Variance

Source        DF          SS          MS         F         P
Regression     1      173.25      173.25   1401.23     0.000
Error        313       38.70        0.12
Total        314      211.95

Unusual Observations
Obs       Age   Paint Ra         Fit  StDev Fit   Residual    St Resid
  6      15.0     8.0000      7.2875     0.0277     0.7125       2.03R
 16      15.0     8.0000      7.2875     0.0277     0.7125       2.03R
 29      16.0     8.0000      7.1630     0.0301     0.8370       2.39R
 30      16.0     8.0000      7.1630     0.0301     0.8370       2.39R
 33      16.0     8.0000      7.1630     0.0301     0.8370       2.39R
 34      16.0     8.0000      7.1630     0.0301     0.8370       2.39R
 38      15.0     8.0000      7.2875     0.0277     0.7125       2.03R
 40      16.0     8.0000      7.1630     0.0301     0.8370       2.39R
 75      15.0     8.0000      7.2875     0.0277     0.7125       2.03R
149      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
162      19.0     6.0000      6.7896     0.0382    -0.7896      -2.26R
185      19.0     6.0000      6.7896     0.0382    -0.7896      -2.26R
192      17.0     6.0000      7.0385     0.0327    -1.0385      -2.97R
202      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
225      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
233      16.0     6.0000      7.1630     0.0301    -1.1630      -3.32R
239      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
244      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
247      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
250      24.0     5.0000      6.1671     0.0531    -1.1671      -3.36RX
261      18.0     6.0000      6.9140     0.0354    -0.9140      -2.61R
273      24.0     6.0000      6.1671     0.0531    -0.1671      -0.48 X
281      23.0     6.0000      6.2916     0.0501    -0.2916      -0.84 X
287      25.0     6.0000      6.0426     0.0562    -0.0426      -0.12 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 14.91  P = 0.0000  DF(pure error) = 289
2 rows with no replicates
```

Regression Analysis

Z: $F^* = 5.8958$, $p = 0.0158$

```
The regression equation is
Paint Rate = 9.06 - 0.201 Age + 0.0103 Age2 -0.000348 Age3

Predictor        Coef        StDev          T         P
```

Constant	9.25895	0.06591	140.49	0.000
Age	-0.20098	0.02508	-8.01	0.000
Age2	0.010337	0.002611	3.96	0.000
Age3	-0.00034843	0.00007770	-4.48	0.000

S = 0.3396   R-Sq = 83.1%   R-Sq(adj) = 82.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	176.080	58.693	508.90	0.000
Error	311	35.869	0.115		
Total	314	211.949			

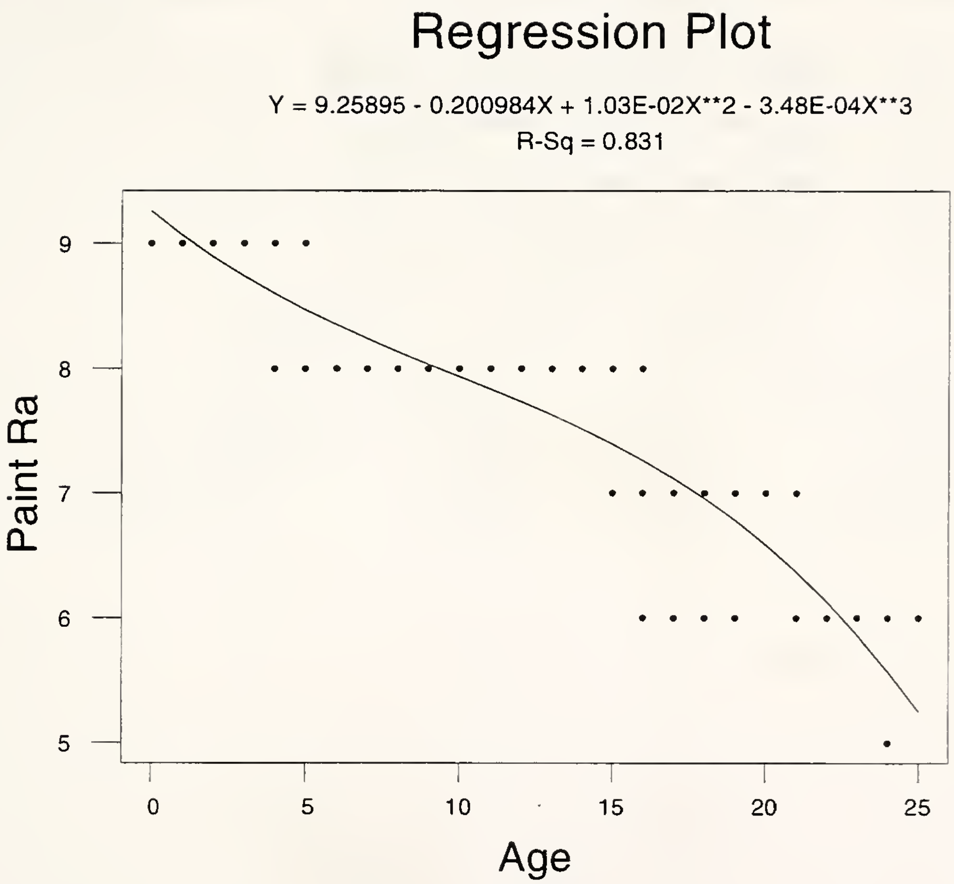
Source	DF	Seq SS
Age	1	173.249
Age2	1	0.511
Age3	1	2.319

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
29	16.0	8.0000	7.2623	0.0359	0.7377	2.18R
30	16.0	8.0000	7.2623	0.0359	0.7377	2.18R
33	16.0	8.0000	7.2623	0.0359	0.7377	2.18R
34	16.0	8.0000	7.2623	0.0359	0.7377	2.18R
40	16.0	8.0000	7.2623	0.0359	0.7377	2.18R
149	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
162	19.0	6.0000	6.7819	0.0427	-0.7819	-2.32R
185	19.0	6.0000	6.7819	0.0427	-0.7819	-2.32R
192	17.0	6.0000	7.1177	0.0372	-1.1177	-3.31R
202	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
225	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
233	16.0	6.0000	7.2623	0.0359	-1.2623	-3.74R
239	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
244	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
247	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
250	24.0	5.0000	5.5726	0.1342	-0.5726	-1.84 X
258	22.0	6.0000	6.1302	0.0796	-0.1302	-0.39 X
261	18.0	6.0000	6.9583	0.0391	-0.9583	-2.84R
273	24.0	6.0000	5.5726	0.1342	0.4274	1.37 X
281	23.0	6.0000	5.8651	0.1037	0.1349	0.42 X
283	22.0	6.0000	6.1302	0.0796	-0.1302	-0.39 X
287	25.0	6.0000	5.2506	0.1714	0.7494	2.56RX

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 14.12 P = 0.0000 DF(pure error) = 289
2 rows with no replicates



Regression Plot
Y = 9.25895 - 0.200984X + 1.03E-02X**2 - 3.48E-04X**3
R-Sq = 0.831
Paint Ra
9
8
7
6
5
0
5
10
15
20
25
Age

S1
State paint(1)

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	66.125592	66.125592	332.80	0.0001
Error	58	11.524408	0.198697		
Corrected Total	59	77.650000			

R-Square	C.V.	Root MSE	RATE Mean
0.851585	6.507359	0.4458	6.8500

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	66.125592	66.125592	332.80	0.0001

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	66.125592	66.125592	332.80	0.0001

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.500528787	60.79	0.0001	0.15627383
AGE	-0.142373972	-18.24	0.0001	0.00780443

General Linear Models Procedure

)ependent Variable: RATE

;ource	DF	Sum of Squares	Mean Square	F Value	Pr > F
Iodel	2	66.199891	33.099946	164.78	0.0001
:rror	57	11.450109	0.200879		
:orrected Total	59	77.650000			

R-Square	C.V.	Root MSE	RATE Mean
0.852542	6.542998	0.4482	6.8500

ource	DF	Type I SS	Mean Square	F Value	Pr > F
GE	1	66.125592	66.125592	329.18	0.0001
1	1	0.074300	0.074300	0.37	0.5455

ource	DF	Type II SS	Mean Square	F Value	Pr > F
GE	1	65.335605	65.335605	325.25	0.0001
1	1	0.074300	0.074300	0.37	0.5455

arameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
NTERCEPT	9.490484836	60.07	0.0001	0.15799522
GE	-0.143085589	-18.03	0.0001	0.00793393
1	0.077639613	0.61	0.5455	0.12766070

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	69.735570	23.245190	164.48	0.0001
Error	56	7.914430	0.141329		
Corrected Total	59	77.650000			

R-Square	C.V.	Root MSE	RATE Mean
0.898076	5.488141	0.3759	6.8500

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	66.125592	66.125592	467.88	0.0001
AGE2	1	3.466438	3.466438	24.53	0.0001
AGE3	1	0.143541	0.143541	1.02	0.3179

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	0.1108375	0.1108375	0.78	0.3796
AGE2	1	0.4070476	0.4070476	2.88	0.0952
AGE3	1	0.1435409	0.1435409	1.02	0.3179

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	8.904326281	40.71	0.0001	0.21871900
