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A Program In Urban Hydrology, Part Ii: An Evaluation Of Rainfall-Runoff For Small Urbanized Watersheds And The Effect Of Urbanization On Runoff

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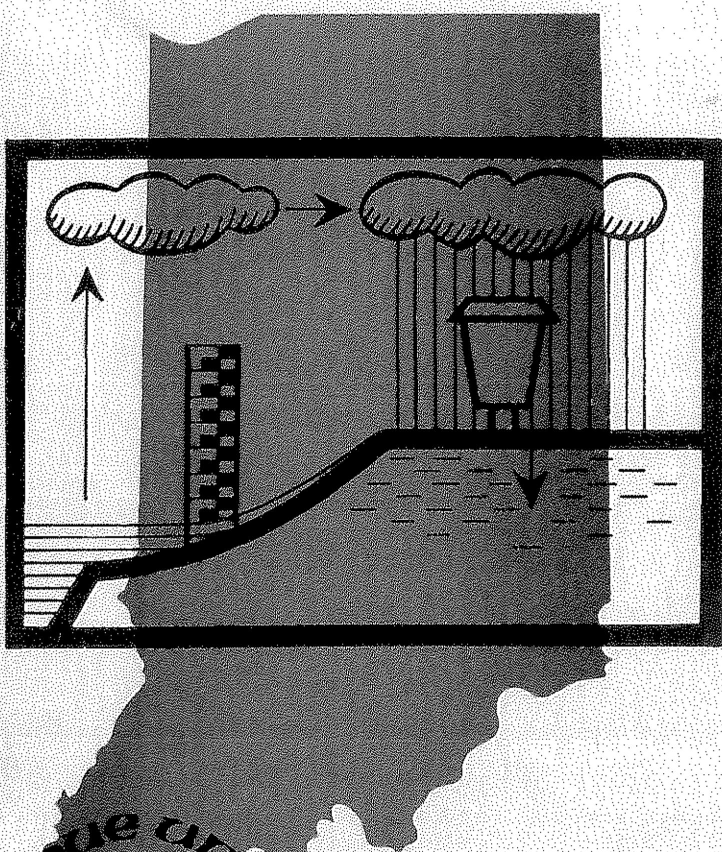
A PROGRAM IN URBAN HYDROLOGY

Part II. An Evaluation of Rainfall - Runoff Models for Small Urbanized Watersheds and the Effect of Urbanization on Runoff



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October 1969

PURDUE UNIVERSITY
WATER RESOURCES RESEARCH CENTER
LAFAYETTE, INDIANA

Water Resources Research Center

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PREFACE

This report covers the work performed under project OWRR-B-002-IND entitled "The Effect of Urbanization in Small Watersheds", between July 1966 and June 1969. It is the second part of a program on Urban Hydrology at Purdue University. The first part was sponsored by the Division of Water, Department of Natural Resources, Indianapolis, Indiana, (formerly Flood Control Commission) from September 1964 to January 1966.

The work done in the first phase was summarized in a discussion by J. W. Delleur and E. B. Vician on "Time in Urban Hydrology" published in the Journal of the Hydraulics Division of the American Society of Civil Engineers of September 1966, pp. 243-251.

The second part was sponsored as a cooperative research project between the Office of Water Resources Research, the Indiana Department of Natural Resources and Purdue University, from July 1966 to June 1969, and it was further supported by Purdue University till September 1969.

This work is to be followed by project OWRR-B-022-IND entitled "Effect of Urbanization on Hydrology of Watersheds" sponsored by the Purdue Research Foundation, Purdue University and the Office of Water Resources Research.

The work reported herein is essentially the doctoral thesis submitted by Mr. P. B. S. Sarma under the supervision of Professors J. W. Delleur and A. R. Rao.

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NOMENCLATURE*

Symbol	Description	Units
A	Area of watershed above the gaging station	Sq. Miles
A_n, n=1,2...	Coefficients	
A_c	Water area of the linear channel	Sq. Ft.
a'_j, j=1,2,...	Area of j^{th} subarea	Sq. Miles
$a, a_0, a_1, a_2, \ldots a_n$	Regression coefficients	
B_m, m=1,2...	Coefficients	
b	A coefficient	
b_1, b_2, b_3	Constants	
C	Runoff coefficient in the rational formula	
$C_0, C_1, C_2 \ldots C_n$	Regression coefficients	
C_T	Translation coefficient	Ft/Sec
C_g	Runoff coefficient in "General Rational Formula"	
C_I	A coefficient to express percentage impervious area (Carter)	
C'_1, C'_2, C'_3, C'_4	Routing coefficients	
D	Differential operator $\frac{d}{dt}$	
D_c	A parameter (Reich)	
DT, dt	Time interval	Hours, Min.

*Most of the units are given as they are used in the present study.

Symbol	Description	Units
$d_{t-(j-1)DT}$	Depth of rainfall excess which occurred at the time (j-1)DT before the time t	Ft.
$e^{(\ )}$	Exponential function	
F_i	Total abstraction loss	
FT	Fourier transform operator	
FT^{-1}	Inverse Fourier transform operator	
$F(\omega)$	Fourier transform of the function f(t)	
f(t)	A function of time	
$f_i(t)$	Infiltration rate at time t	
f_j, j=1,2,...	Function expressing a relationship	
f_o	Initial rate of infiltration	
f_c	Final rate of infiltration	
H(t)	Unit step function	
h(t)	Unit impulse response or IUH	
I, I(t)	Inflow rate, Excess rainfall	cfs, IN/Hr.
$\overline{I}$	Average of inflow rate, Average of excess rainfall	cfs, IN/Hr.
I_{30}	30 minute intense rainfall rate	IN/Hr.
I_m	Maximum rate of rainfall	IN/Hr.
ISE	Integral square error	
IUH	Instantaneous Unit Hydrograph	cfs
i	Running index	
j	Running index, imaginary unit ($\sqrt{-1}$)	
K	Storage coefficient of a linear reservoir	Hrs.

Symbol	Description	Units
$\overline{K}$	Average value of optimum storage coefficient K (Viessman)	Hrs.
K_1	Optimum value of storage coefficient (Criterion 1)	Hrs.
K_2	Optimum value of storage coefficient (Criterion 2)	Hrs.
K_D	Storage coefficient in double routing method	Hrs.
K_N	Parameter of the Nash model	
L	Length of the main stream or channel	Miles
L_j	Length of the main stream of j^{th} subarea	Miles
L_{ca}	Distance along the main stream from the watershed outlet to a point opposite the center of gravity of the watershed	Miles
L_d	Length of drain	Ft.
L_w	Maximum length of travel in the watershed	Miles
L_f	Impervious length factor (Riley)	
L_o	Overland flow length	Ft.
M	An integer	
M_m m=1,2,...	m^{th} moment about the axis of time	
M_{1Q}	First moment of direct runoff hydrograph	Hrs.
M_{2Q}	Second moment of direct runoff hydrograph	$(Hrs.)^2$
m	Number of items (Running index)	
N	An integer	
n	Number of items (reservoirs, variable, coefficients, etc.)	

Symbol	Description	Units
n_1	A parameter (Gray)	
n_r	Retardation coefficient	
n_s	Mannings 'n'	
p	Intensity of uniform rainfall	In/Hr.
P_T	Total rainfall	Inches
P_E	Volume of excess rainfall	Inches
Q	Outflow, discharge	cfs
$\overline{Q}$	Mean discharge	cfs
$\overline{Q}_A$	Mean annual discharge	cfs
Q'	Outflow from first reservoir in double routing method	cfs
Q_p	Peak discharge	cfs
Q_p'	Peak discharge in Fig. 21	
Q_{po}	Observed peak discharge	cfs
Q_{pi}	Peak discharge of IUH	1/Time
Q_{pc}	Computed peak discharge	cfs
$Q_O(I)$ I=1,2,...N	Observed discharge	cfs
$Q_C(I)$ I=1,2,...N	Computed discharge	cfs
q	Overland flow discharge	Sq. Ft/Sec
q'	Outflow from first linear reservoir in double routing method	cfs
q_{um}	Peak discharge of unit hydrograph	cfs/sq. mile
q_{wm}	Mean weighted discharge (Askew)	
Q_0	Discharge at the time at which the rainfall ends	
q_i	Peak discharge of the inlet hydrograph (Bock)	

Symbol	Description	Units
R	Linear correlation coefficient	
R_s	Special correlation coefficient	
S	Storage	cft
$\overline{S}$	Mean basin slope	Ft/Mile
$\overline{S}_c$	Mean channel slope	%
$\overline{S}_g$	Mean gutter slope	%
S_o	Average slope of overland flow	%
S_w	Weighted slope of watershed	%
T	Time of translation or travel time	Hours
T_b	Time elapsed from the beginning of rainfall to the end of period of maximum rainfall intensity	Min.
T_c	Time of concentration	Hours,Min.
T_I	Time to centroid of excess rainfall hyetograph	Hours,Min.
T_p	Time to peak discharge of hydrograph	Hours,Min.
T_{po}	Time to peak of observed hydrograph	Hours,Min.
T_{pc}	Time to peak of regenerated hydrograph	Hours,Min.
T_{pi}	Time to peak discharge of IUH	Hours,Min.
T_Q	Time to centroid of direct runoff hydrograph	Hours,Min.
T_R	Duration of excess rainfall	Hours,Min.
T_{Re}	Recurrence interval	Years
T_L	Time Lag (same as T_4)	Hours,Min.
T_s	Duration of intense rainfall	Seconds
T_1,T_2,T_3,T_4,T_5	Various definitions of time lag	Hours,Min.

Symbol	Description	Units
$\overline{T}_1, \overline{T}_2, \overline{T}_3, \overline{T}_4, \overline{T}_5$	Average value of time lag	Hours, Min.
t	Time	Hours, Min.
$t' = (t-T_R)$	Time elapsed after end of rainfall	Hours, Min.
t_1'	Integration limit	
T_{2U}	Same as T_2 but applied to unit hydrograph	Hours
T_{2UU}	Value of T_{2U} for urban watersheds	Hours
T_{2UR}	Value of T_{2U} for rural watersheds	Hours
U	Percentage of impervious area	%
$U(\Delta t_o, t)$	Ordinate of unit hydrograph of duration Δt_o, at any time t	cfs
V	Mean velocity of flow in the drain	Ft/min.
V_R	Volume of total runoff	Inches
W	Percentage of direct runoff to total runoff	
$X, X(t), X_1, X_2$	Input to the system	
X_0	Coefficient in Muskingum Equation	
$Y, Y(t)$	Output from the system	
Δt_o	Duration of unit hydrograph	Hours
$\delta(\ )$	Dirac-Delta Function	
$\Gamma(\ )$	Gamma Function	
γ'	A parameter (Gray)	
θ	Urban factor (Espey, et.al.)	
Φ	System operator	
ω	Frequency	Rad/Sec
$\omega(t)$	Ordinate of Time-Area-Concentration diagram at time t	

ABSTRACT

Urban and suburban development changes the quantity and the time distribution of runoff. This research was focused on the effect of urban development on the rainfall-runoff relationships.

The effect of urbanization on runoff would be relatively simple to evaluate if the rainfall and runoff data for both the urban and the pre-urban conditions of the watersheds were available. Thus the evaluation of the changes in runoff characteristics caused by urbanization is not possible by direct data comparison and analysis. However, the analysis of the data from watersheds which are in the same region but in different stages of urbanization does reveal the effects of urbanization on runoff characteristics and this comparison of analyses was the general approach used in the present study.

The data for the study were obtained in part from watersheds in West Lafayette, Indiana. These are the Ross Ade upper watershed, the Ross Ade lower watershed, the Purdue Swine Farm upper watershed, and the Purdue Swine Farm lower watershed. Hydrologic data from several other urbanized watersheds in Indiana and in Texas were also used.

The analytical approach adopted was the linear (time invariant, lumped) system analysis. The following conceptual linear systems were used in the analysis of the data: the single linear reservoir model, the double routing method, the Nash model, the single linear-reservoir with linear-channel model. The parameters of the instantaneous unit

hydrographs of the four conceptual models were determined and were optimized for two of the models. In addition, the system kernel functions were determined by the Fourier transform method. The regeneration performances of all these models were then tested.

The single linear reservoir model was selected to simulate the rainfall-runoff process on small urban watersheds (less than 5 sq. miles) based on its superior regeneration performance. Similarly, the Nash model was selected for simulation of the rainfall-runoff process on larger watersheds (between 5 and 20 sq. miles).

The parameters of the single linear reservoir model and of the Nash model were then studied in detail. The variation of these parameters and their relationships with the physiographic characteristics of the watersheds, including the urbanization factor and the storm characteristics, were studied by the techniques of regression analysis. From this analysis, the effects of urbanization on the parameters of the conceptual models and on the time distribution of runoff were deduced. The effect of urbanization on the frequency of annual maximum floods was also studied.

The effects of urbanization on the time lag, the magnitude of the peak discharge, the time to peak discharge, and on the frequency of peak discharge were quantitatively deduced. Tentative relationships were also developed to predict changes in the time distribution of runoff, in the peak discharge, in the time to peak discharge and in the frequency of peak discharge for watersheds which are urbanized to different degrees.

In the next phase of the research the analysis of linear models will be extended, and nonlinear models will be considered in the hydrologic simulation of the larger watersheds in order to develop design methods and criteria for predicting runoff from areas with varying degrees of urbanization.

CHAPTER I

INTRODUCTION

Due to increased industrialization in all nations of the world there has been a tendency for populations to concentrate in the regions where industries are located. This movement of population from rural areas has caused a phenomenal increase in urbanization. In the United States, the percentage of urban population has increased from 41 percent of the total population in 1910 to 66 percent in 1960 and is estimated to increase to 80 percent by the year 2000.[1*] This rapid concentration of population in certain areas causes heavy demand for water for domestic, industrial, and recreational purposes with the consequent increase in the construction of water supply and drainage facilities. The expenditures involved in these constructions are heavy, as illustrated by the average annual $2.5 billion investment in the United States for the urban drainage facilities.[2] Further, because of the increased residential and commercial facilities such as buildings, pavements and parking lots, the built-up or "impervious" areas in the watershed increase. Consequently, the magnitudes and frequencies of flood peaks also increase. Designs of drainage facilities which do not account for this increased runoff are inadequate, and may result in heavy damage and loss of property.

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[*] Superscripted numbers in the text refer to the entries in the Bibliography.

In view of the heavy expenditures involved in providing drainage facilities, it is imperative to properly design these facilities considering not only the estimated runoff from the watersheds in their non-urbanized state, but also by considering the increase in runoff due to urbanization. A comprehensive understanding of the rainfall-runoff process and the effects of increased urbanization on the rainfall-runoff process is presently lacking. In many instances, the procedures which are presently used for hydrologic design of urban drainage structures have not been changed since the turn of the century.[3,4] An understanding of the effects of urbanization on runoff is obviously essential to develop better methods of analysis and realistic design criteria for engineering design of urban drainage systems.

Professional organizations such as the American Society of Civil Engineers (ASCE), and governmental agencies such as the Office of the Water Resources Research (OWRR) and U.S. Geological Survey (USGS) have recognized the inadequacy of the present methods of analysis and design procedures adopted in urban drainage systems and in watersheds which are being urbanized. A Task Force to study "Effects of Urban Development on Flood Discharges" has been formed by the ASCE with the following objectives:[5,6]

> "To seek out information pertaining to changes in runoff characteristics of watersheds due to urban development and to the effects of such changes on the concentration of flood waters in stream channels; to compile a bibliography of works and papers that provide such information; to prepare an inventory of investigations being conducted on this subject or pertinent parts thereof; and to identify areas in which research is needed to broaden knowledge of runoff rates for flood control or protective purposes."

The OWRR has sought to support and organize research which is relevant to important urban water resources problems.[7] A concentrated effort to focus on the problems of urban hydrology, and to identify research areas to alleviate them has been sponsored by the OWRR and is being conducted by the ASCE Urban Hydrology Research Council.[8]

In most of these explorations of the needs for research in urban hydrology, the following observations emerge frequently:

(1) Reliable data from urban watersheds with detailed information of time and space distribution of rainfall and runoff are not presently available. Similarly, information about infiltration and evaporation from urban watersheds is also lacking. Except for a few very small watersheds, data of the above mentioned type are not available.

(2) Hydrologic system analysis techniques have not been widely used for analysis of urban watershed data, nor have any general conceptual or mathematical models been developed for use in urban hydrologic practice. Although several sporadic attempts have been made in these directions, the methods developed have not been tested widely.

(3) Research on the effects of urbanization on runoff from watersheds is in its early stage.

These remarks have also been repeatedly pointed out by many others.[8,9]

The need for research on the effects of urbanization on runoff from small watersheds was recognized and as a result the present study was begun at Purdue University in 1966. This study has been conducted with the following objectives:

(1) To establish hydrologic stations and initiate collection of hydrologic data from four small watersheds which are in different stages of urbanization, in West Lafayette, Indiana,

(2) To develop a library of rainfall excess and direct runoff data for small watersheds which are in different stages of urbanization and which are located principally in Indiana,

(3) To analyze the data obtained in objectives 1 and 2 by using techniques of linear system analysis and to investigate the possibility of modelling the excess rainfall-direct runoff process for watersheds in different stages of urbanization,

(4) To investigate the behavior of the parameters of the linear system models developed in objective 3, as the watershed undergoes changes from rural to urban conditions, and also to develop methods to estimate the model parameters for the case of ungaged watersheds,

(5) To investigate the effects of urbanization on runoff from small watersheds; more specifically to study the changes in

i) Time lag,

ii) Time to peak discharge,

iii) Shape of the instantaneous unit hydrograph,

iv) Magnitude of peak discharge,

v) Frequency of peak discharge,

(6) To develop methods to predict

i) The time distribution of runoff,

ii) Peak discharge,

iii) Time to peak discharge,

iv) Frequency of peak discharge,

for both gaged and ungaged watersheds on which changes in urbanization might take place.

CHAPTER II

LITERATURE REVIEW

System Models of the Rainfall-Runoff Process

The hydrologic cycle is a concept of the process of circulation and distribution of water in the upper mantle of the Earth and in atmosphere. Precipitation and runoff are components of the hydrologic cycle. The part of the precipitation which flows over and in the upper soil stratum of the ground surface is called as "runoff". Essentially, runoff is the flow as it appears at any location of the stream in a watershed. "Direct runoff" is the part of runoff from which the ground water flow and the base flow are excluded. Direct runoff results from "excess rainfall". In the present study, direct runoff due to excess rainfall is used, whereas runoff due to snow melt and other sources are not considered.

The central problem in surface water hydrology is the determination of time distribution of runoff caused by a storm event. Transformation of rainfall into runoff is a complex phenomenon as it is affected by the interaction of several processes such as interception, evaporation, surface detention and infiltration, which are listed by Chow.[10] Because of lack of understanding of many of these processes and the interaction among them, pioneering hydrologic investigations were limited to the development of methods to determine only the magnitudes of peak runoff. Consequently, in the course of time, several empirical formulas to predict the magnitudes of peak runoff have resulted. One of the major drawbacks of empirical formulas is the subjective selection of coefficients and parameters which are to be used with them.

Besides the magnitude of peak runoff, time distribution of runoff must also be known for efficient design of water supply, drainage and other works. Although the rainfall-runoff process is complicated, the effective rainfall-direct runoff process has been traditionally thought to be simpler. Consequently a good deal of attention has been concentrated on simulating the effective rainfall-direct runoff process as a "system".

There is no unique definition of the word "system". However, the following definition[11] is adequate for most engineering purposes:

> "Any structure, device, scheme, or procedure, real or abstract, that interrelates in a given time reference, an input, cause, or stimulus, or matter, energy, or information and an output, effect, or response, of information, energy, or matter."

Apart from this, several other definitions which usually describe particular classes of systems are also in extensive usage in engineering practice.

A system is defined as a "dynamic system" if the input and the output are functions of time, in contrast to the "static system" in which the input and the output are independent of time. In a "distributed" system the input and/or the output are functions of both time and space. If the spatial distribution of input and output are either unimportant or are ignored to simplify the analysis, such systems can be modeled as "lumped systems" in which the input and output are functions of time only.

Apart from the classification of systems based on space-time distribution of inputs and outputs, they are also classified as being either linear or nonlinear. A system can be represented by an equation of the form,

$$Y = \Phi(X) \tag{1}$$

where, Y is the output from the system,

X is the input to the system,

and Φ is called the "system operator".

Let X_1 and X_2 be two inputs to the system and Φ be the operator acting on the inputs to produce the responses, $\Phi(X_1)$ and $\Phi(X_2)$.

If, $$\Phi(X_1) + \Phi(X_2) = \Phi(X_1 + X_2) \tag{2}$$

and $$\Phi(b_1X_1) = b_1\Phi(X_1) \tag{3}$$

where b_1 is a constant,

then, the operator Φ is called a linear operator and the system is said to be a linear system. Eqs. 2 and 3 imply the principles of superposition and the proportionality, which are the essential features of linear systems. If the operator Φ in a system does not satisfy the superposition and proportionality properties, then the system is a nonlinear system.

Linear systems can be characterized by their response to a unit impulse function or to a unit step function. The unit impulse function or the Dirac-Delta function is defined as

$$\delta(t) \begin{cases} = 0 & \text{if } t \neq 0 \\ = \infty & \text{if } t = 0 \end{cases}$$

and

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \qquad (4)$$

The unit step function is defined as

$$H(t) \begin{cases} = 1 & t > 0 \\ = 0 & \text{Otherwise} \end{cases} \qquad (5)$$

Unit pulse function is defined as

$$\delta(t,b_2) \begin{cases} = 1/b_2 & 0 \leqslant t \leqslant b_2 \\ = 0 & \text{Otherwise} \end{cases} \qquad (6)$$

The unit impulse response, the unit step response and the unit pulse response of a system which is initially in an unexcited equilibrium condition, are defined as the outputs respectively due to unit impulse, unit step and unit pulse input. If $X(t)$ represents the input, $Y(t)$ represents the output, and $h(t)$ represents the unit impulse response, the input-output relation for a lumped linear system can be expressed by the convolution (Duhamel) integral

$$Y(t) = \int_{o}^{t} X(\tau)\ h(t - \tau)d\tau \qquad (7)$$

Thus the response of a lumped linear system to any input can be evaluated if the unit impulse response function is known.

Watershed System

In the analysis and synthesis of rainfall-runoff process, watersheds are conceived as systems with rainfall as input to the system and runoff as output from the system. The watershed complex, which transforms rainfall to runoff is usually referred to as the "watershed system". As both rainfall and runoff are functions of time and space, watershed systems are by definition distributed dynamic systems. Consequently, mathematical models for the rainfall-runoff process should be distributed dynamic models. These are complicated and such models have not been completely formulated and tested so far. However, in order to simplify the structure of the model the spatial distribution of rainfall is ignored and the time distribution of runoff is considered only at a point, so that a lumped dynamic system model is used to simulate the rainfall-runoff process. Use of lumped system models is closer to reality for urban basins than for rural basins because the former are often smaller in area and are more uniform in their characteristics.

Systems can also be treated as deterministic or stochastic systems. In the former method, attempts are made to develop relationships among the model parameters, the rainfall characteristics and physiographic characteristics of the watershed. This analysis is conducted using observed data. These relationships are then used to predict future runoff. On the other hand, in the stochastic systems, statistical measures of hydrologic variables are used to generate future events to which probability levels are attached. Long term records, which in many instances are not available, are needed to estimate the parameters of stochastic models, in order to obtain a proper representation of their stochastic nature.

Several deterministic system models have been proposed to simulate the rainfall-runoff process. The application of system analysis and synthesis techniques to mathematically model the rainfall-runoff process and which are relevant to the present study are discussed in the following paragraphs.

Storage Modelling

In formulating mathematical models of rainfall-runoff process, transformation of rainfall to runoff can be considered as a process involving either storage or translation action only or as a combination of both. Models formulated only on the basis of storage effects are called storage models. In storage modelling, a relationship among storage, inflow and outflow is postulated. A simple relationship such as

$$S = K\,Q \tag{8}$$

where, S is the storage,

Q is the outflow at any time,

K is a constant called the storage coefficient,

can be considered to describe the relationship between the storage and the outflow. This storage equation (Eq. 8), when combined with the hydrologic continuity equation

$$I - Q = \frac{dS}{dt} \tag{9}$$

where I is the inflow, yields the linear differential equation

$$I - Q = K\,\frac{dQ}{dt}\,. \tag{10}$$

Thus, lumped linear systems models of the rainfall-runoff process, can be described by linear differential equations such as Eq. 10.

Derivatives of rainfall and runoff can also be included in the storage equation, although Eq. 8 shown above is the simplest storage equation. Kulandaiswamy[12] proposed the following general relationship for storage, in terms of inflow, outflow, and their derivatives,

$$S = \sum_{n=o}^{N} A_n(Q,I) \frac{d^nQ}{dt^n} + \sum_{m=o}^{M} B_m(Q,I) \frac{d^mI}{dt^m} \qquad (11)$$

where, $A_n(Q,I)$ and $B_m(Q,I)$ may be constant or functions of inflow and outflow, and N and M are integers.

By combining the Eq. 11 with the hydrologic continuity equation (Eq. 9), the differential equation for the rainfall-runoff process can be obtained. The parameters A_n and B_m were assumed to be constants and the resulting linear differential equation was used by Kulandaiswamy to relate effective rainfall and direct runoff. For the special case of $N = 3$ and $M = 2$, the outflow from the watershed system was expressed by the differential equation:

$$Q(t) = \left[\frac{-B_1D^2 - B_0D + 1}{A_2D^3 + A_1D^2 + A_0D + 1} \right] [I(t)] \qquad (12)$$

or $$Q(t) = \Phi[I(t)]$$

where D is the differential operator d/dt.

Further, the parameters, A_n, B_m can be expressed as functions of the mean values $\overline{Q}$ and $\overline{I}$, so that they can be estimated and used for prediction.

Rao[13] considered the watershed as a lumped nonlinear system in a study of excess rainfall-direct runoff relationship, in which a storage equation similar to Eq. 11 was used. The parameters A_n and B_m were treated as functions of Q and I. They were approximated by a) a quadratic function of the inflow and outflow, b) a linear function and c) a constant. The resulting models were tested using both laboratory and field data. Several other nonlinear storage models which in general can be derived from Eq. 11 have also been proposed.[14,15]

Conceptual Models and Other Methods

The lumped linear system analysis is the main tool used in the present study. Hence, literature pertaining to lumped linear system analysis methods, as they are used to model rainfall-runoff process, will be briefly reviewed.

The impetus for development and application of lumped linear system analysis methods to rainfall-runoff process originates from the Unit Hydrograph theory.[16] The unit hydrograph is a hydrograph resulting from a one-inch rainfall excess generated uniformly over the basin area at a uniform rate during a specified period of time. Thus a unit hydrograph is associated with a specified unit duration of storm. The assumptions underlying the unit hydrograph theory and the limitations of these assumptions have been discussed in detail elsewhere[10] and hence will not be repeated here. However, it should be noted that the principles of superposition and proportionality are the basic principles of the unit hydrograph theory.

For a given drainage basin, the ordinate at time t of the unit hydrograph having a unit duration Δt_0 is represented by $U(\Delta t_0,t)$ where t is any time after the beginning of rainfall excess. Assuming that the effective rainfall consists of "n" blocks each of same duration Δt_0 and of uniform rainfall of different intensities I_i ($i = 1, 2, \ldots, n$), the ordinate of the direct runoff hydrograph for the given storm can be expressed as

$$Q(t) = \sum_{i=1}^{n} U[\Delta t_0, \{t - (i-1)\Delta t_0\}] I_i \, \Delta t_0 \qquad (13)$$

It must be noted that the unit hydrograph theory requires uniform rainfall at least for the finite duration Δt_0.

A later development in the unit hydrograph theory is the concept of the Instantaneous Unit Hydrograph. The Instantaneous Unit Hydrograph (IUH) is defined as the unit hydrograph of duration $\Delta t_0 = 0$, or mathematically,

$$U(0,t) = \lim_{\Delta t_0 \to 0} U(\Delta t_0, t) \tag{14}$$

and correspondingly by letting Δt_0 in Eq. 13 approach zero, the following equation results:

$$Q(t) = \lim_{\Delta t_0 \to 0} \sum_{i=1}^{n} U[\Delta t_0, \{t - (i-1)\Delta t_0\}] I_i \, \Delta t_0 \tag{15}$$

It can be seen that Eq. 15 is a special case of Eq. 7 in discrete form. Hence, the IUH is the same as the unit impulse response of a lumped linear system. The concept of lumped linear system analysis is thus implied in the unit hydrograph theory. In the concept of IUH, the duration of rainfall which is a basic factor in unit hydrograph analysis, is eliminated, thus removing one more variable from the analysis with consequent simplification.

The impulse response function of a lumped linear system describing rainfall-runoff process can be obtained by making use of, (1) Conceptual models, (2) Transform methods, (3) Methods using orthogonal functions, etc.

As conceptual models and transform methods are used in the present study a brief review of only these two methods is presented here. A detailed discussion of the other methods can be found elsewhere.[17,18,19]

Conceptual Models. In a conceptual model, the transformation of excess rainfall to surface runoff is considered as a combination of translation and storage effects. Using the principle of linearity, these separate elements are combined to formulate a conceptual model of the excess rainfall-direct runoff system. The linear hydrologic components used in the formulation of these models are, 1) linear reservoir, and 2) linear channel, which are defined below:

A linear reservoir is a fictitious reservoir in which the storage S is directly proportional to the outflow Q or

$$S = K\, Q \tag{8}$$

The linear reservoir thus represents pure storage action. By solving the differential equation (Eq. 10), the IUH of a linear reservoir can be shown to be

$$h(t) = \frac{1}{K} e^{-t/K} \tag{16}$$

In a linear channel, the time T required to translate a discharge Q of any magnitude through the given channel reach of length L is constant. Thus when an inflow hydrograph is routed through a linear channel, the shape of the inflow hydrograph will remain the same but it will be lagged by the time of translation T. Therefore in a linear channel if the input function is f(t), the output function will be f(t - T), thus providing the action of pure translation. For a linear channel, at any given section, the water area A_c and the discharge Q are related by

$$A_c = C_T\, Q$$

where C_T is called translation coefficient and is assumed to be a constant in most of the conceptual models. The IUH of a linear channel can be shown to be

$$h(t) = \delta(t - T) \tag{17}$$

The simplest conceptual model of rainfall-runoff process can be formulated by considering only the linear channel or the linear reservoir. If the watershed is considered analogous only to a linear channel, which receives spatially varied flow, then the watershed can be divided into "n" subareas by isochrones (contours of equal travel time). The area enclosed by the adjacent isochrones is plotted against the travel time to the outlet to obtain the "Time-Area-Concentration" diagram. Alekhin[20] derived the following expression for the runoff at the outlet of the basin by considering the basin as a linear channel,

$$d_t = \sum_{j=1}^{n} \frac{a'_j}{A} d_{t-(j-1)DT} \tag{18}$$

where A is the area of the watershed,

a'_j is the area of the j^{th} subarea,

d_t is the depth of excess rainfall or direct runoff at time t,

$d_{t-(j-1)DT}$ is the depth of excess rainfall which occurred at the time (j-1)DT before the time of consideration, t.

