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Abstract: This study deals with robust design optimization strategies in aerodynamics, by considering geometric parameters as uncertainty factors, with application to transonic airfoil design. Mean and standard deviation of the aerodynamic coefficients are considered as cost functions for a multi-objective optimization problem. Statistical moments are evaluated using Monte-Carlo simulations (MCS), on the basis of Radial Basis Functions (RBF) surrogate models. The multi-objective optimization is achieved using the Multiple-Gradient Descent Algorithm (MGDA), which permits to find a descent direction for all criteria simultaneously, starting from a set of initial design points. The airfoil shape parameterization is carried out using the PARSEC approach to define significant design parameters. Moreover, the ANOVA (ANalysis Of Variance) technique is used to identify the most relevant parameters for aerodynamic criteria. The proposed approach is illustrated on a practical robust design problem involving four statistical cost functions.

Key-words: Robust design optimization, parameterization, surrogate models, Multi-gradient descent algorithm

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Résumé : Cette étude concerne les stratégies d'optimisation robuste en aérodynamique, en considérant des paramètres géométriques comme variables aléatoires, avec application à la conception de profils transsoniques. La moyenne et l'écart-type des coefficients aérodynamiques sont considérés comme des fonctions de coûts pour un problème d'optimisation multiobjectif. Les moments statistiques sont évalués par simulations de Monte-Carlo (MCS), sur la base de modèles approchés de type fonctions à bases radiales. L'optimisation multiobjective est réalisée à l'aide de l'algorithme de descente à gradient multiple, qui permet de trouver une direction de descente pour tous les critères simultanément, à partir d'un ensemble de points de conception initiaux. La paramétrisation de la forme des profils est effectuée par l'approche PARSEC pour définir des paramètres de conception ayant une signification pour l'aérodynamique. De plus, la technique ANOVA (analyse de la variance) est utilisée pour identifier les paramètres les plus importants pour les critères aérodynamiques. L'approche proposée est illustrée pour un problème pratique de conception robuste impliquant quatre fonctions de coûts statistiques.

Mots clés : conception robuste, paramétrization, modèles approchés, algorithme de descente à gradient multiple

1. Introduction

In the last decades, Computational Fluid Dynamics (CFD) has known a significant development and is now used for practical aerodynamic shape optimization. The optimization methodologies have also matured with the industrial context and Multidisciplinary Design Optimization (MDO) is now widely used and essential in the aerospace field. Presently, a major issue is robust design, for which uncertainties are taken into account during the design phase.

There are various approaches in aerodynamic shape optimization, which are based on different points of view. Usually in aerodynamic shape design, optimum design parameters are determined to minimize or maximize cost functions. In this classical deterministic approach, the optimized solutions would cause performance losses due to uncertainty factors such as manufacturing tolerance, neglected details, structural deformation, icing, as well as operational parameters. There is another viewpoint, for which stability of cost functions with respect to design variables or operational parameters is also important while optimizing the cost functions. Robust design mentioned above aims at considering off-design conditions as fluctuations of operational parameters [1-4].

The objective of this study is to take into account the uncertainty related to geometrical variables during the design optimization procedure. Therefore, design variables are regarded as uncertainty factors at the same time. In this report, uncertainty analysis and robust optimization applied to aerodynamic airfoil design in transonic flow are presented. Stochastic models are introduced to quantify the impact of uncertainty factors on aerodynamic coefficients. The parameterization of airfoil geometries, which is related to quantifications of uncertainty factors, is also an important issue. Many parameterization methods have been proposed so far, such as NURBS using control points, CST [5], PARSEC [6], etc. To consider manufacturing process, NURBS using control points would be more practical. However, to investigate uncertainty factors from aerodynamic viewpoint, PARSEC technique is preferred since the related design parameters have a significant role for aerodynamic performance.

2. Problem statement

A general description of the problem is provided in this section. To be as general as possible, we make the assumption that uncertainty arises from geometrical design variables x as well as operational variables a. Thus, we consider the design variables and operational variables as random variables X and A, characterized by two uncorrelated probability density functions f_X and f_A defined over Ω_X and Ω_A. For the sake of simplicity, we assume that A is perfectly known and X is Gaussian with mean μ_X, which will be adjusted during the optimization, and fixed standard deviation (Std) σ_X.

In a statistical framework, we intend to solve the design problem by accounting for statistical moments of the cost functional, such as the expectation μ_j and the Std σ_j:

$$\begin{aligned} \mu_j &= E\left[j(x,a)\right] = \int_{\Omega_X}\int_{\Omega_A} j(\mathbf{x},\mathbf{a}) f_X(\mathbf{x}) f_A(\mathbf{a}) d\mathbf{x} d\mathbf{a} \\ \sigma_j &= \sqrt{E\left[(j(x,a)-\mu_j)^2\right]} = \sqrt{\int_{\Omega_X}\int_{\Omega_A} \left(j(\mathbf{x},\mathbf{a})-\mu_j\right)^2 f_X(\mathbf{x}) f_A(\mathbf{a}) d\mathbf{x} d\mathbf{a}} \end{aligned} \tag{1}$$

A typical multi-criterion strategy would consist in minimizing mean and Std, with respect to x:

$$\begin{aligned} &Minimize \quad \mu_j(\mathbf{x}) \\ &Minimize \quad \sigma_j(\mathbf{x}) \end{aligned} \tag{2}$$

Thus, the robust counterpart of a classical optimization with a cost function j, is the multi-objective optimization with two cost functions μ_j and σ_j. The estimation of statistical moments (1) and (2) are presented in Section 3.2.

To solve the multi-objective optimization problem, the Multiple-Gradient Descent Algorithm (MGDA) developed in Opale Project-Team [7] is employed. Details on the underlying theory of MGDA can be found in [7]. Some practical details to obtain the Pareto front solutions and some applications are presented in [8]. Basically, MGDA is an extension of the classical steepest-descent method in case of several criteria. It consists in combining the descent directions related to the different cost functions to define a direction, which improves all criteria simultaneously. Here, we just mention two key elements of MGDA: the calculation of the search direction and the step length.

1. The search direction vector is determined by
   - Explicit geometric features when only two cost functions are considered [7, 8]
   - Gram-Schmidt Orthogonalization of the gradient vector set when the number of cost function is more than two [9]

2. The step length is determined by the golden section method.

Note that the use of Gram-Schmidt Orthogonalization requires a number of objective functions lower than that of design variables.