AGE	0.082658870	0.89	0.3796	0.09333875
AGE2	-0.012551592	-1.70	0.0952	0.00739592
AGE3	0.000161970	1.01	0.3179	0.00016072

General Linear Models Procedure

ependent Variable: RATE

ource	DF	Sum of Squares	Mean Square	F Value	Pr > F
odel	4	70.305109	17.576277	131.61	0.0001
rror	55	7.344891	0.133543		
orrected Total	59	77.650000			

R-Square	C.V.	Root MSE	RATE Mean
0.905410	5.334832	0.3654	6.8500

ource	DF	Type I SS	Mean Square	F Value	Pr > F
GE	1	66.125592	66.125592	495.16	0.0001
GE2	1	3.466438	3.466438	25.96	0.0001
GE3	1	0.143541	0.143541	1.07	0.3044
L	1	0.569539	0.569539	4.26	0.0436

ource	DF	Type II SS	Mean Square	F Value	Pr > F
GE	1	0.2433591	0.2433591	1.82	0.1826
GE2	1	0.6117683	0.6117683	4.58	0.0368
GE3	1	0.2606442	0.2606442	1.95	0.1680
L	1	0.5695387	0.5695387	4.26	0.0436

arameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
NTERCEPT	8.780668720	39.75	0.0001	0.22088028
GE	0.125666302	1.35	0.1826	0.09309071
GE2	-0.015737932	-2.14	0.0368	0.00735302
GE3	0.000222005	1.40	0.1680	0.00015891
L	0.223489259	2.07	0.0436	0.10821971

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	70.464732	10.066390	72.85	0.0001
Error	52	7.185268	0.138178		
Corrected Total	59	77.650000			

R-Square	C.V.	Root MSE	RATE Mean
0.907466	5.426618	0.3717	6.8500

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	66.125592	66.125592	478.55	0.0001
AGE2	1	3.466438	3.466438	25.09	0.0001
AGE3	1	0.143541	0.143541	1.04	0.3128
Z1	1	0.569539	0.569539	4.12	0.0475
AGE*Z1	1	0.043709	0.043709	0.32	0.5762
AGE2*Z1	1	0.001177	0.001177	0.01	0.9268
AGE3*Z1	1	0.114737	0.114737	0.83	0.3664

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	0.2678078	0.2678078	1.94	0.1698
AGE2	1	0.6213582	0.6213582	4.50	0.0387
AGE3	1	0.3119026	0.3119026	2.26	0.1390
Z1	1	0.0523017	0.0523017	0.38	0.5411
AGE*Z1	1	0.0903061	0.0903061	0.65	0.4225
AGE2*Z1	1	0.1095097	0.1095097	0.79	0.3774
AGE3*Z1	1	0.1147372	0.1147372	0.83	0.3664

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	8.765813443	32.17	0.0001	0.27244959
AGE	0.153673721	1.39	0.1698	0.11038443
AGE2	-0.018255036	-2.12	0.0387	0.00860858
AGE3	0.000278222	1.50	0.1390	0.00018518
Z1	0.278062638	0.62	0.5411	0.45196492
AGE*Z1	-0.187303091	-0.81	0.4225	0.23168948
AGE2*Z1	0.016775893	0.89	0.3774	0.01884426
AGE3*Z1	-0.000374774	-0.91	0.3664	0.00041128

Macro is running ... please wait

Polynomial Regression(S1):

Y = 8.90433 + 8.27E-02X - 1.26E-02X**2 + 1.62E-04X**3
R-Sq = 0.898

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	3	69.7356	23.2452	164.476	0
Error	56	7.9144	0.1413		
Total	59	77.6500			

SOURCE	DF	Seq SS	F	P
Linear	1	66.1256	332.797	0
Quadratic	1	3.4664	24.5207	6.90E-06
Cubic	1	0.1435	1.01565	0.317887

Macro is running ... please wait

Polynomial Regression

Y = 9.05508 - 7.04E-03X - 5.17E-03X**2
R-Sq = 0.896

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	69.5920	34.7960	246.138	0
Error	57	8.0580	0.1414		
Total	59	77.6500			

SOURCE	DF	Seq SS	F	P
Linear	1	66.1256	332.797	0
Quadratic	1	3.4664	24.5207	6.90E-06

Regression Analysis

Z: F* = 0.37 , P = 0.5455

The regression equation is
Paint Rate = 9.50 - 0.142 Age

Predictor	Coef	StDev	T	P
Constant	9.5005	0.1563	60.79	0.000
Age	-0.142374	0.007804	-18.24	0.000

S = 0.4458 R-Sq = 85.2% R-Sq(adj) = 84.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	66.126	66.126	332.80	0.000
Error	58	11.524	0.199		
Total	59	77.650			

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
16	0.0	9.0000	9.5005	0.1563	-0.5005	-1.20 X
33	1.0	9.0000	9.3582	0.1490	-0.3582	-0.85 X
35	1.0	9.0000	9.3582	0.1490	-0.3582	-0.85 X
45	2.0	9.0000	9.2158	0.1419	-0.2158	-0.51 X
46	2.0	9.0000	9.2158	0.1419	-0.2158	-0.51 X
57	25.0	5.0000	5.9412	0.0761	-0.9412	-2.14R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 2.30 P = 0.0181 DF(pure error) = 44
4 rows with no replicates

Regression Analysis

The regression equation is
Paint Rate = 9.06 - 0.0070 Age - 0.00517 Age2

Predictor	Coef	StDev	T	P
Constant	9.0551	0.1596	56.74	0.000
Age	-0.00704	0.02811	-0.25	0.803
Age2	-0.005173	0.001045	-4.95	0.000

S = 0.3760 R-Sq = 89.6% R-Sq(adj) = 89.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	69.592	34.796	246.14	0.000
Error	57	8.058	0.141		
Total	59	77.650			

Source	DF	Seq SS
Age	1	66.126
Age2	1	3.466

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
16	0.0	9.0000	9.0551	0.1596	-0.0551	-0.16 X
54	20.0	6.0000	6.8452	0.0628	-0.8452	-2.28R
55	20.0	6.0000	6.8452	0.0628	-0.8452	-2.28R
56	29.0	5.0000	4.5006	0.1949	0.4994	1.55 X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 0.71 P = 0.7404 DF(pure error) = 44
4 rows with no replicates

Regression Analysis

The regression equation is
Paint Rate = 8.90 + 0.0827 Age - 0.0126 Age2 +0.000162 Age3

Predictor	Coef	StDev	T	P
Constant	8.9043	0.2187	40.71	0.000
Age	0.08266	0.09334	0.89	0.380
Age2	-0.012552	0.007396	-1.70	0.095
Age3	0.0001620	0.0001607	1.01	0.318

S = 0.3759 R-Sq = 89.8% R-Sq(adj) = 89.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	69.736	23.245	164.48	0.000
Error	56	7.914	0.141		
Total	59	77.650			

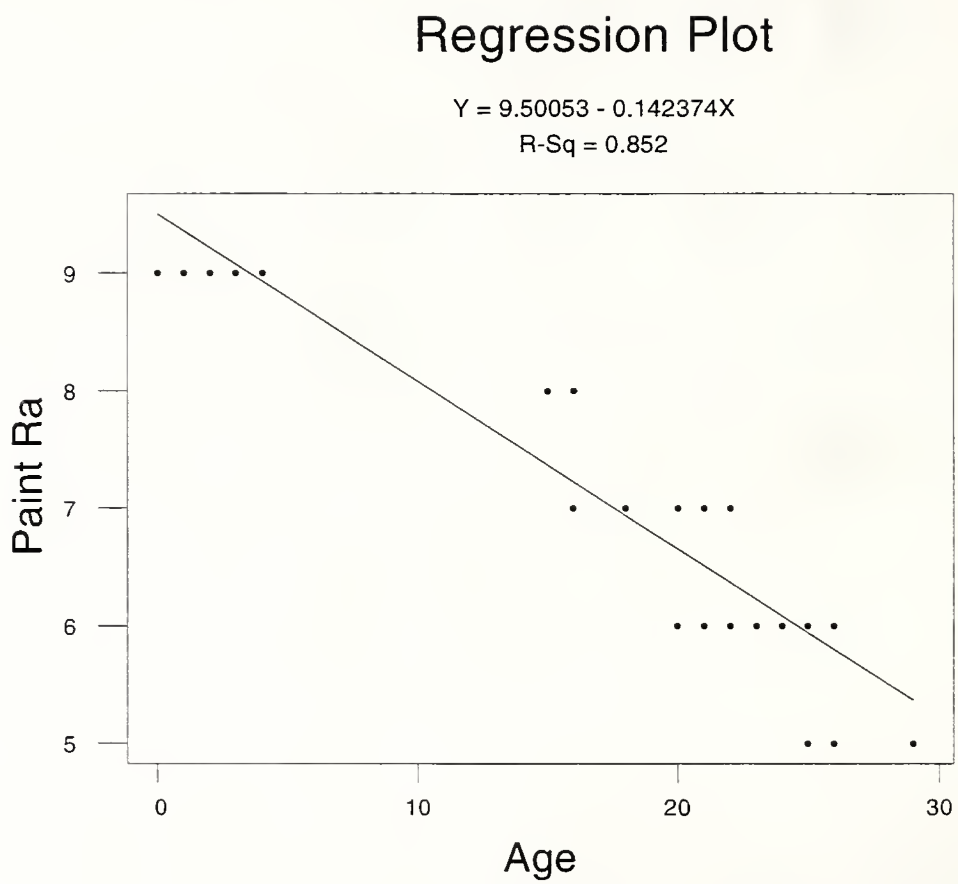
Source	DF	Seq SS
Age	1	66.126
Age2	1	3.466
Age3	1	0.144

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
16	0.0	9.0000	8.9043	0.2187	0.0957	0.31 X
54	20.0	6.0000	6.8326	0.0640	-0.8326	-2.25R
55	20.0	6.0000	6.8326	0.0640	-0.8326	-2.25R
56	29.0	5.0000	4.6958	0.2747	0.3042	1.19 X

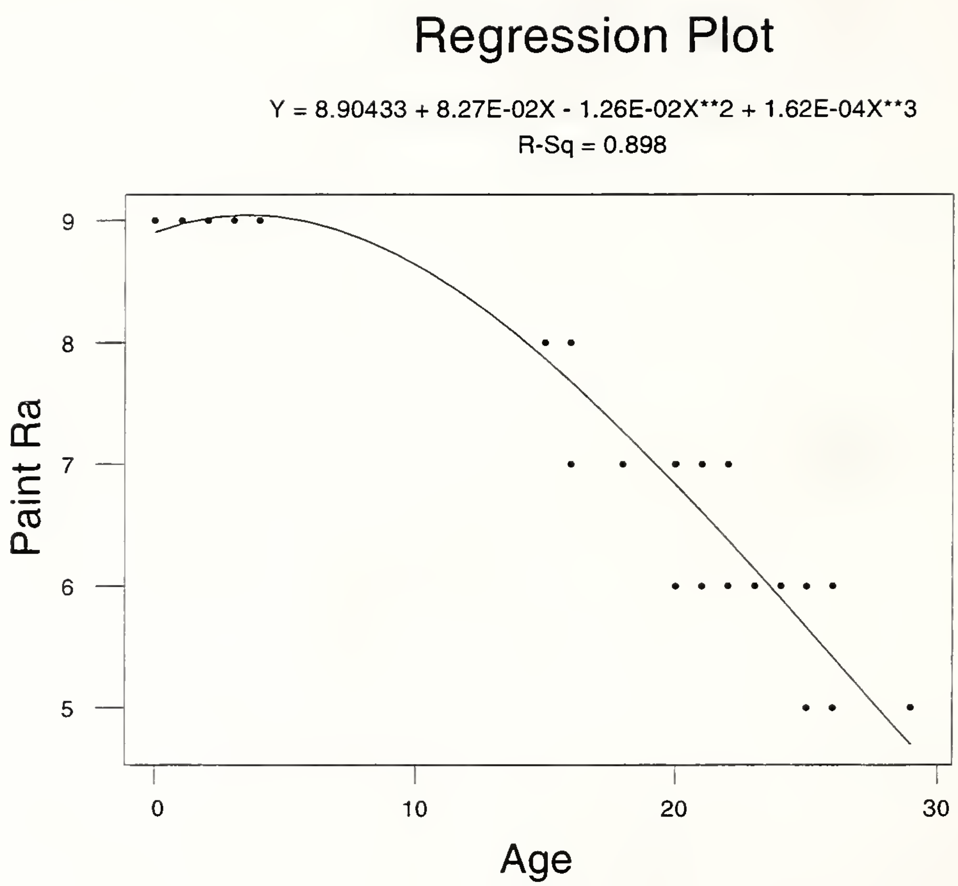
R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 0.69 P = 0.7486 DF(pure error) = 44
4 rows with no replicates



Regression Plot
Y = 9.50053 - 0.142374X
R-Sq = 0.852
9
8
7
6
5
Paint Ra
0
10
20
30
Age


Regression Plot
Y = 9.05508 - 7.04E-03X - 5.17E-03X**2
R-Sq = 0.896
Paint Ra
9
8
7
6
5
0
10
20
30
Age


Regression Plot
Y = 8.90433 + 8.27E-02X - 1.26E-02X**2 + 1.62E-04X**3
R-Sq = 0.898
9
8
7
6
5
Paint Ra
0
10
20
30
Age

General Linear Models Procedure

ependent Variable: RATE