This formula is generally called "The Genetic Runoff Formula" in Russian hydrologic literature. Dooge[21] has demonstrated that the IUH, resulting from modelling the watershed as a linear channel only, has the same shape as the time-area-concentration curve. This can also be seen from the resemblance between the Eqs. 18 and 15.

Zoch[22] considered a small elemental area of the watershed for which he assumed that "at any time, the discharge is proportional to the rainfall remaining with the soil at that time." This assumption is the same as the concept of linear reservoir. For each elemental area of the watershed, Zoch developed the two following equations for discharge, the first of which applies for the period of rainfall and the second for the period after cessation of rainfall.

$$Q = I(t)(1 - e^{-t/K}) \qquad 0 \le t \le T_R \tag{19}$$

$$Q = Q_o \, e^{-t'/K} \qquad t \ge T_R \tag{20}$$

where T_R is duration of excess rainfall,

$t' = t - T_R$, time elapsed after the end of rainfall,

and $Q_o = Q\Big|_{t=T_R}$

These equations were then integrated over the area of the watersheds for different shapes of the time-area-concentration curves. The complete analysis for the cases of rectangular, triangular and semi-elliptical shapes of the time-area-concentration curves has been presented. Zoch's method is equivalent to routing the time-area-concentration curve through a single linear reservoir for a rainfall of finite duration T_R. Alternatively stated, Zoch's model is a series combination of a linear channel and linear reservoir.

Clark[23] suggested that the IUH can be derived by routing the time-area-concentration curve through a single element of storage. Physically, this is equivalent to Zoch's[22] formulation except that in this case, the rainfall duration is reduced to zero. However, Clark did not derive an explicit mathematical expression for the IUH. Instead, he derived the IUH by routing the time-area-concentration curve through a reservoir for which, the storage equation is represented by the special form of the Muskingum equation. However, Clark did not outline any procedure to derive the time-area-concentration curve.

O'Kelly[24] applied Clark's method for several watersheds in Ireland. He observed that the smoothing involved in routing is sufficient to permit the replacement of the time-area-concentration curve by an isosceles triangle. In a closing note to the discussion on his paper,[24] O'Kelly derived mathematical expressions for IUH for the Clark's model. However, these expressions were not applied by him to analysis of hydrologic data.

Nash[25] suggested that the IUH can be obtained by routing the instantaneous input through a series of n successive linear reservoirs of equal storage coefficient K_N. The model proposed by Nash does not involve the translation effect, and the expression for IUH is given by

$$h(t) = \frac{1}{K_N} \frac{e^{-t/K_N}}{\Gamma(n)} (t/K_N)^{n-1} \qquad (21)$$

where Γ is the Gamma function.

Dooge[21] derived a general equation for the IUH by separating the storage and translation effects in a watershed. In this method, each element of inflow contributed by the linear channel (represented by the ordinate of the time-area-concentration curve), is routed through the same sequence of linear reservoirs. The reservoirs may not have the same value for the storage coefficient. The IUH is expressed by

$$h(t) = \frac{1}{T_c} \int_o^t \frac{\delta(t - \tau)}{\prod_{j=1}^{n} (1 + K_j D)} \omega(\tau/T_c) d\tau \qquad (22)$$

where T_c is the classical "Time of Concentration",

n is the number of subareas of the watershed,

$\omega(\tau/T_c)$ is the ordinate of dimensionless time-area-concentration curve, D is the differential operator d/dt.

Solution of Eq. 22 becomes complicated if the model involves linear reservoirs with different storage coefficients, particularly if the number of subareas is large.

A technique of successive routing through partial storage of two equal linear reservoirs, has been proposed by Holtan and Overton.[26,27] This model is based only on the recession constant. The storage coefficient of each of the two reservoirs is assumed to be equal to half the recession constant. Thus, the lag effect is equally distributed between the two linear reservoirs. This particular conceptual model is a special case of Nash model.

Dawdy and O'Donnell[28] have presented an "over-all" model for the hydrologic cycle. The model is similar in concept to the Stanford Watershed model[29] but is deliberately kept simpler to facilitate parameter sensitivity studies. The model consists of four elements representing surface, channel, soil moisture, and ground water storages. Each of these elements were modelled separately considering the inputs and outputs to the elements. The nine parameters governing the model are optimized by using the "Hill-climbing" technique. The model is under continued investigation.[30,31]

Transform Methods. In all the cases of linear lumped system analysis of excess rainfall and direct runoff, in which conceptual models are used, the model structure is formulated first and then the parameters of the model are evaluated by using excess rainfall and direct runoff data. Evaluation of the impulse response function by using transform methods constitutes an alternative approach to "identify" the excess rainfall-direct runoff system. The basic assumption in these methods of analysis is that the excess rainfall-direct runoff process is linear.

The fact that the convolution operation in the time domain is equivalent to multiplication operation in the frequency domain forms the central concept of these transform methods. Either the Fourier Transform or the Laplace Transform can be used to transform the convolution integral from time domain to frequency domain.

A brief outline of the transform method using Fourier Transform is given below, as this method has been used in this study.

The Fourier Transform of a function f(t) is defined as

$$FT[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \qquad (23)$$

and the Inverse Fourier Transform is

$$FT^{-1}[F(\omega)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \qquad (24)$$

where FT is the Fourier Transform operator,

$F(\omega)$ is the Fourier Transform of the function f(t),

j is the imaginary unit = $\sqrt{-1}$

ω is the frequency in radians per second,

and FT^{-1} is the Inverse Fourier Transform operator.

Application of the Fourier Transform to the convolution integral (Eq. 7),

$$FT[Y(t) = \int_{0}^{t} X(\tau)\ h(t - \tau)d\tau] \qquad (25)$$

results in

$$\left.\begin{aligned} Y(\omega) &= X(\omega)\,H(\omega) \\ &\text{or} \\ H(\omega) &= Y(\omega)/X(\omega) \end{aligned}\right\} \qquad (26)$$

Thus,

$$\left.\begin{matrix}\text{the Fourier Transform of} \\ \text{the kernel function } h(t)\end{matrix}\right] = \frac{FT[\text{output}]}{FT[\text{input}]}$$

Blank and Delleur[32] developed a method of solving Eq. 26 for $H(\omega)$, in which effective rainfall and direct runoff data specified at discrete time intervals are used. The response function $h(t)$, can be obtained by taking the inverse transformation of $H(\omega)$, using Eq. 24.

Analysis of Rainfall-Runoff Process in Urban Watersheds

Although the general principles underlying the rainfall-runoff process are the same for rural or non-urban watersheds and urban watersheds, urban watersheds usually have different characteristics in comparison with rural watersheds. The urban watershed areas are usually smaller, and also, the stream channels in urban watersheds are more uniform. Furthermore, the storm sewers induce swift conveyance in urban watersheds. Consequently the urban watershed response will usually be much faster in comparison with the rural watershed response. In view of these and other differences urban hydrologic analysis is usually somewhat different from the hydrologic analysis of nonurbanized watersheds. The literature of urban hydrologic analyses pertaining to the present study will therefore be briefly reviewed.

Rational Formula

In the hydrologic design of drainage works in urban areas, the most popular empirical formula which is used to compute the peak discharge due to a storm is the Rational Formula, which is given by

$$Q_p = C\ I\ A \qquad (27)$$

where Q_p is peak discharge in cfs,

C is a runoff coefficient which depends upon the characteristics of the drainage basin,

I is the intensity of uniform rainfall,

and A is the area of the drainage basin in acres.

There have been several attempts to improve the Rational Formula ever since its introduction in 1887. Metcalf and Eddy[33] used a method called the "Zone Principle" in which the drainage area is divided into zones by isochrones or contours of equal travel time. Each zone is assigned an "appropriate" value of runoff coefficient, the magnitude of which depends upon the imperviousness of the zone, and the distance of the zone from the outlet. An average runoff coefficient applicable for the entire watershed is then estimated and used in the Rational Formula. Gregory and Arnold[34] developed a "General Rational Formula" by considering factors such as watershed shape, slope and pattern of drainage network. The General Rational Formula has the form,

$$Q_p = C_g \frac{a_0\ T_{Re}^{a_1}\ A}{T_c^{a_2}} \qquad (28)$$

where C_g is the runoff coefficient,

a_0, a_1, a_2 are coefficients which depend on geographic location of the watershed,

T_{Re} is the recurrence interval in years for a rainfall of intensity I in inches per hour equaled or exceeded for a duration of T_c minutes,

T_c is the duration in minutes of rainfall of intensity I and is equal to the time of concentration,

and A is the drainage area of the watershed in acres.

Mehn[35] developed a method to evaluate the "composite runoff coefficient" which is similar to the runoff coefficient 'C' of the Rational Formula with the exception that the composite runoff coefficient is applicable only to urban watersheds. In order to compute the composite runoff coefficient, the watershed was divided into subareas and the individual subareas were assigned different values of runoff coefficients. The magnitudes of runoff coefficients depend upon the physiographic characteristics of the subareas and were obtained from the ASCE manual of Engineering Practice.[36] These runoff coefficients were weighted and the weighted average value was adopted as the composite runoff coefficient instead of the coefficient C in the Rational Formula to compute the peak discharge from the entire area. Mehn's method is similar to the procedure suggested by Metcalf and Eddy,[33] but the watershed is grouped into subareas and runoff coefficients are assigned to these subareas in a different way to compute the composite runoff coefficient.

The frequency of peak runoff obtained by using the Rational Formula is assumed to be the same as the frequency of the rainfall intensity which is selected to compute the peak runoff. An investigation to check this assumption was undertaken by Schaake, et.al.[37,38] From the analysis of data obtained from six small urban watersheds (each of area less than 150 acres) located in Baltimore, Maryland area, frequencies of both the rainfall intensity and the peak runoff were found to be log-normally distributed. Empirical equations for computing the values of C and the "rainfall intensity averaging time", were also derived in terms of the physiographic characteristics of the watershed.

The accuracy of prediction of peak discharges by using the Rational Formula or its variations depends on the appropriate estimation of the values of the coefficient C, which in turn depends on the judgment of the designer. Thus the results obtained from the Rational Formula have considerable variation. However, the Rational Formula still remains popular in the hydrologic design of urban drainage facilities.[3,4]

Hydrograph Synthesis by Routing

Empirical formulas such as the Rational Formula yield only peak discharge estimates which are not too reliable. This drawback, and also the necessity of knowing the time distribution of runoff gave rise to methods of hydrograph analysis in urban watersheds. Horner and Flynt[39] were among the first to use hydrograph methods in the design of storm sewers. They measured the temporal variations in rainfall and runoff on three small (less than 5 acres), heavily urbanized areas in St. Louis, Missouri. Assuming that the abstractions from the rainfall are zero, the "100 Percent Runoff" hydrograph was computed for each storm on a drainage basin by using the unit hydrograph method. The "runoff factor", defined as the ratio of accumulated actual runoff to the accumulated "100 Percent Runoff", was computed for different times during the storm. The runoff factor at any time was found to be appreciably different from the ratio of instantaneous value of actual and 100 percent runoff, at the corresponding time. Further, the ratio between the total observed runoff and the total rainfall was found to vary widely for a given drainage area. The ratio between the total observed runoff and the total rainfall was also observed to be affected by antecedent precipitation and seasonal climatic conditions.

Horner and Jens[40] attempted to synthesize the hydrograph by first computing the excess rainfall distribution for each subarea of a watershed. The infiltration rates were estimated by using Horton's equation,

$$f_i(t) = f_c + (f_o - f_c)e^{-b_3 t} \tag{29}$$

where $f_i(t)$ is the rate of infiltration at time t,

f_c is the final constant rate of infiltration,

b_3 is a constant dependent on the soil type and vegetation,

and f_o is initial rate of infiltration.

The direct runoff hydrograph for each subarea was then computed by using Horton's equation of overland flow,

$$q = I \tanh^2[0.922t(I/n_r L_o)^{0.5} S_o^{0.25}] \tag{30}$$

where q is the overland flow at any time t in inches per hour,

n_r is the retardation coefficient representing the surface roughness,

S_o is the average overland flow slope expressed in percentage,

and L_o is the effective length of overland flow in feet.

These direct runoff hydrographs resulting from various subareas were suitably lagged and superposed to obtain the hydrographs of direct runoff at the outlet of the watershed.

Hicks[41] suggested a graphical method called as the "Peak Rate Method" of synthesizing direct runoff hydrographs. By analyzing the data of effective rainfall of 10 year frequency and different intensities and times of concentrations, charts were developed to compute direct runoff hydrographs. These direct runoff hydrographs were supposed to yield the runoff from a completely impervious area and were called "Basic runoff hydrographs". Then, by using a trial and error procedure in which the conduit storage was accounted for, the peak discharges of basic runoff hydrographs were computed. A table of peak discharges of basic runoff hydrographs for different times of concentration was prepared along with charts for "runoff factors". Runoff factors, defined as the ratio of volume of runoff to volume of rainfall, were computed by analyzing data from experimental watersheds which had different land-use classifications and soil types. The peak runoff rate from a given effective rainfall for any drainage area is computed by multiplying the basic peak rate with the appropriate runoff factor taken from the charts. Although the runoff hydrographs can also be computed by this method, the main emphasis is on computation of peak discharges.

For larger times of concentration, a method of "Summing Hydrographs" which is an extension of the "peak rate method" was suggested by Hicks. In the "Summing Hydrographs" method, the watershed was divided into subareas for each of which the direct runoff hydrographs were first obtained by the peak rate method. Then, the hydrographs from all the subareas which drain to a junction point were combined. The resulting combined hydrograph was routed to the next downstream junction point in the basin. The other combined hydrographs from other subareas in the basin which drain to the same downstream junction point were similarly routed. The routing process was continued to obtain the direct runoff hydrograph at the outlet of the drainage basin. The

"Summing Hydrographs" method was found to be more useful for large drainage areas with extensive sewer development.

A program of critical investigation of existing methods for design of storm sewers was conducted for the city of Chicago after the second World War. As a part of this program, Tholin and Keifer[42] developed methods to synthesize runoff hydrographs from urban watersheds. In Tholin and Keifer's method, direct runoff hydrographs for various times of travel were developed by routing the excess rainfall through the gutters, the laterals and the main sewers. From these hydrographs, a number of charts for peak runoff as a function of percentage of impervious area, time of travel, rainfall magnitude and physiographic characteristics of the watershed were prepared. These charts cover the maximum probable range of each of the independent variables such as percentage of imperviousness, and are used as design charts to predict peak rate of runoff from a given storm for a drainage area. Prior knowledge of infiltration and depression storage is necessary to use this method.

The Storm Drainage Research project was initiated at the Johns Hopkins University in 1949. The objectives of the project were to develop methods of accurate measurement of rainfall and runoff especially in small urban watersheds and to develop methods of predicting runoff hydrographs from urban watersheds by using the given rainfall information and the data of physiographic characteristics of the urban watersheds. Initially, four completely paved watersheds, all of area less than an acre, and which had longitudinal slopes of one to three percent, were instrumented. This program of data collection from urban watersheds was later extended so that currently data are being collected from 29 urban watersheds of areas ranging from 0.1 acre to 153 acres. The percentage of built-up or impervious area in these watersheds varies from 9 to 100 percent, and these watersheds are all located in Baltimore, Maryland.

As a part of the Johns Hopkins project, Bock[43] introduced the "Inlet Method" for the prediction of peak runoff at the outlet or at any desired point of the watershed. The inlet hydrograph is assumed to be triangular, with a peak discharge of q_i. The base time of inlet hydrograph is assumed to be $2T_b$ (in minutes), where T_b is defined as the time elapsed from the beginning of rainfall to the end of the period of maximum rainfall intensity. The routing relation used in this method is,

$$Q_p = q_i\left(\frac{2T_b}{2T_b + 0.8}\right)\frac{L_d}{V} \qquad (31)$$

where L_d is the length of the drain from the inlet to the outlet or to the desired point in the watershed, in feet,

and V is the mean velocity of flow in the drain in feet per minute.

Using the above equation, the inflow to the individual inlets were routed to a common outlet point by suitably lagging and superposing the outflows to obtain the combined outflow. The Inlet Method, according to Bock, yielded good peak discharge estimates in 80 percent of the 56 storms tested in comparison with 56 percent good peak estimatesobtained by the Rational Formula. In a continuation of the work of Bock, Viessman and Geyer[44] developed the following relationship for peak discharge in sewers in fully paved areas:

$$Q_p = \frac{0.769}{n_s} P_E^{0.09} T_R^{0.16} I_m^{0.88} A^{0.95} \bar{S}_g^{0.17} \qquad (32)$$

where n_s is the selected value of Manning's 'n'

I_m is the maximum rate of rainfall in inches per hour,

A is the watershed area in acres,

$\bar{S}_g$ is the mean gutter slope,

and P_E is the volume of excess rainfall.

Further, the time of rise T_P (in seconds), defined as the time from the beginning of surface runoff to the peak of runoff hydrograph, was observed to be related to the duration of the intense rainfall T_s (in seconds), by the following relationship:

$$T_P = 77.8 + 1.011\ T_s$$

In the first of his two studies, Willeke[45,46] used the Muskingum Routing Method to compute the runoff from urban watersheds. After separating the base flow by the Φ-Index method, a trial and error procedure was adopted to determine the coefficients in the Muskingum Storage Equation. Willeke concluded that a model based on storage equation of the type of Eq. 8 yielded a better reproduction of the observed runoff hydrographs for small urban areas and also that his model was relatively insensitive to the different methods of baseflow separation.

In his second study, Willeke[46] applied his method of analysis to data from four small urban watersheds. Willeke postulated that the runoff from small urban watersheds could be satisfactorily modelled by the simple storage-discharge relationship given by Eq. 8. Hence the watershed response is a function of time lag only, where the time lag is defined as the time interval between the centers of mass of effective rainfall hyetograph and direct runoff hydrograph. The storage equation (Eq. 8) was combined with the hydrologic continuity equation and the resulting differential equation (Eq. 10) was solved in its finite difference form. Willeke concluded that the time lag was essentially a constant, and that there was no significant correlation between time lag and storm characteristics for the data from impervious watersheds studied. For pervious watersheds, however, the time lag varied considerably from storm to storm and the total abstraction, F_i (in inches) was found to be linearly related to the watershed slope, $\overline{S}$(%), by the equation

$$F_i = 0.162 - 0.039\ \overline{S}$$

Unit Hydrograph Methods

Eagleson[47] applied the unit hydrograph methods to study rainfall-runoff relationships in urban watersheds. The "Volumetric runoff coefficient" which was defined as the "ratio of total volume of runoff to the total volume of rainfall", was found to be a constant for the data used in the analysis. By using the volumetric runoff coefficient, Eagleson computed the rainfall excess and thereby derived the 10 minute unit hydrographs. The unit hydrograph characteristics were then related to the physiographic characteristics of the watershed. The relationship between the unit hydrograph peak discharge per square mile of the watershed, q_{um}, and the mean basin slope $\overline{S}$, was found to be

$$q_{um} = (2.13 \times 10^5)\overline{S}$$

Eagleson observed that for watersheds with appreciable channel storage, q_{um} was a decreasing function of excess rainfall. The unit hydrograph base width, the widths at 50% and 75% of q_{um} were plotted against the maximum unit hydrograph discharge, q_{um}.

Viessman[48,49,50] also used the unit hydrograph method to analyze rainfall-runoff process in urban watersheds. The excess rainfall was obtained by using a combination of an initial abstraction deduction and the Φ-Index method. For all the storms on a watershed, one minute unit hydrographs were derived, so that the outflow Q at any time t was given by

$$Q = I(1 - e^{-1/k})\ e^{\frac{(1-t)}{k}} \tag{33}$$

The optimum value of the storage constant k was computed for each storm by minimizing the sum of the squares of the difference between the observed and computed discharges, and also by equating the times to peak of the observed and computed peak discharges.

Viessman concluded that the optimum values of the storage constant varied considerably. However, the hydrographs regenerated by using the average value of the optimum storage coefficients, $\overline{K}$, agreed very well with the observed direct runoff hydrographs. Also, the optimum storage coefficients were not found to be significantly correlated with the rainfall characteristics. For each watershed, the average value of the optimum storage constant $\overline{K}$ was found to be related to both the average time lag $\overline{T}_4$ and the geophysical characteristics of the basin by the following equations:

$$\overline{K} = 1.565\ \overline{T}_4 - 3.575 \tag{34}$$

and

$$\overline{K} = 0.015 \left[\frac{n_s\ L_d}{0.015\sqrt{S_g}}\right]^{0.66} \tag{35}$$

Instantaneous Unit Hydrograph Methods

The possibility of modelling rainfall-runoff process on very small impervious areas (less than 1 acre) by means of conceptual models was investigated by Eagleson and March.[51] For purposes of comparison the instantaneous unit hydrographs were derived by the "Direct Method"[52] and also by using conceptual models proposed by Zoch,[22] Nash[25] and Singh.[53] It was observed that actual direct runoff hydrographs were satisfactorily reproduced by using the Instantaneous Unit Hydrographs derived by the direct method, although there was considerable variation in the shape of the IUH. Hence it was concluded that no single IUH can be used to obtain the runoff from a watershed for all storms. Another conclusion of this study was that among the three conceptual linear models considered, the Zoch model provided better regeneration of run-off than either the Nash model or the Singh model.

Delleur and Vician[54] have used two conceptual models in their analysis of data from urban watersheds in West Lafayette, Indiana. The storage coefficient K, of the single linear reservoir model, which was the first conceptual model used in the analysis, was determined by a trial procedure. From the data analyzed, it was reported that a value of K which is equal to 0.8 times the observed time lag, gave better re-generation of the runoff hydrograph than the cases in which K was assumed to be equal to the observed time lag or its average value. The second conceptual model which was a series combination of a linear channel and a linear reservoir, was used in an attempt to represent both lag and storage effects in the watershed. The travel time re-quired for obtaining the time-area-concentration curve was estimated by calculating the actual velocities of flow in the storm sewers. The linear-reservoir-channel model consistently predicted lesser peak discharges than the single linear reservoir model for the data analyzed.

Estimation of Parameters of Linear Models

The models of rainfall-runoff process are developed with the ultimate objective of using them for prediction of runoff from storms. The parameters of these models may be estimated by using the storm and/or the physiographic characteristics. Previous research in parameter estimation of linear conceptual models was based on essentially the two following aspects. The unit hydrographs or the instantaneous unit hydrographs or their dimensionless graphs were described in terms of a set of parameters. These parameters were then related, in most of the instances, to the physiographic characteristics of the watershed. A very brief review of some of these investigations in parameter estimation are given next.

Dimensionless graphs, which were obtained by an analysis of unit hydrographs, were developed by Gray[55] for several watersheds. The dimensionless graph represents a modified form of the unit hydrograph in which the basic shape is retained. A two parameter Gamma distribution function of the form

$$Q(t/T_p) = 25.0 \frac{(\gamma')^{nl}}{\Gamma(nl)} e^{-\gamma'(t/T_p)} (t/T_p)^{nl-1}$$

was fitted to these dimensionless graphs. The parameter γ' was found to be related to the parameter nl, and to the time to peak discharge T_p by the following relationships:

$$\gamma' = n1-1$$

and $$\gamma' = C_0 + C_1 \, T_p$$

where, C_0 and C_1 are constants.

The ratio T_p/γ' was designated as the storage factor, and was correlated to the stream length L, and the channel slope S_c, by the relationship:

$$T_p/\gamma' = C_2 \; (L/\sqrt{S_c})^{C_3}$$

Thus by knowing C_2, C_3, L and S_c, the parameters γ' and n1 of the Gamma distribution function can be estimated. Hence the dimensionless graph and the unit hydrograph applicable to a watershed can be determined and used for prediction of runoff.

Reich[56] developed a method of predicting the runoff hydrograph by fitting the following three-parameter equation to the time distribution of total runoff which has Q_p, T_p, and $(T_Q - T_p)$ as its parameters.

$$Q = Q_p \; e^{-t/(T_Q - T_p)} (1 - t/T_p)^{-T_p/(T_Q - T_p)}$$

where t is the time measured from the time of occurrence of peak discharge, and

$(T_Q - T_p)$ is the time interval between the time to peak discharge and the centroid of the runoff hydrograph, both of which are measured from the beginning of runoff hydrograph.

The parameter Q_p was related to the physiographic characteristics of the watershed and the 30 minute intense rainfall I_{30}, by the relationship

$$Q_p = C_0 + C_1 \, I_{30} + C_2 \, L_{ca} + C_3 \, L$$

where L_{ca} is the distance along the main stream from the watershed outlet to a point opposite the center of gravity of the watershed.

The mean basin slope $\bar{S}$ and the mean channel slope $\bar{S}_c$ were related to the parameter (T_Q-T_p) by the relationship

$$(T_Q-T_p) = C_0\ D_c^5 / (\bar{S}_c^{\ C_1}\ \bar{S}^{C_2})$$

where D_c is a factor which depends on the soil type, land use and topography. Further, the parameter T_R was graphically related to the volume of total runoff V_R, which was related to the infiltration capacity f_c, the total rainfall P_T, and the time of concentration T_c, by the relationship

$$V_R = C_4 + C_5\ f_c + C_6\ T_c + C_7\ P_T$$

Thus if the magnitudes of the total rainfall, the infiltration capacity, the 30 minute intense rainfall and the physiographic characteristics of the watershed are available, the total runoff hydrograph can be calculated by Reich's method.

Nash[57] estimated the parameters of the IUH and of the unit hydrograph by computing moments of the IUH and relating them to the area A, length of overland flow L_o, and the slope S_o of overland flow by the following equations:

$$M_1 = 27.6\ \ A^{0.3}\ \ S_o^{-0.3} \tag{36}$$

$$M_2 = 0.41\ \ L_0^{-0.1} \tag{37}$$

From Eqs. 36 and 37, and the equations for the moments of the IUH, parameters n and K_N of the Nash model can be determined.

In another study based on the Nash model in the dimensionless form the following equations for estimating the time to peak T_{pi} and the parameter K_N of the IUH, were given by Wu, Delleur and Diskin.[58]

$$T_{pi} = 31.4\ \ A^{1.08}\ \ L^{-1.23}\ \ S^{-0.67}$$

$$K_N = 783\ \ A^{0.94}\ \ L^{-1.48}\ \ S^{-1.47}$$

The parameter n of the Nash model is obtained by using the relationship

$$T_{pi} = (n-1)\ K_N$$

The value of Q_p, which is needed, is computed by using the rational formula. The runoff hydrograph can be obtained by using the appropriate dimensionless instantaneous hydrograph (Q/Q_{pi} vs t/T_{pi}) which corresponds to the estimated value of the parameter n.

In all these methods, the parameters of the unit hydrograph or of the IUH or of the dimensionless graph are implicitly assumed to be unique for a watershed. Hence a unique set of parameters which is used for a watershed to describe the unit hydrograph or the IUH, has been related mainly to the physiographic characteristics. Although the storm characteristics were used to a certain extent by Reich,[56] the several factors such as infiltration capacity, the factor D_c, etc., which were used by him cannot be estimated readily.

Effects of Urbanization on Runoff From Small Watersheds

Urbanization is human inhabitation and the consequent controlled development of previously uninhabited land. Apart from the increase in population density which is of interest to civil engineers in general, construction of buildings and facilities which change the watershed response to the rainfall is of particular importance to the hydrologist. Because of the increased number of buildings and of larger paved areas, the average local infiltration rates decrease thus resulting in higher volumes and rates of runoff. The time lag between rainfall excess and of the runoff hydrograph also decreases mainly due to the improvement of channels and the construction of sewer systems in urbanizing areas.

Although some work has been reported, quantitative evaluation of effects of urbanization on runoff is still in its initial stages as the techniques needed for such studies are still evolving. In most of these studies, the percentage built-up or "impervious" area in the watershed has been used to characterize the stage of urbanization. Savini and Kammerer[59] were among the first to discuss the effects of urbanization on watershed response, although their emphasis was on the effect of urbanization on water resources of the region in general.

Hydrologic data for urbanized watersheds are usually unavailable for the rural conditions of the past. Because of this lack of availability of hydrologic data for the watersheds before and after urbanization, evaluation of effects of urbanization on runoff of a particular watershed by analysis and comparison of observed data is not possible. Hence, in order to study the effects of urbanization, runoff data from an urban watershed and from a rural watershed which is in proximity of the urban watershed, are used. Then, some of the runoff characteristics as obtained from the data of the two watersheds are compared. Most of the previous investigators have used time lag and magnitude and frequency of peak discharges to study the effects of urbanization on runoff. Almost all such studies have revealed that, compared to rural watersheds, the average time lag for urban watershed is lesser by 60 to 70 percent, the peak discharges are higher by three to five times and that the frequency of peak discharge is relatively higher. However, the quantitative results in these studies vary widely because the data used for the studies have been drawn from different regions. Further, the results from different investigations cannot be easily compared as the definitions used are usually different. For example, one of the most commonly studied parameters is the time lag, which is defined in different ways by various investigators. Various definitions of time lag used by several investigators are summarized in Table No. 1. However, some of the important investigations of effects of urbanization on runoff, which are pertinent in the context of the present study are discussed below.

Time Lag

Carter[60] based his studies of the effects of urbanization on runoff on the time lag T_4 (Table No. 1). The average time lag $\overline{T}_4$ in hours for a watershed was expressed by Carter as

$$\overline{T}_4 = a_0(L/\sqrt{S_W})^{a_1} \tag{38}$$

where L is in miles, S_W is the weighted slope of the main stream expressed in feet per mile, a_0 and a_1 are coefficients. Carter's results were based on the analysis of data from 22 streams in and around Washington, D.C. Curves 1 and 2 in Fig. 2 show the average time lag, $\overline{T}_4$, as a function of the ratio $(L/\sqrt{S_W})$ for different basins which are qualitatively grouped by Carter as partly sewered basins and natural basins with no sewer construction. In comparing his results with those of Snyder,[61] Carter observed that the time of concentration T_c, in Snyder's study was expressed by Eq. 38 but with different values of the coefficients a_0 and a_1. However, the coefficient 'a_0' in the equations of Carter and Snyder had different values, although the values of the exponent a_1 were almost equal. Snyder's results shown by Curve 3 in Fig. 2 were obtained from the data of completely sewered areas in

Table 1. Definitions of Time Lag

Symbol	Definition*	Reference
T_1	Time from centroid of excess rainfall to the peak of direct runoff hydrograph	47,65,75
T_2	Time from beginning of continuous excess rainfall to the peak of direct runoff hydrograph	66,69,73
T_3	Time from beginning of continuous excess rainfall to the centroid of direct runoff hydrograph	47,64
T_4	Time from the centroid of excess rainfall to the centroid of direct runoff hydrograph	45,46,54, 60,63,70,101
T_5	Time from the centroid of excess rainfall to the mid-volume of direct runoff	26,27

*See Fig. 1. Numbers in the 'Reference' column refer to entries in bibliography.