The approach used for uncertainty qualification and the strategy proposed for robust design are described herein. First of all, we define the search domain for the design variables, which will also impact the evaluation of the statistical moments of the cost functions, as explained later. Then, initial sampling points are determined by Latin Hypercube Sampling (LHS) in the selected search domain. The aerodynamic coefficients Cl and Cd are obtained by CFD calculations using the Num3sis platform, which is developed in OPALE team [10]. In classical optimization cases, these aerodynamic coefficients are considered directly as the cost functions. Alternatively, in robust optimization cases, the mean and Std of each aerodynamic coefficient are considered as objective functions. In this study, for the sake of simplicity, design variables only are considered as uncertain factors. Thus, uncertainty related to operational conditions is not addressed here. As first step to investigate geometric uncertainty factors from aerodynamic viewpoints, the design variables are represented by PARSEC parameterization [6]. Then, surrogate models are constructed for each cost function to estimate the statistical moments. Before solving the optimization problem, the contributions of each design variable to the statistical moments of the cost functions are quantified by ANOVA (ANalysis Of VAriance). The motivation to use ANOVA is to determine if some design variables are negligible and to select a relevant design space. The details on the application of ANOVA are described later.

Flow charts of classical optimization and robust optimization are shown in Figures 1 and 2, respectively. Details on each method in the flow charts are presented in the next section. Firstly, initial sampling points **x** are generated by LHS. The parameterization of airfoils as design variables is conducted by PARSEC. CFD calculations are carried out to obtain aerodynamic coefficients. Then, a response surface is constructed for each cost function to lead the optimization process. Then, for the classical optimization, MGDA is used to update the design parameters and new cost function values for each new point are obtained by CFD calculations. This process is iterated until convergence to their Pareto front.

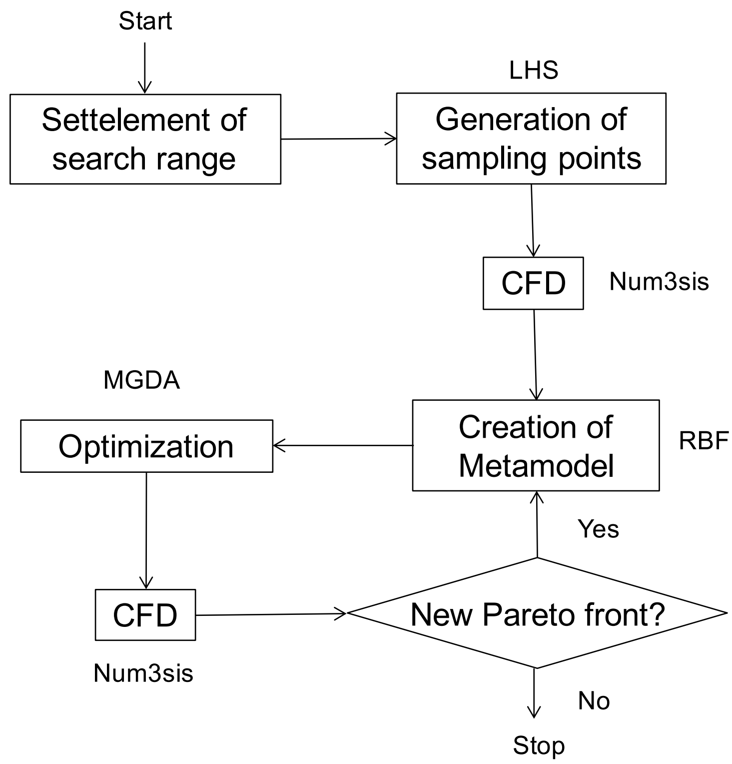

FIGURE 1: Flow chart of classical optimization

In the robust optimization process, two steps are added, compared to the classical optimization process. First, the statistical moments of the aerodynamic coefficients are calculated by MCS on the basis of surrogate models. Second, ANOVA is introduced for a possible reconsideration of the design space and modification of the search range.

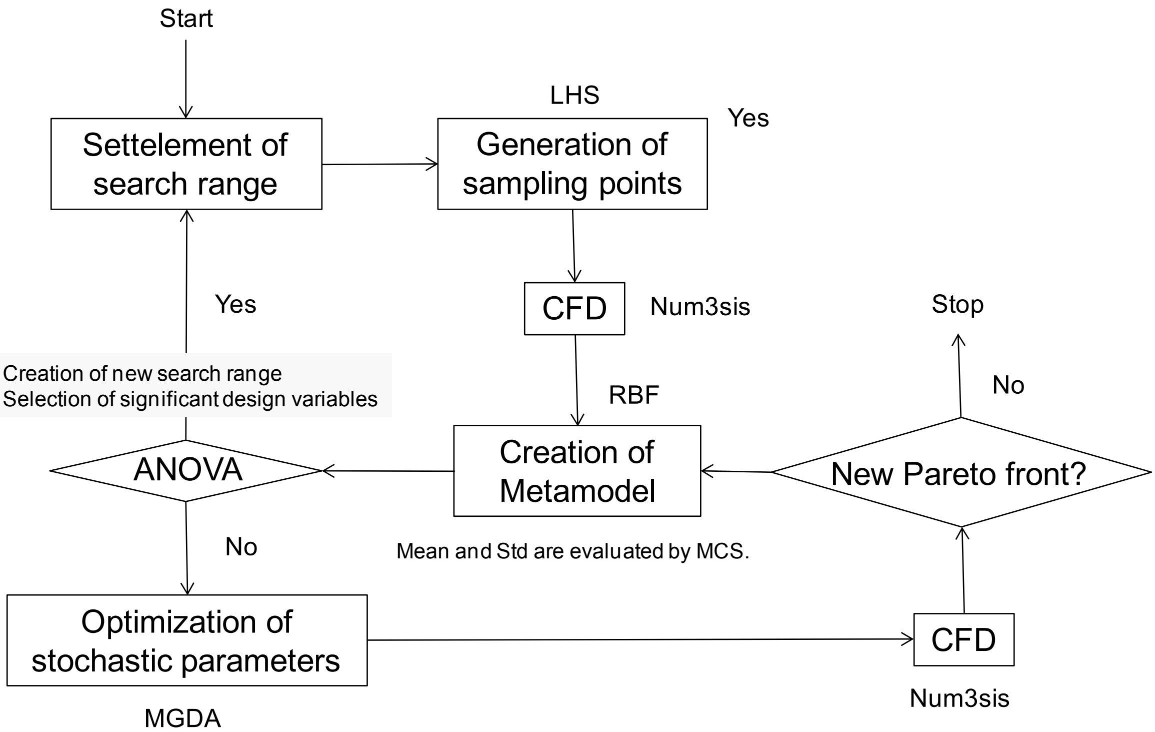

FIGURE 2: Flow chart of robust optimization

3. Methods

3.1 Definition of geometric parameters as both design variables and uncertainty factors

For the purpose of working with aerodynamically significant geometric features, we used the PARSEC method [6]. PARSEC is one of the conventional parameterizations for transonic airfoils to control the local shock wave on the upper surface. It has been applied to aerodynamic shape optimization problems [11-14]. It has been modified to capture the suction peak at the leading edge in transonic flow or in supersonic flow [13,14]. In this framework, the shapes of two-dimensional transonic airfoils are controlled by 11 parameters and are described by the following polynomial terms.