ource	DF	Sum of Squares	Mean Square	F Value	Pr > F
odel	1	254.52948	254.52948	1598.24	0.0001
rror	275	43.79543	0.15926		
orrected Total	276	298.32491			

R-Square	C.V.	Root MSE	RATE Mean
0.853196	5.087075	0.3991	7.8448

ource	DF	Type I SS	Mean Square	F Value	Pr > F
GE	1	254.52948	254.52948	1598.24	0.0001

ource	DF	Type II SS	Mean Square	F Value	Pr > F
GE	1	254.52948	254.52948	1598.24	0.0001

arameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
NTERCEPT	9.276541532	215.23	0.0001	0.04309968
GE	-0.156019671	-39.98	0.0001	0.00390264

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	254.68150	127.34075	799.46	0.0001
Error	274	43.64341	0.15928		
Corrected Total	276	298.32491			

R-Square	C.V.	Root MSE	RATE Mean
0.853705	5.087496	0.3991	7.8448

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	254.52948	254.52948	1597.98	0.0001
AGE2	1	0.15202	0.15202	0.95	0.3295

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	202.46711	202.46711	1271.12	0.0001
AGE2	1	0.15202	0.15202	0.95	0.3295

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.268165327	210.89	0.0001	0.04394771
AGE	-0.154199101	-35.65	0.0001	0.00432503
AGE2	-0.000056974	-0.98	0.3295	0.00005832

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	258.05711	86.01904	583.18	0.0001
Error	273	40.26780	0.14750		
Corrected Total	276	298.32491			

R-Square	C.V.	Root MSE	RATE Mean
0.865020	4.895732	0.3841	7.8448

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	254.52948	254.52948	1725.61	0.0001
AGE2	1	0.15202	0.15202	1.03	0.3109
AGE3	1	3.37561	3.37561	22.89	0.0001

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	2.8874888	2.8874888	19.58	0.0001
AGE2	1	3.4450888	3.4450888	23.36	0.0001
AGE3	1	3.3756105	3.3756105	22.89	0.0001

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.027775168	137.45	0.0001	0.06567823
AGE	-0.075278707	-4.42	0.0001	0.01701412
AGE2	-0.004885682	-4.83	0.0001	0.00101093
AGE3	0.000053545	4.78	0.0001	0.00001119

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	254.77530	127.38765	801.48	0.0001
Error	274	43.54961	0.15894		
Corrected Total	276	298.32491			

R-Square	C.V.	Root MSE	RATE Mean
0.854020	5.082026	0.3987	7.8448

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	254.52948	254.52948	1601.42	0.0001
Z1	1	0.24582	0.24582	1.55	0.2147

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	248.74084	248.74084	1565.00	0.0001
Z1	1	0.24582	0.24582	1.55	0.2147

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.248280634	189.96	0.0001	0.04868574
AGE	-0.155418060	-39.56	0.0001	0.00392866
Z1	0.062366077	1.24	0.2147	0.05014843

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	258.14737	64.53684	436.91	0.0001
Error	272	40.17754	0.14771		
Corrected Total	276	298.32491			

R-Square	C.V.	Root MSE	RATE Mean
0.865323	4.899224	0.3843	7.8448

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	254.52948	254.52948	1723.15	0.0001
AGE2	1	0.15202	0.15202	1.03	0.3112
AGE3	1	3.37561	3.37561	22.85	0.0001
Z1	1	0.09026	0.09026	0.61	0.4351

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	2.9472396	2.9472396	19.95	0.0001
AGE2	1	3.2970008	3.2970008	22.32	0.0001
AGE3	1	3.2322704	3.2322704	21.88	0.0001
Z1	1	0.0902590	0.0902590	0.61	0.4351

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.014731777	132.94	0.0001	0.06781008
AGE	-0.076260043	-4.47	0.0001	0.01707248
AGE2	-0.004804587	-4.72	0.0001	0.00101696
AGE3	0.000052662	4.68	0.0001	0.00001126
Z1	0.038002863	0.78	0.4351	0.04861589

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	258.17460	43.02910	289.36	0.0001
Error	270	40.15031	0.14870		
Corrected Total	276	298.32491			

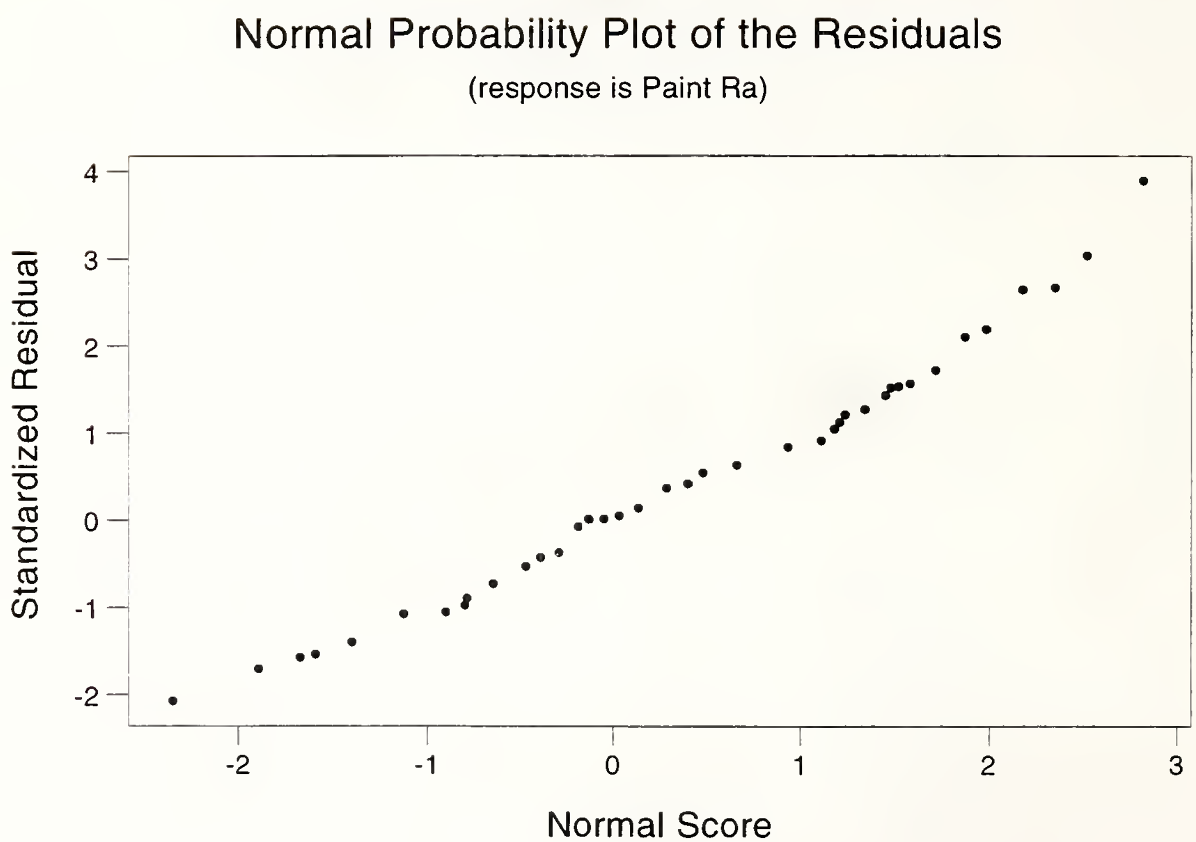
R-Square	C.V.	Root MSE	RATE Mean
0.865414	4.915669	0.3856	7.8448

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	254.52948	254.52948	1711.64	0.0001
AGE2	1	0.15202	0.15202	1.02	0.3129
AGE3	1	3.37561	3.37561	22.70	0.0001
Z1	1	0.09026	0.09026	0.61	0.4366
AGE2*Z1	1	0.00953	0.00953	0.06	0.8004
AGE3*Z1	1	0.01770	0.01770	0.12	0.7304

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	2.0735807	2.0735807	13.94	0.0002
AGE2	1	2.5513219	2.5513219	17.16	0.0001
AGE3	1	2.5021819	2.5021819	16.83	0.0001
Z1	1	0.1013223	0.1013223	0.68	0.4098
AGE2*Z1	1	0.0237829	0.0237829	0.16	0.6895
AGE3*Z1	1	0.0176980	0.0176980	0.12	0.7304

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.001146261	118.72	0.0001	0.07581545
AGE	-0.073517380	-3.73	0.0002	0.01968757
AGE2	-0.004921304	-4.14	0.0001	0.00118812
AGE3	0.000053927	4.10	0.0001	0.00001315
Z1	0.062124733	0.83	0.4098	0.07526178
AGE2*Z1	-0.000570534	-0.40	0.6895	0.00142663
AGE3*Z1	0.000022192	0.34	0.7304	0.00006433


Regression Plot
Y = 9.07656 - 0.101248X - 2.03E-03X**2 - 2.87E-05X**3
R-Sq = 0.865
Paint Ra
9
8
7
6
5
4
0
10
20
30
Age


Normal Probability Plot of the Residuals
(response is Paint Ra)
Standardized Residual
4
3
2
1
0
-1
-2
-2
-1
0
1
2
3
Normal Score

Regression Analysis(S2):

The regression equation is
Paint Rate = 9.03 - 0.0753 Age - 0.00489 Age2 +0.000054 Age3

Predictor	Coef	StDev	T	P
Constant	9.02778	0.06568	137.45	0.000
Age	-0.07528	0.01701	-4.42	0.000
Age2	-0.004886	0.001011	-4.83	0.000
Age3	0.00005354	0.00001119	4.78	0.000

S = 0.3841   R-Sq = 86.5%   R-Sq(adj) = 86.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	258.057	86.019	583.18	0.000
Error	273	40.268	0.148		
Total	276	298.325			

Source	DF	Seq SS
Age	1	254.529
Age2	1	0.152
Age3	1	3.376

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
21	16.0	6.0000	6.7919	0.0350	-0.7919	-2.07R
31	23.0	5.0000	5.3633	0.0891	-0.3633	-0.97 X
52	15.0	8.0000	6.9800	0.0333	1.0200	2.67R
81	16.0	6.0000	6.7919	0.0350	-0.7919	-2.07R
86	27.0	5.0000	4.4875	0.1396	0.5125	1.43 X
106	16.0	6.0000	6.7919	0.0350	-0.7919	-2.07R
135	14.0	8.0000	7.1632	0.0329	0.8368	2.19R
137	19.0	7.0000	6.2010	0.0502	0.7990	2.10R
161	26.0	5.0000	4.9935	0.3840	0.0065	1.52 X
215	14.0	8.0000	7.1632	0.0329	0.8368	2.19R
218	14.0	8.0000	7.1632	0.0329	0.8368	2.19R
229	20.0	7.0000	5.9963	0.0583	1.0037	2.64R