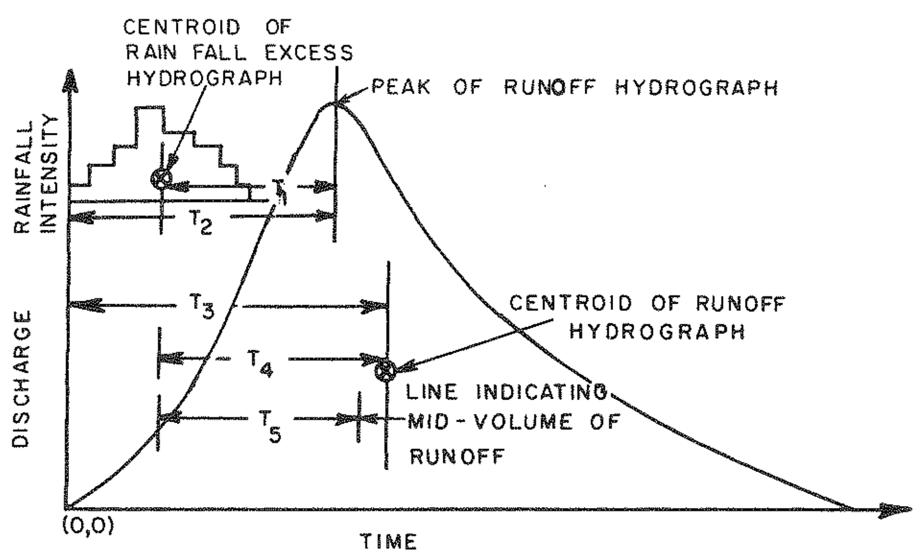


FIGURE I. DEFINITIONS OF TIME LAG

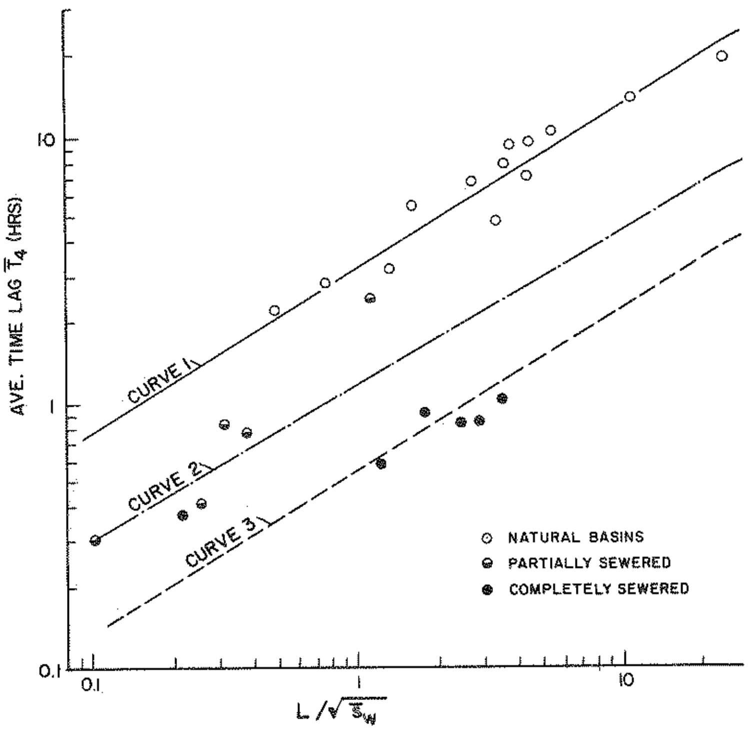


FIGURE 2. VARIATION OF AVERAGE TIME LAG WITH $L/\sqrt{\bar{S}_W}$ (AFTER CARTER[60]).

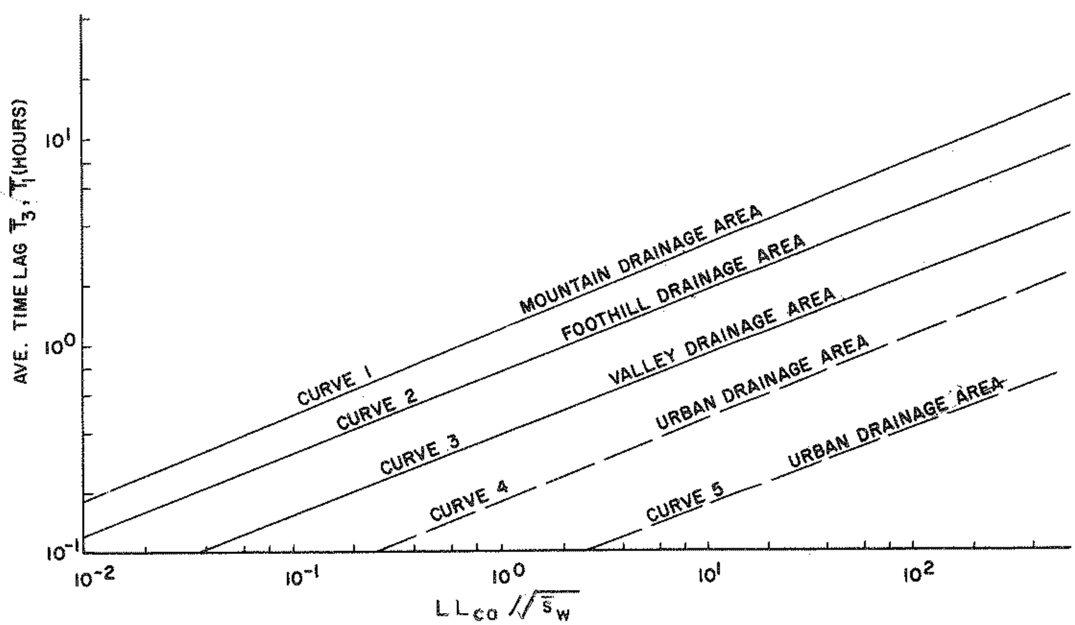


FIGURE 3. VARIATION OF AVERAGE TIME LAG WITH $LL_{ca}/\sqrt{\bar{S}_W}$ (AFTER EAGLESON[47]).

Virginia and other states. From Fig. 2 it may be noticed that compared to the average time lag values of the natural basins, the average time lag values of the partially sewered basins and the completely sewered basins were respectively lesser by about 66 percent and 85 percent. However, Carter made no attempt to express the average time lag as a function of the percentage of impervious area in the watershed.

Based on an analysis of data of floods for 81 watersheds, 59 of which were in Washington, D.C. area, Anderson[62] reported that the time lag T_4, was approximately constant for all average or larger floods, provided that the basin was wet prior to the storm.

Wiitala[63] applied the Carter's Equation (Eq. 38) for two watersheds near Detroit, Michigan. One of the two watersheds (The Plum Brook Watershed, 22.9 square miles) was in rural condition and the other (The Red Run Watershed, 36.5 square miles), was completely sewered and had approximately 25 percent impervious cover. He observed that because of urbanization the magnitude of $\overline{T}_4$ of the Red Run Watershed was lesser by as much as 70 percent compared to $\overline{T}_4$, of the Plum Brook Watershed.

Linsley, Kohler and Paulhus[64] presented the following equation for the average time lag $\overline{T}_3$ (Table No. 1), as a function of the watershed characteristics:

$$\overline{T}_3 = a_0[L\ L_{ca}/\sqrt{S}]^{a_1} \qquad (39)$$

where L_{ca} is the distance along the main stream from the basin outlet to a point opposite the center of gravity of the basin in miles, a_0, a_1 are the regression coefficients, and $\overline{T}_3$ is the average value of the time lag T_3. The Eq. 39 was obtained from the analysis of data from undeveloped watersheds in California. Curves 1, 2 and 3 shown in Fig. 3 represent average time lag ($\overline{T}_3$) values for watersheds in mountainous terrain, in foothills and in valleys respectively. In his study of unit hydrographs of urban watersheds, Eagleson[47] compared the values of average time lag $\overline{T}_1$ of urban watersheds with the results presented by Linsley, et.al. Data from urban watersheds in Louisville, Kentucky were analyzed by Eagleson to obtain average values of T_1 and T_3, which are shown as curves 4 and 5 in Fig. 3. Results show that the time lag values of the urban drainage areas were lesser relative to the time lag of mountainous, foothill and valley areas.

Van Sickle,[65] in his study of effects of urbanization on runoff from watersheds near Houston, Texas, classified the watersheds as 1) cultivated, some urban, no storm sewers, 2) more urban, some storm sewers, no channel improvement, 3) extensive urban, storm sewers, no channel improvement, and 4) extensive urban, storm sewers, considerable channel improvement. Based on the comparison of values of average time lag $\overline{T}_1$, for the watersheds of classes 1 through 4 above, Van Sickle concluded that urbanization can cause as much as 67 to 92 percent reduction in the average time lag.

Espey, Morgan and Masch,[66,67] and Espey,[68] have assumed the 30 minute unit hydrograph to be representative of watershed response. Characteristics of the 30 minute unit hydrographs which were derived by using data from urban and rural watersheds were compared by using relationships obtained by regression analysis techniques. Espey, et.al., observed that the length of the stream and slope of the stream were significant in characterizing the unit hydrograph parameters for rural as well as for urban watersheds. The time of rise of the unit hydrograph, (T_{2U}) which is defined as the time from the beginning of runoff to the peak runoff was used as a factor for evaluating the effects of urbanization on runoff. The following equation for the times of rise of unit hydrograph of rural watersheds (T_{2UR}) and of urban watersheds (T_{2UU}) have been derived:

$$T_{2UR} = 1.24 \ (L/\sqrt{\overline{S}})^{0.36} \quad (40)$$

and

$$T_{2UU} = 20.8 \ L^{0.29} \ (\overline{S})^{-.11} \ I^{-0.61} \quad (41)$$

However, the predicted time of rise was found to be more than the actual time of rise for some urban watersheds. In order to correct this descrepancy a correction factor for urban watersheds, θ, which is the ratio between the observed and predicted times of rise was defined and used.

A limited study, in which the data obtained from Espey, et.al., was used to investigate the effects of urbanization on unit hydrograph characteristics, has been reported by Riley and Dhruvanarayana.[69] The analysis was carried out using analog simulation techniques and the "representative" 30 minute unit hydrographs derived by Espey, et.al. An "Equivalent Rural Watershed" which is defined as a hypothetical rural watershed which has the same unit hydrograph as the corresponding urban watershed, has been used for comparison. Two parameters, the length and slope of the equivalent rural watershed, which were expressed in terms of the equations for the unit hydrograph parameters developed by Espey,[66,67] et.al., were defined as follows:

$$L_{WER} = [T_{2U} \ [\overline{S}_U \ L_{WU}]^{C_0} \ /C_1]^{C_2}$$

$$\bar{S}_{ER} = [\bar{S}_U \; L_{WU}] \;/L_{WER}$$

where L_W is the maximum length of travel on the watershed, and subscripts U and ER represent urban and equivalent rural watersheds respectively. An "impervious length factor" (L_f) which is defined as the ratio,

$$L_f = \left[\frac{\sum a_j \; L_j}{\sum a_j \; L_W}\right] /L_W$$

where a_j represents area of the j^{th} individual impervious area in the drainage basin, and L_j is the distance of the j^{th} individual impervious area from the basin outlet measured along the stream, in miles, was also used for comparison.

The various components of the hydrologic cycle were simulated on the analog computer. The parameters of the analog model were then related to the impervious length factor. The variation of the factors L_{ER}, S_{ER}, time lag T_{2U} and the peak of urban unit hydrograph, as a function of percentage imperviousness and impervious length factor are shown in Fig. 4.

Time lag has thus been used as the significant parameter both in urban hydrologic studies and in studies of effect of urbanization on runoff. Investigations of Snyder,[61] Linsley,[64] et.al., Carter,[60] Wiitala,[63] Viessman,[48,49,50] Eagleson,[51] Espey,[66,67,68] et.al., and Schaake[70,71] are based on the assumption that average time lag was affected mainly by watershed characteristics. Relationships for average time lag in terms of physiographic characteristics of watersheds have been proposed by many of these investigators. However, time lag values vary widely from storm to storm and consequently are affected not only by the watershed characteristics but also by the storm

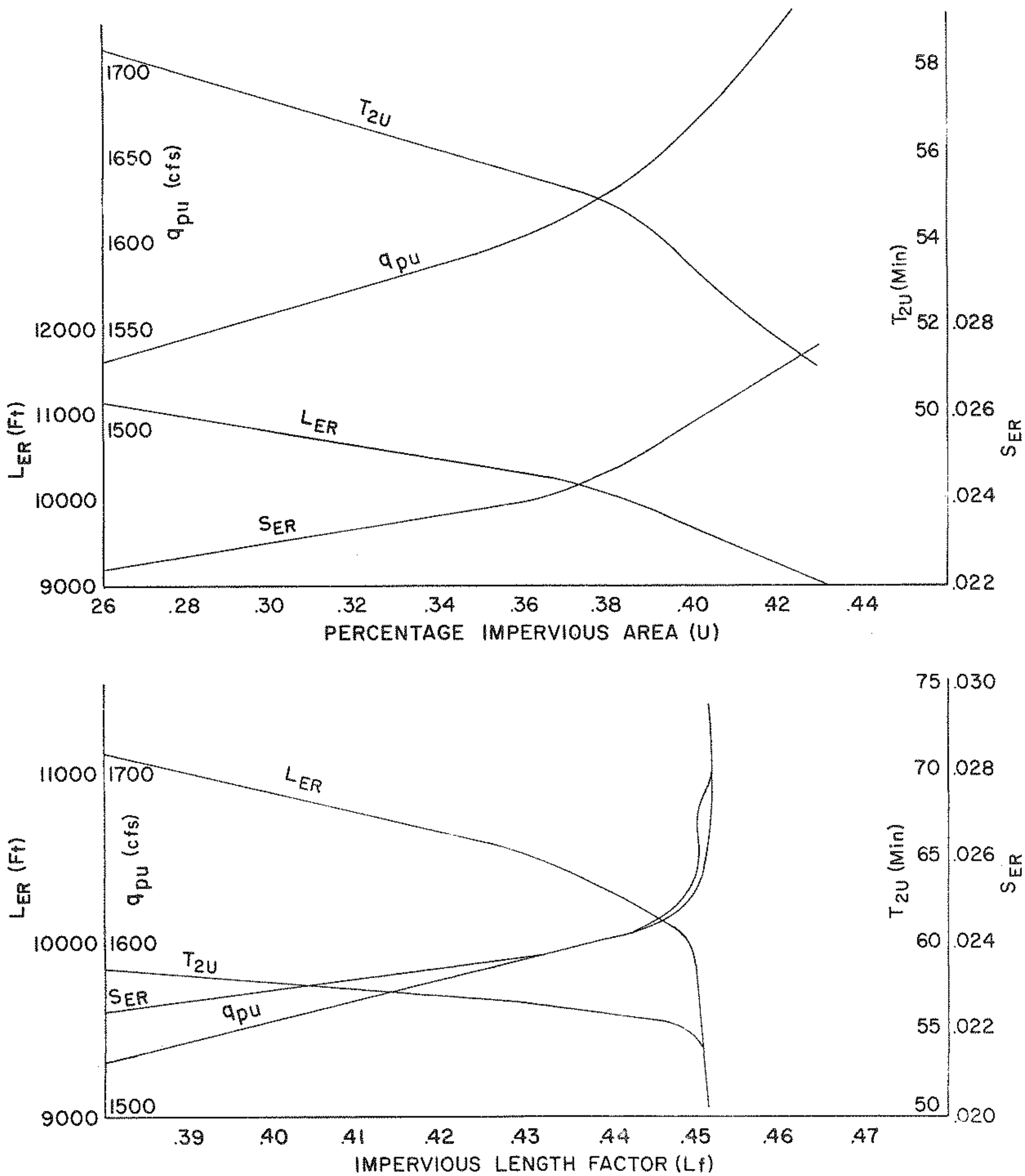


FIGURE 4. VARIATION OF CHARACTERISTICS OF UNIT HYDROGRAPH WITH PARAMETERS U AND L_f. (AFTER RILEY, et. al.[69])

characteristics. Changes in soil moisture level in the basin from storm to storm might also contribute to the variation in the time lag. This variation in time lag values has been recognized by Viessman and Razaq[72] and Landreth[73] among others, although no general relationships which consider the variation in time lag have been proposed so far.

Peak Discharge and its Frequency

Apart from time lag which has been intensively studied and widely used to characterize the effects of urbanization on runoff, changes in magnitude and frequency of peak floods due to urbanization of watersheds have also received considerable attention. Carter,[60] in his study of changes in magnitude and frequency of peak floods caused by urbanization of watersheds, assumed that the average rainfall-runoff coefficient of 0.3, which was obtained by rainfall-flood volume studies for watersheds near Washington, D.C., is applicable to peak floods also. Further, the effect of changes in impervious area was assumed to be independent of the size of the flood, and that 75 percent of rainfall volume on impervious surfaces reaches the stream.

Using the above mentioned assumptions, Carter[60] applied regression techniques to relate the annual peak flood $\overline{Q}$, which corresponds to a definite recurrence interval, to the area of the watershed A (in square miles) and average time lag $\overline{T}_4$ (in hours) as

$$\overline{Q} = C_I \, a_0 \, A^{a_1} \, \overline{T}_4^{\,a_2} \tag{42}$$

In Eq. 42, a_0, a_1, a_2, are regression coefficients whose values depend on the recurrence interval, and the coefficient C_I is related to the percentage of impervious area U by the relationship

$$C_I = \frac{(0.30 + 0.0045\ U)}{0.30} \tag{43}$$

By this analysis, Carter concluded that effects of sewer construction, channel improvement and other features of urbanization on peak floods were more significant than the effects of changes in percentage of impervious areas. Also, the maximum effect of complete suburban development in watersheds larger than 4 square miles in area, was found to be an increase in peak discharge by 80 percent.

In a later report, Carter and Thomas[74] modified the coefficient C_I defined above. Based on additional data from drainage basins near Washington, D.C., they redefined the coefficient C_I as

$$C_I = \frac{W - 0.01\ W\ U + 0.0075\ U}{W} \tag{44}$$

where W is the percentage of direct runoff to total runoff.

Anderson[62] conducted an analysis similar to Carter's and presented charts to estimate the magnitudes of peak floods, which have recurrence intervals ranging up to 100 years for watersheds in which various degrees of urban or suburban development has occurred.

Stall and Smith[75] compared unit hydrographs derived from data obtained from two small watersheds in Champaign, Illinois, one of which was almost entirely agricultural land and the other with 38.1 percent of impervious area. The mean basin slopes and the stream channel shapes of the two watersheds were generally similar. Comparison of unit hydrographs showed that the peak of the unit hydrograph of the urban watershed was about four times that of the rural watershed. The average time lag $\overline{T}_2$ was about four times greater for the rural watershed than for the urban watershed.

In the study of Espey,[66] et.al., discussed earlier, it was concluded that because of urbanization, the magnitudes of peak discharges increase by 51 percent compared to those of rural watersheds and that the "unit yield" defined as the volume of runoff per unit area of the watershed increases by about 200 percent relative to the unit yield of rural watersheds.

Critique and Motivation

From the preceding literature review, the methods of analysis of the urban rainfall-runoff process and the methods used to study the hydrologic effects of urbanization seem to be still evolving. Relative performance of different types of hydrologic models when they are used to analyze data from urban watersheds, their use in quantitative evaluation of the effects of urbanization on runoff, and the related topics have not yet been fully explored.

From many studies it has been noticed that the rainfall-runoff process is not linear.[11,76,77] However, the basic underlying assumption of the commonly used methods in urban hydrology, such as the routing and the unit hydrograph methods, is that the rainfall-runoff process is linear. The unit hydrograph and the IUH are not unique for any watershed but vary from storm to storm on the same basin.[32,76,78] The physiographic characteristics of the watershed and the soil moisture in the watershed antecedent to the storm also contribute to the variation of the unit hydrograph and the IUH. Unless the variations in the unit hydrograph due to the storm and the physiographic characteristics can be clearly identified and separated, the changes in the unit hydrographs due to urbanization alone cannot be accurately evaluated.

Although there has been some success in explaining the variation of unit hydrographs,[76] the process is still subjective. Consequently, it was decided to use the instantaneous unit hydrographs instead of the unit hydrographs for further analysis in the present study.

Analyses of data for very small impervious areas (which are usually less than 1 acre in area) by Willeke,[46] Viessman,[48,50] and Eagleson et.al.,[51] indicate that the rainfall-runoff process in small impervious areas can be simulated by the linear conceptual models. Delleur and Vician[54] have also used conceptual models in actual urban watersheds which are not totally impervious. However, other than the results reported by Eagleson et.al.,[51] the information about the relative regeneration performance of various conceptual models when they are used to analyze actual urban rainfall-runoff data, is not available. As various conceptual models can be formulated by combinations of the linear elements such as the linear reservoir and the linear channel the question of suitability of any of these models to simulate the urban rainfall-runoff process arises. Further, it is possible that for watersheds of different degrees of urbanization different conceptual models may be suitable to accurately simulate the rainfall-runoff process. Thus an investigation of the regeneration performance of a number of linear models and nonlinear models is necessary.

Variation of the parameters of these models with the physiographic characteristics including the percentage built-up area in the watershed, and the storm characteristics should be investigated with a view to establish the relationships among them. Using these relationships, provided they are meaningful and accurate enough, the effects of changes in the built-up areas on runoff should be investigated.

Based on these considerations, it was decided to investigate the relative regeneration performance of several linear (lumped, time invariant) system models, to serve as a guide for the selection of a suitable model or models to simulate the rainfall-runoff process in urbanized watersheds. It is proposed to relate the parameters of the selected model(s) with the important factors which affect the rainfall-runoff process and then to employ the model to investigate the effects of urbanization on runoff.

CHAPTER III

COLLECTION AND PREPARATION OF DATA FOR ANALYSIS

Quantitative evaluation of effects of urbanization on runoff would be relatively simple if rainfall and runoff data for watersheds were available for both their pre-urban and urban conditions. For most watersheds such hydrologic data are not usually available and hence evaluation of changes in runoff characteristics due to urbanization by direct comparison and data analysis is not possible. However, analysis of data from watersheds in the same region but which are in different stages of urbanization does reveal the effects of urbanization on runoff characteristics and this is the general approach used in the present study. To conduct such an analysis hydrologic data from several watersheds with different degrees of urbanization are needed. Hydrologic data from the following urban watersheds have been collected for analysis:

1) Ross Ade (upper), West Lafayette, Indiana,
2) Ross Ade (lower), West Lafayette, Indiana,
3) Purdue Swine Farm (upper), West Lafayette, Indiana,
4) Purdue Swine Farm (lower), West Lafayette, Indiana,
5) Pleasant Run at Arlington Avenue, Indianapolis, Indiana,
6) Pleasant Run at Brookville Road, Indianapolis, Indiana,
7) Waller Creek at 38th Street, Austin, Texas,
8) Waller Creek at 23rd Street, Austin, Texas.

Data from the following rural watersheds are also used in the analysis:

1) Little Eagle Creek at Speedway, Indiana,

2) Lawrence Creek at Fort Benjamin Harrison, Indiana,

3) Bear Creek near Trevlac, Indiana,

4) Bean Blossom Creek near Bean Blossom, Indiana,

5) Wilbarger Creek, Austin, Texas.

Data collection from the Purdue Swine Farm watersheds in West Lafayette was initiated as part of the present study and is being continued. The Ross Ade upper and Ross Ade lower watersheds were commissioned as part of a previous study.[79] The data from other watersheds in Indiana and Texas were acquired from the records and reports of the U.S. Geological Survey. Some of the pertinent details, watershed location and position of gaging stations, and instruments used for gaging, etc., are presented in Table No. 2 for all the watersheds from which the data has been acquired for analysis. The physiographic characteristics of West Lafayette watersheds and the instrumentation used to measure hydrologic data from these watersheds are described below in detail. Similar details for other watersheds can be obtained from other sources.[32,80,81]

Hydrologic Stations in West Lafayette, Indiana

The Ross Ade and the Purdue Swine Farm watersheds are located in West Lafayette, Indiana. The Ross Ade Watershed has been divided into two sub-watersheds as the Ross Ade upper and the Ross Ade lower watersheds (Fig. 5). The Ross Ade upper watershed has an area of 29 acres, and includes a fully developed residential area northeast of Northwestern Avenue, West Lafayette. The Ross Ade lower watershed has an

Table 2. Some Details About the Watersheds and the Gaging Stations

No.	Name of Watershed	Details of the Watershed	Location of Stream Gaging Station	Location of Rainfall Gaging Station(s)	Recording Instruments and Type of Records	Agency from Which Data Were Obtained
1	Ross Ade (upper)	West Lafayette, Indiana; includes fully developed residential area northeast of Purdue University Campus (Fig. 6)	Longitude: 86° 55' 0" Latitude: 40° 26' 15"	At the same location as stream gaging station	1) Columbus-type weir to measure flow. 2) Stevens A-35 20-inch water level recorder. 3) A raingage with a rainfall receiver of 16.2 inch diameter. 4) Temperature sensing element. Continuous trace of water stage, accumulated rainfall and temperature are recorded simultaneously on the same chart of Stevens A-35 recorder which is driven by a synchronous motor at a speed of 144 inches/day. Records available from water year 1964.	Data collected as part of this study.
2	Ross Ade (lower)	West Lafayette, Indiana; Ross Ade (upper) watershed and part of Purdue University Campus and sport grounds are included (Fig. 7)	Longitude: 86° 55' 30" Latitude: 40° 25' 10" At the gravel pit on the southern edge of Purdue University Campus	At upper and lower Ross Ade stream gaging stations	1) Columbus-type weir to measure flow. 2) Stevens A-35 10-inch water level recorder run by a Telechron Synchronous motor at a speed of 144 inches/day. 3) A raingage with a rainfall receiver of 22.62 inch diameter. Accumulated rainfall is recorded by a Friez weighing type recorder run at a speed of 1 inch/hour.	-do-
3	Purdue Swine Farm (upper)	West Lafayette, Indiana; Residential area of Barbarry Heights, Avondale Subdivision approx. 25% developed, Cumberland School, etc. are in the watershed (Fig. 12)	Longitude: 86° 54' 30" Latitude: 40° 27' 40" North of Cumberland School, 60 ft. downstream of the point where the main storm sewer discharges into an open ditch	At the same location as stream gaging station	1) Two Parshall flumes, one of 12-inch throat width to measure flows accurately up to 5 cfs and the other of 8 ft. throat width for flows higher than 5 cfs. 2) A Stevens Duplex 2A-35, 20-inch water level recorder run by a Telechron Synchronous motor at a speed of 144 inches/day. 3) A raingage with a rainfall receiver of 22.62 inch diameter. Continuous trace of water stage in the two Parshall flumes, and the accumulated rainfall, are recorded on the same chart of the Stevens Duplex 2A-35 recorder. Records available from water year 1968.	-do-
4	Purdue Swine Farm (lower)	West Lafayette, Indiana; Includes the Purdue Swine Farm (upper) watershed and the area between the Cumberland School and the Kalberer Road which is farm land to be urbanized in the near future (Fig. 12)	Longitude: 86° 54' 15" Latitude: 40° 28' 0" About 200 feet south of Kalberer Road	At the upper and lower stream gaging stations	1) Two Parshall flumes, one of 18-inch throat width to measure flows accurately up to 8 cfs and the other of 10 ft. throat width for flows higher than 8 cfs. 2) A raingage with a rainfall receiver of 22.62 inch diameter. 3) A temperature sensing element. 4) A Duplex 2A-35 water level recorder run by a Telechron Synchronous motor at a speed of 144 inches/day. Continuous trace of water stage in the two Parshall flumes, the rainfall and the temperature are recorded on the same 20-inch chart of the Duplex 2A-35 recorder.	-do-
5	Pleasant Run at Arlington	Indianapolis, Indiana; watershed includes northeastern parts of the city of Indianapolis.	Longitude: 86° 03' 50" Latitude: 39° 46' 33"	Raingage station is located in the northwestern corner of the watershed at the intersection of Shadeland Road and New York Central Railroad.	1) Stevens A-35 type 10-inch water level recorder running at a speed of 4.8 inches/day. Records available from water year 1959.	U.S. Geological Survey, Indianapolis, Indiana[81]

Table 2 (continued)