$$Z = \sum_{n=1}^{6} a_n X^{n-0.5} \tag{3}$$

Figure 3 depicts these 11 parameters. Z_{UP}, Z_{LO} and X_{UP}, X_{LO} represent the maximum thickness and their positions in the chord direction, respectively. Z_{XXUP} and Z_{XXLO} are curvatures at the maximum thickness positions. r is a radius, which represents the curvature at the leading edge. Z_{TE} and ΔZ_{TE} are the position of the trailing edge and its thickness, respectively. α_{TE} and β_{TE} are the angle representations of the camber and thickness at the trailing edge, respectively. These 11 parameters are considered as aerodynamically significant ones in transonic airfoil design. The coefficients a_n $(n = 1, \cdots, 6)$ in Eq. (3) are uniquely determined by solving the following six-equation system obtained by geometric considerations:

$$\begin{cases} Z(1) = Z_{TE} + 0.5\Delta Z_{TE} \\ Z(Xup) = Zup \\ \left.\dfrac{dZ}{dX}\right|_{X=1} = \tan(\alpha_{TE} - 0.5\times\beta_{TE}) \\ \left.\dfrac{dZ}{dX}\right|_{X=Xup} = 0 \\ \left.\dfrac{d^2Z}{dX^2}\right|_{X=Xup} = Z_{XXup} \\ -\dfrac{\left[1+\left(\left.\dfrac{dZ}{dX}\right|_{X=0}\right)^2\right]^{1.5}}{\left.\dfrac{d^2Z}{dX^2}\right|_{X=0}} = r_{le} \end{cases} \quad \begin{cases} Z(1) = \sum_{n=1}^{6} a_n \\ Z(Xup) = \sum_{n=1}^{6} a_n Xup^{n-0.5} \\ \left.\dfrac{dZ}{dX}\right|_{X=1} = \sum_{n=1}^{6}(n-0.5)\cdot a_n \\ \left.\dfrac{dZ}{dX}\right|_{X=Xup} = \sum_{n=1}^{6}(n-0.5)\cdot a_n Xup^{(n-1.5)} \\ \left.\dfrac{d^2Z}{dX^2}\right|_{X=Xup} = \sum_{n=1}^{6}(n-1.5)\cdot(n-0.5)\cdot a_n Xup^{(n-2.5)} \\ -\dfrac{\left[1+\left(\left.\dfrac{dZ}{dX}\right|_{X=0}\right)^2\right]^{1.5}}{\left.\dfrac{d^2Z}{dX^2}\right|_{X=0}} = -\dfrac{\left[\left(\dfrac{dX}{dT}\right)^2+\left(\dfrac{dZ}{dT}\right)^2\right]^{1.5}}{\left.\dfrac{dX}{dT}\dfrac{d^2Z}{dT^2}\right|_{T=0} - \left.\dfrac{dZ}{dT}\dfrac{d^2X}{dT^2}\right|_{T=0}} = \dfrac{a_1^2}{2} \end{cases} \tag{4}$$

Then, the airfoil coordinates are determined by Eq. (3), for arbitrary X ($0 \le X \le 1$). The most significant design variables for aerodynamic coefficients will be quantified by using ANOVA, shown in Section 3.3.

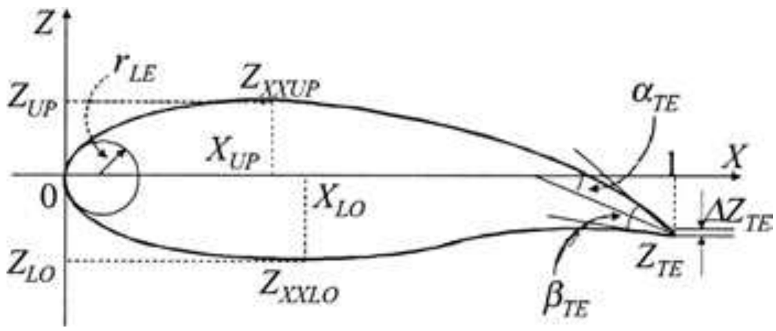

FIGURE 3: Description of PARSEC geometric parameters

3.2 Surrogate models for statistical moments estimation and MGDA

In order to have a reasonable computational cost, the mean and Std of cost functions are estimated using response surfaces (also called surrogate models), which are constructed using Radial Basis Functions (RBFs). The surrogate model $f(\mathbf{x})$, at arbitrary design variables $\mathbf{x}$, is represented by the following equation:

$$f(\mathbf{x}) = \boldsymbol{\beta}^{\mathrm{T}}\mathbf{g}(\mathbf{x}) = \sum_{i=1}^{m}\beta_i g_i, \quad \text{where} \quad g_i = \phi(r_i(\mathbf{x})) \tag{5}$$

Here, $\boldsymbol{\beta}$ is a vector uniquely determined for a given cost function and set of the sampling points. m is the number of sampling points, ϕ is a radial basis function, determined by the Euclidean distance between $\mathbf{x}$ and the sampling point. Details on the determination of $\boldsymbol{\beta}$ can be found in [15]. Here, we used Gaussian radial basis function: $\phi = \exp(-r^2)$.

The gradient of the cost function, which has to be used for MGDA, is analytically evaluated by the following equation:

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \boldsymbol{\beta}^{\mathrm{T}} \frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \sum_{i=1}^{m} \beta_i \phi'(r_i(\mathbf{x})) \frac{\partial r_i}{\partial \mathbf{x}}, \quad \text{where} \quad \frac{\partial r_i}{\partial \mathbf{x}} = \frac{(\mathbf{x} - \mathbf{x}_i)^{\mathrm{T}}}{r_i(\mathbf{x})} \tag{6}$$

A more detailed discussion on gradient and Hessian of surrogated models based on RBF can be found in [15].

Mean and Std of the cost functions are evaluated by using the constructed surrogate models. According to Eq. (1), the mean μ_f and Std σ_f could be evaluated by numerical integration. For example, the integral of Eq. (5) is represented as:

$$\int f(\mathbf{x}) d\mathbf{x} = \sum_{i=1}^{m} \beta_i \int \phi(r_i(\mathbf{x})) d\mathbf{x} \tag{7}$$

A difficulty arises when the number of parameters is large, since this integral is defined in a space of high-dimension. To reduce the computational cost, two possibilities could be envisaged:

1. Use of Monte Carlo Simulation (MCS), whose accuracy does not depend on the dimension
2. Reduce the number of design variables and use classical numerical integration rules

The first approach is based on random numbers yielding a noisy estimation of statistical moments. Thus, there is a possibility that MGDA converges to a local minimum. The second approach requires defining a selection of dominant design variables, which is also related to ANOVA, described in the next section. In this report, we present results obtained by using MCS to estimate the statistical moments. Therefore, the mean μ_f and Std σ_f are estimated by random sampling points $\mathbf{x}_i$ as follows:

$$\mu_f = \frac{1}{N} \sum_{i=1}^{N} f(\mathbf{x}_i) \tag{8}$$

$$\sigma_f = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(f(\mathbf{x}_i) - \mu_f\right)^2} \tag{9}$$

where N is the number of the sampling points $\mathbf{x}_i$. The sampling points are determined using a random number generator. To generate a Gaussian distribution, we employ the Box-Muller algorithm to transform a uniform distribution. Finally, the evaluated mean μ_f and Std σ_f are dependent on the following three factors:

- The selected design space
- The statistical moments of design variables
- The number of the random sampling points used in MCS

To select a relevant design space, ANOVA is used and described in the next section.