231	20.0	7.0000	5.9963	0.0583	1.0037	2.64R
235	16.0	6.0000	6.7919	0.0350	-0.7919	-2.07R
237	16.0	6.0000	6.7919	0.0350	-0.7919	-2.07R
268	10.0	9.0000	7.8400	0.0344	1.1600	3.03R
270	12.0	9.0000	7.5134	0.0338	1.4866	3.89R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 11.11 P = 0.0000 DF(pure error) = 252
3 rows with no replicates

Regression Analysis

The regression equation is
Paint Rate = 9.28 - 0.156 Age

Predictor	Coef	StDev	T	P
Constant	9.27654	0.04310	215.23	0.000
Age	-0.156020	0.003903	-39.98	0.000

S = 0.3991   R-Sq = 85.3%   R-Sq(adj) = 85.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	254.53	254.53	1598.24	0.000
Error	275	43.80	0.16		
Total	276	298.32			

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
31	23.0	5.0000	5.6881	0.0590	-0.6881	-1.74 X
52	15.0	8.0000	6.9362	0.0330	1.0638	2.67R
86	27.0	5.0000	5.0640	0.0736	-0.0640	-0.16 X
135	14.0	8.0000	7.0923	0.0305	0.9077	2.28R
161	26.0	5.0000	5.2200	0.0699	-0.2200	-0.56 X
215	14.0	8.0000	7.0923	0.0305	0.9077	2.28R
218	14.0	8.0000	7.0923	0.0305	0.9077	2.28R
223	22.0	5.0000	5.8441	0.0555	-0.8441	-2.14R
229	20.0	7.0000	6.1561	0.0486	0.8439	2.13R
231	20.0	7.0000	6.1561	0.0486	0.8439	2.13R
268	10.0	9.0000	7.7163	0.0242	1.2837	3.22R
270	12.0	9.0000	7.4043	0.0264	1.5957	4.01R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Pure error test - F = 11.99 P = 0.0000 DF(pure error) = 252
3 rows with no replicates

P3
paint (3)

Regression Analysis(Paint 3):

The regression equation is
Paint Rate = 8.88 - 0.123 Age

Predictor	Coef	StDev	T	P
Constant	8.8752	0.1642	54.06	0.000
Age	-0.12252	0.01098	-11.16	0.000

S = 0.3766 R-Sq = 91.9% R-Sq(adj) = 91.2%

Analysis of Variance

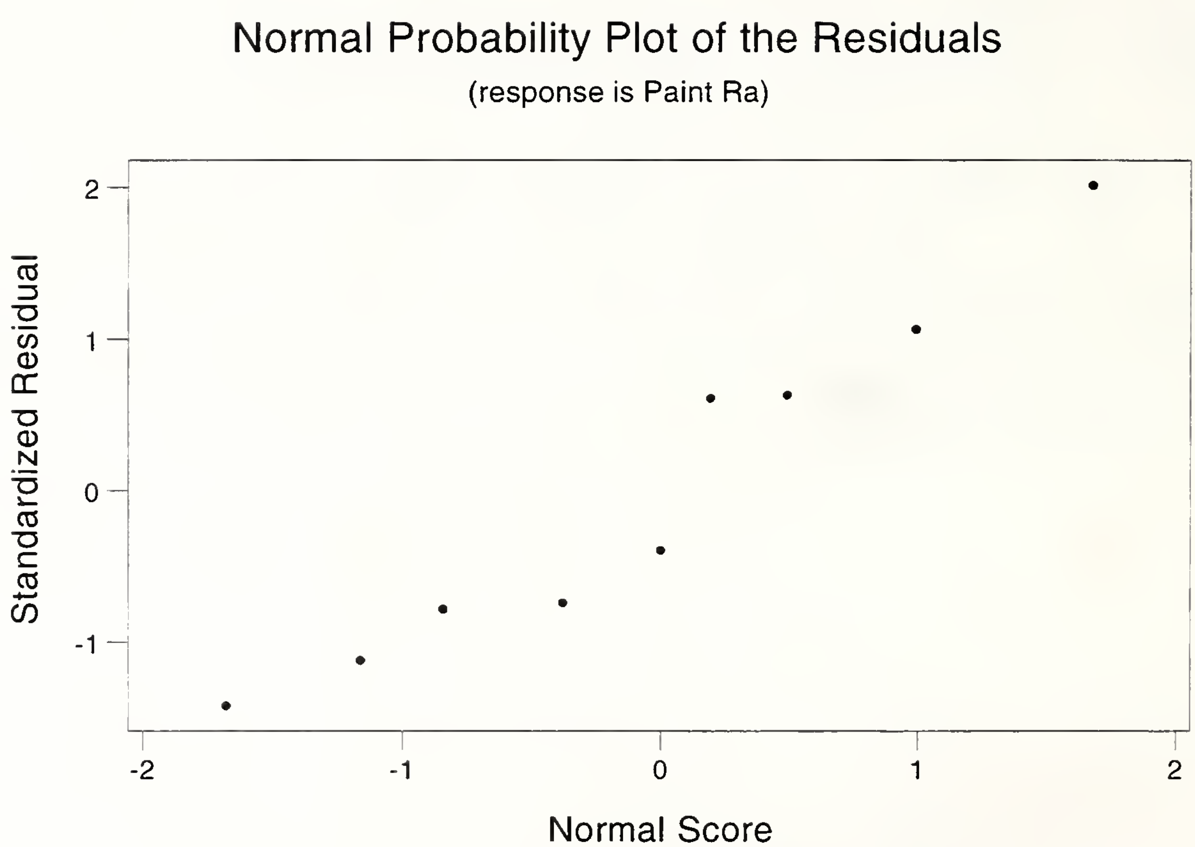
Source	DF	SS	MS	F	P
Regression	1	17.671	17.671	124.60	0.000
Error	11	1.560	0.142		
Total	12	19.231			

Unusual Observations

Obs	Age	Paint Ra	Fit	StDev Fit	Residual	St Resid
8	33.0	5.000	4.832	0.258	0.168	0.61 X
9	21.0	7.000	6.302	0.147	0.698	2.01R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Sum of squares for pure error is (nearly) zero.
Cannot do pure error test.


Normal Probability Plot of the Residuals
(response is Paint Ra)
Standardized Residual
2
1
0
-1
-2
-1
0
1
2
Normal Score


Regression Plot
Y = 8.87520 - 0.122517X
R-Sq = 0.919
Paint Ra
9
8
7
6
5
0
10
20
30
Age

General Linear Models Procedure

Dependent Variable: RATE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	66.199891	33.099946	164.78	0.0001
Error	57	11.450109	0.200879		
Corrected Total	59	77.650000			

R-Square	C.V.	Root MSE	RATE Mean
0.852542	6.542998	0.4482	6.8500

Source	DF	Type I SS	Mean Square	F Value	Pr > F
AGE	1	66.125592	66.125592	329.18	0.0001
Z1	1	0.074300	0.074300	0.37	0.5455

Source	DF	Type II SS	Mean Square	F Value	Pr > F
AGE	1	65.335605	65.335605	325.25	0.0001
Z1	1	0.074300	0.074300	0.37	0.5455

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	9.490484836	60.07	0.0001	0.15799522
AGE	-0.143085589	-18.03	0.0001	0.00793393
Z1	0.077639613	0.61	0.5455	0.12766070

Worksheet size: 100000 cells

Macro is running ... please wait

Regression

The regression equation is
y = 8.08 - 0.116 x

Predictor	Coef	StDev	T	P
Constant	8.08193	0.08592	94.07	0.000
x	-0.11616	0.01686	-6.89	0.000

S = 0.5195   R-Sq = 43.8%   R-Sq(adj) = 42.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	12.809	12.809	47.47	0.000
Error	61	16.461	0.270		
Total	62	29.270			

Macro is running ... please wait

Polynomial Regression

Y = 8.03525 - 1.32E-02X - 1.13E-02X**2
R-Sq = 0.461

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	13.4899	6.74496	25.6464	8.92E-09
Error	60	15.7799	0.26300		
Total	62	29.2698			

SOURCE	DF	Seq SS	F	P
Linear	1	12.8093	47.4688	3.58E-09
Quadratic	1	0.6807	2.58810	0.112919

Macro is running ... please wait

Polynomial Regression

Y = 8.04753 - 0.113698X + 1.70E-02X**2 - 1.86E-03X**3
R-Sq = 0.465

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	3	13.5979	4.53263	17.0640	4.28E-08
Error	59	15.6719	0.26563		
Total	62	29.2698			

SOURCE	DF	Seq SS	F	P
Linear	1	12.8093	47.4688	3.58E-09
Quadratic	1	0.6807	2.58810	0.112919
Cubic	1	0.1080	0.406518	0.526209

Regression Analysis

The regression equation is
Paint Rating = 8.08 - 0.116 3CoatAge

Predictor	Coef	StDev	T	P
Constant	8.08193	0.08592	94.07	0.000
3CoatAge	-0.11616	0.01686	-6.89	0.000

S = 0.5195 R-Sq = 43.8% R-Sq(adj) = 42.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	12.809	12.809	47.47	0.000
Error	61	16.461	0.270		
Total	62	29.270			

Unusual Observations

Obs	3CoatAge	Paint Ra	Fit	StDev Fit	Residual	St Resid
1	11.0	5.0000	6.8041	0.1454	-1.8041	-3.62R
2	7.0	6.0000	7.2688	0.0904	-1.2688	-2.48R
4	7.0	6.0000	7.2688	0.0904	-1.2688	-2.48R
13	0.0	7.0000	8.0819	0.0859	-1.0819	-2.11R

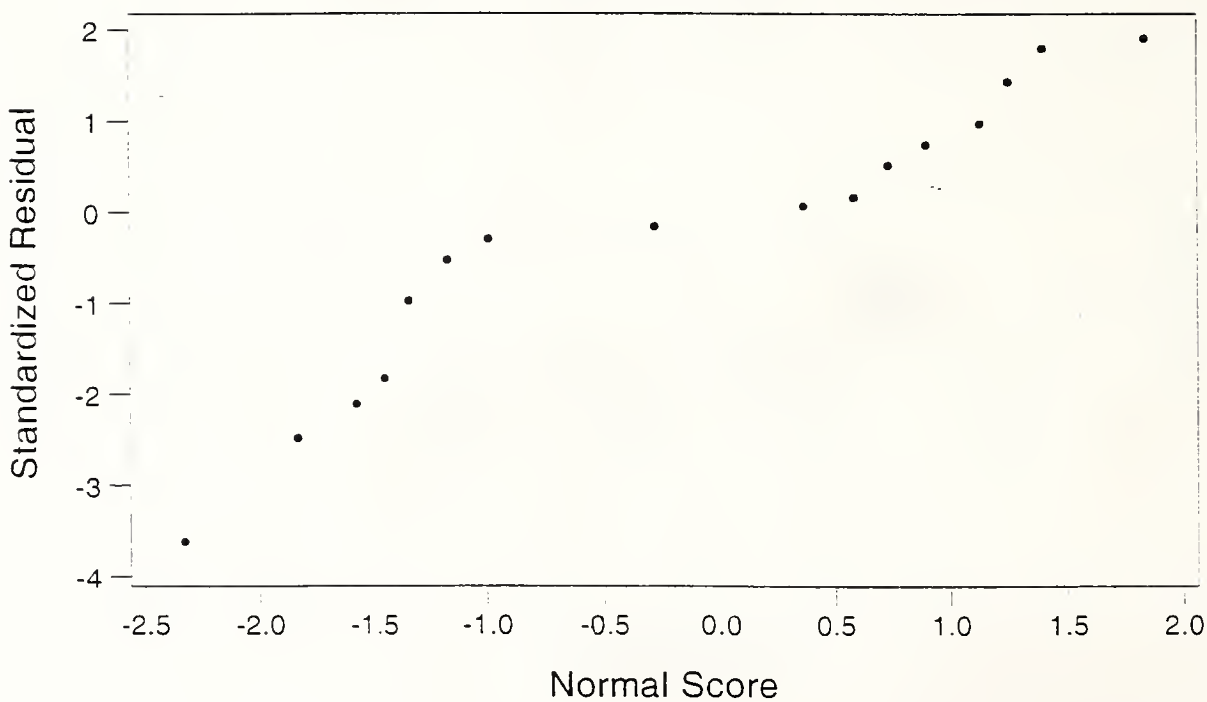
R denotes an observation with a large standardized residual

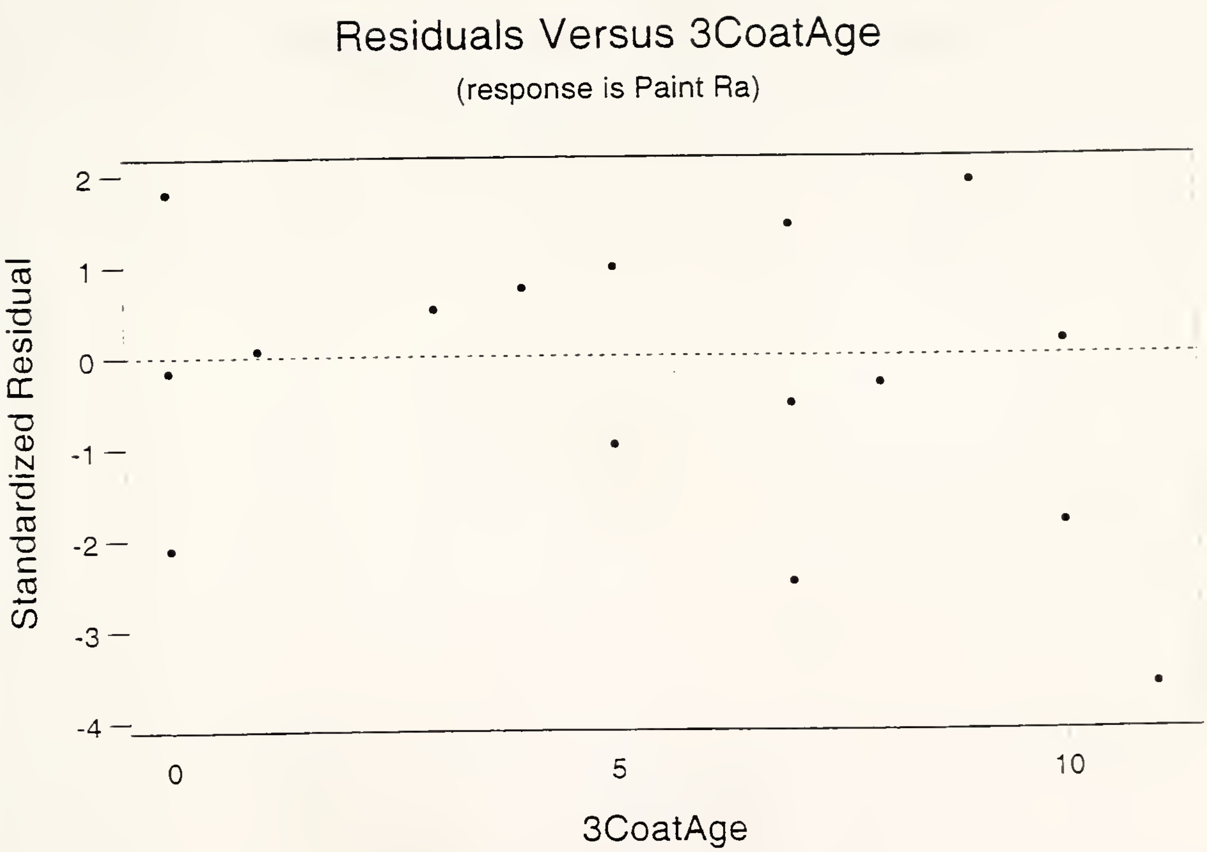
Pure error test - F = 8.31 P = 0.0000 DF(pure error) = 53
2 rows with no replicates

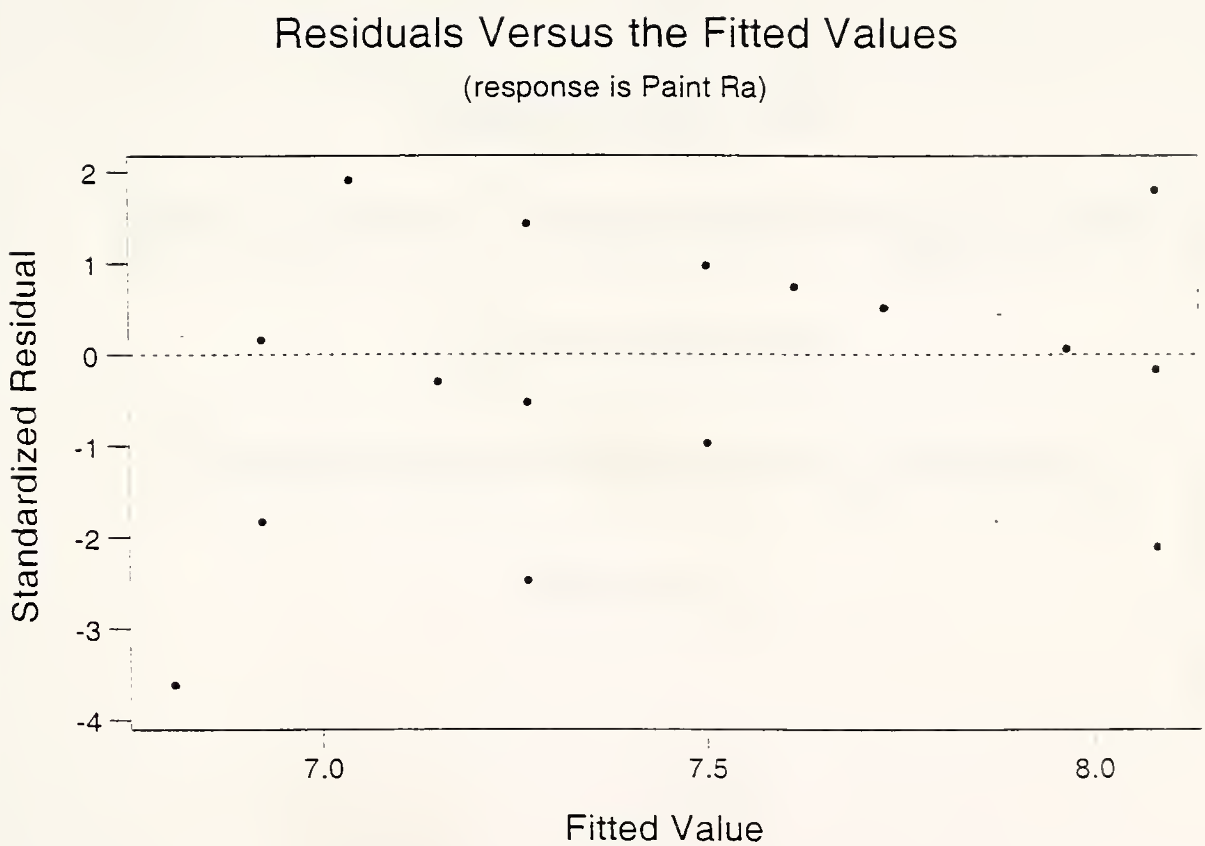

Regression Plot
Y = 8.08193 - 0.116163X
R-Sq = 0.438
9
8
7
6
5
Paint Ra
0
5
10
3CoatAge

Normal Probability Plot of the Residuals

(response is Paint Ra)




Residuals Versus 3CoatAge
(response is Paint Ra)
Standardized Residual
2
1
0
-1
-2
-3
-4
0
5
10
3CoatAge



Residuals Versus the Fitted Values
(response is Paint Ra)
Standardized Residual
2
1
0
-1
-2
-3
-4
7.0
7.5
8.0
Fitted Value

Appendix B:
Non-Linear Programming (NLP) Calculations (Transition Probability Matrix Values)

Transition Probabilities for Paint Type (1): Interstate

Age	P(1)	P(2)	P(3)	P(4)
0.0 - 4.0	*0.943*	*0.768*	*0.701*	*0.999*
5.0 - 8.0	*0.938*	*0.759*	*0.671*	*0.496*
9.0-12.0	*0.887*	*0.923*	*0.899*	*0.928*
13.0-16.0	*0.829*	*0.802*	*0.887*	*0.961*
17.0-20.0	*0.891*	*0.84*	*0.816*	*0.815*
21.0-24.0	*0.809*	*0.748*	*0.746*	*0.794*
25.0-28.0	*0.612*	*0.559*	*0.59*	*0.694*
29.0-32.0	*0.0001*	*0.0001*	*0.0001*	*0.12*

Transition Probabilities for Paint Type (1): State