6	Pleasant Run at Brookville Road	Indianapolis, Indiana; Includes Pleasant Run upstream of Arlington Avenue and part of the residential area between Arlington Avenue and Brookville Road. Pleasant Run is a tributary to White River.	Longitude: 86° 05' 43" Latitude: 39° 45' 52"	Raingage station is located in the northwestern corner of the watershed at the intersection of Shadeland Road and New York Central Railroad.	1) Stevens A-35 type 10-inch water level recorder running at a speed of 4.8 inches/day. Records available from water year 1959.	U.S. Geological Survey, Indianapolis, Indiana[81]
7	Little Eagle Creek	Speedway, Indiana; watershed includes parts northwest of the city of Indianapolis and covers mostly rural areas except for the residential area of Speedway. Little Eagle Creek is a tributary to White River.	Longitude: 86° 13' 41" Latitude: 39° 47' 15" The stream gaging station is located at the southeastern corner of the Speedway at 16th Street.	There is no raingage within the boundaries of the watershed. Rainfall data from the raingage located in the Pleasant Run watershed and the data from the raingage located at the nearby airport are to be used.	-do-	-do-
8	Lawrence Creek	Ft. Benjamin Harrison, Indiana; watershed includes rural area 3 miles northeast of Pleasant Run watershed. Lawrence Creek is a tributary of Fall Creek.	Longitude: 86° 01' 25" Latitude: 39° 52' 09" Stream gaging station is located at Ft. Benjamin Harrison, Marion County, Indiana.	No raingaging station is located within the watershed boundaries. Rainfall data from the raingaging station in Pleasant Run watershed and from the raingage station at the nearby airport are to be used	A Stevens A-35, 10-inch water level recorder running at a speed of 4.8 inches/day records the stream stage. Records available from water year 1953.	1) U.S. Geological Survey, Indianapolis, Indiana[81] 2) Blank and Delleur[32]
9	Bear Creek	Near Trevlac, Indiana; The watershed includes rural area in Brown County, Indiana. Bear Creek is a tributary to Wabash River.	Longitude: 86° 20' 45" Latitude: 39° 16' 40" On left bank, 15 ft. west of County Road, 1.1 miles northwest of Trevlac	There are no raingaging stations within the boundaries of the watershed. Rainfall data from nearby raingaging stations outside the boundaries are to be used	A Stevens A-35, 10-inch water level recorder, running at a speed of 4.8 inches/day records the stream stage. Records available from water year 1953.	-do-
10	Bean Blossom Creek	Near Bean Blossom, Indiana; The watershed includes parts of rural area in Brown County, Bean Blossom	Longitude: 86° 29' 57" Latitude: 39° 14' 30" Near Bean Blossom in Brown County	-do-	A Stevens A-35, 10-inch water level recorder running at a speed of 4.8 inches/day. Records available from water year 1953.	-do-
11	Waller Creek at 38th Street	Austin, Texas; Waller Creek lies about 2 miles east of the Balcones escarpment in Austin, Texas. The watershed includes mainly residential and business area. Soil is predominantly clay, of variable thickness and underlain the Austin chalk limestone which outcrops along most of the channel. Waller Creek is a tributary to Colorado River.	Longitude: 97° 43' 36" Latitude: 30° 17' 49" At a point 200 feet upstream of the bridge at the 38th Street in Austin, Travis County, Texas.	One recording and two non-recording raingaging stations are located within the watershed.	A Stevens A-35 type, 10-inch water level recorder records the stream stage. Records available from water year 1955.	U.S. Geological Survey, Austin, Texas[80]
12	Waller Creek at 23rd Street	Austin, Texas; The watershed includes the area upstream of 38th Street, Austin and the area between 38th Street and 23rd Street, Austin, Texas. The type of soil and nature of development in the watershed are similar to those of 38th Street, Austin.	Longitude: 97° 44' 01" Latitude: 30° 17' 08" On San Jacinto Boulevard, 50 ft. upstream from bridge on east 23rd Street, Austin, Travis County, Texas.	There are three recording and three non-recording raingages located inside the watershed.	-do-	-do-
13	Wilbarger Creek	Wilbarger Creek watershed lies north of Pflugerville, 13 miles north of Austin, Texas. The creek flows in a southeasterly direction to the Colorado River. The drainage area is rural. Soil is predominantly clay of variable thickness and is underlain by the Austin chalk limestone which outcrops at many places in the channel.	Longitude: 97° 36' 02" Latitude: 30° 27' 16" Stream gaging station is located 131 ft. downstream from Pfluger Lane and 1.6 miles northeast of Pflugerville, Travis County, Austin, Texas.	Three recording raingaging stations are located in the watershed.	A Stevens A-35 type, 10-inch water level recorder records the stream stage. Records available from water year 1963.	U.S. Geological Survey, Austin, Texas[80]

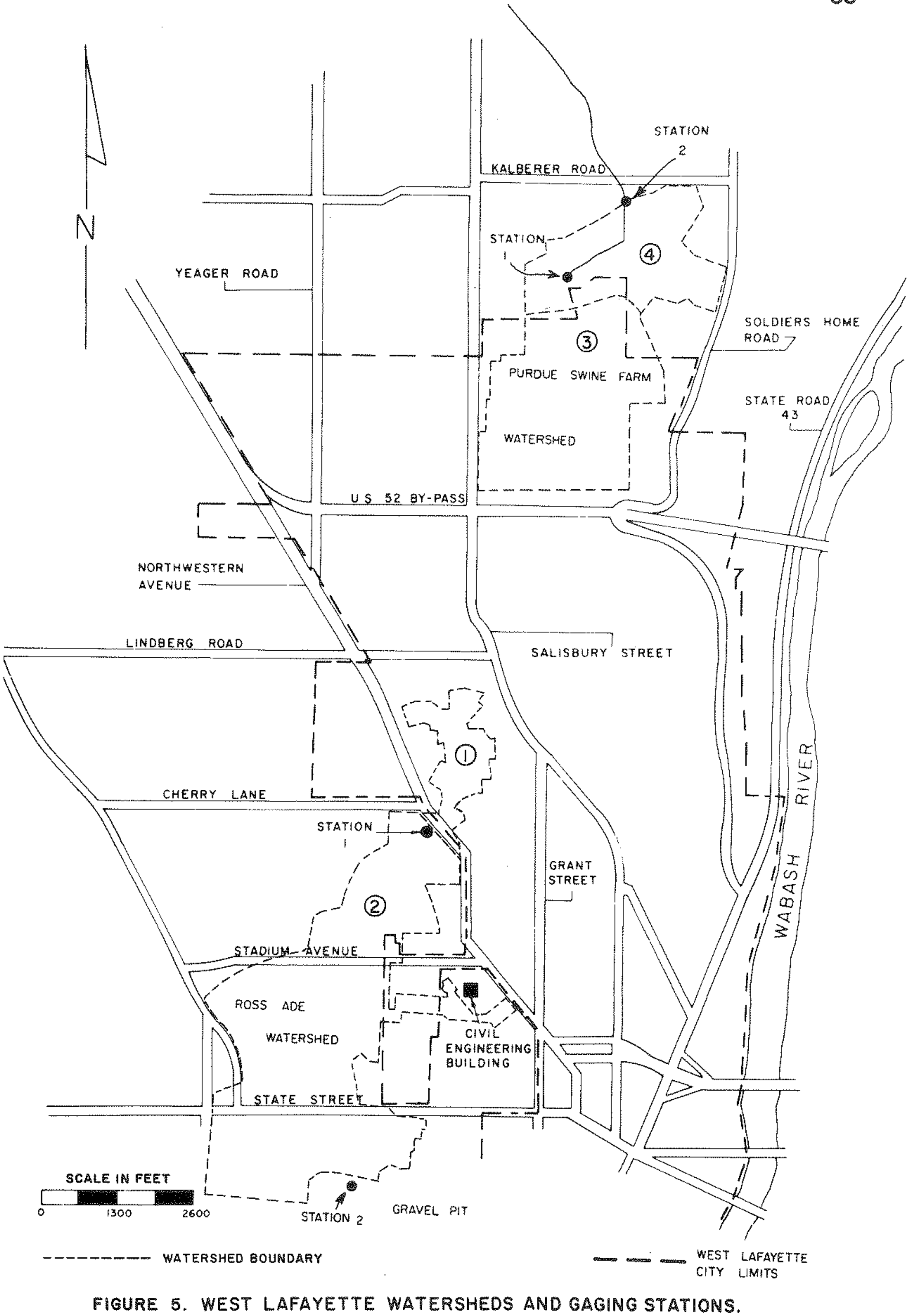


FIGURE 5. WEST LAFAYETTE WATERSHEDS AND GAGING STATIONS.

area of 392 acres in which the Ross Ade upper watershed and parts of Purdue University Campus are included. The Purdue Swine Farm watershed has a total area of 292 acres and is sub-divided into two sub-watersheds as upper and lower watersheds (Fig. 5). The Purdue Swine Farm upper watershed covers an area of 178 acres and includes Barbarry Heights and Avondale subdivisions, which are partly developed residential areas. In the Purdue Swine Farm lower watershed, in addition to the Purdue Swine Farm upper watershed, the area north of Cumberland School and south of Kalberer Road, which consists mostly of farm land are included. Thus the West Lafayette watersheds have considerable variation in stages of urbanization, as they include areas which range from farm land to fully developed urban areas.

Ross Ade Watershed

The Ross Ade Watershed extends in a generally north-south direction, and the soil type in the watershed ranges from poorly drained soils such as Crosby silt loam in the northern parts to excessively drained soils such as Warsaw silt loam in the southern parts. A main 72-inch diameter sewer known as "Ross Ade Drain", which has a slope of 0.77 percent carries the storm runoff from the Ross Ade Watershed and drains into a gravel pit located at the southern edge of the watershed. Two gaging stations were established on the Ross Ade Drain by the U.S. Geological Survey, and data acquisition from these stations was started in September 1964. The watershed, drainage system and location of gaging stations for the upper and lower Ross Ade watersheds are respectively shown in Figures 6 and 7.

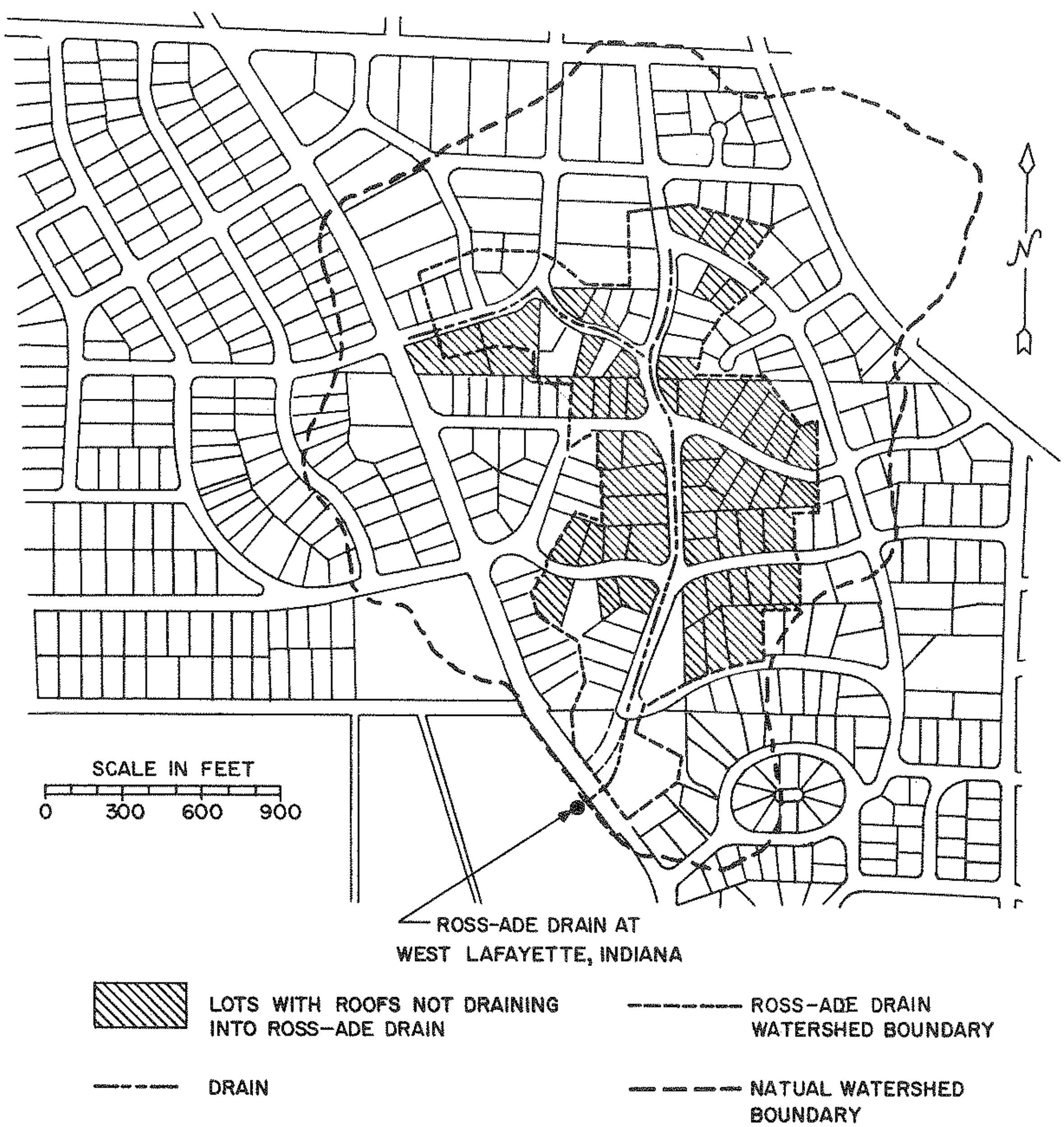


FIGURE 6 ROSS-ADE UPPER WATERSHED

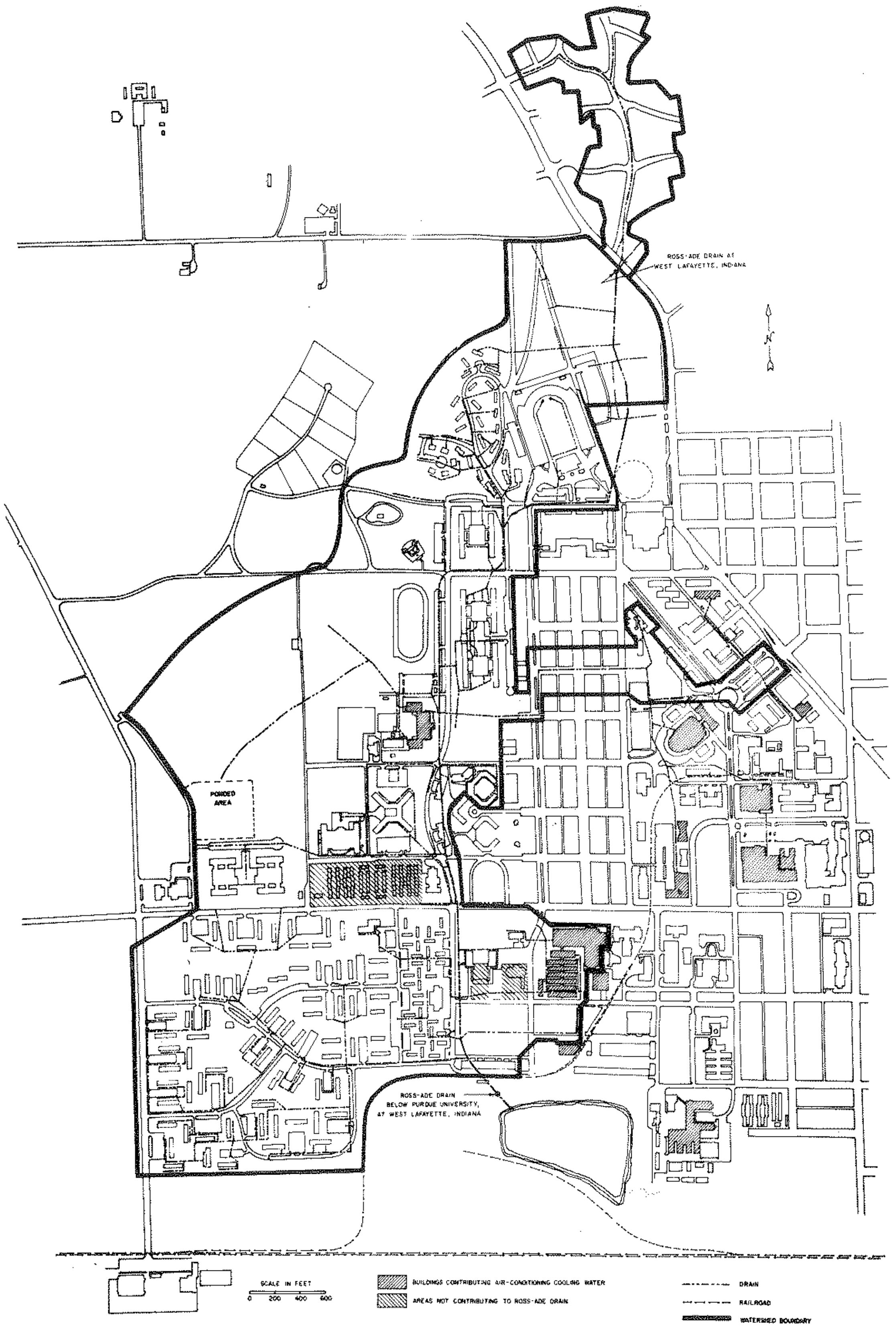


FIGURE 7 ROSS-ADE LOWER WATERSHED

Instrumentation at both the Ross Ade gaging stations is being continually improved since its installation in 1964. Details of the instruments which are currently used in these stations are given below. Rainfall and runoff data from the Ross Ade upper watershed are recorded at a location just west of Northwestern Avenue in the northeast corner of the Purdue University football field. A Stevens A-35 continuous stage recorder with 20 inch chart, a recording raingage, a temperature recording unit, and an electric tape gage are used in this station. At the site of the gaging station, the sewer drains into a concrete flume which is 6 feet wide, 10 feet deep and 29 feet long, at the downstream end of which a columbus type deep notch weir with a crest length of 6 feet is located. A 4-foot square stilling well is connected to the flume by a two-inch diameter pipe from a point 6 feet upstream of the weir. A 15-inch diameter float in the stilling well is connected to a Stevens A-35 stage recorder through an arrangement of pulleys, and the Stevens A-35 stage recorder is driven by a Telechron Synchronous motor at the rate of 144 inches per day so that the stage values can be read at 1/4 minute intervals.

The raingage has a 16-inch diameter rainfall receiver mounted at 8 feet above the ground level. The 16-inch rainfall receiver yields a record which is magnified 4 times that of the record obtained by the standard 8-inch diameter rainfall receiver, thus facilitating better estimate of rainfall magnitudes. The rainfall receiver is connected to a standard 8-inch diameter collecting tank by a 1/2-inch copper tube. A 6-inch diameter float in the collecting tank is in turn connected to the stage recorder through a set of pulleys, thus facilitating simultaneous rainfall and stage recording on the same chart. An 18-inch long temperature sensor, mounted 8 feet above the ground level, is also connected to the stage recorder through a Stevens temperature sensing unit. Thus the rainfall, the stage and the air temperature are all recorded on the same chart of the stage recorder.

The electric tape gage is a contact type, non-recording, stage measuring device. It consists of a metal sensor attached to a steel tape which is connected to a voltmeter through a dry battery power pack. Instantaneous stage values can be measured up to 1/100 of a foot. This electric tape gage provides a check for the stage value recorded by the stage recorder.

All the recording instruments mentioned above are housed in a concrete instrument shelter located below the ground level and on the top of the stilling well, which is located to a side of the measuring flume. A manhole opening at the top of the instrument shelter and a steel ladder provide access to the instrument shelter. Stage recorder, electric tape gage, and collecting tank are shown in Fig. 8 whereas raingage receiver and temperature sensor are shown in Fig. 9.

Runoff from the Ross Ade upper watershed and the lower watershed which includes part of Purdue University Campus is collected by the Ross Ade drain which drains to a gravel pit located at the southern edge of the Ross Ade lower watershed just outside the Purdue University Campus (Fig. 7). Some details of the station are shown in Fig. 10. The recording instruments provided at this station are a Stevens A-35 stage recorder with a 10-inch chart, a recording raingage and an electric tape gage. A combination of columbus type deep notch weir located

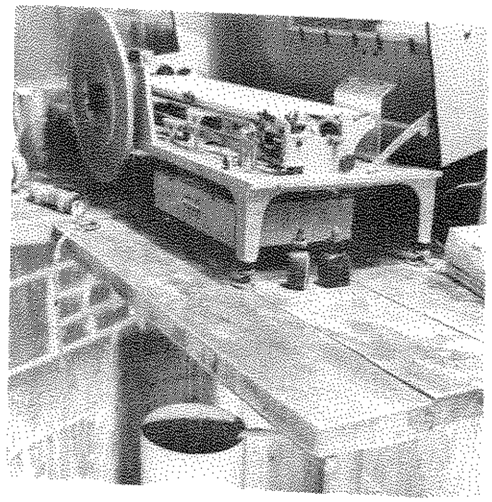


Figure 8. Recording Instruments at Ross Ade Upper Watershed Gaging Station

Figure 9. General View of Ross Ade Upper Watershed Gaging Station

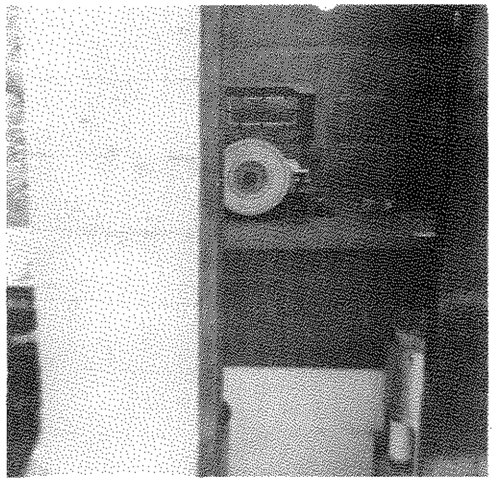


Figure 10. Recording Instruments at Ross Ade Lower Watershed Gaging Station

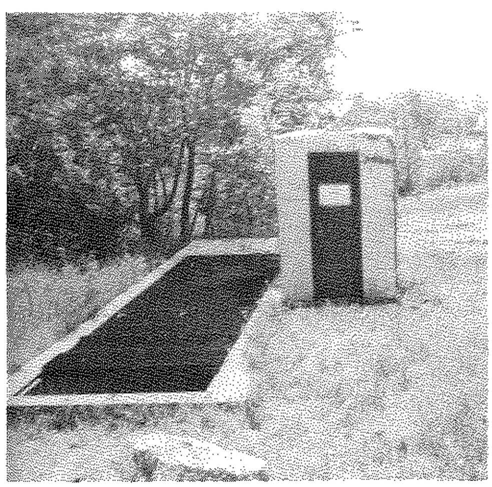


Figure 11. General View of Ross Ade Lower Watershed Gaging Station

at the downstream end of a concrete flume and a stilling well identical to the one used to measure the runoff from Ross Ade upper watershed are used at this gaging station also. A 15-inch diameter float in the stilling well is connected to the stage recorder through an arrangement of pulleys. The Stevens A-35 recorder is driven by a Telechron Synchronous motor at the rate of 144 inches per day, so that the stage values can be read at 1/4 minute intervals.

The raingage has a 22.624 inch diameter rainfall receiver and is mounted at about 10 feet above the ground level (Fig. 11). The special size of the rainfall receiver is designed to magnify the record obtained by a standard 8-inch diameter raingage by a factor of eight. A Freiz-Weighing type rainfall recorder with a chart mounted on a revolving drum moving at a speed of about 1 inch per hour is used to record the rainfall. A 1/2-inch diameter rubber tube is used to connect the rainfall receiver to the rainfall recorder. Thus, at this gaging station, rainfall and runoff stage are recorded by separate instruments on separate charts in contrast with the Ross Ade upper watershed gaging station where both rainfall and runoff are recorded on the same chart.

Purdue Swine Farm Watersheds

The Purdue Swine Farm Watershed located in West Lafayette, Indiana at about 4 miles north of Purdue University Campus extends in a south-north direction (Fig. 12). The terrain is flat with a gentle slope in the northern direction, with slowly permeable Crosby silt loam in the upper reaches and Brookston silt loam in the downstream region.

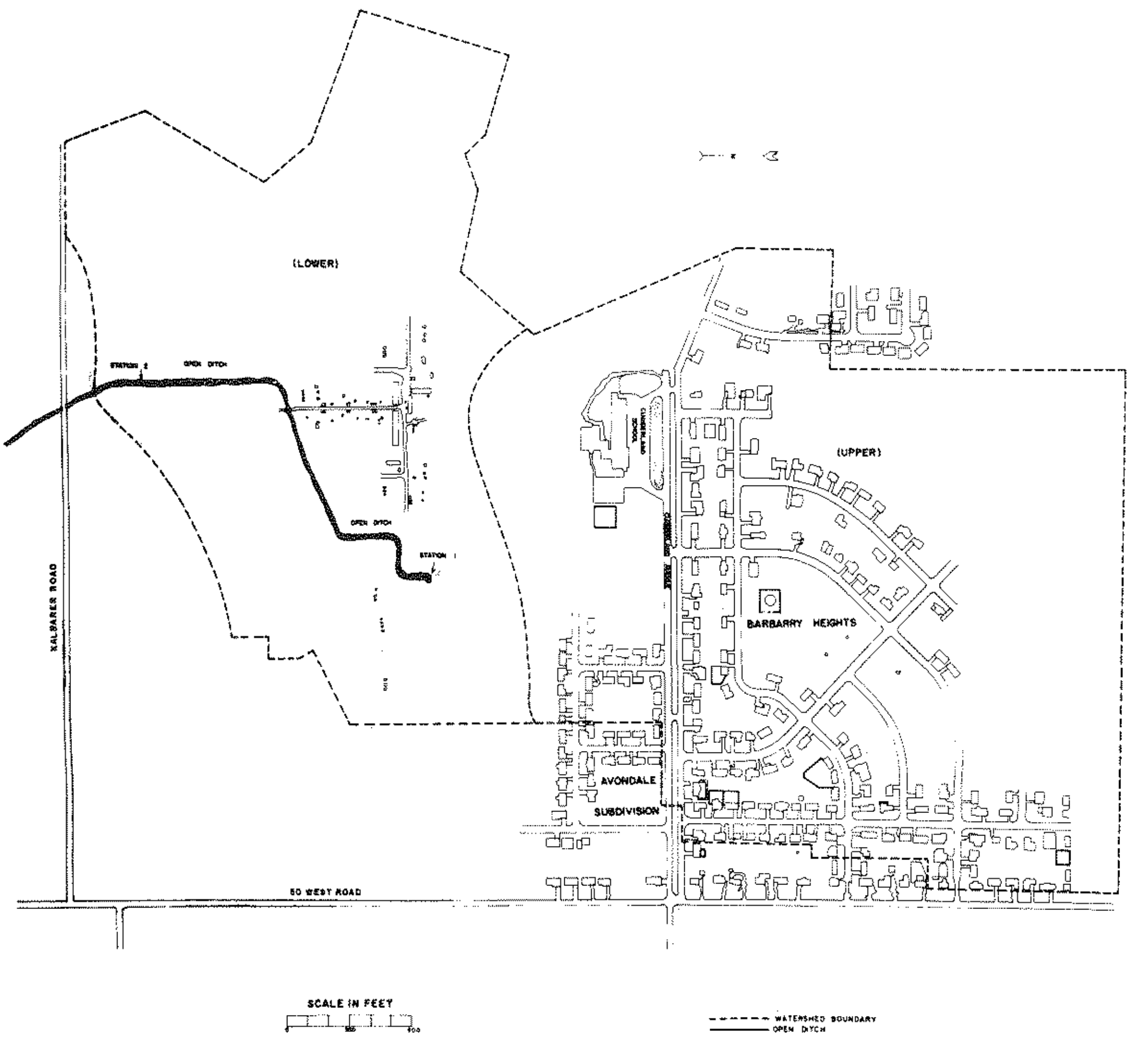
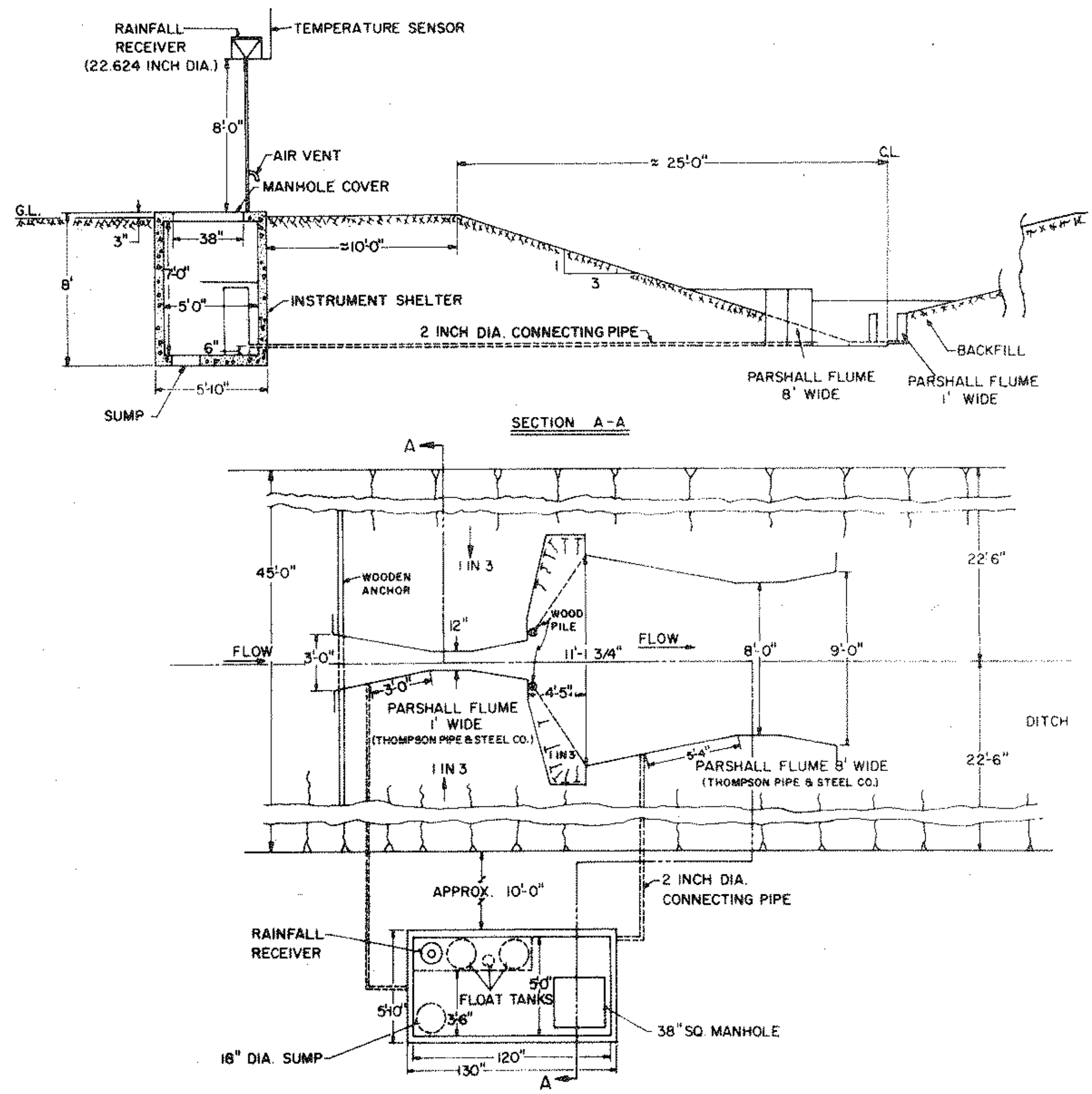


FIGURE 12 PURDUE SWINE FARM WATERSHEDS

A 72-inch diameter, corrugated galvanized iron main sewer drains the area of Barbarry Heights and Avondale subdivisions into an open ditch at about 500 feet northwest of Cumberland School. This area which is drained by the 72-inch sewer is called the Purdue Swine Farm upper watershed. The area situated to the north of Cumberland School and south of Kalberer Road drains into an open ditch, thus forming the Purdue Swine Farm lower watershed. As a part of the present study two gaging stations were designed and constructed on the Purdue Swine Farm watershed. The general layout of the gaging stations and details of construction are shown in Fig. 13.