3.3 ANOVA technique

ANOVA is used to quantify the influence of design variables to each cost function [16]. First of all, we calculate the total mean and total variance for a cost function $F(x_1, \cdots, x_n)$:

$$\mu_{total} = \int \cdots \int F(x_1, \cdots, x_n)\, dx_1 \cdots dx_n \tag{10}$$

$$\sigma_{total}^2 = \int \cdots \int \left[F(x_1, \cdots, x_n) - \mu_{total}\right]^2 dx_1 \cdots dx_n \tag{11}$$

To evaluated the effect of an arbitrary design variable x_i to the objective function, the partial mean and variance of the design variable x_i are evaluated by the following equations:

$$\mu_i(x_i) = \int \cdots \int F(x_1, \cdots, x_n)\, dx_1 \cdots dx_{i-1} dx_{i+1} \cdots dx_n - \mu_{total} \tag{12}$$

$$\sigma^2(x_i) = \int \left[\mu_i(x_i)\right]^2 dx_i \tag{13}$$

Finally, the effect of the design variable x_i to the objective function $F(x_1, \cdots, x_n)$ is considered to be:

$$\frac{\sigma^2(x_i)}{\sigma_{total}^2} \tag{14}$$

Thus, this technique permits to quantify the influence of the design variables, for each objective function. In practical use, ANOVA is applied to surrogate models, the integrals in Eqs. (10) - (12) being evaluated by MCS.

4. Application to aerodynamic design of transonic airfoils

In this section, we present the application of the proposed method to transonic airfoil design. All of the CFD calculations used to evaluate aerodynamic coefficients are inviscid analyses based on two-dimensional compressible Euler equations. The grids are generated using the Gmsh software, for each new configuration. The Mach number and angle of attack as operational parameters are fixed to 0.8 and 4 degrees, respectively. For these conditions, airfoils are expected to generate strong shock waves from their leading edges.

4.1 Classical optimization

At first, some classical design optimization cases are briefly introduced. The objective functions are the lift coefficient Cl and drag coefficient Cd. The design variables are the 11 PARSEC parameters shown in Fig. 3. Based on the NACA0012 airfoil as baseline, the search range is set as shown in Table 1. Thus, the following multi-criterion shape optimization problem is:

$$\begin{array}{ll} Maximize & C_l(\mathbf{x}) \\ Minimize & C_d(\mathbf{x}) \end{array} \quad \mathbf{x} \in R^{11} \tag{15}$$

In the design space, 40 sampling points are generated randomly by LHS. Figure 4 shows Cl-Cd distribution for this initial sampling, for optimized designs obtained after three iterations of the cycle shown in Fig. 1, and for baseline NACA0012 for comparison. It can be observed that most of the initial points converge to the higher lift-to-drag ratio (Cl/Cd) line, indicated in red. The pressure fields for the airfoil with the highest lift-to-drag ratio and for NACA0012 airfoil are also shown in the figure.

As reference, ANOVA results obtained from the surrogate models based on the CFD calculations are presented in Fig. 5. It is confirmed that the camber and thickness effects at the trailing edge, represented by α, β and the curvature of the upper surface Z_{XXUP}, are the most significant variables for both aerodynamic coefficients. These parameters control the shape at the trailing edge, which influences aerodynamic performance much in transonic flow, and the shape of the suction surface, which influences the shock wave generated. In fact the optimized airfoil, shown in Fig. 4, underlines the significance of these three parameters. In particular, it can be seen that the generated shock wave is controlled by the curvature of the upper element Z_{XXUP}.

Table 1: PARSEC parameters for classical optimization.

	X_{UP}	X_{LO}	Z_{UP} $*10^{-2}$	Z_{LO} $*10^{-2}$	Z_{TE}	ΔZ_{TE} $*10^{-3}$	α_{TE} (deg)	β_{TE} (deg)	Z_{XXUP}	Z_{XXLO}	r_{LE}
NACA0012	0.2947	0.2947	6.001	-6.001	0.000	2.520	0.000	15.97	-0.4622	0.4622	0.1250
Design Range	0.2500	0.2500	5.000	-7.000	-0.010	0.000	-4.000	14.00	-0.5000	0.4000	0.1000
(Min, Max)	0.3500	0.3500	7.000	-5.000	0.010	5.000	0.000	18.00	-0.4000	0.5000	0.1500