Age	P(1)	P(2)	P(3)	P(4)
0.0 - 4.0	*0.998*	*0.479*	*0.999*	*0.476*
5.0 - 8.0	*0.922*	*0.943*	*0.001*	*0.927*
9.0-12.0	*0.933*	*0.862*	*0.554*	*0.692*
13.0-16.0	*0.917*	*0.804*	*0.647*	*0.788*
17.0-20.0	*0.842*	*0.724*	*0.673*	*0.818*
21.0-24.0	*0.683*	*0.632*	*0.633*	*0.677*
25.0-28.0	*0.0001*	*0.064*	*0.062*	*0.265*
29.0-32.0	*0.0001*	*0.0001*	*0.0001*	*0.0001*

Transition Probabilities for Paint Type (2): Interstate

Age	P(1)	P(2)	P(3)	P(4)
0.0 - 4.0	*0.958*	*0.0001*	*0.0001*	*0.0001*
5.0 - 8.0	*0.892*	*0.909*	*0.505*	*0.648*
9.0-12.0	*0.831*	*0.942*	*0.999*	*0.999*
13.0-16.0	*0.933*	*0.865*	*0.83*	*0.571*
17.0-20.0	*0.885*	*0.819*	*0.749*	*0.661*
21.0-24.0	*0.629*	*0.684*	*0.49*	*0.466*
25.0-28.0	*0.0001*	*0.0001*	*0.0001*	*0.0001*

Transition Probabilities for Paint Type (2): State

Age	P(1)	P(2)	P(3)	P(4)
0.0 - 4.0	*0.935*	*0.766*	*0.301*	*0.0001*
5.0 - 8.0	*0.923*	*0.815*	*0.553*	*0.703*
9.0-12.0	*0.888*	*0.797*	*0.611*	*0.879*
13.0-16.0	*0.844*	*0.74*	*0.674*	*0.85*
17.0-20.0	*0.799*	*0.617*	*0.548*	*0.824*
21.0-24.0	*0.0001*	*0.0001*	*0.999*	*0.21*
25.0-28.0	*0.0001*	*0.0001*	*0.0001*	*0.0001*

Transition Probabilities for Paint Type (3)

Age	P(1)	P(2)	P(3)	P(4)
0.0 - 4.0	*0.794*	*0.996*	*0.0001*	*0.0001*
5.0 - 8.0	*0.748*	*0.952*	*0.633*	*0.053*
9.0-12.0	*0.999*	*0.851*	*0.872*	*0.744*
13.0-16.0	*0.923*	*0.889*	*0.851*	*0.772*
17.0-20.0	*0.864*	*0.863*	*0.837*	*0.797*
21.0-24.0	*0.841*	*0.807*	*0.784*	*0.81*
25.0-28.0	*0.83*	*0.694*	*0.656*	*0.782*
29.0-32.0	*0.219*	*0.283*	*0.336*	*0.528*

Transition Probabilities for 3-Coat System OZEU

Age	P(1)	P(2)	P(3)	P(4)
0.0 - 4.0	0.0001	0.856	0.981	0.33
5.0 - 8.0	1	0.944	0.782	0.829
9.0-12.0	1	0.897	0.826	0.866
13.0-16.0	1	0.867	0.811	0.863
17.0-20.0	1	0.808	0.766	0.834
21.0-24.0	1	0.632	0.639	0.758
25.0-28.0	1	0.0001	0.129	0.36

Appendix C:
Markov Decision Process (MDP) Calculations

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(0 - 4 years Range)

V1	***0.3284***		***0.943***	***0.057***	***0***	***0***	***0***	
V2	***0.9686***		***0***	***0.768***	***0.232***	***0***	***0***	
V3	***1.4326***		***0***	***0***	***0.701***	***0.299***	***0***	
V4	***1.965***		***0***	***0***	***0***	***0.999***	***0.001***	
V5	***220.3***		***1***	***0***	***0***	***0***	***0***	
State 1								
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0.309681	0.05521	0	0	0	0.364891	1
State 2								
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0.743885	0.332363	0	0	1.076248	1
2	25	0	0.9686	0	0	0	25.9686	
State 3								
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	1.004253	0.587535	0	1.591788	1
2	25	0	0	1.21771	0.29475	0	26.51246	
3	110	0	0	1.4326	0	0	111.4326	
State 4								
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	1.963035	0.2203	2.183335	1
2	90	0	0	0	1.965	0	91.965	
3	180	0	0	0	1.965	0	181.965	
State 5								
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	0.3284	0	0	0	0	220.3284	4
The Result								
State I	1	2	3	4	5			
Decision K	*1*	*1*	*1*	*1*	*4*			
Policy Y1	*1*	*1*	*1*	*1*	*4*			

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(5 - 8 years Range)

V1	*38.1933*		*0.938*	*0.062*	*0*	*0*	*0*	
V2	*105.54*		*0*	*0.759*	*0.241*	*0*	*0*	
V3	*155.8056*		*0*	*0*	*0.671*	*0.329*	*0*	
V4	*208.4249*		*0*	*0*	*0*	*0.496*	*0.504*	
V5	*254.3739*		*1*	*0*	*0*	*0*	*0*	
State 1								
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	35.82532	6.54348	0	0	0	42.3688	1
State 2								
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	80.10486	37.54915	0	0	117.654	1
2	25	0	105.54	0	0	0	130.54	
State 3								
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	104.5456	68.57179	0	173.1173	1
2	25	0	0	132.4348	31.26374	0	188.6985	
3	110	0	0	155.8056	0	0	265.8056	
State 4								
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	103.3788	128.2044	231.5832	1
2	90	0	0	0	208.4249	0	298.4249	
3	180	0	0	0	208.4249	0	388.4249	
State 5								
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	38.1933	0	0	0	0	258.1933	4

The Result					
State I	1	2	3	4	5
Decision K	1	1	1	1	4
Policy Y1	1	1	1	1	4

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(9 - 12 years Range)

V1	***8.8077***		***0.887***	***0.113***	***0***	***0***	***0***	
V2	***17.4681***		***0***	***0.923***	***0.077***	***0***	***0***	
V3	***42.6748***		***0***	***0***	***0.899***	***0.101***	***0***	
V4	***89.6218***		***0***	***0***	***0***	***0.928***	***0.072***	
V5	***227.9269***		***1***	***0***	***0***	***0***	***0***	

State 1 K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	7.81243	1.973895	0	0	0	9.786325	1

State 2 K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	16.12306	3.28596	0	0	19.40902	1
2	25	0	17.4681	0	0	0	42.4681	

State 3 K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	38.36465	9.051802	0	47.41645	1
2	25	0	0	36.27358	13.44327	0	74.71685	
3	110	0	0	42.6748	0	0	152.6748	

State 4 K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	83.16903	16.41074	99.57977	1
2	90	0	0	0	89.6218	0	179.6218	
3	180	0	0	0	89.6218	0	269.6218	

State 5 K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	8.8077	0	0	0	0	228.8077	4

The Result

State I	1	2	3	4	5
Decision K	1	1	1	1	4
Policy Y1	1	1	1	1	4

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(13 - 16 years Range)

V1	*11.7264*		*0.829*	*0.171*	*0*	*0*	*0*	
V2	*19.3459*		*0*	*0.802*	*0.198*	*0*	*0*	
V3	*30.2022*		*0*	*0*	*0.887*	*0.113*	*0*	
V4	*59.8996*		*0*	*0*	*0*	*0.961*	*0.039*	
V5	*230.5538*		*1*	*0*	*0*	*0*	*0*	
State 1								
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	9.721186	3.308149	0	0	0	13.02933	1
State 2								
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	15.51541	5.980036	0	0	21.49545	1
2	25	0	19.3459	0	0	0	44.3459	
State 3								
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	26.78935	6.768655	0	33.55801	1
2	25	0	0	25.67187	8.98494	0	59.65681	
3	110	0	0	30.2022	0	0	140.2022	
State 4								