One of the two gaging stations is located just downstream of the above mentioned sewer outlet to gage runoff from Purdue Swine Farm upper watershed and is designated as gaging station number one. The stream flow is measured by two steel Parshall flumes. One of the two Parshall flumes has a throat width of 12 inches and is used to measure flows accurately up to 5 cfs and the second Parshall flume has a throat width of 8 feet and flows larger than 5 cfs are measured by using this flume. Two 2-inch diameter inlet pipes, one from each of the Parshall flumes, are connected to two separate 18-inch diameter stilling wells. These stilling wells are located at about 25 feet from the flumes. Each of the stilling wells is provided with a 15-inch diameter float which in turn is connected to a Stevens Duplex 2A-35, 20-inch chart stage recorder through a pulley system. A raingage with a rainfall receiver of 22.624-inches diameter is located at 8 feet above the ground level. A 1/2-inch diameter rubber tube is used to connect the rainfall receiver to a standard 8-inch diameter collecting tank which is provided



SCALE : 1" = 8'

FIGURE 13. GENERAL LAYOUT AND DETAILS OF INSTRUMENTATION - PURDUE SWINE FARM WATERSHEDS

with a 6-inch diameter float. The float in turn is connected to the stage recorder through a set of pulleys, thus allowing the rainfall to be recorded on the same chart simultaneously with the stage record. All the recording instruments and the stilling wells are located in an instrument shelter (5' x 7' x 10') below the ground level. Instruments in the underground instrument shelter, rainfall receiver, and the relative positions of the Parshall flumes are shown in Figs. 14 and 15.

The second of the two gaging stations on the Purdue Swine Farm watershed in which runoff from both upper and lower Purdue Swine Farm watersheds is gaged is called gaging station number two, and is located about 150 feet upstream of the culvert at the Kalberer Road (Fig. 12). A Stevens Duplex 2A-35, 20-inch chart stage recorder, a recording raingage and a temperature recording unit are located in this gaging station. Stream flow is measured by using two steel Parshall flumes, one of which has a throat width of 18 inches and is used to measure flows accurately up to 8 cfs whereas the second Parshall flume has a throat width of 10 feet to measure flows larger than 8 cfs. The connections of Parshall flumes to stilling wells, arrangement of raingage unit and the instrument shelter, are exactly same as for the gaging station number one. In addition, an 18-inch long temperature sensing element is located 8 feet above the ground level. The temperature is recorded by a Stevens temperature recording unit used with the stage recorder. The water stages in the two Parshall flumes, rainfall and temperature are recorded simultaneously on the same chart. Instruments in the instrument shelter are shown in Fig. 16, whereas Parshall flumes, rainfall receiver, temperature sensor and the Evaporation Station are shown in Fig. 17.

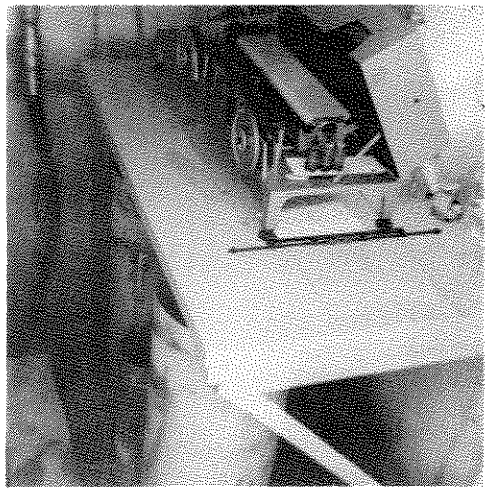


Figure 14. Recording Instruments at Purdue Swine Farm Upper Watershed Gaging Station

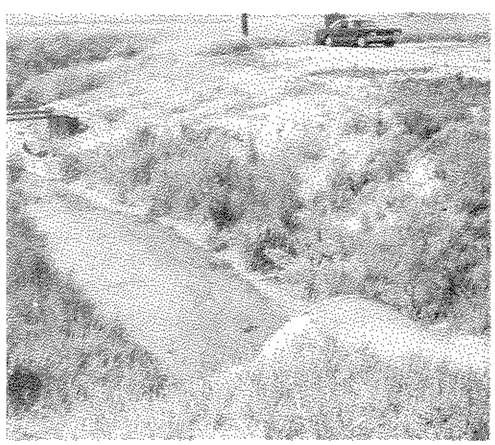


Figure 15. General View of Purdue Swine Farm Upper Watershed Gaging Station

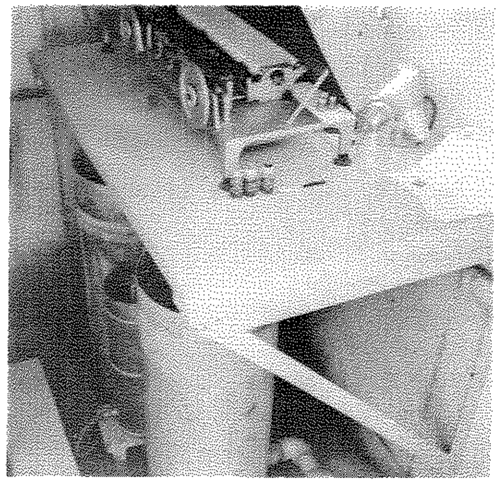


Figure 16. Recording Instruments at Purdue Swine Farm Lower Watershed Gaging Station

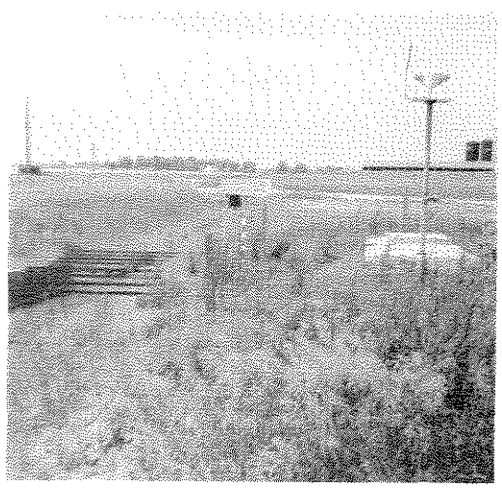


Figure 17. General View of Purdue Swine Farm Lower Watershed Gaging Station

In addition to the above mentioned recording instruments, an "Evaporation Station", consisting of a 4-foot diameter evaporation pan, a maximum-minimum thermometer, an integrating anemometer to measure evaporation, temperature variation and wind velocities, respectively, has been installed at the gaging station number two (Fig. 18).

Laboratory Tests. Laboratory tests were conducted on one of the Parshall flumes which is used to measure the runoff from Purdue Swine Farm watersheds in order to check the rating curve supplied by the manufacturer. The 12-inch Parshall flume was selected for testing and was calibrated in the laboratory. The layout of the testing facility is shown in Fig. 19. The inflow was measured by a 10-inch diameter orifice meter, and the discharge was measured by a 4-foot long sharp-crested rectangular weir and the head upstream of the throat in the Parshall flume was also measured. The observed readings are presented in Table 3 and the calibration results are presented in Fig. 20. The rating curves for the Parshall flume agreed satisfactorily with the rating curve supplied by the manufacturer. As a part of this study, the dynamic response of float type stage recorders was also tested by Blank and Delleur.[32]

Physiographic Characteristics of the Watersheds

Apart from the rainfall-runoff data collection conducted as a part of the present study, or the collection of data from the records of U.S. Geological Survey, some of the physiographic characteristics of the watersheds mentioned earlier (Table 2) were obtained. These are: the area of the watershed (A), the length of the main stream (L), the

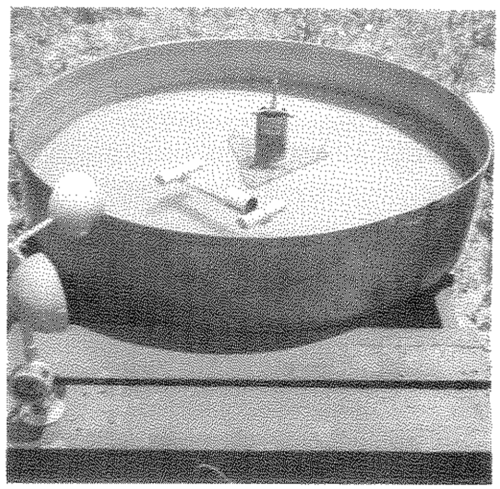


Figure 18. Evaporation Station at the Purdue Swine Farm Lower Watershed Gaging Station

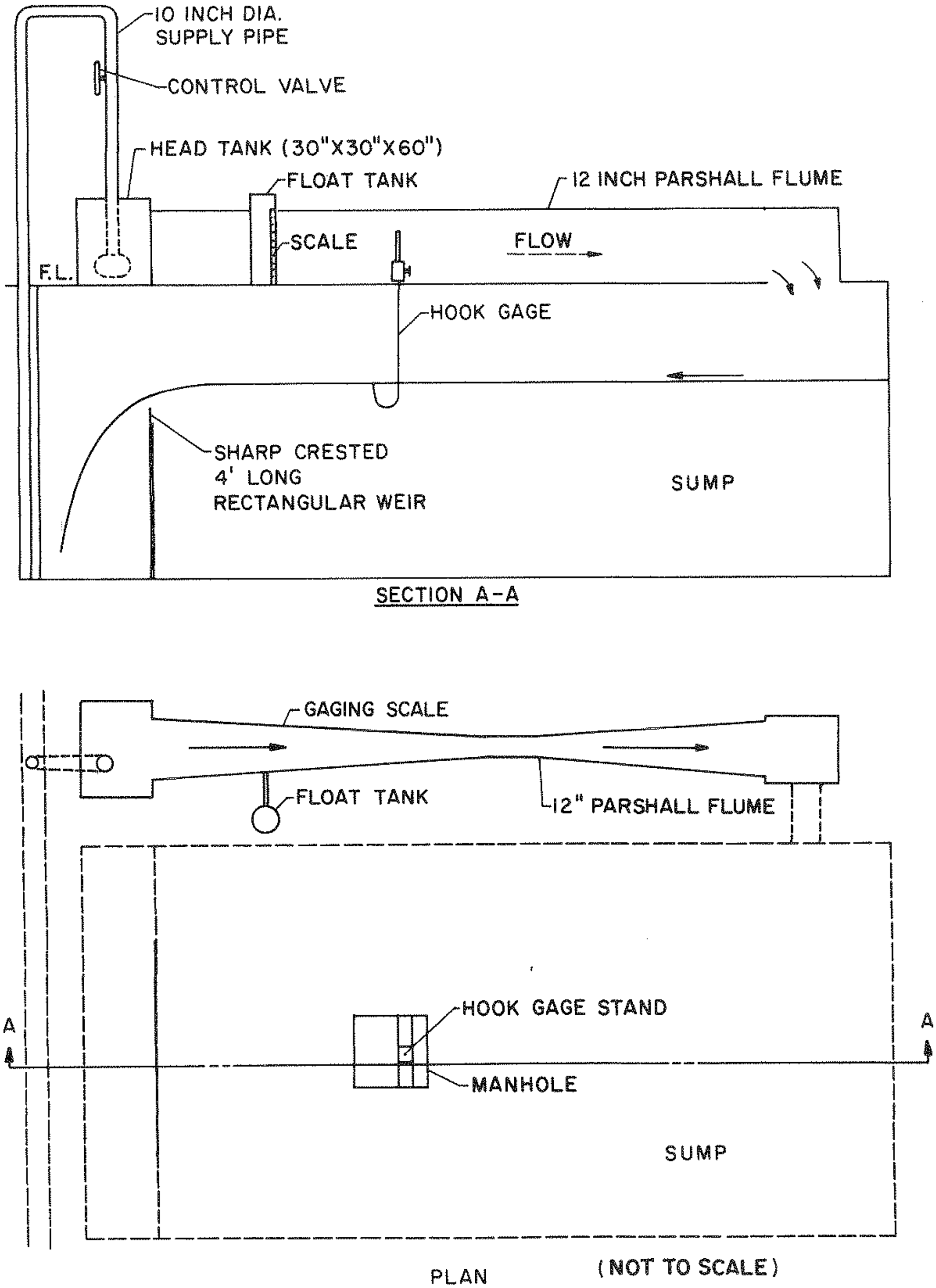


FIGURE 19. EXPERIMENTAL SETUP FOR CALIBRATION OF THE PARSHALL FLUME

Table 3. Calibration Results for the 12-inch Parshall Flume

No.	Float Reading (Ft.)	Hook Gage Reading (Rectangular Weir) (Ft.)	Parshall Flume Scale Reading (Ft.)	Discharge From Rectangular Weir (cfs)	Discharge From Parshall Flume** (cfs)
1	0.15	0.167	0.19	0.30	0.30
2	0.20	0.185	0.24	0.44	0.46
3	0.25	0.204	0.29	0.62	0.61
4	0.30	0.224	0.34	0.76	0.77
5	0.35	0.247	0.40	0.93	0.99
6	0.40	0.265	0.43	1.14	1.11
7	0.45	0.290	0.48	1.33	1.31
8	0.50	0.306	0.53	1.15	1.39
9	0.55	0.325	0.58	1.71	1.62
10	0.60	0.346	0.63	1.91	1.98
11	0.65	0.363	0.68	1.94	2.23
12	0.70	0.385	0.73	1.95	2.48
13	0.75	0.410	0.78	2.40	2.74
14	0.80	0.431	0.83	2.70	3.02
15	0.85	0.452	0.88	3.05	3.29
16	0.90	0.474	0.92	3.65	3.52
17	0.95	0.493	*	3.90	*
18	1.00	0.506	*	4.00	*

*Water Surface was very rough so that accurate reading of the scale was not possible.

**As per manufacturer rating curve.

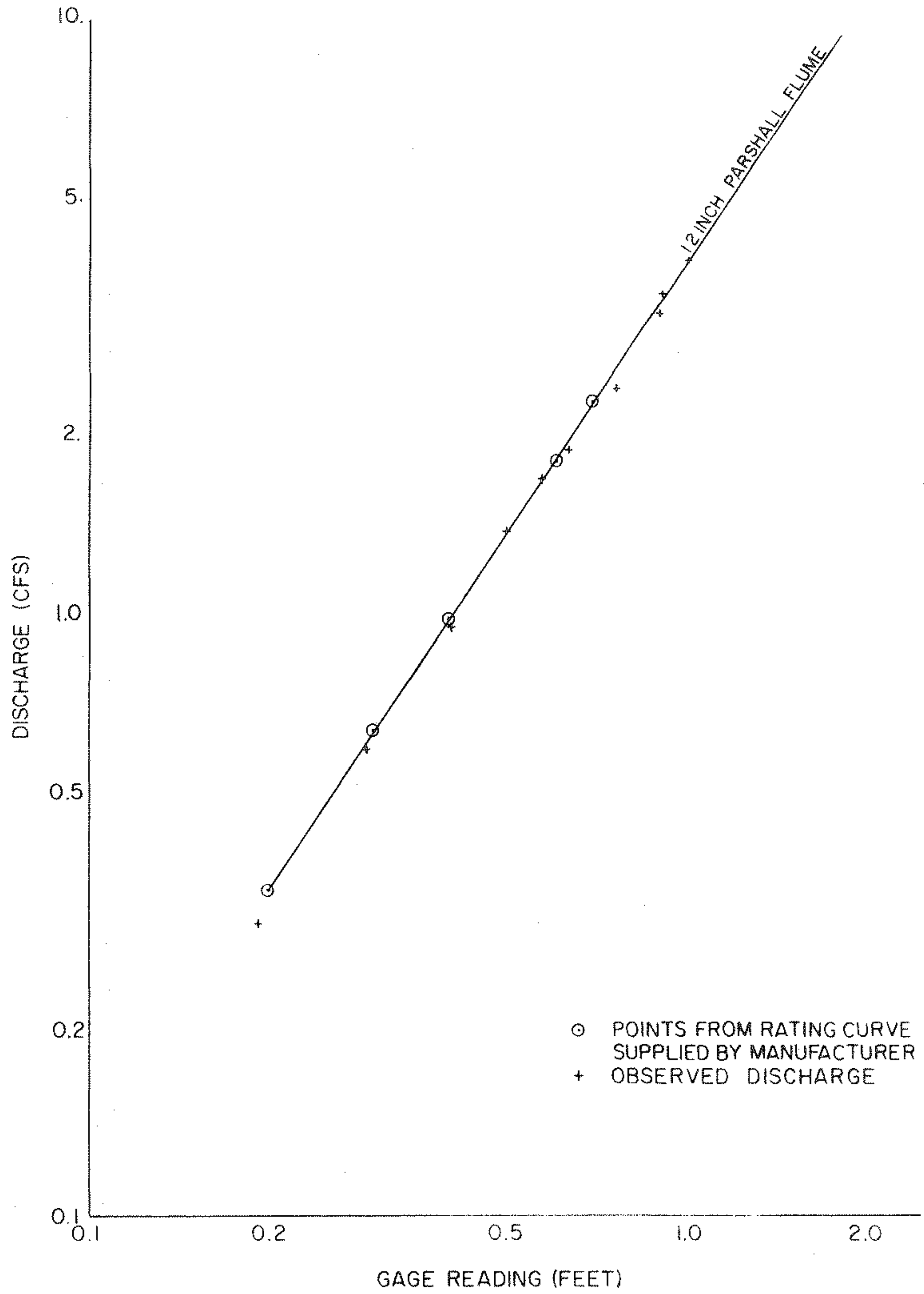


FIGURE 20. CALIBRATION CURVE FOR PARSHALL FLUME.

mean basin slope ($\bar{S}$), and the percentage of impervious area in the watershed (U), which were obtained from other sources or were determined from topographic maps. Each of these characteristics are defined below, and the method by which each was evaluated is also described.

Area. The area enclosed by the water divide is defined as the watershed area. For the West Lafayette watersheds, the watershed boundaries or the water divide lines were established by using topographic maps and aerial photographs. The area within the water divide was measured by a planimeter. In sewered areas, runoff from only a part of the area within the water divide might be drained by the sewer system. For example, in the Ross Ade upper watershed, the sewered area does not coincide with the area in the water divide as shown in Fig. 6. In such instances only the area, the runoff from which is contributed to the sewers, is considered as the area of the watershed. For other watersheds, the values of the drainage areas were obtained from the records of U.S. Geological Survey.[80,81]

Main Stream Length. Main stream length is the length measured along the main stream on a topographic map, from the starting point of the stream to the gaging station. For all the watersheds, the main stream length was determined by using a "Map Measure" and the resulting values were checked with the values given in the records of U.S. Geological Survey, and were found to be in close agreement.

Mean Basin Slope. The mean basin slope for a watershed has been defined and determined in several ways. In one of these methods the stream profile (elevation against distance along the stream) is plotted and the area under this profile is determined. The mean basin slope, $\bar{S}$, is then computed to be the slope of a straight line which has the same area under it as the area under the profile.[82] In another method, the mean basin slope is determined by dividing the stream into n equal reaches and the slopes of these n reaches are computed. The mean basin slope is then obtained by the formula[83]

$$\bar{S} = \left[\frac{n}{\frac{1}{\sqrt{S_1}} + \frac{1}{\sqrt{S_2}} + \ldots + \frac{1}{\sqrt{S_n}}}\right]^2$$

where S_1, S_2, ...S_n are the slopes of the individual reaches of the stream. If this formula is used for watersheds with relatively small slopes, errors in computing the mean basin slope become greatly amplified.

The mean basin slope, as used in this study is the difference in elevations of the extreme upstream point and the outlet of the watershed divided by the distance between these points. The mean basin slope for each watershed under study, was determined according to the above definition and was expressed in feet per mile.

Impervious Area. For the purposes of this study, the "impervious area" is defined as the area in the watershed which does not contribute to direct infiltration. Paved roads, parking lots, building roofs, lawns, and side walks have been considered impervious areas. No attempt has been made to distinguish the relative effectiveness of each type of surface and all surface types mentioned above have been classified as impervious areas. Aerial photographs of scale 660-ft. to an inch were used to obtain the percentage of impervious area in each of the watersheds in West Lafayette and Indianapolis. For watersheds in Texas, impervious area data were available from other sources.[80]

In general, the built-up areas are not uniformly distributed within the watershed. Also, measuring all the impervious areas would be time consuming and tedious. Hence, the following procedure was adopted to obtain an estimate of the percentage of impervious area in a watershed:

1) From an inspection of the aerial photographs, the watershed area was subdivided into 4 or 5 subareas, such that each subarea represented a particular characteristic urbanization density. The area of each of these subareas was determined by a planimeter.

2) From each subarea, one or two "sample-areas", which are representative of the development in that subarea were selected.

3) The aerial photographs of these "sample-areas" were magnified 8 times their original scale.

4) From each of these magnified pictures the total area of paved roads, parking lots, roof tops, lawns, foot paths and side walks was estimated by superposing these pictures onto a transparent centimeter graph paper and counting the number of squares occupied by these impervious areas on the graph paper. The ratio of the impervious area to the total area in each of the magnified pictures of the "sample" was then computed.

5) The percentage of impervious area for the subarea under study was computed by taking the arithmetic average value of the percentage of impervious area in the "sample-areas", computed in step 4.

6) Thus, percentage of impervious area for each subarea was obtained by adopting the procedure described in steps 2 through 5.

7) Each subarea was assigned a weight proportional to its area.

8) The percentage of impervious area in the entire watershed was obtained by computing the weighted average value of the percentage of impervious area in each of the subareas.

For each of the watersheds under study, the values of area of the drainage basin A, length of the main stream L, mean basin slope $\overline{S}$, and the percentage of impervious area U, are presented in Table 4.

Processing Hydrologic Data for Analysis

Runoff Data

The runoff hydrographs are affected by the factors such as antecedent moisture condition of soil in the watershed, areal distribution and movement of storm. Hydrographs were selected for analysis such that all the hydrographs were affected by these factors to approximately the same extent. Hydrographs which satisfied the following criteria were selected for analysis:

1) The storms which were relatively isolated,
2) Storms which exhibited approximate uniform spatial distribution over the entire watershed,
3) Stage hydrographs of storms which had a well defined rising limb culminating in a single sharp peak followed by unsustained recession.

Although it is desirable to use a large number of hydrographs for analysis, many hydrographs do not satisfy the above criteria. On an

Table 4. Physiographic Characteristics of the Watersheds

Watershed Number	Name of the Watershed	Area of the Watershed A(Sq. Miles)	Length of Stream L(Miles)	Mean Basin Slope $\bar{S}$(Ft/Mile)	Percentage Impervious Area U(%)	No. of Storms Used for Analysis
1	Ross Ade (upper)	0.0455	0.6613	112.0	38.0	21
2	Ross Ade (lower)	0.6125	2.1765	112.0	37.4	10
3	Purdue Swine Farm (upper)	0.2776	0.6439	3.9	21.3	4
4	Purdue Swine Farm (lower)	0.4562	1.067	3.40	13.3	2
5	Pleasant Run (Arlington)	7.580	3.822	12.67	10.5	13
6	Pleasant Run (Brookville)	10.10	5.644	14.26	15.5	12
7	Little Eagle Creek	19.31	11.10	23.23	2.1	8
8	Lawrence Creek	2.86	1.705	32.21	0.	12
9	Bear Creek	7.00	3.864	39.07	0.	5
10	Bean Blossom Creek	14.60	6.44	32.74	0.	7
11	Waller Creek (38th Street)	2.31	4.371	47.0	27.0	17
12	Waller Creek (23rd Street)	4.13	5.23	47.0	37.0	12
13	Wilbarger Creek	4.61	3.17	45.94	0.	8

average, about ten hydrographs for each watershed were selected for analysis. For the Indianapolis and Texas watersheds, the stage hydrographs and the appropriate rating tables for the selected storms were obtained from the records of U.S. Geological Survey. The stage hydrographs were digitized and were put on IBM cards to obtain hydrographs with ordinate spacings of 60 or 30 or 0.5 minutes interval. The ordinate spacing depended on the type of the stage recording machine used for data collection.

Rainfall Data

Rainfall data in the form of a continuous record of cumulative rainfall were available for most of the watersheds used in the present study. The density of rainfall gaging stations in the state of Indiana is about one station per 250 square miles.[32] Hence particularly for the small rural watersheds in Indiana selected for this study, no rainfall gaging station was located within the watershed. In such cases, hourly rainfall data from the rainfall gaging stations within the vicinity of the watershed boundaries were obtained from the U.S. Weather Bureau. The arithmetic averages of the rainfall from the nearby stations were used to obtain the total rainfall mass curves for the watershed under study.

As excess rainfall and direct runoff are used in the present study, the direct runoff hydrograph and the excess rainfall hyetograph were determined from the data of total runoff hydrograph and total rainfall. Methods used to obtain the direct runoff hydrograph and the excess rainfall are described below.

Base Flow Separation

The total runoff hydrograph can be subdivided into three parts as the direct runoff, the interflow and the ground water flow. At present, it is not possible to distinctly isolate these three components of runoff although separation of ground water flow or base flow from the observed hydrograph is much easier than separation of interflow and direct runoff. Consequently the total hydrograph was separated into base flow and direct runoff components. The methods which are used for separation of base flow are still empirical and hence the distinction between direct runoff and base flow is arbitrary. Further, experience has shown that the shape and the volume of the direct runoff hydrograph are relatively insensitive to the method of base flow separation. Several empirical methods to separate base flow are in practice[32,82,84] and the following method was adopted in this study for the separation of base flow.

In a hydrograph, let A and B respectively represent the points at which the direct runoff begins and ends (Fig. 21). Location of the point A is easy in isolated storms and the value of the peak discharge Q_p' is determined by drawing a horizontal line through the point A. The point B is then located on the recession limb of the hydrograph such that the discharge at B, Q_B, is one hundredth of the peak discharge Q_p'. The points A and B are joined by a straight line which represents the base flow separation line. The ordinates of the direct runoff hydrograph are then obtained by subtracting the ordinates below the straight line AB from the corresponding ordinates of the total runoff hydrograph. This method of separation of base flow compared favorably with the other methods in practice.[14,32,53]

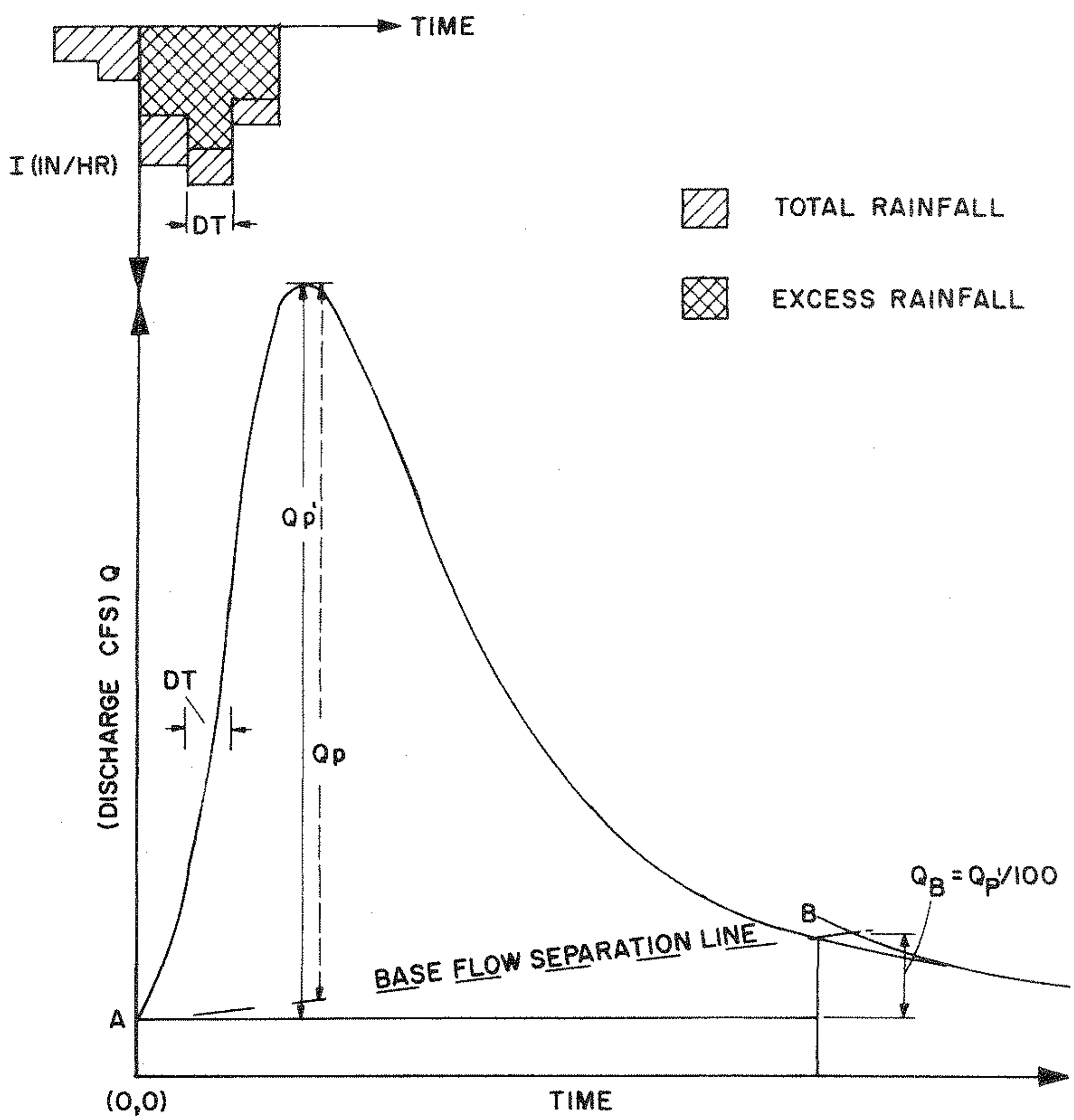


FIGURE 21. BASEFLOW SEPARATION AND EXCESS RAINFALL DISTRIBUTION

Base Flow Separation

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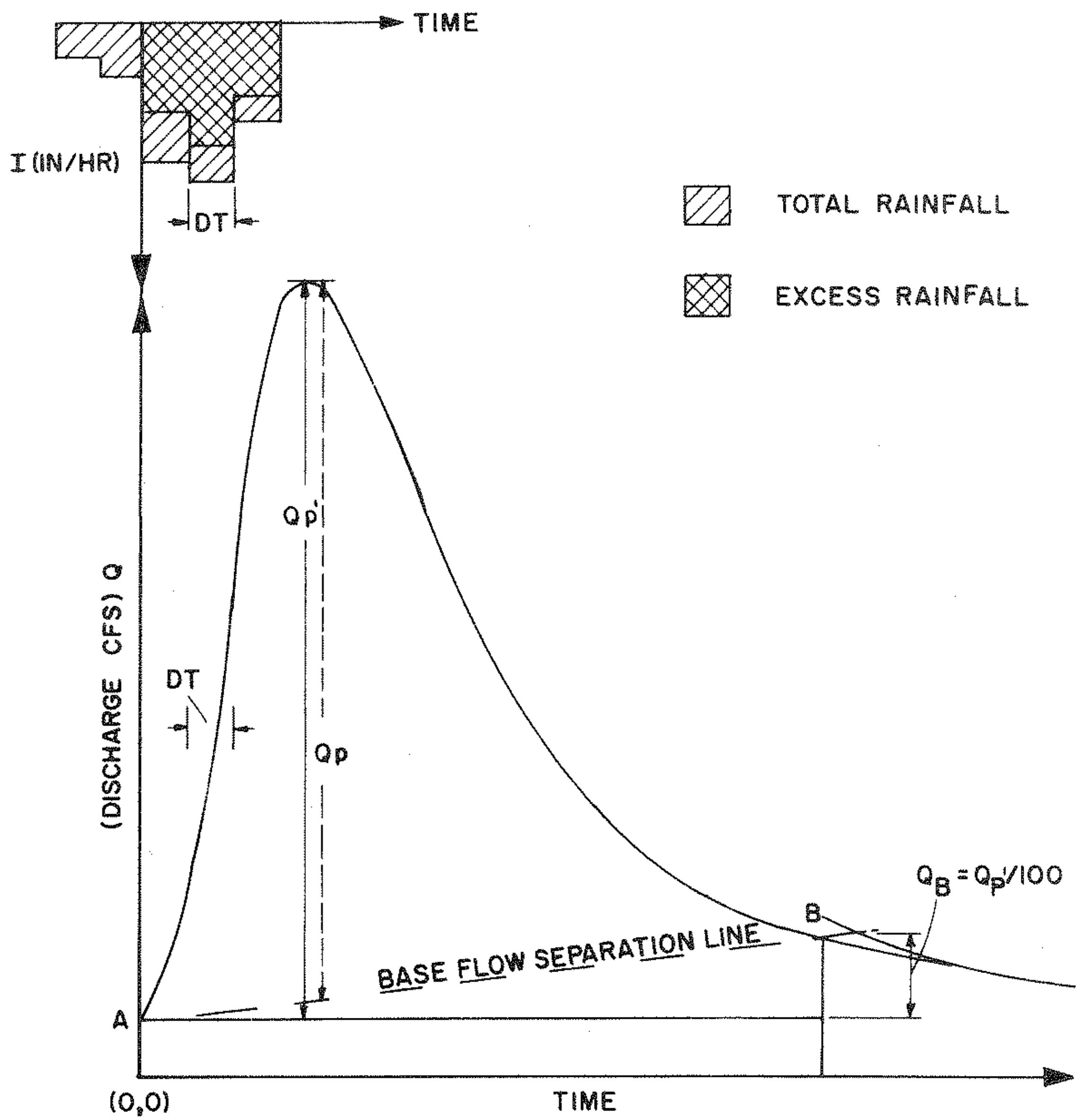


FIGURE 21. BASEFLOW SEPARATION AND EXCESS RAINFALL DISTRIBUTION

Determination of Excess Rainfall

Only a part of the rainfall contributes to direct runoff whereas infiltration, evaporation, interception, and temporary surface detention account for the rest. Infiltration, which is the predominant loss, is higher at the beginning of rainfall and tends to decrease as an exponential decay function as rainfall progresses. The time rate of infiltration which depends on, among other factors, antecedent moisture condition, areal distribution of rainfall, cannot be computed exactly. Consequently, the time distribution of excess rainfall is often estimated arbitrarily. However, the volume of excess rainfall must be, and hence was, adjusted to be equal to the volume of direct runoff. Quite often, in obtaining the effective rainfall hyetographs, the time distribution is altered. In the present study the excess rainfall is estimated assuming that the sum of the interception, evaporation and depression storage is linearly related to the rainfall intensity.[85] The rainfall which occurred before the time of commencement of direct runoff was assumed to be initial abstraction. The time of rise was taken as the beginning time of excess rainfall. The ordinates of the excess rainfall hyetograph were obtained by multiplying the corresponding ordinates of the total rainfall hyetograph by the ratio of the volume of direct runoff to the volume of the total rainfall. Thus the time distribution of excess rainfall and the total rainfall are similar after the time of rise (Fig. 21).