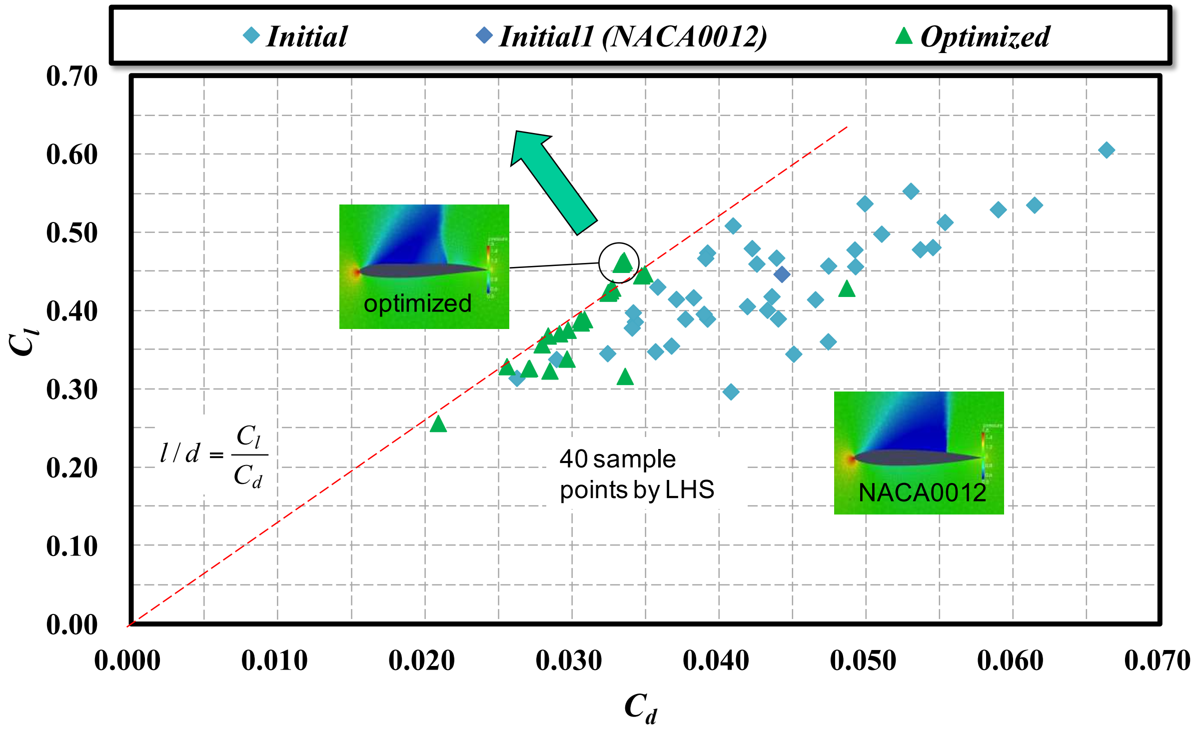

FIGURE 4: Cl - Cd distributions for classical optimization

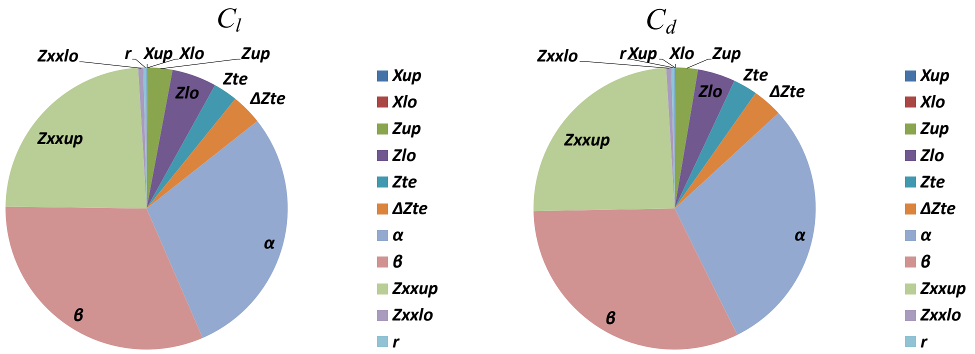

FIGURE 5: ANOVA result for classical optimization

4.2 Robust optimization

As explained in Section 2, robust optimization is treated as multi-objective optimization of statistical moments of the cost functions. Note that in this context, the design variables are considered as uncertainty factors at the same time. Therefore, we put an additional step to the classical optimization process. We reconsider the design space using ANOVA, after CFD calculations of randomly selected sampling points. This step is intended for:

- Neglecting low-influence parameters
- Generating new parameter ranges

This strategy "feedback of settlement of search range" is shown in Fig. 2.
In this study, the following three robust optimization cases are finally discussed:

1. Robust optimization with initial design space (named "Initial")
2. Robust optimization by neglecting low-influence parameters using ANOVA (named "Selected")
3. Robust optimization with new parameter ranges using ANOVA (named "Range")

In these three cases, the cost functions are the mean and Std of Cl (μ_{Cl} , σ_{Cl}) and Cd (μ_{Cd} , σ_{Cd}) calculated by Eqs. (8) and (9). The number and the range of the design variables are dependent on the cases. In robust optimization, the aim is to take into account design fluctuations and to investigate their effects on aerodynamics. Based on the baseline NACA0012 airfoil, the search range is defined in Table 2. As for the classical optimization, the Mach number and angle of attack are fixed to 0.8 and 4 degrees, respectively. 40 sampling points are randomly generated by LHS in the selected design space. The results obtained for these three cases are discussed in the following sections. Then, a summary of the contributions of these three cases to robust optimization design is described.

Table 2. Initial PARSEC parameters for robust optimization.

	X_{UP}	X_{LO}	Z_{UP} *10^{-2}	Z_{LO} *10^{-2}	Z_{TE}	ΔZ_{TE} *10^{-3}	α_{TE} (deg)	β_{TE} (deg)	Z_{XXUP}	Z_{XXLO}	r_{LE}
NACA0012	0.2947	0.2947	6.001	-6.001	0.000	2.520	0.000	15.97	-0.4622	0.4622	0.1250
Design Range (Min, Max)	0.2447	0.2447	5.001	-7.001	-0.010	0.000	-1.000	14.97	-0.5122	0.4122	0.1000
	0.3447	0.3447	7.001	-5.001	0.010	5.040	1.000	16.97	-0.4122	0.5122	0.1500

4.2.1 Robust optimization with initial design space

Robust optimization is firstly conducted in the initial design space defined in Table 2. The following multi-objective optimization problem is solved:

$$\begin{array}{ll} Maximize & \mu_{C_l}(\mathbf{x}) \\ Minimize & \mu_{C_d}(\mathbf{x}) \\ Minimize & \sigma_{C_l}(\mathbf{x}) \\ Minimize & \sigma_{C_d}(\mathbf{x}) \end{array} \quad \mathbf{x} \in R^{11} \tag{16}$$

ANOVA results are obtained from the CFD simulations at the sampling points. Figure 6 shows the resulting analyses for the 4 objective functions. The characteristics of these 4 cost functions are similar to each other. It can be easily observed that the parameters α and β, which are related to the geometry at the trailing edge, and Z_{XXUP}, the curvature of the upper surface, are the most significant variables, like in the case of the classical optimization in the previous section. Two parameters related to the airfoil thickness Z_{UP} and Z_{LO} follow them. Regarding the other parameters, the influence of the thickness of the trailing edge ΔZ_{TE} is much lower for Std of the aerodynamic coefficients.

These ANOVA results are helpful for uncertainty quantifications. As presented in Section 2, we use the ANOVA results to reconsider the design space. These modifications are detailed in the following sections.