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	57.56352	8.991598	66.55511	1
2	90	0	0	0	59.8996	0	149.8996	
3	180	0	0	0	59.8996	0	239.8996	
State 5								
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	11.7264	0	0	0	0	231.7264	4

The Result

State I	1	2	3	4	5
Decision K	1	1	1	1	4
Policy Y1	1	1	1	1	4

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(17 - 20 years Range)

V1	*27.9045*		*0.891*	*0.109*	*0*	*0*	*0*	
V2	*56.3495*		*0*	*0.84*	*0.16*	*0*	*0*	
V3	*95.4811*		*0*	*0*	*0.816*	*0.184*	*0*	
V4	*153.1388*		*0*	*0*	*0*	*0.815*	*0.185*	
V5	*245.1141*		*1*	*0*	*0*	*0*	*0*	
State 1								
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	24.86291	6.142096	0	0	0	31.00501	1
State 2								
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	47.33358	15.27698	0	0	62.61056	1
2	25	0	56.3495	0	0	0	81.3495	
State 3								
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	77.91258	28.17754	0	106.0901	1
2	25	0	0	81.15894	22.97082	0	129.1298	
3	110	0	0	95.4811	0	0	205.4811	
State 4								
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	124.8081	45.34611	170.1542	1
2	90	0	0	0	153.1388	0	243.1388	
3	180	0	0	0	153.1388	0	333.1388	
State 5								
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	27.9045	0	0	0	0	247.9045	4

The Result

State I	1	2	3	4	5
Decision K	1	1	1	1	4
Policy Y1	1	1	1	1	4

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(21 - 24 years Range)

V1	*53.0988*		*0.809*	*0.191*	*0*	*0*	*0*		
V2	*83.9882*		*0*	*0.748*	*0.252*	*0*	*0*		
V3	*121.02*		*0*	*0*	*0.746*	*0.254*	*0*		
V4	*173.9596*		*0*	*0*	*0*	*0.794*	*0.206*		
V5	*267.7889*		*1*	*0*	*0*	*0*	*0*		
State 1									
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection	
1	0	42.95693	16.04175	0	0	0	58.99868	1	
State 2									
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection	
1	0	0	62.82317	30.49703	0	0	93.3202	1	
2	25	0	83.9882	0	0	0	108.9882		
State 3									
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection	
1	0	0	0	90.28089	44.18574	0	134.4666	1	
2	25	0	0	102.867	26.09394	0	153.9609		
3	110	0	0	121.02	0	0	231.02		
State 4									
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection	
1	0	0	0	0	138.1239	55.16451	193.2884	1	
2	90	0	0	0	173.9596	0	263.9596		
3	180	0	0	0	173.9596	0	353.9596		
State 5									
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection	
4	220	53.0988	0	0	0	0	273.0988	4	

The Result					
State I	1	2	3	4	5
Decision K	1	1	1	1	4
Policy Y1	1	1	1	1	4

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(25 - 28 years Range)

V1	*116.3918*		*0.612*	*0.388*	*0*	*0*	*0*	
V2	*149.7228*		*0*	*0.559*	*0.441*	*0*	*0*	
V3	*187.4459*		*0*	*0*	*0.59*	*0.41*	*0*	
V4	*238.2442*		*0*	*0*	*0*	*0.694*	*0.306*	
V5	*324.7526*		*1*	*0*	*0*	*0*	*0*	
State 1								
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	71.23178	58.09245	0	0	0	129.3242	1
State 2								
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	83.69505	82.66364	0	0	166.3587	1
2	25	0	149.7228	0	0	0	174.7228	
State 3								
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	110.5931	97.68012	0	208.2732	1
2	25	0	0	159.329	35.73663	0	220.0656	
3	110	0	0	187.4459	0	0	297.4459	
State 4								
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	165.3415	99.3743	264.7158	1
2	90	0	0	0	238.2442	0	328.2442	
3	180	0	0	0	238.2442	0	418.2442	
State 5								
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	116.3918	0	0	0	0	336.3918	4
The Result								
State I	1	2	3	4	5			
Decision K	1	1	1	1	4			
Policy Y1	1	1	1	1	4			

Application of Markov Decision Process (MDP) for Interstate Paint type (1)
(29 - 32 years Range)

V1	*341.0915*		*0.0001*	*0.9999*	*0*	*0*	*0*	
V2	*378.9943*		*0*	*0.0001*	*0.9999*	*0*	*0*	
V3	*421.109*		*0*	*0*	*0.0001*	*0.9999*	*0*	
V4	*467.9036*		*0*	*0*	*0*	*0.12*	*0.88*	
V5	*526.9823*		*1*	*0*	*0*	*0*	*0*	
State 1								
K	C1k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0.034109	378.9564	0	0	0	378.9905	1
State 2								
K	C2k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0.037899	421.0669	0	0	421.1048	
2	25	0	378.9943	0	0	0	403.9943	2
State 3								
K	C3k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0.042111	467.8568	0	467.8989	
2	25	0	0	357.9427	70.18554	0	453.1282	2
3	110	0	0	421.109	0	0	531.109	
State 4								
K	C4k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
1	0	0	0	0	56.14843	463.7444	519.8929	1
2	90	0	0	0	467.9036	0	557.9036	
3	180	0	0	0	467.9036	0	647.9036	
State 5								
K	C5k	P1*V1	P2*V2	P3*V3	P4*V4	P5*V5	Value	Selection
4	220	341.0915	0	0	0	0	561.0915	4

The Result

State I	1	2	3	4	5
Decision K	1	2	2	1	4
Policy Y1	1	1	1	1	4

Appendix D:

Economic Analysis Calculations

Interstate paint type (1) Scenario # 1

n =	i =	F =			
30	0.035	220	$A = A(A/F,i,N) + A(A/P,i,N) = F \dfrac{i * L}{(1+i)^N - 1}$	+	$A(A/P,i,N)$
Inflation factor =		2.806794			
AI11 =	***$33.57***	includes the first value.			

Interstate paint type (1) Scenario # 2

n =	i =	F =	
10	0.035	25	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	195		
Inflation factor =		1.410599	
AI12 =	***$15.27***	includes the first value.	

Interstate paint type (1) Scenario # 3

n =	i =	F =	
18	0.035	50	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	170		
Inflation factor =		1.857489	
AI13 =	***$19.70***	includes the first value.	

Interstate paint type (1) Scenario # 4

n =	i =	F =	
18	0.035	110	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	110		
Inflation factor =		1.857489	
AI14 =	***$23.68***	includes the first value.	

Interstate paint type (1) Scenario # 5

n =	i =	F =	
24	0.035	180	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	$310.62		
Inflation factor =		2.283328	
PV1 =	$78.83		
PV2 =	$11.79		
AI15 =	***$28.43***	includes the first value.	

Interstate paint type (2) Scenario # 1

n =	i =	F =	
26	0.035	180	$A = A(A/F,i,N) + A(A/P,i,N) = F \dfrac{i * L}{(1+i)^N - 1} + A(A/P,i,N)$

Inflation factor = 2.445959

AI21 = ***$26.07*** includes the first value.