To facilitate further computations, the time interval between the successive ordinates of direct runoff hydrograph and the time interval between the successive ordinates of excess rainfall was kept the same and designated as DT. The required computations for the determination of the time distribution of direct runoff and excess rainfall, were carried out on a CDC 6500 digital computer and the results were punched on IBM cards for further use.

CHAPTER IV

EXCESS RAINFALL-DIRECT RUNOFF MODELS USED IN THE STUDY

General Considerations

The ability of lumped linear systems to satisfactorily represent the rainfall-runoff process in small urban watersheds has been the impetus for using conceptual models of lumped linear systems in the present study. Since the basis of the formulation of linear conceptual models is the existence of a linear storage-discharge relationship, a preliminary study of the storage-discharge relationship was conducted using the observed data from the West Lafayette watersheds. Some of the typical storage-discharge curves obtained by using the data from West Lafayette watersheds are presented in Fig. 22. The study of storage-discharge curves indicated that the linear storage-discharge relationship assumption might be adequate. Consequently, the linear system analysis techniques were pursued further.

Many types of lumped linear system models can be formulated by combinations of the linear elements which represent the storage and lag effects, and quite a number of such models have been constructed. The abundance of models, and the lack of comparison between their prediction performances over a variety of conditions, present to the hydrologist the problem of the choice of any particular model in preference to the others. When a conceptual model is selected arbitrarily to

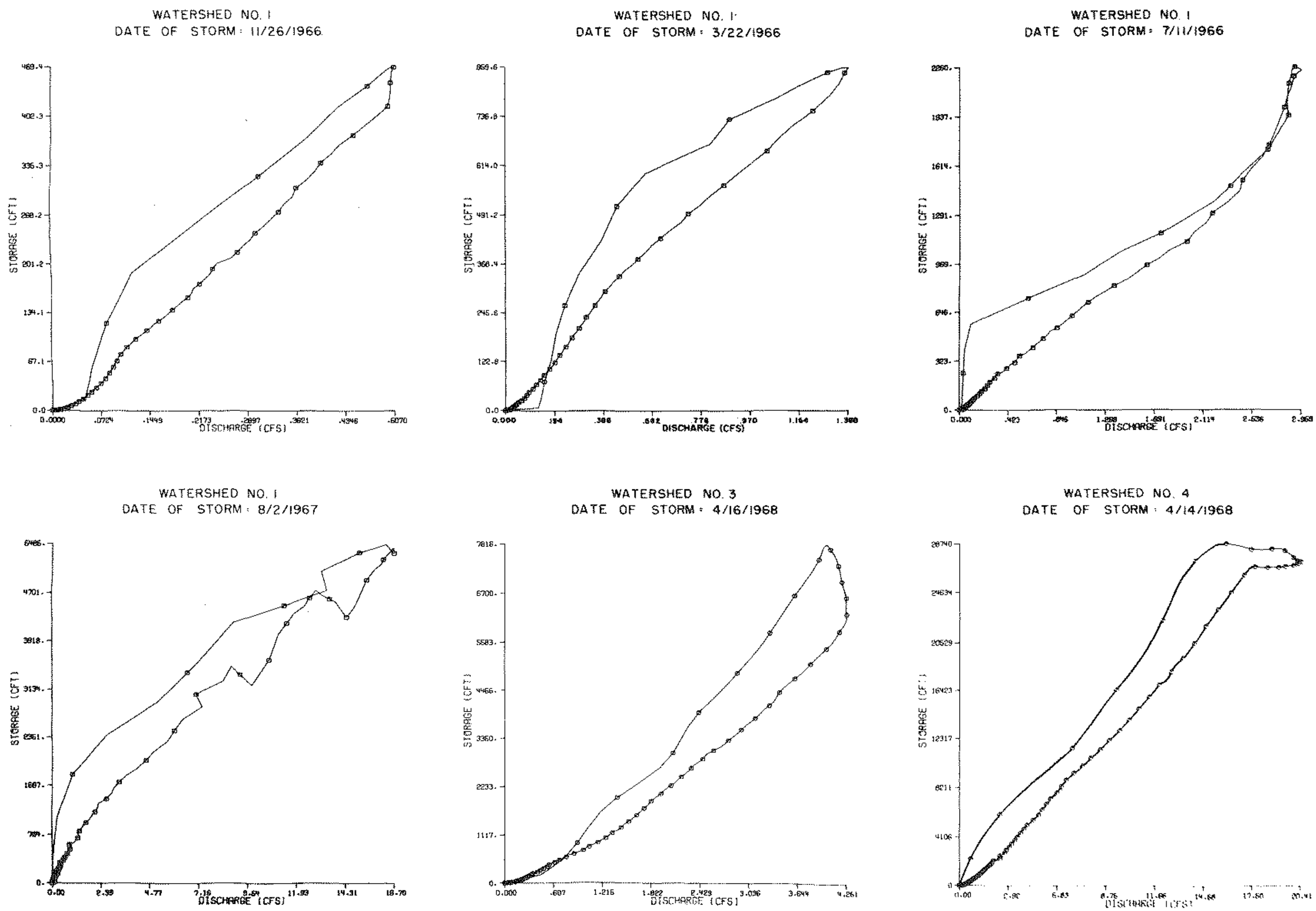


FIGURE 22. TYPICAL STORAGE-DISCHARGE CURVES

simulate the system, one cannot be certain that it is the best possible choice. Therefore in an attempt to guide the choice of a linear system model to study the effects of urbanization on runoff, it was decided to analyze the observed data by using some of the linear models which have been used rather widely in urban hydrology.

The relative performances of the models were tested by regenerating the data which were used to obtain the model parameters. "Regeneration", as used in this study, involves the following computations: 1) The parameters of the model under consideration are determined by using excess rainfall-direct runoff data. 2) The IUH of the model is computed by using the parameters of the model which were determined in step 1, and then, 3) the runoff hydrograph is computed or "regenerated" by using the convolution integral (Eq. 7). Models which exhibited sustained superior capacity for regeneration are then selected and used to estimate the effects of urbanization on runoff.

The following five methods were analyzed to compare their relative performance. The first four are conceptual models in which the system structure is prescribed apriori: 1) the single linear reservoir model,[46,48,49,50] 2) the Nash model,[25] 3) the double routing model proposed by Holtan and Overton,[26,27] and 4) a model which consists of combination of a linear channel and a linear reservoir such as the one proposed in principle by Clark.[23] The fifth method, which does not prescribe the system structure apriori, is that of system identification by evaluating the kernel function in the convolution integral (Eq. 7). This was done by the method of the Fourier transforms, which had been tested for rural watershed data.[32]

For each of the linear models investigated, the mathematical expression for the IUH has been either derived or is indicated. The method of evaluating the parameters of the model is discussed next. Then the parameters are computed by using observed data and the regeneration performances of the models are compared.

There is no unique method of comparing the observed and the regenerated direct runoff hydrographs. Visual inspection, accurate regeneration of peak and time to peak of the direct runoff hydrograph, are some of the criteria which are usually employed by hydrologists. However, there are some statistical measures such as 1) the Correlation Coefficient,[86] 2) Integral Square Error,[51] and 3) Special Correlation Coefficient,[51] which are defined below and which can be used to compare the observed and regenerated hydrographs. These were employed in the present study for quantitative comparison of observed and regenerated direct runoff hydrographs.

The Correlation Coefficient

$$R = \frac{N\sum_{i=1}^{N} Q_O(i)Q_C(i) - \left(\sum_{i=1}^{N} Q_O(i)\right)\left(\sum_{i=1}^{N} Q_C(i)\right)}{\sqrt{\left[N\sum_{i=1}^{N}(Q_O(i))^2 - \left(\sum_{i=1}^{N} Q_O(i)\right)^2\right]\left[N\sum_{i=1}^{N}(Q_C(i))^2 - \left(\sum_{i=1}^{N} Q_C(i)\right)^2\right]}} \tag{45}$$

The Integral Square Error

$$ISE = \frac{\left[\sum_{i=1}^{N}(Q_O(i) - Q_C(i))^2\right]^{1/2}}{\sum_{i=1}^{N} Q_O(i)} \times 100 \tag{46}$$

The Special Correlation Coefficient

$$R_s = \left[\frac{2\sum_{i=1}^{N} Q_O(i)Q_C(i) - \sum_{i=1}^{N} (Q_C(i))^2}{\sum_{i=1}^{N} (Q_O(i))^2} \right]^{1/2} \tag{47}$$

In Eqs. 45, 46, and 47, $Q_O(i)$ and $Q_C(i)$ are, respectively the i^{th} values of observed and regenerated outflows and N is the number of values in the outflow series.

Details about these statistical measures and the criteria used in the present study to evaluate the regeneration performance of the various models based on these statistical measures are given in Appendix A-1.

Single Linear Reservoir Model

The single linear reservoir model is based on the concept of the watershed behaving as a reservoir, in which the storage is linearly related to the outflow (Eq. 8).

$$S = KQ \tag{8}$$

The storage equation (Eq. 8) is combined with the hydrologic continuity equation (Eq. 9) to obtain the differential equation which governs the conceptual model of a linear reservoir as

$$I - Q = K \frac{dQ}{dt} \tag{10}$$

The integration of Eq. 10 using the initial condition that $Q = 0$ at $t = 0$, results in

$$Q = I(t)(1-e^{-t/K}) \tag{19}$$

If the inflow terminates at time T_R after the beginning of the outflow, and if Q_0 is the outflow at time T_R, then Eq. 10 becomes

$$-Q = K \frac{dQ}{dt'} \tag{48}$$

where $t' = t - T_R$

The integration of Eq. 48 using the condition that at $t' = 0$, $Q = Q_0$, yields

$$Q = Q_0 \, e^{-t'/K} \tag{20}$$

For an inflow I which fills the reservoir of storage S_0 instantaneously ($T_R = 0$), Eq. 20 becomes

$$Q = \frac{S_0}{K} \, e^{-t/K} \tag{49}$$

and for unit input or unit storage, the IUH is then given by

$$h(t) = \frac{1}{K} \, e^{-t/K} \tag{16}$$

The outflow Q(t) due to any given excess rainfall input I(t) can now be obtained by using the convolution integral (Eq. 7) which now takes the form

$$Q(t) = \int_0^t I(\tau) \, \frac{1}{K} \, e^{-(t-\tau)/K} \, d\tau \tag{50}$$

Equation 8 can also be viewed as a special case of the Muskingum equation[87]

$$S = K[x_0 I + (1 - x_0) Q] \tag{51}$$

where x_0 is a coefficient which can assume values between 0 and 1.0; with $x_0 = 0$, Eq. 51 reduces to Eq. 8. Equation 10 can also be solved by a numerical technique such as the one used in the "Muskingum Method",[87] in which the following recurrence equation (Eq. 52) is used.

$$Q_2 = C_1'(I_1 + I_2) + C_2' \, Q_1 \tag{52}$$

where I_1 is the inflow at the beginning of the interval DT,

I_2 is the inflow at the end of the interval DT,

Q_1 is the outflow at the beginning of the interval DT,

Q_2 is the outflow at the end of the interval DT,

and the "Routing Coefficients" C_1', C_2' are defined as

$$C_1' = \frac{0.5DT}{K+0.5DT}$$

$$C_2' = \frac{K-0.5DT}{K+0.5DT}$$

Eq. 52 can be used to compute the outflow at successive times DT apart and thus the outflow hydrograph can be obtained. It is obvious that in the routing methods such as the Muskingum Method outlined above, the IUH derivation is unnecessary.

The Parameter K and its Determination

The storage coefficient K used in the single linear reservoir model has units of time. The storage coefficient K can be shown to be equal to the time lag T_4, which is defined as the time interval between the centers of mass of excess rainfall and direct runoff.

Taking moments of inflow and outflow in Eq. 10, about the axis $t = 0$, we can write

$$\int_o^\infty I\ t\ dt - \int_o^\infty Q\ t\ dt = \int_o^\infty K\ t\ \frac{dQ}{dt}\ dt \qquad (53)$$

The right hand side of Eq. 53 can be written as

$$\int_o^\infty K\ t\ \frac{dQ}{dt}\ dt = K \int_o^\infty t\ dQ = K\ t\ Q\Big|_o^\infty - \int_o^\infty Q\ dt$$

$$= -K \int_o^\infty Q\ dt$$

as $Q = 0$ at $t = \infty$.

Hence Eq. 53 becomes

$$\int_o^\infty I\ t\ dt - \int_o^\infty Q\ t\ dt = -K \int_o^\infty Q\ dt$$

or

$$\frac{\int_0^\infty I\ t\ dt}{\int_0^\infty Q\ dt} - \frac{\int_0^\infty Q\ t\ dt}{\int_0^\infty Q\ dt} = -K \tag{54}$$

Also, by continuity at $t = \infty$, the total inflow volume is equal to the total outflow volume

or

$$\int_0^\infty I\ dt = \int_0^\infty Q\ dt \tag{55}$$

Hence from Eqs. 54 and 55,

$$\frac{\int_0^\infty I\ t\ dt}{\int_0^\infty I\ dt} - \frac{\int_0^\infty Q\ t\ dt}{\int_0^\infty Q\ dt} = -K$$

or

$$T_I - T_Q = -K$$

where T_I is the time interval from $t = 0$ up to the centroid of inflow

and T_Q is the time interval from $t = 0$ up to the centroid of the outflow.

Hence,

$$T_4 = T_Q - T_I = K \tag{56}$$

Thus, for the conceptual model of a linear reservoir, the storage coefficient is same as the time lag T_4.

If the excess rainfall-direct runoff process were to be a linear process, then the value of the parameter K, which can be estimated by using excess rainfall and corresponding direct runoff data and Eq. 56, would be a constant for all the storms. However, when the storage coefficient K was computed from the observed data, values of K were found to vary from storm to storm. This variation of K has also been observed previously by various investigators.[51,54] An average value of K for each watershed was determined by using the values of K obtained from data from that watershed and was used as the representative value for that particular watershed in regeneration of direct runoff hydrographs. This regeneration proved to be unsatisfactory. Consequently for regeneration the time lag value for each storm was used as the parameter K in Eq. 16, instead of the average value of the time lag. A preliminary study of regenerated hydrographs so obtained indicated that the regeneration could be improved by a slight change in the value of K. The changes in the value of K, however, were to be made according to some well defined criteria. Two criteria were established to optimize the value of the parameter K, one of which was to minimize the sum of the squares of the deviations of the observed and the regenerated hydrograph ordinates, or to minimize the quantity $\sum_{i=1}^{N} (Q_0(i) - Q_C(i))^2$. The second criterion selected for optimization of K was to minimize the deviation between the peak values of observed and regenerated hydrographs, or, to minimize the quantity

$$\left[\left(\frac{Q_{po} - Q_{pc}}{Q_{po}}\right)^2 + \left(\frac{T_{po} - T_{pc}}{T_{po}}\right)^2\right]^{1/2}$$

where Q_{po} and Q_{pc} are respectively the magnitudes of peak discharge of the observed and regenerated hydrographs whereas T_{po} and T_{pc} are the times corresponding to Q_{po} and Q_{pc}.

The first of the above mentioned two criteria, yields an optimum value of K, designated as K_1, which ensures the regeneration of the time distribution of the outflow with the minimum deviation between the observed and regenerated hydrographs. The value of K designated as K_2, obtained by using the second criterion, yields the best estimate of the magnitude of the peak discharge and of the time to the peak discharge. Any combination of these two criteria will yield neither the overall best fit for the time distribution of the hydrograph nor the best matching of the peak discharges. Hence no attempt to combine the two criteria was made. The optimum values of K_1 and K_2 bear a definite relationship with the observed value of time lag. This relationship is shown in Fig. 23 for some of the watersheds. In most cases the values of K_2 were noticed to be consistently less than those of K_1.

The observed hydrograph and the regenerated hydrographs obtained by using the values of the storage coefficient obtained by: 1) assuming K to be equal to T_4, (Method 1) 2) using the value of K_1, (Method 2) and 3) using the value of K_2, (Method 3) are presented in Fig. 24. The results of regeneration for each of these three methods are presented in Table 5. This analysis was conducted for all the storms on the watersheds mentioned in Table 4, and typical results are shown in Table 5 and Fig. 24. As expected, method 2 yielded superior regeneration of the time distribution of runoff, compared to methods 1 and 3. Consequently, method 2 was considered further for the study of effects of urbanization on the time distribution of runoff.

Nash Model

A conceptual model consisting of a cascade of n linear reservoirs each of which has a storage coefficient of K_N, was proposed by Nash[25] and used in the present study. The expression for IUH of the Nash model is

$$h(t) = \frac{1}{K_N} \frac{e^{-t/K_N}}{\Gamma(n)} (t/K_N)^{n-1} \tag{21}$$

where $\Gamma(\cdot)$ is the gamma function.

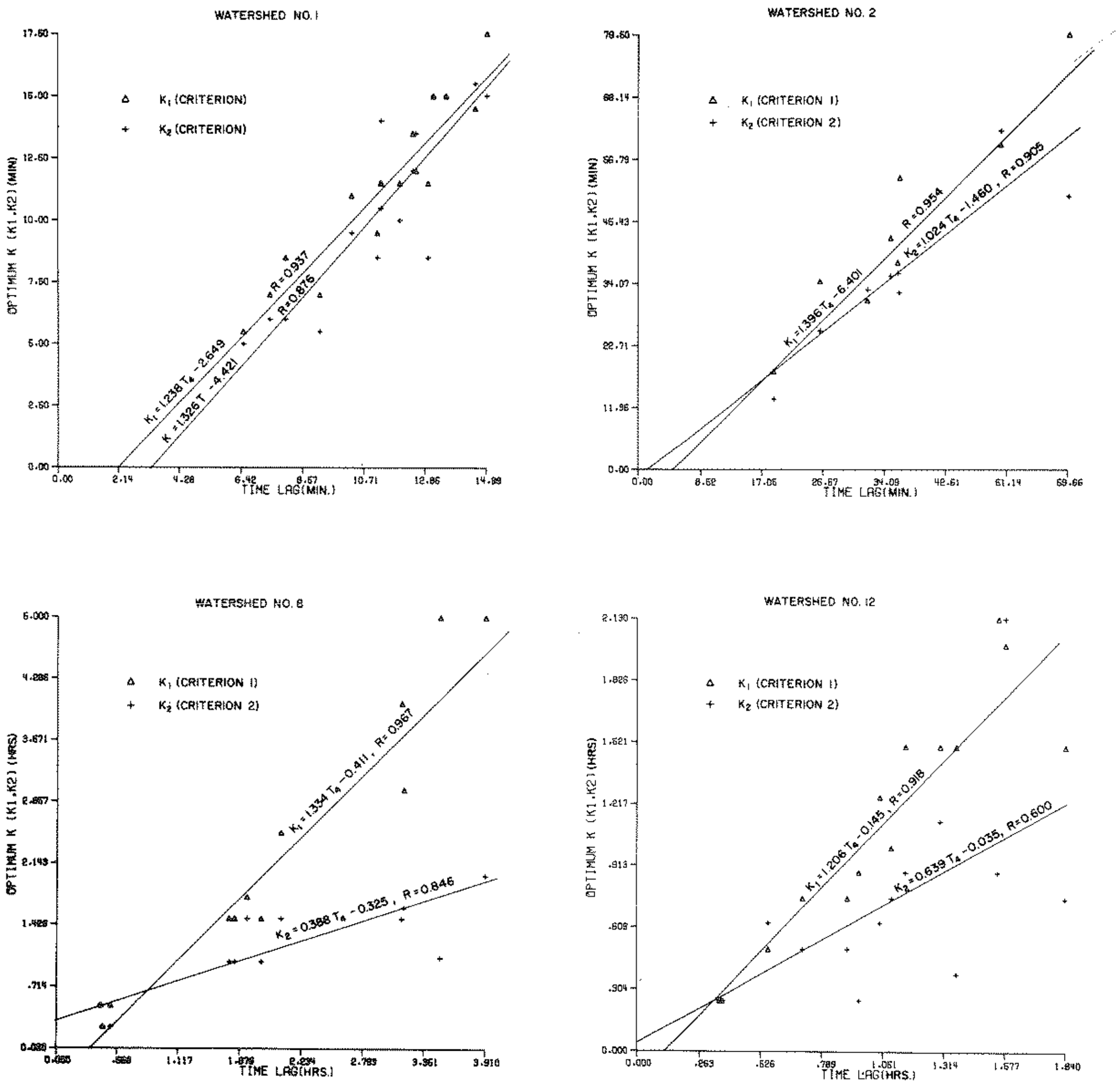


FIGURE 23. COMPARISON OF OPTIMUM VALUES OF K_1 AND K_2 WITH TIME LAG T_4

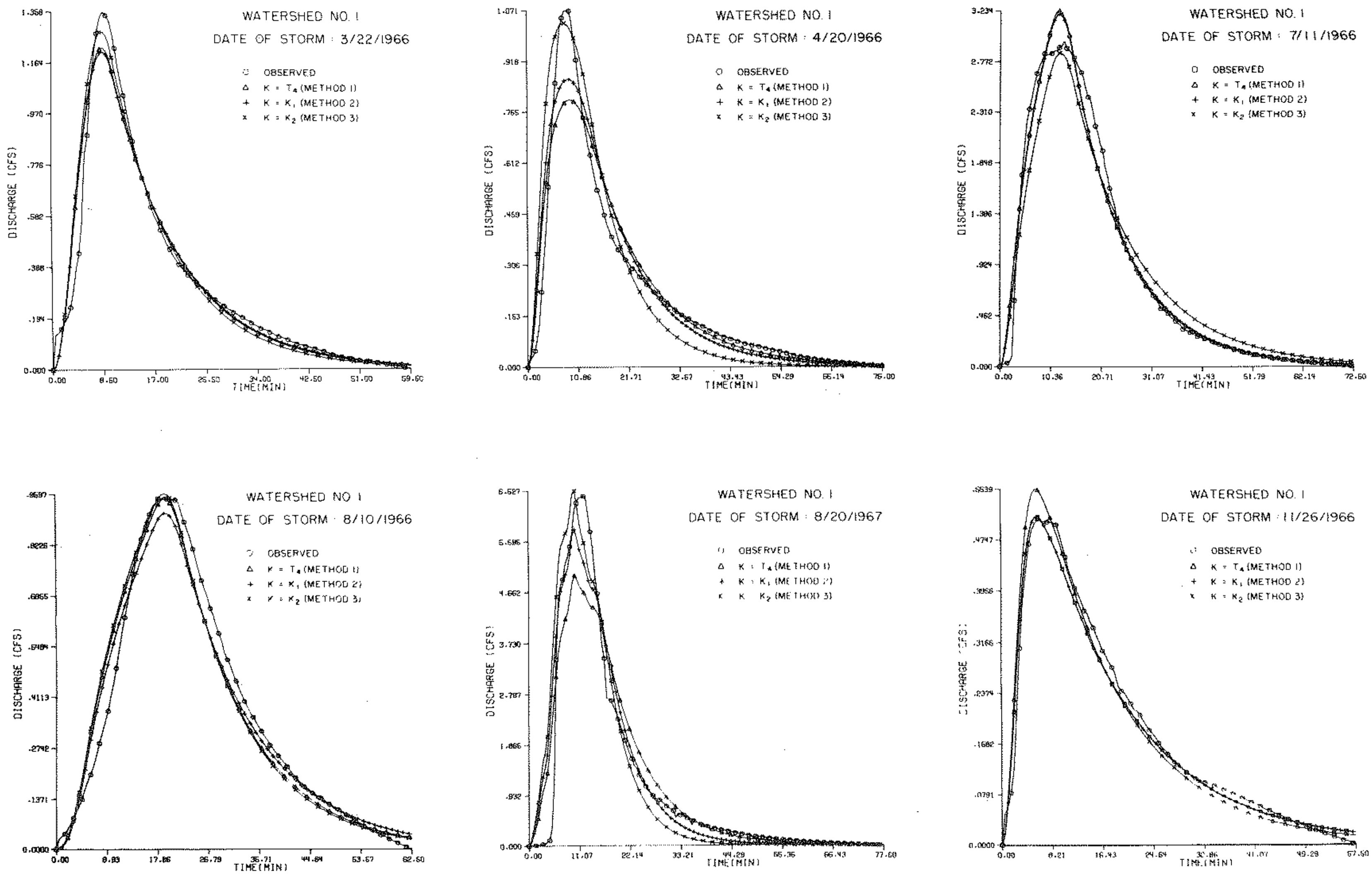


FIGURE 24. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE SINGLE LINEAR RESERVOIR MODEL

TABLE 5. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE SINGLE LINEAR RESERVOIR MODEL

	OBSERVED	METHOD 1	METHOD 2	METHOD 3
MARCH 22 1966 ROSS ADE(UPPER)				
PEAK DISCHARGE	1.36	1.22	1.21	1.28
TIME TO PEAK	8.50	8.50	8.50	8.50
STORAGE COEFFT.	11.30	11.30	11.50	10.50
SIG.ERROR SQ.		.54	.54	.62
INTEG.SQ.ERROR.		1.82	1.82	1.96
CORR.COEFF.		.98	.98	.98
SPE.COR.COEFF.		.99	.99	.99
NOVEMBER 26 1966 ROSS ADE(UPPER)				
PEAK DISCHARGE	.51	.55	.51	.51
TIME TO PEAK	6.00	5.50	6.00	6.00
STORAGE COEFFT.	13.57	13.57	15.00	15.00
SIG.ERROR SQ.		.08	.04	.04
INTEG.SQ.ERROR.		1.36	1.03	1.03
CORR.COEFF.		.99	.99	.99
SPE.COR.COEFF.		.99	1.00	1.00
APRIL 20 1966 ROSS ADE(UPPER)				
PEAK DISCHARGE	1.07	.80	.86	1.03
TIME TO PEAK	8.00	9.00	9.00	8.00
STORAGE COEFFT.	12.94	12.94	11.50	8.50
SIG.ERROR SQ.		.82	.72	1.34
INTEG.SQ.ERROR.		2.67	2.50	3.40
CORR.COEFF.		.97	.97	.96
SPE.COR.COEFF.		.98	.98	.97
AUG. 10 1966 ROSS ADE(UPPER)				
PEAK DISCHARGE	.95	.94	.91	.96
TIME TO PEAK	19.00	19.00	19.00	19.00
STORAGE COEFFT.	12.41	12.41	13.50	12.00
SIG.ERROR SQ.		.43	.35	.50
INTEG.SQ.ERROR.		1.44	1.30	1.55
CORR.COEFF.		.98	.99	.98
SPE.COR.COEFF.		.99	.99	.99
AUG. 20.1967 ROSS ADE(UPPER)				
PEAK DISCHARGE	6.43	4.99	5.81	6.53
TIME TO PEAK	12.00	10.00	10.00	10.00
STORAGE COEFFT.	9.17	9.17	7.00	5.50
SIG.ERROR SQ.		47.43	36.27	50.23
INTEG.SQ.ERROR.		3.80	3.40	4.00
CORR.COEFF.		.96	.96	.95
SPE.COR.COEFF.		.97	.97	.96
JULY 11 1966 ROSS ADE(UPPER)				
PEAK DISCHARGE	2.96	3.23	3.20	2.86
TIME TO PEAK	13.00	12.00	12.00	12.50
STORAGE COEFFT.	11.32	11.32	11.50	14.00
SIG.ERROR SQ.		3.12	3.56	6.01
INTEG.SQ.ERROR.		1.30	1.37	1.03
CORR.COEFF.		.99	.99	.96
SPE.COR.COEFF.		.99	.99	.96

Computation of the Parameters of the Nash Model

The IUH of the Nash model can be described in terms of two parameters K_N and n, which must be determined before a storm event can be regenerated by using this model. Nash[25] suggested that the two parameters (K_N, n) can be evaluated by computing the first and second moments of the excess rainfall and direct runoff distributions. The m^{th} moment of the IUH (M_m) about the origin of time (t = 0) is

$$M_m = \frac{1}{K_N} \frac{1}{\Gamma(n)} \int_o^\infty e^{-t/K_N} (t/K_N)^{n-1} t^m \, dt$$

$$= \frac{K_N^m}{\Gamma(n)} \int_o^\infty e^{-r'} (r')^{n-1+m} \, dr'$$

where

$$r' = t/K_N \text{ and } dr' = \frac{dt}{K_N} .$$

Thus

$$M_n = \frac{K_N^m}{\Gamma(n)} \Gamma(n+m) \tag{57}$$

From Eq. 57 the first moment (m = 1) can be written as

$$M_1 = \frac{K_N}{\Gamma(n)} \Gamma(n+1) = n \, K_N \tag{58}$$

and the second moment (m = 2) is,

$$M_2 = \frac{K_N^2 \, \Gamma(n+2)}{\Gamma(n)} = n(n + 1) \, K_N^2 . \tag{59}$$

By using Eqs. 58 and 59 in conjunction with excess rainfall and direct runoff data, Nash derived the following equations which can be solved to obtain the values of n and K_N,

$$M_1 = n \, K_N = M_{1Q} - M_{1I} \tag{60}$$

$$M_2 = n(n+1)\, K_N^{\;2} = M_{2Q} - M_{2I} - 2n\, K_N\, M_{1I} \qquad (61)$$

where M_{1Q} is the first moment of direct runoff hydrograph,

M_{1I} is the first moment of excess rainfall hyetograph,

M_{2Q} is the second moment of direct runoff hydrograph,

and M_{2I} is the second moment of excess rainfall hyetograph,

The parameters n and K_N were determined in the present study by using Eqs. 60 and 61 for all the data. The values of n and K_N were found to vary from storm to storm, and this variation has also been frequently reported by several previous investigators.[32,58,88] As it was obvious that a unique set of n and K_N values was not applicable to a watershed no attempt was made to estimate such a unique set of values. Instead, by using data from a watershed, the n and K_N values were estimated for each storm, and then were used for regeneration of the same storm by using Eqs. 7 and 21. From the analysis of data of all the storms on all the watersheds mentioned in Table 4, it was observed that the regeneration performance of the Nash model was good for data from watersheds larger than about 5 square miles. Typical results of regeneration which are obtained by analyzing the data from watersheds 5 and 6 are presented in Table 6 and Fig. 25.