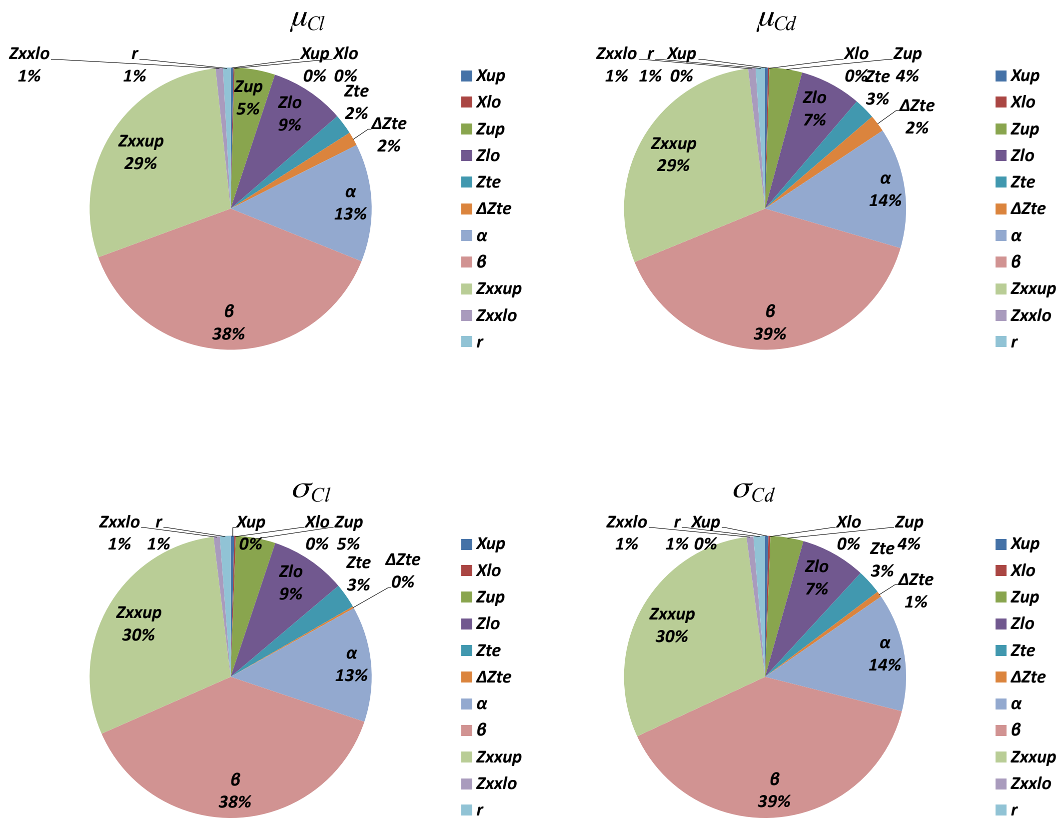

FIGURE 6: ANOVA results for initial design space in robust optimization

4.2.2 Robust optimization by dominant design variables

One possible strategy to extract dominant parameters is to consider if the percentage of each design parameter is greater than that of the equally divided value (around 9% for 11 parameters). The three design parameters α , β and Z_{XXUP} are selected using this strategy. In this study, we choose to select the top 5 dominant parameters, according to Fig. 6 (α , β , Z_{XXUP}, Z_{UP} and Z_{LO}) because using MGDA the number of design variables must be greater than the number of objective functions.

New sampling points are generated by LHS and CFD calculations are carried out to obtain new ANOVA results. The other 6 parameters are fixed to their respective baseline value. Therefore, the multi-objective optimization problem is changed as follows:

$$\begin{array}{ll} Maximize & \mu_{C_l}(\mathbf{x}) \\ Minimize & \mu_{C_d}(\mathbf{x}) \\ Minimize & \sigma_{C_l}(\mathbf{x}) \\ Minimize & \sigma_{C_d}(\mathbf{x}) \end{array} \quad \mathbf{x} \in R^5 \qquad (17)$$

Table 3 shows the 5 selected design variables and their respective design ranges (the same as the initial ones). Figure 7 presents the analyses of ANOVA results for the 4 objective functions with respect to the newly selected design variables. It can be observed that the contributions of the parameters are different from previously. For instance, the contributions of β to

Std of both coefficients is greater than those to the mean. On the other hand, the characteristics of Z_{LO} indicate the opposite modification, especially for Std of Cd.

Table 3. Selected dominant PARSEC parameters for robust optimization.

	Z_{UP} $*10^{-2}$	Z_{LO} $*10^{-2}$	α_{TE} (deg)	β_{TE} (deg)	Z_{XXUP}
NACA0012	6.001	-6.001	0.000	15.97	-0.4622
Design Range (Min, Max)	5.001	-7.001	-1.000	14.97	-0.5122
	7.001	-5.001	1.000	16.97	-0.4122

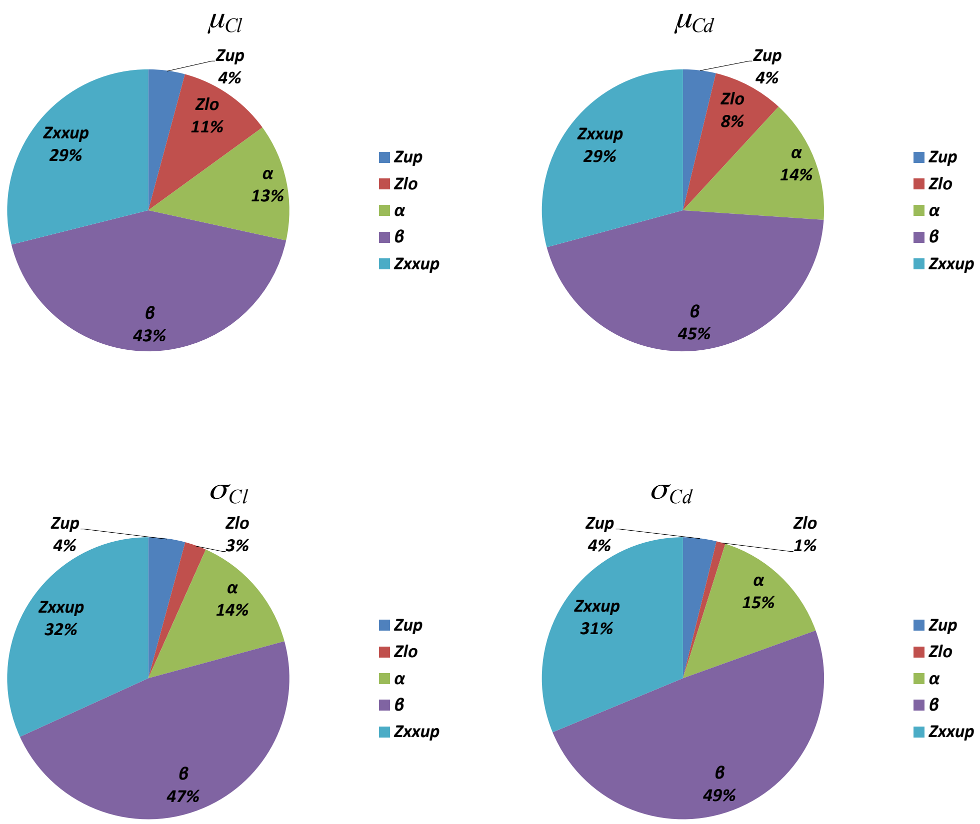

FIGURE 7: ANOVA results for selected 5 dominant design variables in robust optimization

4.2.3 Robust optimization by new ranges of design variables

The previous studies have underlined the fact that a very small amount of design variables, in the context of PARSEC parameterization, have a very strong impact on the aerodynamic coefficient variations, and their moments if geometrical uncertainties are considered.