Interstate paint type (2) Scenario # 2

n =	i =	F =	
9	0.035	20	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	160		

Inflation factor = 1.362897

AI22 = ***$12.32*** includes the first value.

Interstate paint type (2) Scenario # 3

n =	i =	F =	
18	0.035	40	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	140		

Inflation factor = 1.857489

AI13 = ***$16.06*** includes the first value.

Interstate paint type (2) Scenario # 4

n =	i =	F =	
18	0.035	100	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	80		
Inflation factor =		1.857489	
AI14 =	***$20.04***	includes the first value.	

Interstate paint type (2) Scenario # 5

n =	i =	F =	
23	0.035	150	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	$257.42		
Inflation factor =		2.206114	
PV1 =	$67.99		
PV2 =	$9.43		
AI15 =	***$22.77***	includes the first value.	

State paint type (1) Scenario # 1

n =	i =	F =	
28	0.035	220	$A = A(A/F,i,N) + A(A/P,i,N) = F \dfrac{i * L}{(1+i)^N - 1} + A(A/P,i,N)$

Inflation factor = 2.620172

AS11 = ***$32.63*** includes the first value.

State paint type (1) Scenario # 2

n =	i =	F =	
14	0.035	25	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	195		

Inflation factor = 1.618695

AS12 = ***$16.36*** includes the first value.

State paint type (1) Scenario # 3

n =	i =	F =	
20	0.035	50	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	170		

Inflation factor = 1.989789

AS13 = ***$20.56*** includes the first value.

State paint type (1) Scenario # 4

n =	i =	F =	
20	0.035	110	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	110		
Inflation factor =		1.989789	
AS14 =	***$24.17***	includes the first value.	

State paint type (1) Scenario # 5

n =	i =	F =	
24	0.035	180	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	$310.62		
Inflation factor =		2.283328	
PV1 =	$78.83		
PV2 =	$11.79		
AS15 =	***$28.43***	includes the first value.	

State paint type (2) Scenario # 1

n =	i =	F =	
25	0.035	180	$A = A(A/F,i,N) + A(A/P,i,N) = F \dfrac{i * L}{(1 + i)^N - 1} + A(A/P,i,N)$

Inflation factor = 2.363245

AS21 = ***$25.81*** includes the first value.

State paint type (2) Scenario # 2

n =	i =	F =	
9	0.035	20	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	160		

Inflation factor = 1.362897

AS22 = ***$12.32*** includes the first value.

State paint type (2) Scenario # 3

n =	i =	F =	
15	0.035	40	$A = A1(A/P,i,N1) + A2(A/P,i,N2)$
P =	140		

Inflation factor = 1.675349

AS23 = ***$15.22*** includes the first value.

State paint type (2) Scenario # 4

n =	i =	F =	
15	0.035	100	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	80		
Inflation factor =		1.675349	
AS24 =	***$19.92***	includes the first value.	

State paint type (2) Scenario # 5

n =	i =	F =	
20	0.035	150	***A = A1(A/P,i,N1) +A2(A/P,i,N2)***
P =	$30.00		
Inflation factor =		1.989789	
AS25 =	***$23.39***	includes the first value.	

3-Coat/Lead

Do Nothing Untill State 5 and then Complete Repaint

Age (yrs)	Cost initial/ft^2	Interest Rate	
25	*$4.00*	*7.00%*	
Inflation Factor		*0.035*	

Item	n	Infl.Factor	Cost
Present Value for initial Cost:	*0*	*1*	*$4.00*
Present Value for Fv after 25 yrs:	*25*	*2.363245*	*$1.74*
Present Value for Fv after 50 yrs:	*50*	*5.5849269*	*$0.76*
Total Present Value TPV/ft^2			***$6.50***
EUAC			***$0.46***

3-Coat/Lead
Spot Repairs at State 3

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*$4.00*	*$4.00*
Present Value for Fv after 15 yrs:	*15*	*1.675348831*	*$1.50*	*$0.91*
Present Value for Fv after 30 yrs:	*30*	*2.806793705*	*$1.50*	*$0.55*
Present Value for Fv after 45 yrs:	*45*	*4.702358551*	*$1.50*	*$0.34*
Total Present Value TPV/ft^2				***$5.80***
EUAC				***0.413112***

3-Coat/Lead
Spot Repairs at State 4

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*$4.00*	*$4.00*
Present Value for Fv after 20 yrs:	*20*	*1.989789*	*$2.50*	*$1.29*
Present Value for Fv after 40 yrs:	*40*	*3.95926*	*$2.50*	*$0.66*
Total Present Value TPV/ft^2				***$5.95***
EUAC				***0.423565***

3-Coat/Zinc

Do Nothing Untill State 5 and then Complete Repaint

Age (yrs)	Cost initial/ft^2	Interst Rate	
25	$2.80	7.00%	
Inflation Factor		0.035	
Item	n	Infl.Factor	Cost
Present Value for initial Cost:	0	1	$2.80
Present Value for Fv after 25 yrs:	25	2.363245	$1.22
Present Value for Fv after 50 yrs:	50	5.584927	$0.53
Total Present Value TPV/ft^2			**$4.55**
EUAC			**$0.32**

3-Coat/Zinc
Spot Repairs at State 3

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*\$2.80*	*\$2.80*
Present Value for Fv after 15 yrs:	*15*	*1.675348831*	*\$1.00*	*\$0.61*
Present Value for Fv after 30 yrs:	*30*	*2.806793705*	*\$1.00*	*\$0.37*
Present Value for Fv after 45 yrs:	*45*	*4.702358551*	*\$1.00*	*\$0.22*
Total Present Value TPV/ft^2				***\$4.00***
EUAC				***0.284905***

3-Coat/Zinc
Spot Repairs at State 4

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*$2.80*	*$2.80*
Present Value for Fv after 20 yrs:	*20*	*1.989789*	*$2.00*	*$1.03*
Present Value for Fv after 40 yrs:	*40*	*3.95926*	*$2.00*	*$0.53*
Total Present Value TPV/ft^2				***$4.36***
EUAC				***0.31036***

Disruption Cost Calculations

User Cost Lane/day	Road of 4-lanes	Full Paint 4 days	Spot Repairs for 3 days	Spot Repairs for 2 days
(1) *$45,000*	*$180,000*	*$720,000*	*$540,000*	*$360,000*
Average Bridge Weight =			*177 tons*	*from data*
Conversion factor =			*115 ft^2/ton*	
User Cost Lane/day $/ton	**Road of 4-lanes $/ton**	**Full Paint 4 days $/ton**	**Spot Repairs for 3 days $/ton**	**Spot Repairs for 2 days $/ton**
$254.24	*$1,016.95*	*$4,067.80*	*$3,050.85*	*$2,033.90*
User Cost Lane/day $/ft²	**Road of 4-lanes $/ft²**	**Full Paint 4 days $/ft²**	**Spot Repairs for 3 days $/ft²**	**Spot Repairs for 2 days $/ft²**
$2.21	*$8.84*	*$35.37*	*$26.53*	*$17.69*

(1) Mr. Mike Long[35], INDOT, gave this approximated number as:
the user cost = $45,000/lane/day.

3-Coat/Lead

Do Nothing Untill State 5 and then Complete Repaint

Age (yrs)	Cost initial/ft^2	Interest Rate	
25	*$4.00*	*7.00%*	

Inflation Factor	*0.035*		
Item	**n**	**Infl.Factor**	**Cost**
Present Value for initial Cost	*0*	*1*	*$39.37*
Present Value for Fv after 25 yrs	*25*	*2.363245*	*$17.14*
Present Value for Fv after 50 yrs	*50*	*5.5849269*	*$7.46*
Total Present Value TPV/ft^2			***$63.98***
EUAC			***$4.56***

3-Coat/Lead
Spot Repairs at State 3

Interest Rate				
7.00%				
Inflation Factor	0.035			

Item	n	Infl.Factor	Cost	Total
Present Value for initial Cost	*0*	*1*	*$4.00*	*$21.69*
Present Value for Fv after 15 yrs	*15*	*1.675348831*	*$1.50*	*$11.65*
Present Value for Fv after 30 yrs	*30*	*2.806793705*	*$1.50*	*$7.07*
Present Value for Fv after 45 yrs	*45*	*4.702358551*	*$1.50*	*$4.30*
Total Present Value TPV/ft^2				***$44.71***
EUAC				***$3.18***

3-Coat/Lead
Spot Repairs at State 4