Double Routing Method

Holtan and Overton[26,27] proposed a "Double Routing" method to compute the direct runoff from a storm. A graphical depiction of the routing procedure is shown in Fig. 26. In this method, the excess rainfall is routed in series through two linear reservoirs having the same storage constant K_D, each reservoir having the storage-discharge relationship,

TABLE 6. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE NASH MODEL

JAN.6,1962 PLEASANTRUN-A ARLINGTON

TIME LAG= 6.972 NASH K= 1.636 NASH N= 4.26

	OBSERVED	REGENERATED
PEAK DISCHARGE	224.27	187.26
TIME TO PEAK	7.00	7.13
STORAGE COEFFT.	6.97	1.64
SIG.ERROR SQ.		52589.37
INTEG.SQ.ERROR		1.82
CORR.COEFFT.		.97
SPE.COR.COEF.		.98

MAY1 1962 PLEASANTRUN-A ARLINGTON

TIME LAG= 4.596 NASH K= 2.780 NASH N= 1.65

	OBSERVED	REGENERATED
PEAK DISCHARGE	453.23	372.12
TIME TO PEAK	2.88	2.50
STORAGE COEFFT.	4.60	2.78
SIG.ERROR SQ.		147652.17
INTEG.SQ.ERROR		1.90
CORR.COEFFT.		.98
SPE.COR.COEF.		.98

OCT.13,1962 PLEASANTRUN-A ARLINGTON

TIME LAG= 2.136 NASH K= .879 NASH N= 2.43

	OBSERVED	REGENERATED
PEAK DISCHARGE	307.98	302.05
TIME TO PEAK	1.38	1.88
STORAGE COEFFT.	2.14	.88
SIG.ERROR SQ.		986708.38
INTEG.SQ.ERROR		7.50
CORR.COEFFT.		.61
SPE.COR.COEF.		.77

JULY25,1964 PLEASANTRUN-B BROOKVILLE

TIME LAG= 1.436 NASH K= .252 NASH N= 5.70

	OBSERVED	REGENERATED
PEAK DISCHARGE	979.78	882.23
TIME TO PEAK	1.50	1.38
STORAGE COEFFT.	1.44	.25
SIG.ERROR SQ.		69466.06
INTEG.SQ.ERROR		2.54
CORR.COEFFT.		.99
SPE.COR.COEF.		.99

MARCH4,1964 PLEASANTRUN-A ARLINGTON

TIME LAG= 3.768 NASH K= 1.333 NASH N= 2.83

	OBSERVED	REGENERATED
PEAK DISCHARGE	430.30	435.65
TIME TO PEAK	4.88	3.88
STORAGE COEFFT.	3.77	1.33
SIG.ERROR SQ.		55727.21
INTEG.SQ.ERROR		1.33
CORR.COEFFT.		.99
SPE.COR.COEF.		.99

MAY11,1967 PLEASANTRUN-A ARLINGTON

TIME LAG= 3.914 NASH K= 2.876 NASH N= 1.36

	OBSERVED	REGENERATED
PEAK DISCHARGE	475.65	418.41
TIME TO PEAK	2.25	2.13
STORAGE COEFFT.	3.91	2.88
SIG.ERROR SQ.		226688.02
INTEG.SQ.ERROR		2.54
CORR.COEFFT.		.97
SPE.COR.COEF.		.98

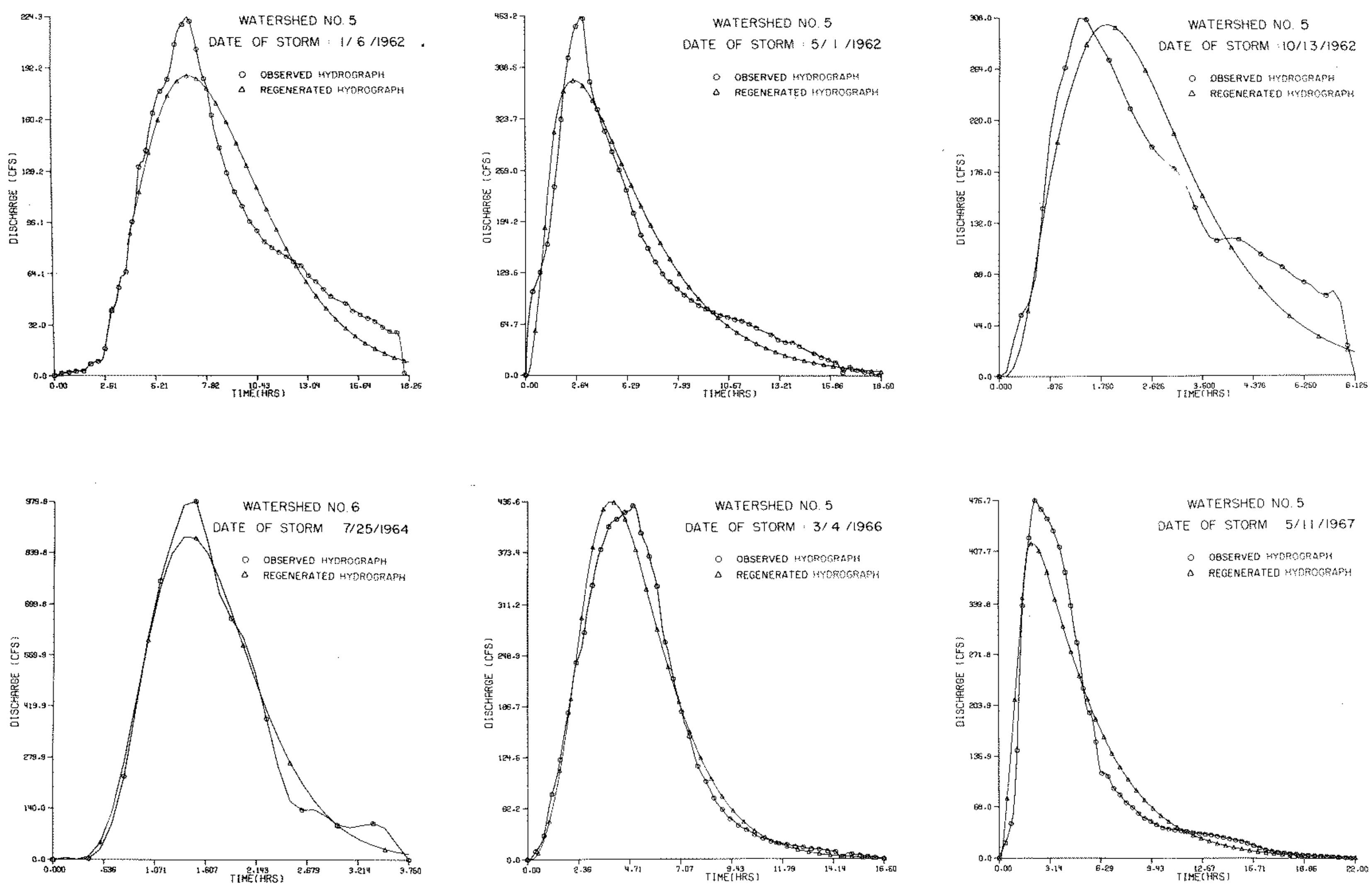


FIGURE 25. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE NASH MODEL

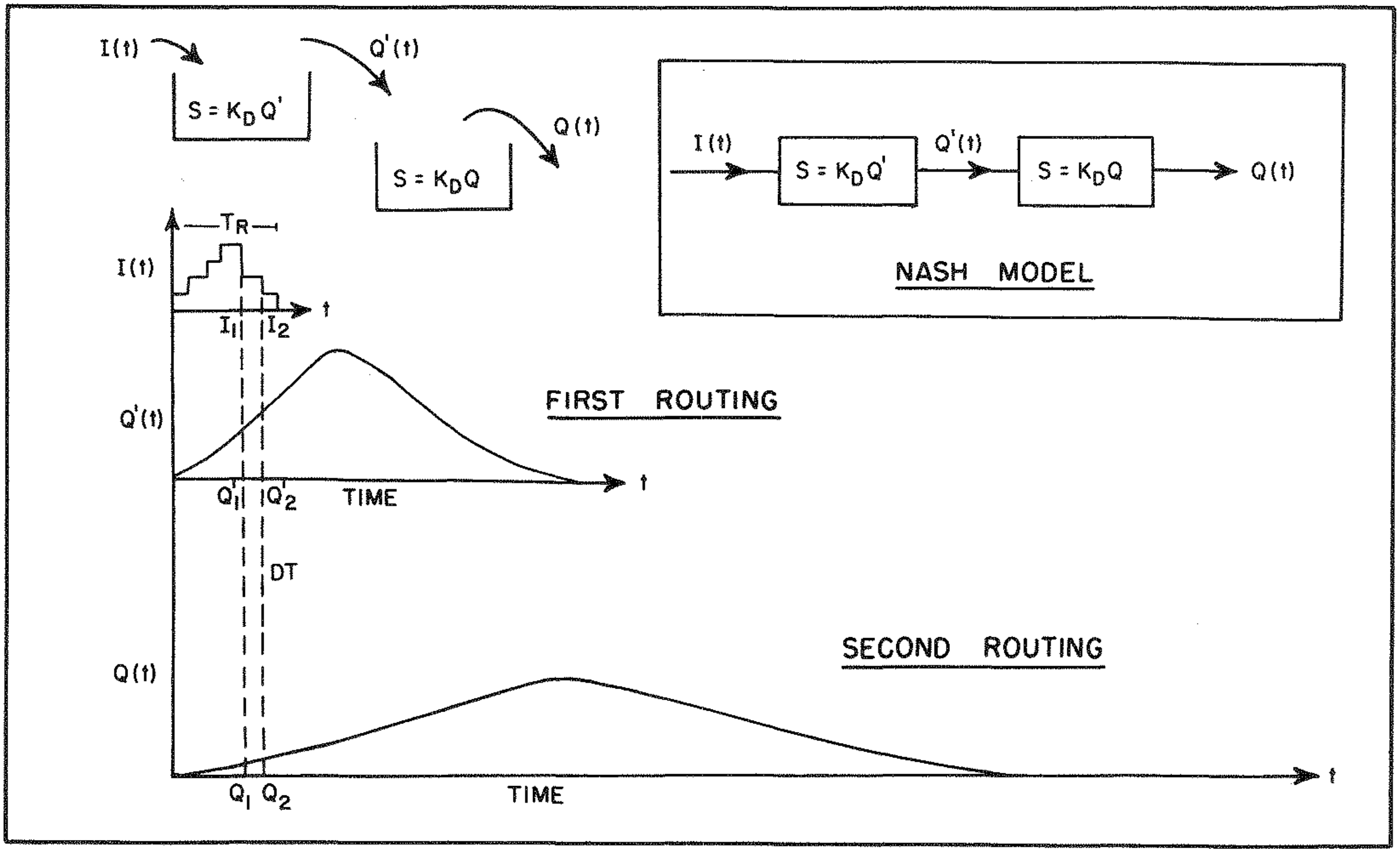


FIGURE 26. ROUTING THROUGH TWO LINEAR RESERVOIRS IN SERIES.

$$S = K_D \, Q \tag{62}$$

Although according to Holtan and Overton the storage coefficient K_D can be assumed to be equal to half of the time lag T_5, the actual values of K_D used by them were obtained from the recession limbs of the runoff hydrographs. The hydrologic continuity equation (Eq. 9) can be combined with Eq. 62 and the resulting equation is

$$I - Q = K_D \frac{dQ}{dt} \tag{63}$$

If Q' is the discharge from the first reservoir, Eq. 63 can be written as

$$I - Q' = K_D \frac{dQ'}{dt} \tag{64}$$

Eq. 64 can be solved for discharge Q' by using numerical techniques with the recurrence relation

$$Q_2' = C_3'(I_1 + I_2) + C_4' \, Q_1' \tag{65}$$

where subscripts 1 and 2 refer to the values at the beginning and end of the interval DT, and C_3' and C_4' are routing coefficients

$$C_3' = \frac{0.5 \, DT}{K_D + 0.5 \, DT} \tag{66}$$

$$C_4' = \frac{K_D - 0.5DT}{K_D + 0.5DT} \tag{67}$$

Further, if the discharge from the first reservoir is applied as input to the second reservoir, the discharge from the second reservoir can be computed by adopting a similar procedure as mentioned above with the inflow I replaced by Q'. The recurrence equation for computing the discharge from the second reservoir is given by

$$Q \;\; = C_3' \, (Q_1' + Q_2') + C_4' \, Q_1 \tag{68}$$

Thus, the outflow from the watershed can be computed for any given rainfall by using Eqs. 65 and 68 in succession.

This numerical technique was used by Holtan and Overton to compute the direct runoff hydrograph for any time distribution of excess rainfall. For the particular case of uniform intensity of excess rainfall, p(IN/Hr.), lasting for a duration T_R, Holtan and Overton[26,27] have presented the following expressions for the discharge Q, peak discharge Q_p, and time to peak T_p, with $K_D = K/2 = T_5/2$:

$$Q(t) = e^{-2(t-T_R)/K}\left[\frac{2}{K}\,Q'(T_R)(t-T_R) + Q(T_R)\right] \qquad (69)$$

$$Q_p = 2\ p\ \exp\left[-\frac{T_R}{K}\coth\ (T_R/K)\right]\sinh\left(\frac{T_R}{K}\right) \qquad (70)$$

and

$$T_p = \frac{T_R}{2}\left[1 + \coth\ (T_R/K)\right] \qquad (71)$$

where $Q'(T_R)$ is the outflow from the first reservoir at $t = T_R$,

$$Q'(T_R) = p\left[1 - e^{-2T_R/K}\right]$$

$Q(T_R)$ is the outflow from the second reservoir at $t = T_R$,

$$Q(T_R) = p\left[1 - e^{-2T_R/K}\ (2\ T_R/K + 1)\right]$$

Double Routing Method as a Special Case of the Nash Model

Although Holtan et.al.,[26,27] used the above mentioned procedure, the "Double Routing" method is actually a special case of the Nash model. If two linear reservoirs are considered in the Nash model, (n = 2), and the storage coefficient $K_N = K_D = T_4/2$, then the IUH for the first linear reservoir is given by

$$h_1(t) = \frac{1}{K_D}\,e^{-t/K_D}$$

If $h_1(t)$ is applied as input to the second linear reservoir, the IUH for this particular model is given by

$$h(t) = \int_o^t \frac{1}{K_D} e^{-\tau/K_D} \frac{1}{K_D} e^{-(t-\tau)/K_D} d\tau$$

$$= \frac{1}{{K_D}^2} t e^{-t/K_D} \tag{72}$$

The discharge Q(t) due to any given rainfall I(t) can now be computed by using the convolution integral (Eq. 7) and Eq. 72 as

$$Q(t) = \int_o^{t_1'} I(\tau) \frac{(t-\tau)}{{K_D}^2} e^{-(t-\tau)/K_D} d\tau \tag{73}$$

where $t_1' = t$ for $t \leq T_R$

$t_1' = T_R$ for $t \geq T_R$

Eq. 73 can be used to compute the outflow hydrograph for any given time distribution of excess rainfall. For the particular case of uniform intensity of excess rainfall p, lasting for a duration T_R, the expression for the time distribution can be obtained from Eq. 73 as shown below. With $K_D = K/2 = T_4/2$, Eq. 72 can be written as

$$h(t) = \left(\frac{2}{K}\right)^2 t e^{-2t/K} \tag{74}$$

For uniform intensity of excess rainfall, I(t) = p, for a duration $t = T_R$ the outflow Q(t) is given by

$$Q(t) = p \int_o^{t_1'} \left(\frac{2}{K}\right)^2 e^{-2(t-\tau)/K} (t-\tau)d\tau$$

$$= \left(\frac{2}{K}\right)^2 p e^{-2t/K} \int_o^{t_1'} e^{2\tau/K} (t-\tau)d\tau$$

$$= \left(\frac{2}{K}\right)^2 p e^{-2t/K}\left\{\int_o^{t_1'} t e^{2\tau/K} d\tau - \int_o^{t_1'} \tau e^{2\tau/K} d\tau\right\}$$

$$= \left(\frac{2}{K}\right)^2 p e^{-2t/K} \left\{t \frac{K}{2} \left[e^{2t_1'/K} - 1\right] - \frac{K}{2}\left[t_1' e^{2t_1'/K} - 0\right] + \right.$$

$$\frac{K}{2}\,\frac{K}{2}\left[e^{2t_1'/K} - 1\right]\Bigg\}$$

$$= p\left(\frac{2}{K}\right)^2 e^{-2t/K}\left\{t\,\frac{K}{2}\left[e^{2t_1'/K} - 1\right]\right.$$

$$\left. - \frac{K}{2}\left[t_1'\, e^{2t_1'/K}\right] + \left(\frac{K}{2}\right)^2\left[e^{2t_1'/K} - 1\right]\right\}$$

$$= p\,\frac{2}{K}\, e^{-2t/K}\left\{\left(t\, e^{2t_1'/K} - t\right) - t_1'\, e^{2t_1'/K}\right.$$

$$\left. + \frac{K}{2}\, e^{2t_1'/K} - \frac{K}{2}\right\}$$

Thus

$$Q(t) = p\,\frac{2}{K}\, e^{-2t/K}\left\{e^{2t_1'/K}\left[t - t_1' + \frac{K}{2}\right] - t - K/2\right\}$$

for $t \leq T_R$, $t_1' = t$

$$Q(t) = p\,\frac{2}{K}\, e^{-2t/K}\left\{e^{2t/K}\left(\frac{K}{2}\right) - t - K/2\right\}$$

$$= p\,\frac{2}{K}\left[\frac{K}{2}\left[1 - e^{-2t/K}\right] - t\, e^{-2t/K}\right]$$

for $T \geq T_R$, $t_1' = T_R$

$$Q(t) = p\,\frac{2}{K}\, e^{-2t/K}\left\{e^{2T_R/K}\left[t - T_R + \frac{K}{2}\right] - t - K/2\right\}$$

$$= e^{-2(t-T_R)/K}\left\{\frac{2t}{K}\, p - \frac{2p}{K}\, T_R + p - \frac{2t}{K}\, p\, e^{-2T_R/K}\right.$$

$$\left. - p\, e^{-2T_R/K}\right\}$$

$$= e^{-2(t-T_R)/K}\left[\frac{2t}{K}\, p\left(1 - e^{-2T_R/K}\right) - \frac{2p}{K}\, T_R\left(1 - e^{-2T_R/K}\right)\right.$$

$$\left. + p\left(1 - e^{-2T_R/K}\right) - \frac{2p}{K}\, T_R\, e^{-2T_R/K}\right]$$

$$= e^{-2(t-T_R)/K} \left[\frac{2}{K} p\left(1 - e^{-2T_R/K}\right)(t-T_R) + p\left(1 - e^{-2T_R/K} - \frac{2T_R}{K} e^{-2T_R/K}\right) \right]$$

Hence

$$Q(t) = e^{-2(t-T_R)/K} \left[\frac{2}{K} Q'(T_R)(t-T_R) + Q(T_R) \right]$$

which is same as Eq. 69. The expressions for peak discharge and time to peak can also be similarly shown to be the same as Eqs. 70 and 71.

Computation of the Parameters in the Double Routing Method

With the number n of linear reservoirs in the conceptual model being specified as two, the only parameter of the model to be determined is the storage coefficient K_D which in turn is half the time lag T_4. Hence, if the time lag value is known, the IUH of this conceptual model can be described. Time lag T_4 can be determined by Eq. 56 as mentioned earlier by using the data of excess rainfall and direct runoff.

Instead of using the value of $T_4/2$ as the storage coefficient K_D, an alternative method of evaluating K_D by using the properties of the recession limb was used by Holtan and Overton. As the recession limb represents only the depletion of water from the watershed, it has been claimed that the recession phase of the hydrograph can be modelled as a single linear reservoir with coefficient K.[26,27] If this assumption is valid, then Eq. 10 serves as the basis for analysis. After the cessation of rainfall, Eq. 10 reduces to Eq. 48, which when integrated between the times t_1 and t_2 yields

$$\ln\left(\frac{Q_1}{Q_2}\right) = \left(\frac{t_2 - t_1}{K}\right) \quad (75)$$

If t_2 and T_1 are so chosen that $t_2 - t_1 = K$,

then

$$Q_2 = Q_1/e \quad (76)$$

In other words, the value of the storage coefficient K or the time lag can be computed by determining the time interval between the values of the discharge Q_1 and (Q_1/e) on the recession limb of the hydrograph. Holtan and Overton[26,27] have used this method to determine the parameter K_D. Although Eq. 75 suggests that a straight line is obtained if the discharge is plotted against time on a semilogarithmic paper with Q on the logarithmic scale, usually the result is not a single straight line, because the recession segment contains surface flow and interflow and ground water flow each having a different lag characteristic. Hence the time lag cannot be uniquely determined from the recession limb of the hydrograph.

The nonuniqueness of the values of K_D obtained from the recession limbs of the observed hydrographs was further studied. Three different regions (1. closest to the peak, 2. near the end, and 3. the middle) of the recession limb of the hydrograph of each storm event were analyzed. From each of these regions, a value of the storage coefficient K was computed by using Eq. 75 and the corresponding value of the parameter K_D was taken as half the value of the storage coefficient K. These three values were designated as K_{D1}, K_{D2} and K_{D3}. The average value of the above three values of K_D was considered as a fourth value of K_D and was designated as K_{D4}. Further, the value of K_D obtained from value of time lag computed by using Eq. 56, was considered as a fifth value of

K_D and was designated as K_{D5}. The values of K_D thus obtained for watersheds 5 and 6 are presented in Table 7 and a comparison of these values of K_D with the corresponding values of T_4 is presented in Fig. 27. All the five values of the parameter K_D were successively used in Eqs. 65 and 68 for regeneration, and the corresponding computations were designated as methods 1, 2, 3, 4 and 5 respectively to indicate the use of the five different values of K_D. Runoff hydrographs were also regenerated by using Eq. 73 for all these five cases. The peak discharge and time to peak discharge of the regenerated hydrographs were plotted against the corresponding observed values for all the above mentioned cases and for all storms on watersheds 5 and 6 in Figs. 28 and 29. Some results of regeneration for watersheds 5 and 6 by using the five different values of K_D are presented in Table 8 and Fig. 30. A comparison of results revealed that for all the storms, the hydrograph which was regenerated by using the value K_{D5}, derived from the time lag T_4, yielded relatively better results among the five cases investigated. As expected from the theory, Table 8 shows that the results of regeneration by using equations 65 and 68 or by equation 73 are essentially the same. Thus it was concluded that the parameter K_D is same as the parameter K_N which can be estimated from T_4 by using Eq. 60.

The Linear-Channel Linear-Reservoir Model

A drainage basin can be modelled by assuming it to be only a channel receiving instantaneous unit excess rainfall. The travel time for the flow from any section of the channel to the outlet of the channel is a function of the velocity of flow. However, the variation of velocity with time and distance in a channel which is part of a natural

Table 7. Values of the Recession Constants

Watershed No.	Time Lag T_4 Hrs.	$2K_{D1}$* Hrs.	$2K_{D2}$* Hrs.	$2K_{D3}$* Hrs.	$2K_{D4}$* Hrs.
5	3.27	4.75	8.00	4.00	5.58
	4.60	4.00	3.38	2.63	3.38
	3.07	3.50	8.00	4.00	5.17
	3.91	6.50	5.50	2.75	4.92
	6.97	7.31	5.13	2.44	4.96
	2.14	3.31	7.19	1.06	1.46
	3.90	7.59	8.56	4.28	6.81
	3.47	7.28	6.44	3.22	5.65
	3.91	7.56	8.13	4.94	6.88
	3.78	2.34	5.81	2.91	3.69
	2.59	8.63	5.75	2.88	5.75
	5.54	13.50	15.87	7.94	12.46
	3.77	2.34	5.81	2.91	3.69
6	6.15	9.75	7.00	3.50	6.75
	5.77	11.62	8.25	4.13	8.00
	2.66	4.88	6.75	3.38	5.00
	4.14	12.19	8.13	4.06	8.13
	3.16	4.85	3.84	6.81	3.91
	2.53	9.25	6.25	3.13	6.21
	1.44	7.59	1.00	0.44	0.48
	2.82	2.88	6.63	3.38	4.29
	3.68	5.66	5.69	3.09	4.81
	3.16	4.28	4.81	3.34	4.15
	2.93	6.97	7.06	3.53	5.85
	2.50	4.44	8.13	4.56	5.71

*For definitions of K_{D1}, etc., see pages 108, 109 and also Table 12.