For this reason, we consider now a modification of the variable ranges. More precisely, the design variables, which have the most significant weight according to a first ANOVA study, will see their range decrease. For example, it can be observed by the ANOVA results using the initial design space, shown in Fig. 6, that the curvature of the upper surface Z_{XXUP} has much more influence to the aerodynamic performance than the lower surface Z_{XXLO}. Therefore, we selected new design ranges for the parameters whose percentages are greater than the percentage corresponding to the equally divided weight (around 9%). Using this approach, the three parameters α, β and Z_{XXUP} are selected. The ranges of these three parameters have to be smaller than the initial ranges. The ranges of α , β and Z_{XXUP} are chosen to be a quarter, a quarter and a half, according to the occupation of the percentages in Fig. 6, respectively. The new parameter ranges are shown in Table 4. The multi-objective optimization problem considered is the same as the first case:

$$\begin{array}{ll} Maximize & \mu_{C_l}(\mathbf{x}) \\ Minimize & \mu_{C_d}(\mathbf{x}) \\ Minimize & \sigma_{C_l}(\mathbf{x}) \\ Minimize & \sigma_{C_d}(\mathbf{x}) \end{array} \quad \mathbf{x} \in R^{11} \tag{18}$$

Only the parameter ranges are different from those of the first case shown in Table 2. Figure 8 represents the ANOVA results. The dominant parameters β and Z_{XXUP} are not changed in the ratio. However, the other minor parameters have a weight closer to the equally divided ratio (around 9%). The parameter α , which is one of the most dominant three parameters in the first case, is now at the same level as the other parameters.

Compared with the results of the 1st case, it can be mentioned that the two parameters β and Z_{XXUP} have still a very strong influence on the statistical moments of aerodynamic coefficients (mean and Std), even when the range of these parameters are small. This indicates that we have to pay much attention to uncertainty related to these two parameters, and that these parameters would make robust design problem more difficult.

Table 4. PARSEC parameters by new ranges for robust optimization.

	X_{UP}	X_{LO}	Z_{UP} *10^{-2}	Z_{LO} *10^{-2}	Z_{TE}	ΔZ_{TE} *10^{-5}	α_{TE} (deg)	β_{TE} (deg)	Z_{XXUP}	Z_{XXLO}	r_{LE}
NACA0012	0.2947	0.2947	6.001	-6.001	0.000	2.520	0.000	15.97	-0.4622	0.4622	0.1250
Design Range	0.2447	0.2447	5.001	-7.001	-0.010	0.000	**-0.500**	**15.72**	**-0.4747**	0.4122	0.1000
(Min, Max)	0.3447	0.3447	7.001	-5.001	0.010	5.040	**0.500**	**16.22**	**-0.4497**	0.5122	0.1500

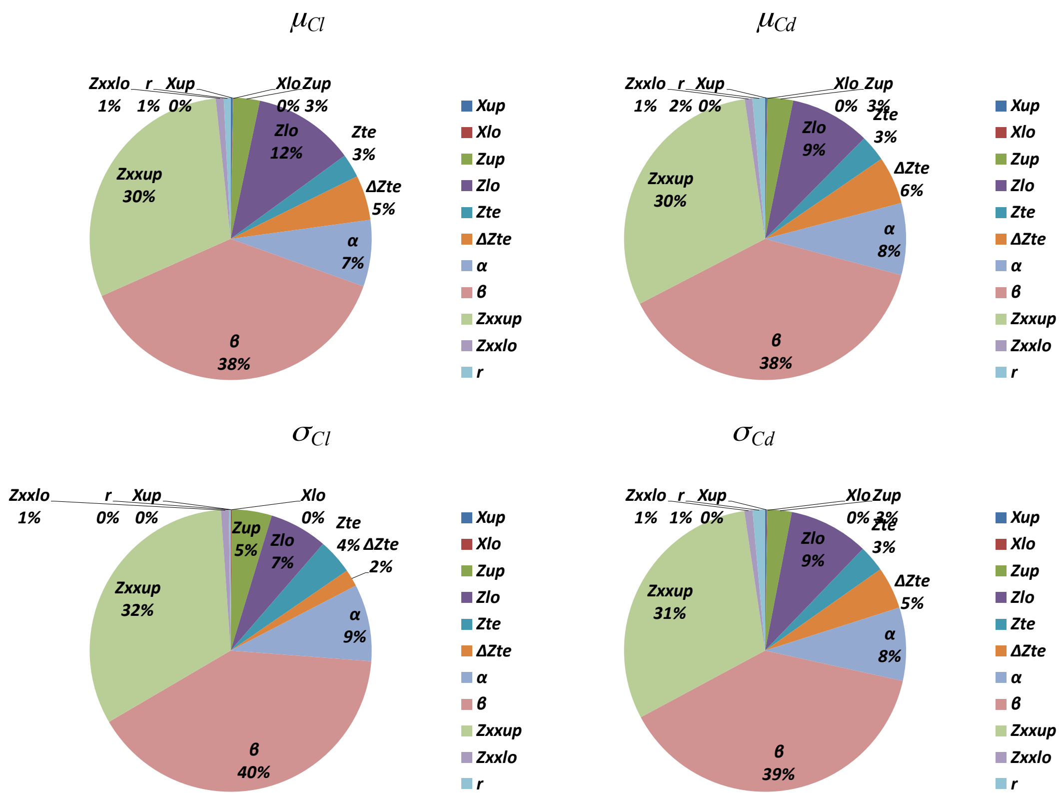

FIGURE 8: ANOVA results for new ranges of design variables in robust optimization

4.2.4 Comparison of robust design optimization results

Finally, the results obtained by these three design exercises are discussed. The comparison of the case1 (Initial) to the case2 (Selected) and case3 (Range) focuses on two points. Firstly, we estimate the effect of the strategy of selecting dominant parameters. Then, we quantify the influence of aerodynamically significant parameters as uncertainty factors.

Figure 9 shows the distribution of the solution with respect to the three dominant parameters α , β and Z_{XXUP} , selected by the ANOVA technique for the original design space (Initial), after three iterations of the design cycle. One observes that the distribution of the solutions is different for the three cases (Initial, Selected and Range). Moreover, there is a cluster of design points for each case, except for "Selected" case. By taking into account the characteristics of the results of classical optimization case, presented in Section 4.1, we regard these clusters of design points as the Pareto solutions in robust optimization cases (even though we evaluate design parameters not cost functions here). Concerning the results of "Selected" case, it is difficult to find the Pareto front compared with other cases. Here we discuss mainly the results of "Initial" and "Range" cases. Some comments about "Selected" case are added later, based on the results of "Initial" and "Range" cases.