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost	*0*	*1*	*$4.00*	*$30.53*
Present Value for Fv after 20 yrs	*20*	*1.989789*	*$2.50*	*$14.93*
Present Value for Fv after 40 yrs	*40*	*3.95926*	*$2.50*	*$7.68*
Total Present Value TPV/ft^2				***$53.13***
EUAC				***$3.78***

3-Coat/Zinc
Do Nothing Untill State 5 and then Complete Repaint

Age (yrs)	Cost initial/ft^2	Interst Rate	
25	$2.80	7.00%	
Inflation Factor		0.035	
Item	n	Infl.Factor	Cost
Present Value for initial Cost	0	1	$38.17
Present Value for Fv after 25 yrs	25	2.363245	$16.62
Present Value for Fv after 50 yrs	50	5.584927	$7.24
Total Present Value TPV/ft^2			***$62.03***
EUAC			***$4.42***

3-Coat/Zinc
Spot Repairs at State 3

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost	*0*	*1*	*$2.80*	*$20.49*
Present Value for Fv after 15 yrs	*15*	*1.675348831*	*$1.00*	*$11.35*
Present Value for Fv after 30 yrs	*30*	*2.806793705*	*$1.00*	*$6.89*
Present Value for Fv after 45 yrs	*45*	*4.702358551*	*$1.00*	*$4.18*
Total Present Value TPV/ft^2				***$42.91***
EUAC				***$3.06***

3-Coat/Zinc
Spot Repairs at State 4

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost	*0*	*1*	*$2.80*	*$29.33*
Present Value for Fv after 20 yrs	*20*	*1.989789*	*$2.00*	*$14.67*
Present Value for Fv after 40 yrs	*40*	*3.95926*	*$2.00*	*$7.54*
Total Present Value TPV/ft^2				***$51.54***
EUAC				***$3.67***

3-Coat/Lead
Do Nothing Untill State 5 and then Complete Repaint

Age (yrs)	Cost initial/ft^2	Interst Rate	
25	*$4.00*	*7.00%*	
Inflation Factor		*0.035*	

Item	n	Infl.Factor	Cost
Present Value for initial Cost:	*0*	*1*	*$720,000.00*
Present Value for Fv after 25 yrs:	*25*	*2.363245*	*$313,506.68*
Present Value for Fv after 50 yrs:	*50*	*5.584927*	*$136,508.94*
Total Present Value TPV/ft^2			***$1,170,015.62***
EUAC			***$83,339.31***

3-Coat/Lead
Spot Repairs at State 3

Interest Rate				
7.00%				
Inflation Factor	*0.035*			

Item	n	Infl.Factor	Cost	Total
Present Value for initial Cost:	*0*	*1*	*$4.00*	*$360,000.00*
Present Value for Fv after 15 yrs:	*15*	*1.675348831*	*$1.50*	*$218,600.47*
Present Value for Fv after 30 yrs:	*30*	*2.806793705*	*$1.50*	*$132,739.34*
Present Value for Fv after 45 yrs:	*45*	*4.702358551*	*$1.50*	*$80,602.45*
Total Present Value TPV/ft^2				***$791,942.26***
EUAC				***$56,409.43***

3-Coat/Lead
Spot Repairs at State 4

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*$4.00*	*$540,000.00*
Present Value for Fv after 20 yrs:	*20*	*1.989789*	*$2.50*	*$277,667.60*
Present Value for Fv after 40 yrs:	*40*	*3.95926*	*$2.50*	*$142,776.47*
Total Present Value TPV/ft^2				***$960,444.07***
EUAC				***$68,411.69***

3-Coat/Zinc

Do Nothing Untill State 5 and then Complete Repaint

Age (yrs)	Cost initial/ft^2	Interst Rate	
25	*$2.80*	*7.00%*	
Inflation Factor		*0.035*	
Item	n	Infl.Factor	Cost
Present Value for initial Cost:	*0*	*1*	*$720,000.00*
Present Value for Fv after 25 yrs:	*25*	*2.363245*	*$313,506.68*
Present Value for Fv after 50 yrs:	*50*	*5.584927*	*$136,508.94*
Total Present Value TPV/ft^2			***$1,170,015.62***
EUAC			***$83,339.31***

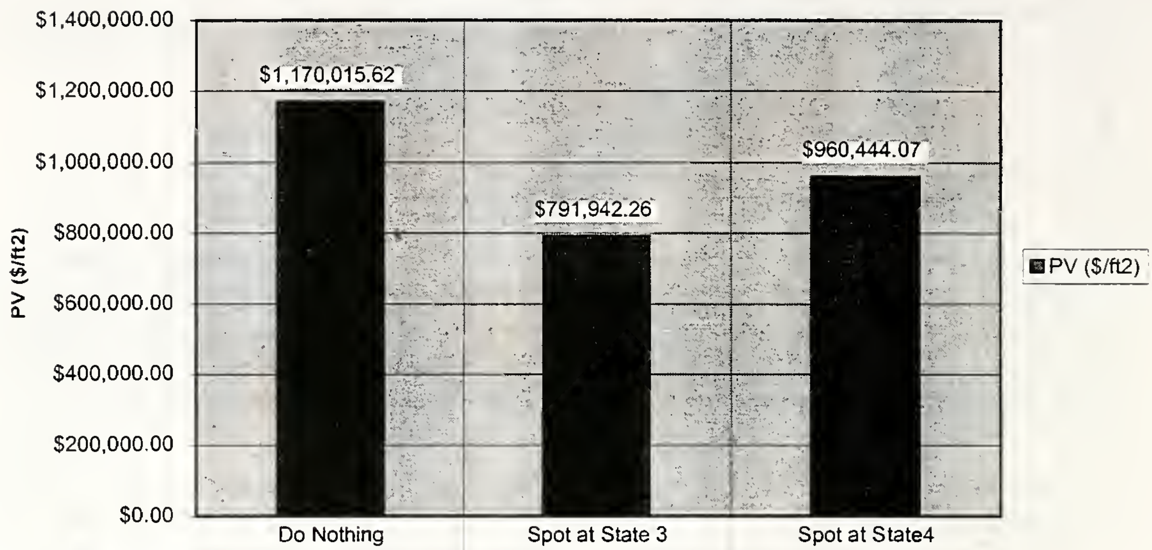
3-Coat/Zinc
Spot Repairs at State 3

Interest Rate				
7.00%				
Inflation Factor	*0.035*			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*$2.80*	*$360,000.00*
Present Value for Fv after 15 yrs:	*15*	*1.675348831*	*$1.00*	*$218,600.47*
Present Value for Fv after 30 yrs:	*30*	*2.806793705*	*$1.00*	*$132,739.34*
Present Value for Fv after 45 yrs:	*45*	*4.702358551*	*$1.00*	*$80,602.45*
Total Present Value TPV/ft^2				***$791,942.26***
EUAC				***$56,409.43***

3-Coat/Zinc
Spot Repairs at State 4

Item	n	Infl.Factor	Cost	Total
Interest Rate 7.00%				
Inflation Factor	0.035			
Item	**n**	**Infl.Factor**	**Cost**	**Total**
Present Value for initial Cost:	*0*	*1*	*$2.80*	*$540,000.00*
Present Value for Fv after 20 yrs:	*20*	*1.989789*	*$2.00*	*$277,667.60*
Present Value for Fv after 40 yrs:	*40*	*3.95926*	*$2.00*	*$142,776.47*
Total Present Value TPV/ft²				***$960,444.07***
EUAC				***$68,411.69***

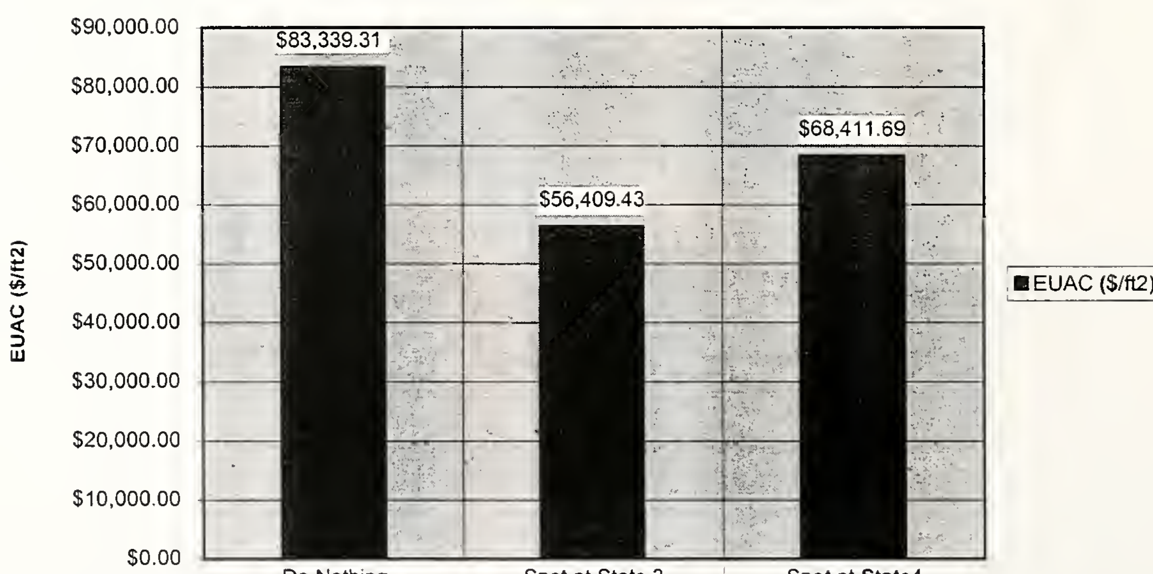
Figure A.1: PV ($/ft2) Comparison of 3-COAT System/ZINC for Disruption Cost Only



	Do Nothing	Spot at State 3	Spot at State4
PV ($/ft2)	$1,170,015.62	$791,942.26	$960,444.07

Different 3-Coat System Scenarios

Figure A.2: EUAC ($/ft2) for 3-COAT System/ ZINC for Disruption Cost Only



	Do Nothing	Spot at State 3	Spot at State4
EUAC ($/ft2)	$83,339.31	$56,409.43	$68,411.69

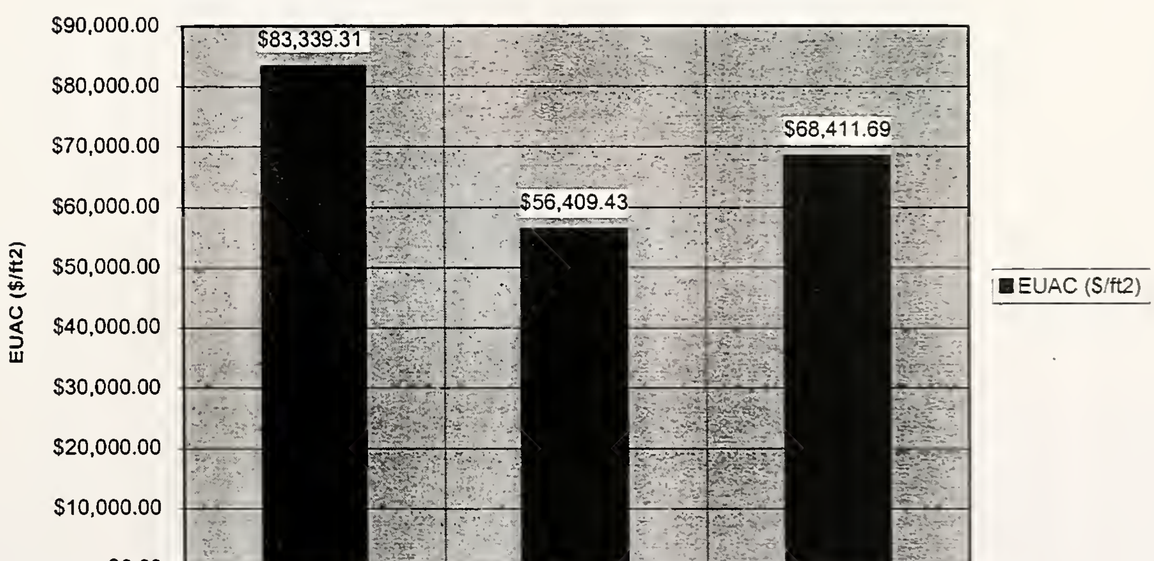
Different 3-Coat System Scenarios

Figure A.3: PV ($/ft2) Comparison of 3 COAT System /Lead for Disruption Cost Only

	Do Nothing	Spot at State 3	Spot at State4
PV ($/ft2)	$1,170,015.62	$791,942.26	$960,444.07

Different 3-Coat System Scenarios

Figure A.4: EUAC ($/ft2) for 3-COAT System / Lead for Disruption Cost Only



	Do Nothing	Spot at State 3	Spot at State4
EUAC ($/ft2)	$83,339.31	$56,409.43	$68,411.69

Different 3-Coat System Scenarios

Appendix E:
Condition Rating Standards

Table E.1: Comparison between INDOT and MDOT Standards Used for Paint Inspection.

Code	MDOT Description*	National Bridge Institute (NBI) Description (INDOT use)**
N	NA	NOT APPLICABLE.
9	New condition.	EXCELLENT CONDITION.
8	Good condition, no repairs needed.	VERY GOOD CONDITION – no problems noted
7	Generally good condition, faint/light rusting, no scale.	GOOD CONDITION – some minor problems.
6	Fair condition, less than 1% light rusting, slight blistering.	SATISFACTORY CONDITION – structural elements show some minor deterioration.
5	Generally fair condition, between 2 to 5% rusting and light scale.	FAIR CONDITION – all primary structural elements are sound but may have minor section loss, cracking, spalling or scour.
4	Marginal condition, between 5 to 10% rusting and/or heavy scale.	POOR CONDITION – advanced section loss, deterioration, spalling or scour.
3	Poor condition, between 10 to 15% rust, minor section loss.	SERIOUS CONDITION – loss of section, deterioration, spalling or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or sheer cracks in concrete may be present.
2	Paint first year, rust over 15% of beam, loss of section at beam ends.	CRITICAL CONDITION – advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored it may be necessary to close the bridge until corrective action is taken.
1	Paint immediately, rust over 20% of beam, loss of section at beam ends.	"IMMINENT" FAILURE CONDITION – major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic but corrective action may put back in light service.
0	NA	FAILED CONDITION – out of service – beyond corrective action.

* Collected from Ms. Glenn Bukoski (MDOT) (bukoskig@state.mi.us)

** Adapted from National Bridge Institute (NBI) Standards report no. FHWA-PD-96-001 page no. 38.