The characteristics of the parameters obtained are different in "Initial" and "Range" cases, except for α . The parameter α , which represents the camber line at the trailing edge, is optimized to around the value 0.5 in both design spaces. The parameters β and Z_{XXUP}, which are related to the thickness of the trailing edge and the curvature of the upper surface, have different characteristics for the two cases “Initial” and “Range”. For "Initial" case, the airfoil is changed to have more thickness and sharper concavity, respectively. On the other hand, the change of these two parameters is moderate for "Range" case.

Figure 10 shows some configurations found, and the associated pressure fields. The airfoil configuration for "Selected" case has not only a different shape, but also has different aerodynamic coefficients from that of "Initial" case, which underlines the fact that the search is carried out in two different design spaces. The aerodynamic coefficients at the design point are not always better than the initial airfoil (NACA0012). However, we emphasize that these airfoils were designed by accounting for mean and Std of the aerodynamic coefficients. The difference between "Initial" and "Range" cases is the degree of robustness of design parameters. The results obtained by "Range" case are more robust with respect to fluctuations of the geometric parameters, because the ranges of the dominant parameters are limited. Concerning the "Selected" case, it is difficult to extract the information about the Pareto front.

In practical applications, geometric uncertainties, for instance in manufacturing process, may not occur according to the PARSEC parameters. As mentioned in the introduction, we use PARSEC parameterization to account for aerodynamic knowledge and make the design process more efficient. For example, the curvature of the upper element is much more significant than that of the lower one. This fact does not change even when the design search area of the upper element becomes smaller, as indicated in the results. The results obtained in this study could be employed for more realistic robust design applications, by using parameterization methods based on manufacturing parameters.

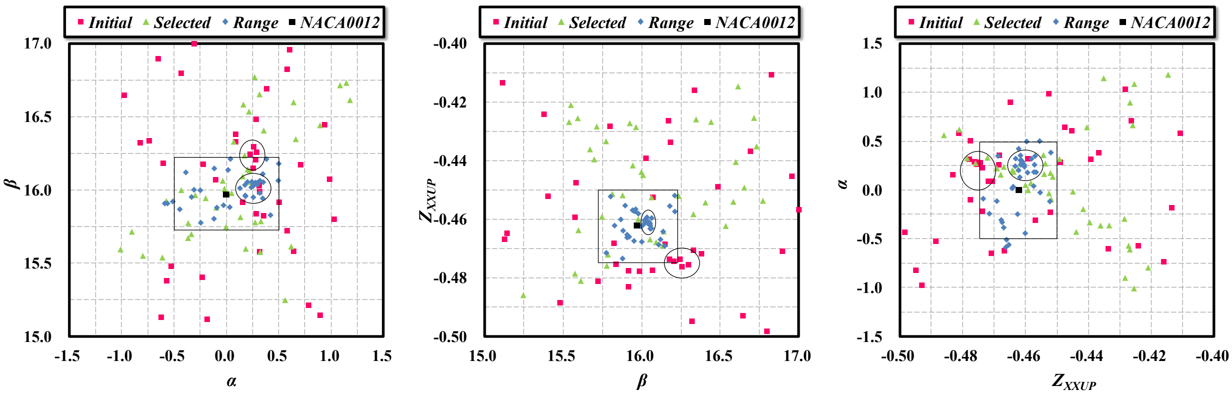

FIGURE 9: Distribution of three dominant design parameters (the square indicates the new design ranges for "Range". the circle indicates a cluster of design points)

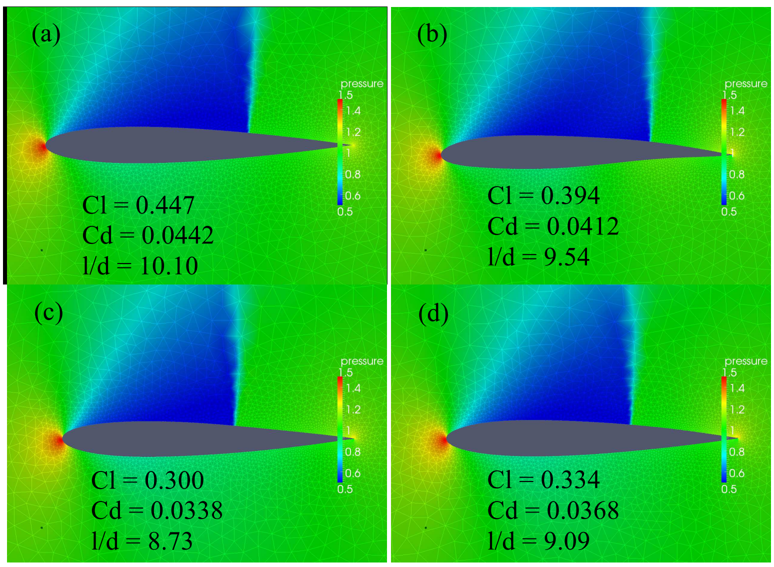

FIGURE 10: Pressure distributions, (a) NACA0012, (b) Optimized using initial design space (Initial), (c) Optimized using selected design parameter (Selected), (d) Optimized using new design range (Range)

Conclusion

To achieve robust design optimization of transonic airfoils, we used mean and standard deviation of aerodynamic coefficients, the geometric parameters being considered as uncertainty factors. The robust design is a multi-objective optimization problem, which is solved using the Multiple-Gradient Descent Algorithm (MGDA) to find Pareto-stationary solutions.

To obtain aerodynamic knowledge in design, we used PARSEC as geometry parameterization and ANOVA to reconsider the geometric parameters within two different strategies. One is the selection of the most significant parameters. The other is the modification of design ranges. The first strategy is rather useful to reduce the number of design variables. The second one is helpful to know more about aerodynamic characteristics in the design process.

As application of these robust design methods, we considered the aerodynamic design of NACA0012 airfoil in transonic flow (Mach number of 0.8 and angle of attack of 4 degrees), for which a strong shock wave on the upper surface is observed. In comparison with the results obtained in the initial design space, the first strategy is not suitable to find the Pareto solutions. The second one yields different results from the initial case. The influence of some of the dominant parameters can be reduced, so that the designed airfoil becomes more robust with respect to the geometric parameters. However, this robust design strategy is based on the assumption that the tolerance of the sensitive parameters is known and controllable. By combining ANOVA results for changing the variable ranges with calculations of statistical moments of aerodynamic coefficients, more practical airfoil design is possible, accounting for geometric parameters with small tolerance.

The proposed method can also produce a database of the influence on aerodynamics of aerodynamically significant parameters. In the future, other parameterization methods considering practical manufacturing errors, should be used based on the aerodynamic knowledge obtained. In this study, surrogate models were constructed to calculate statistical moments using Monte Carlo simulations. Considering the balance between accuracy and computational costs, it should be noted that there are still margins also in the surrogate model constructions.

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