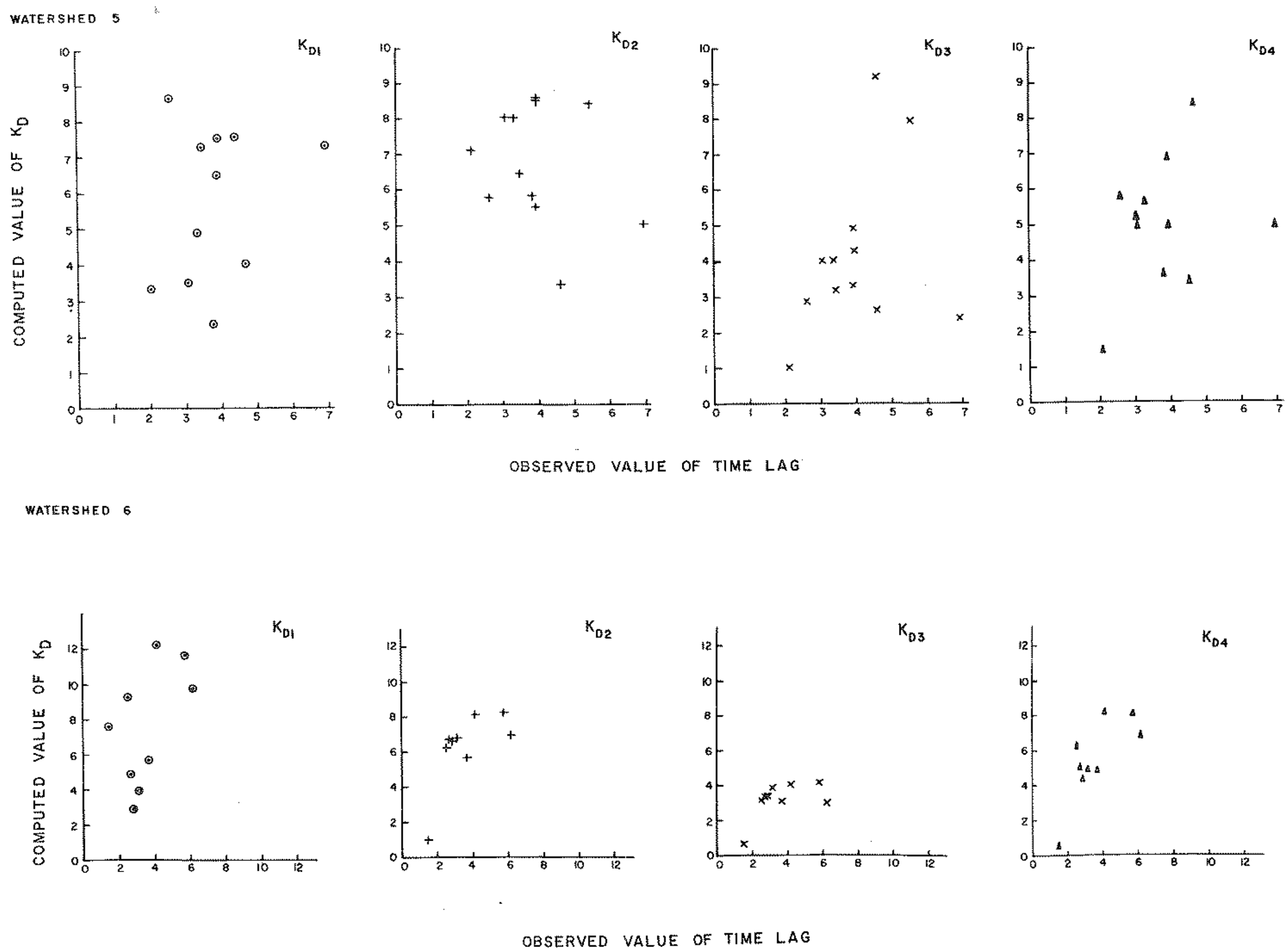


FIGURE 27. COMPARISON OF VALUES OF K_D WITH TIME LAG T_4

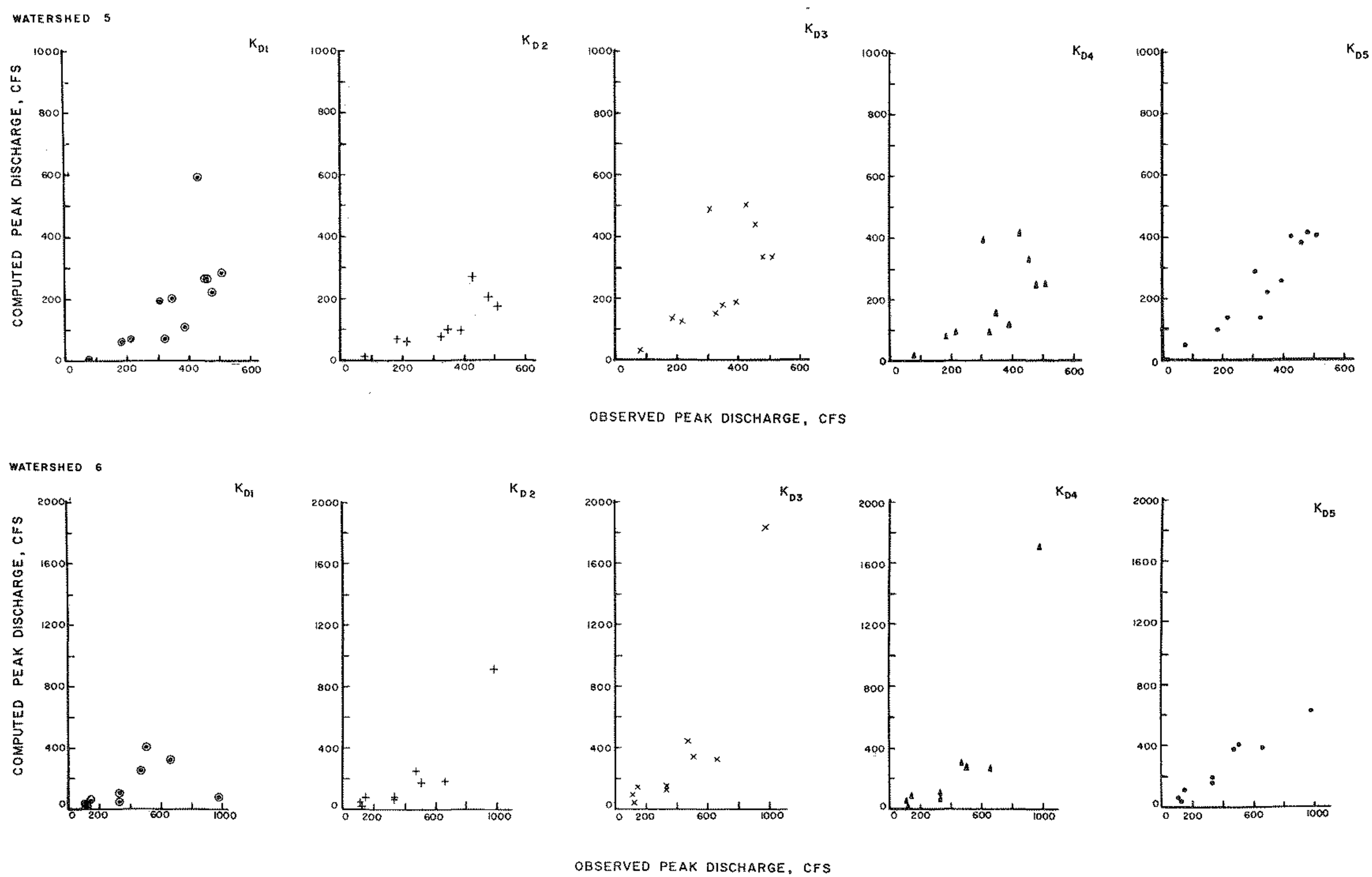


FIGURE 28. COMPUTED AND OBSERVED VALUES OF PEAK DISCHARGE FOR DIFFERENT VALUES OF K_D

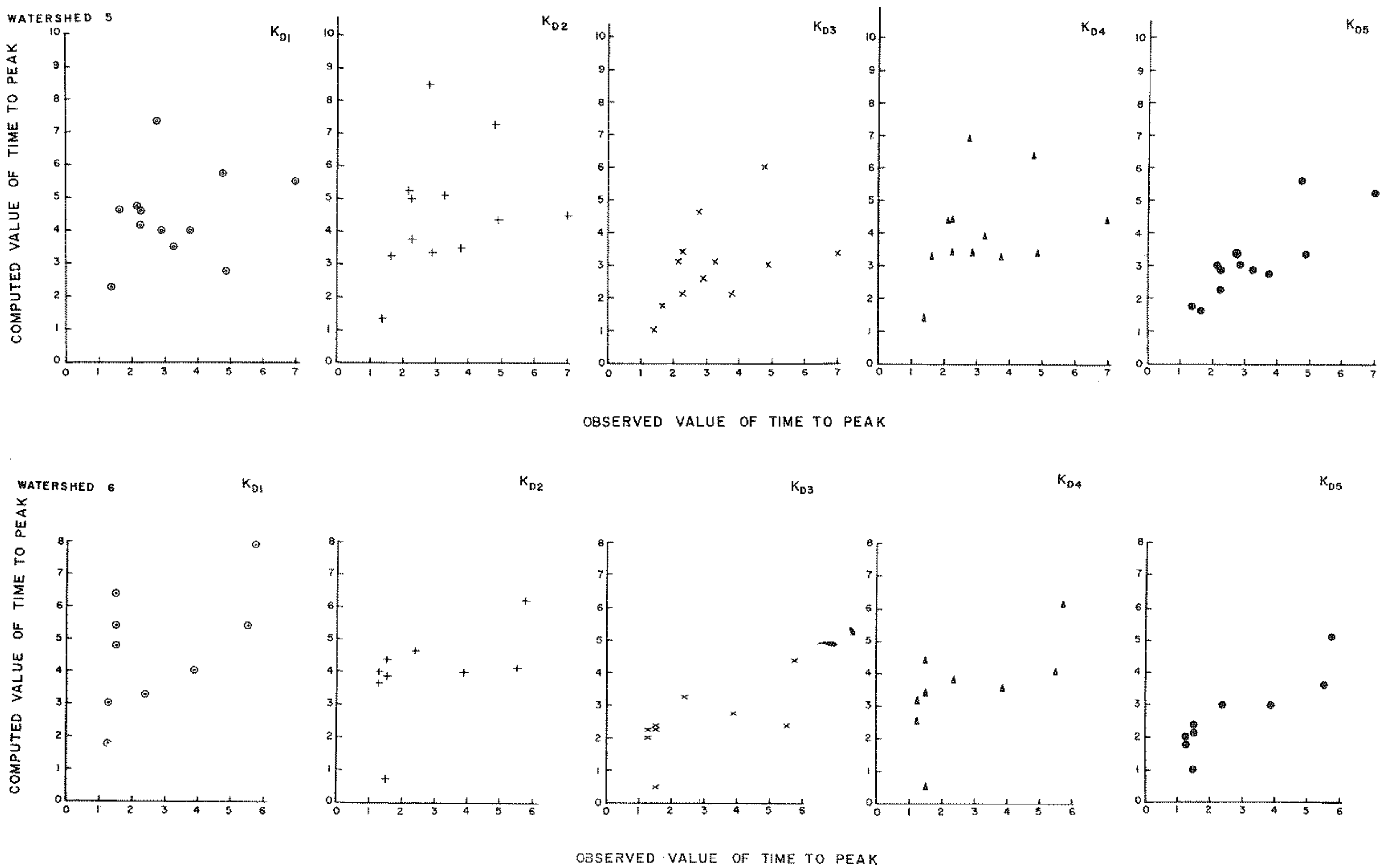


FIGURE 29. COMPUTED AND OBSERVED VALUES OF TIME TO PEAK DISCHARGE FOR DIFFERENT VALUES OF K_D

TABLE 8. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING DIFFERENT VALUES OF K_D IN THE DOUBLE ROUTING METHOD

	OBSERVED	METHOD 1 **		METHOD 2		METHOD 3		METHOD 4		METHOD 5	
JAN.6.1962	PLEASANTRUN-A	A	B	A	B	A	B	A	B	A	B
PEAK DISCHARGE	224.27	145.52	145.60	201.12	201.26	350.64	351.51	207.03	207.18	152.14	152.23
TIME TO PEAK	7.00	5.50	5.50	4.50	4.50	3.38	3.38	4.38	4.38	5.25	5.25
STORAGE COEFFT.	6.97	7.31	7.31	5.13	5.13	2.44	2.44	4.96	4.96	6.97	6.97
SIG.ERROR SQ.		162834.5	162715.8	411530.2	411898.6	2341955.2	2319258.4	458415.6	458839.1	171254.9	171191.3
INTEG.SQ.ERROR		3.38	3.38	5.37	5.37	12.81	12.75	5.67	5.67	3.46	3.46
CORR.COEFF.		.881	.881	.675	.675	.055	.039	.650	.650	.861	.861
SPE.COR.COEFF.		.947	.947	.861	.860	.691	.680	.843	.843	.944	.944
MAY1 1962	PLEASANTRUN-A										
PEAK DISCHARGE	453.23	265.29	265.45	328.22	328.43	442.80	443.21	330.74	330.95	378.25	378.53
TIME TO PEAK	2.88	4.00	4.00	3.38	3.38	2.63	2.63	3.38	3.38	3.00	3.00
STORAGE COEFFT.	4.60	6.59	6.59	5.31	5.31	3.91	3.91	5.27	5.27	4.60	4.60
SIG.ERROR SQ.		733473.9	732995.0	309670.2	309779.4	187311.8	189714.6	298388.3	298530.0	163739.0	164643.0
INTEG.SQ.ERROR		4.48	4.47	2.91	2.91	2.26	2.28	2.86	2.86	2.12	2.12
CORR.COEFF.		.831	.831	.929	.929	.983	.982	.931	.931	.966	.966
SPE.COR.COEFF.		.919	.919	.967	.966	.980	.980	.968	.968	.982	.982
OCT.13.1962	PLEASANTRUN-A										
PEAK DISCHARGE	307.98	194.70	194.91	190.52	194.91	488.60	490.60	388.43	388.93	289.28	289.94
TIME TO PEAK	1.38	2.25	2.25	.75	2.25	1.00	1.00	1.38	1.38	1.75	1.75
STORAGE COEFFT.	2.14	3.31	3.31	.50	0.00	1.06	1.06	1.46	1.46	2.14	2.14
SIG.ERROR SQ.		164635.4	163942.3	24460.3	163942.3	720853.6	731075.0	243319.1	246803.7	30873.6	31033.9
INTEG.SQ.ERROR		5.63	5.62	17.92	5.62	11.79	11.67	6.85	6.90	2.44	2.45
CORR.COEFF.		.796	.796	.909	.796	.760	.761	.902	.902	.967	.967
SPE.COR.COEFF.		.940	.940	.926	.940	.698	.693	.909	.908	.989	.989
JULY25.1964	PLEASANTRUN-B										
PEAK DISCHARGE	979.78	72.17	637.98	900.15	910.30	1833.74	1963.36	1695.44	1809.91	635.01	637.98
TIME TO PEAK	1.50	4.75	1.00	.75	.75	.50	.50	.50	.50	1.00	1.00
STORAGE COEFFT.	1.44	7.59	0.00	1.00	1.00	.44	.44	.48	.48	1.44	1.44
SIG.ERROR SQ.		254645.1	1488034.0	3146315.0	3223005.9	465412.0	983928.4	930487.3	371060.4	1469241.7	1488034.0
INTEG.SQ.ERROR		8.39	12.15	17.67	17.88	37.88	38.56	35.82	36.42	12.07	12.15
CORR.COEFF.		.545	.750	.489	.484	-.177	-.174	-.123	-.119	.756	.750
SPE.COR.COEFF.		.755	.879	.721	.712	1.100	1.136	.988	1.021	.881	.879
MARCH4.1964	PLEASANTRUN-A										
PEAK DISCHARGE	430.30	590.99	592.48	271.81	271.98	502.67	503.44	411.79	412.19	404.15	404.55
TIME TO PEAK	4.88	2.75	2.75	4.38	4.38	3.00	3.00	3.38	3.38	3.38	3.38
STORAGE COEFFT.	3.77	2.34	2.34	5.81	5.81	2.91	2.91	3.69	3.69	3.77	3.77
SIG.ERROR SQ.		1802861.3	1805879.9	754620.7	753600.1	769936.9	772267.9	235209.4	235585.6	218005.4	218271.4
INTEG.SQ.ERROR		7.60	7.61	4.92	4.91	4.97	4.97	2.74	2.76	2.64	2.64
CORR.COEFF.		.779	.778	.926	.925	.885	.885	.960	.960	.964	.964
SPE.COR.COEFF.		.811	.811	.926	.926	.924	.924	.977	.977	.979	.979
MAY11.1967	PLEASANTRUN-A										
PEAK DISCHARGE	475.65	220.79	220.95	205.73	205.97	334.91	335.15	242.53	242.67	418.27	418.66
TIME TO PEAK	2.25	4.75	4.63	5.00	5.00	3.38	3.38	4.38	4.38	2.88	2.88
STORAGE COEFFT.	3.91	7.56	7.56	8.13	8.13	4.94	4.94	6.88	6.88	3.91	3.91
SIG.ERROR SQ.		2158972.8	2157589.3	2411052.1	2409519.8	722794.6	721175.1	1820161.9	1818740.7	202112.2	201251.1
INTEG.SQ.ERROR		8.03	8.03	8.49	8.49	4.65	4.64	7.38	7.37	2.46	2.45
CORR.COEFF.		.662	.662	.605	.605	.905	.905	.730	.730	.973	.973
SPE.COR.COEFF.		.782	.783	.753	.753	.933	.933	.820	.820	.982	.982

* A DOUBLE ROUTING METHOD

B NASH MODEL (n=2)

** FOR DETAILS OF METHOD 1, etc. SEE PAGE 108 AND TABLE 12.

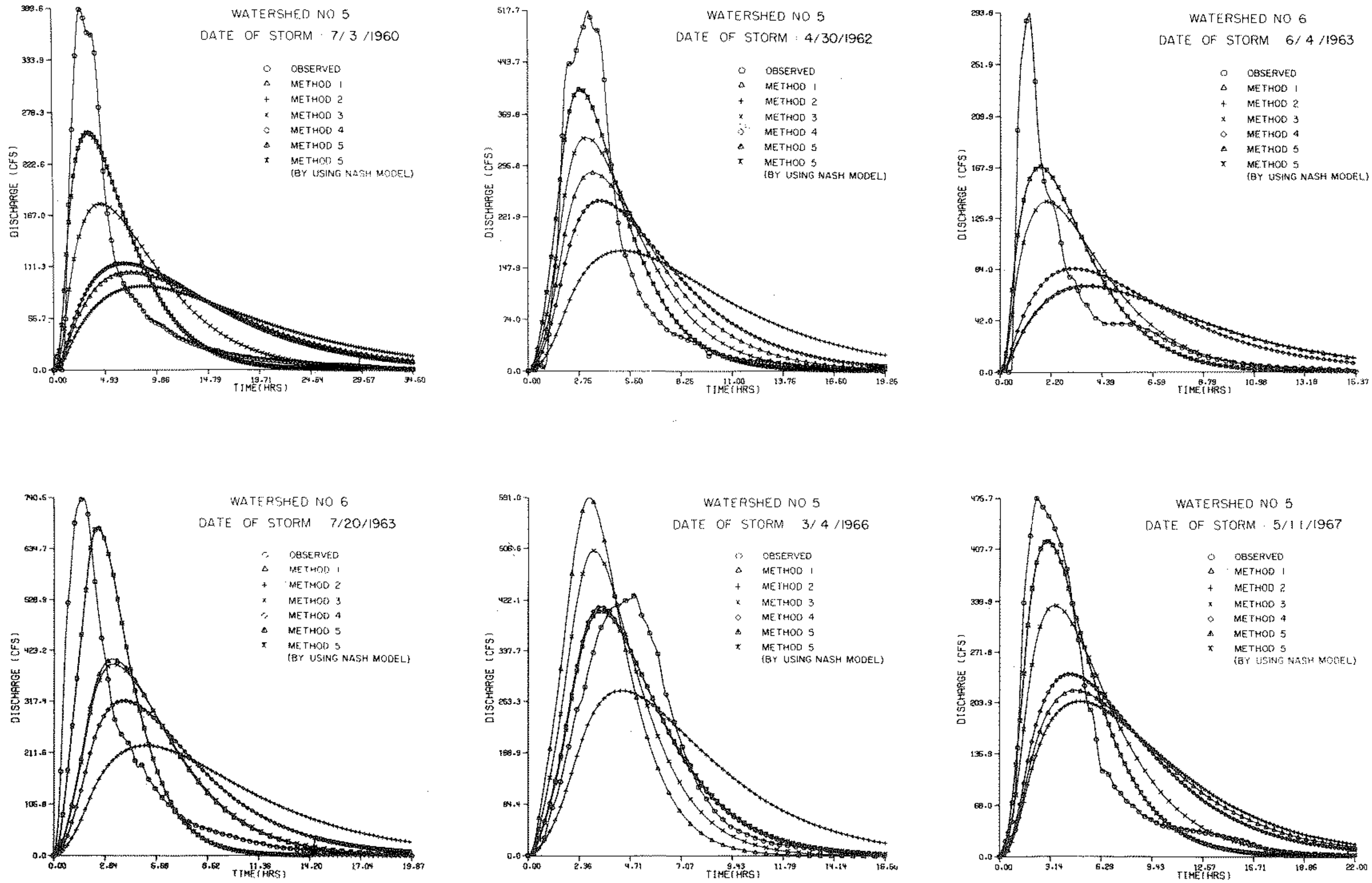


FIGURE 30. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE DOUBLE ROUTING METHOD

drainage network is complicated and cannot be estimated accurately. Dooge[21] assumed that the velocity in a fictitious channel to be independent of time, and hence the channel becomes a "linear-channel" in which the mutual interference of flows from individual areas is eliminated. He also proposed the division of the watershed by isochrones or contours of equal travel time to the outlet. The diagram called an area-distance diagram is obtained by plotting the length of the isochrones against their distance along the channel. By using a suitable velocity function, the area-distance diagram can be converted into an area-time diagram. The time-area-concentration diagram is obtained if the ordinates of the area-time diagram are divided by the total area of the watershed and the resulting ordinates are plotted against time. The time-area-concentration diagram, $\omega(t)$, has the following properties:

$$\omega(t) = \begin{cases} \frac{1}{A} \frac{dA}{dt} & 0 < t < T_c \\ 0 & \text{Otherwise} \end{cases} \tag{77}$$

and

$$\int_0^\infty \omega(\tau) d\tau = 1 \tag{78}$$

In a conceptual model which consists of a series combination of a linear channel and a linear reservoir, the translation effects of the linear channel and the storage effects of linear reservoir, are combined. Since the order of the linear operations is immaterial, the flow may be considered as first passing through a linear channel and then through a linear reservoir. Considering the time-area-concentration diagram $\omega(t)$ as inflow to the linear reservoir with a storage coefficient K, the IUH of conceptual model can be written as

$$h(t) = \int_{o}^{t'<T_c} \frac{1}{K} e^{-(t-\tau)/K} \omega(\tau)d\tau \qquad (79)$$

Eq. 79 can be shown to be a special case of the general expression for IUH derived by Dooge[21] (Eq. 22). If the conceptual model proposed by Dooge[21] involves only one linear reservoir, Eq. 22 reduces to

$$h(t) = \frac{1}{T_c} \int_{o}^{t\leq T_c} \frac{1}{K} e^{-(t-\tau)/K} \omega(\tau/T_c)d\tau$$

$$= \int_{o}^{t\leq T_c} \frac{1}{K} e^{-(t-\tau)/K} \left[\frac{1}{T_c} \omega(\tau/T_c)\right] d\tau$$

$$= \int_{o}^{t\leq T_c} \frac{1}{K} e^{-(t-\tau)/K} \omega(\tau)d\tau$$

(because, by definition, $\frac{1}{T_c} \omega(\tau/T_c) = \omega(\tau)$), which is same as Eq. 79.

Dimensionless IUH

In order to analyze data from actual watersheds using the model, the IUH of which is given by Eq. 79, the time-area-concentration diagram has to be derived by determining the isochrones. However, no reliable method is available to accurately determine the isochrones for any watershed. O'Kelly[24] suggested that the smoothing involved in the routing is sufficient to permit replacement of the time-area-concentration diagram by a well defined geometrical shape like a rectangle or an isosceles triangle, with a base equal to T_c and which satisfies the Eqs. 77 and 78. Mathematical expressions for IUH for the cases of rectangular and equilateral triangular shapes of the time-area-concentration diagram were also presented by O'Kelly.[24] Dooge[21] adopted O'Kelly's assumptions and presented curves of dimensionless instantaneous unit hydrographs for the cases of rectangular and equilateral triangular shapes of time-area-concentration diagrams. Dooge and O'Kelly have routed the time-area-concentration diagrams through a single linear reservoir to obtain the IUH. However, the IUH thus obtained was not used for regeneration either by Dooge or O'Kelly. Singh[53] performed a similar analysis by routing time-area-concentration diagrams of different shapes through a series combination of two linear reservoirs. Singh[53] derived the instantaneous unit hydrographs for the cases of 1) rectangular shape, and triangular shapes with peaks at 2) beginning, 3) middle, and 4) the end of time T_c, and 5) parabolic shape, of the time-area-concentration diagram, and used the IUH so obtained in each case for regeneration.

In order to investigate the validity of the simplification mentioned above, the expressions for the IUH ordinates were derived by using Eq. 79 for four different shapes of time-area-concentration diagrams, namely rectangular, isosceles triangular, the triangular shape with peaks occurring at the beginning and the triangular shape with peak occurring at the end of the time T_c. The expressions for IUH for the four cases are presented in Table 9. Values of the peak and time to peak of the dimensionless instantaneous unit hydrographs for the four different cases mentioned above for different values of K/T_c were computed and the results for a range of 0.1 to 10 for the ratio K/T_c, are presented in Table 10. It was observed that, 1) the shape of the IUH changes with the ratio K/T_c, 2) for a given value of K/T_c, the peak of the dimensionless IUH is very nearly the same for all the four different shapes of the time-area-concentration diagrams considered, 3) in all the four cases considered the magnitude of the peak of the IUH

Table 9. Dimensionless Instantaneous Unit Hydrographs

For the Linear-Channel Linear-Reservoir Model

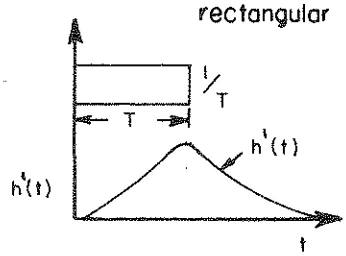
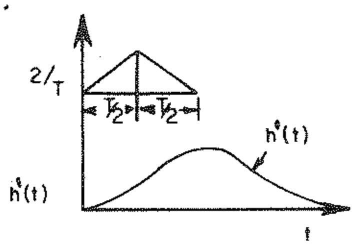
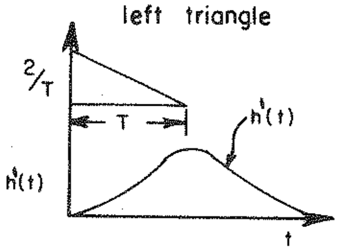
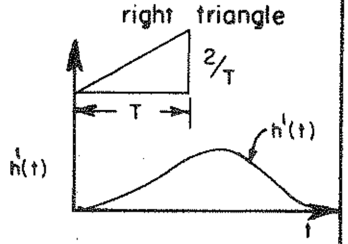
1. rectangular 	$h'(t) = h(t)T = \begin{cases} [1-e^{-t/K}] & \text{for } 0 \le t \le T \\ [e^{-(t-T)/K} - e^{-t/K}] & \text{for } T \le t \le \infty \end{cases}$
2. isosceles triangular 	$h'(t) = h(t)T = \begin{cases} \frac{4}{T}\,[(t-K)+K\,e^{-t/K}] & \text{for } 0 \le t \le T/2 \\ \frac{4}{T}\,e^{-t/K}[K-2Ke^{T/2K}+e^{t/K}(T-t)+Ke^{t/K}] & \text{for } T/2 \le t \le T \\ \frac{4K}{T}\,e^{-t/K}[1-e^{T/2K}]^2 & \text{for } T \le t \le \infty \end{cases}$
3. left triangle 	$h'(t) = h(t)T = \begin{cases} \frac{2}{T}\,[(T-t)+K-(T+K)e^{-t/K}] & \text{for } 0 \le t \le T \\ \frac{2}{T}\,e^{-t/K}[-T+K(e^{T/K}-1)] & \text{for } T \le t \le \infty \end{cases}$
4. right triangle 	$h'(t) = h(t)T = \begin{cases} \frac{2}{T}\,[t-K+K\,e^{-t/K}] & \text{for } 0 \le t \le T \\ \frac{2}{T}\,e^{-t/K}[e^{T/K}(T-K)+K] & \text{for } T \le t \le \infty \end{cases}$

Table 10. Variation of Magnitude of Peak Discharge and Time to Peak Discharge of Dimensionless IUH with K/T_c

K/T_c	Time-Area-Concentration Diagram: Rectangle		Isosceles Triangle		Left Triangle		Right Triangle*	
	Q_{pi}'	T_{pi}'	Q_{pi}'	T_{pi}'	Q_{pi}'	T_{pi}'	Q_{pi}'	T_{pi}'
0.10	1.000	1.000	1.71	0.600	1.502	0.200	1.800	1.000
0.50	0.865	1.000	1.012	0.700	0.896	0.600	1.135	1.000
1.00	0.632	1.000	0.671	0.800	0.614	0.700	0.736	1.000
2.00	0.393	1.000	0.401	0.900	0.378	0.800	0.426	1.000
2.50	0.330	1.000	0.333	0.900	0.317	0.800	0.352	1.000
3.00	0.286	1.000	0.289	0.900	0.274	0.900	0.302	1.000
4.00	0.221	1.000	0.221	0.900	0.215	0.900	0.230	1.000
5.00	0.181	1.000	0.181	1.000	0.177	0.900	0.187	1.000
10.00	0.095	1.000	0.095	1.000	0.094	1.000	0.097	1.000

*See Table 9 for definition.

increases with the decreasing value of K/T_c and vice versa, (Fig. 31), and 4) the time to peak of IUH for all the four cases, decreases with a decrease in the ratio K/T_c, (Fig. 32).

Computation of the Parameters of the Model

In order to apply the dimensionless instantaneous unit hydrographs obtained above, for data from actual watersheds the values of K and T_c are to be estimated. The value of K can be estimated by Eq. 56 as mentioned earlier. The time of concentration, T_c, is defined as the time required for the surface runoff from the remotest part of the watershed to reach the point under consideration. In natural watersheds accurate evaluation of time of concentration is not possible. However, different methods to approximately evaluate T_c are available.[20,89] Based on the premise that the time of concentration T_c is the time taken for the last drop of the excess rainfall to reach the outlet of the watershed, Snyder[61] suggested that T_c can be estimated as the time elapsed between the end of rainfall and the point of inflection on the recession limb of the hydrograph.

Clark[23] suggested that the time base T_c of the time-area diagram can be determined as the time between the cessation of excess rainfall and the time at which the rate of discharge decrease is greatest in relation to the prevailing discharge. Both of these[23,61] definitions were used to determine the value of T_c for each storm event. It was found that the value of T_c obtained by using the definitions of both Snyder and Clark varied from storm to storm on a given watershed. As some of the values of T_c obtained by using Clark's definition were unrealistic, only the T_c values computed by using Snyder's definition

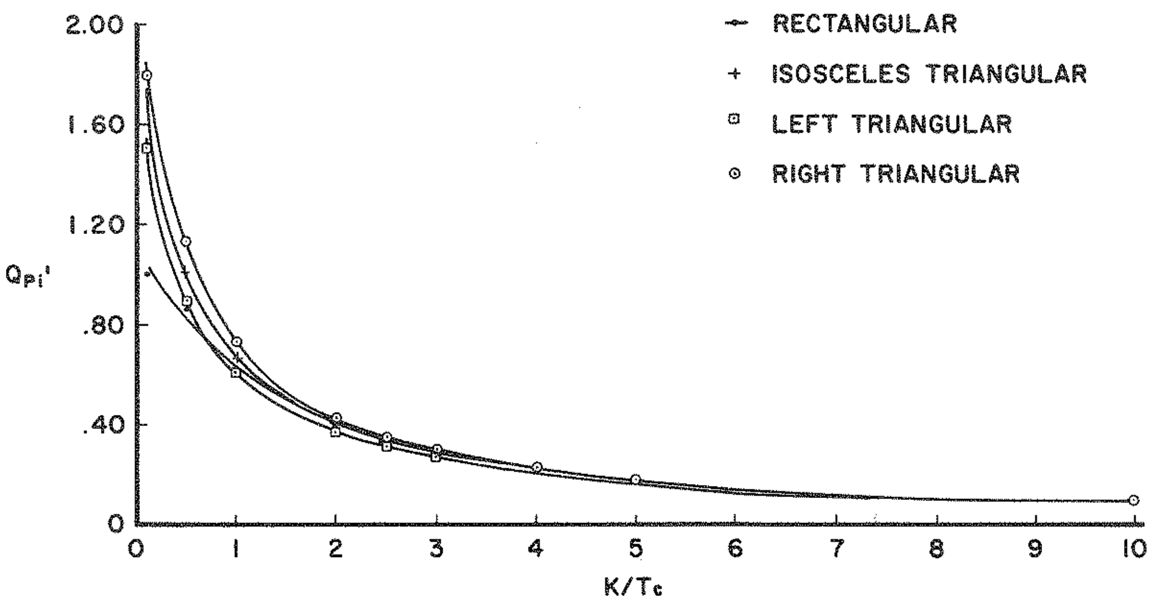


FIGURE 31. VARIATION OF PEAK OF DIMENSIONLESS IUH WITH K/T_c

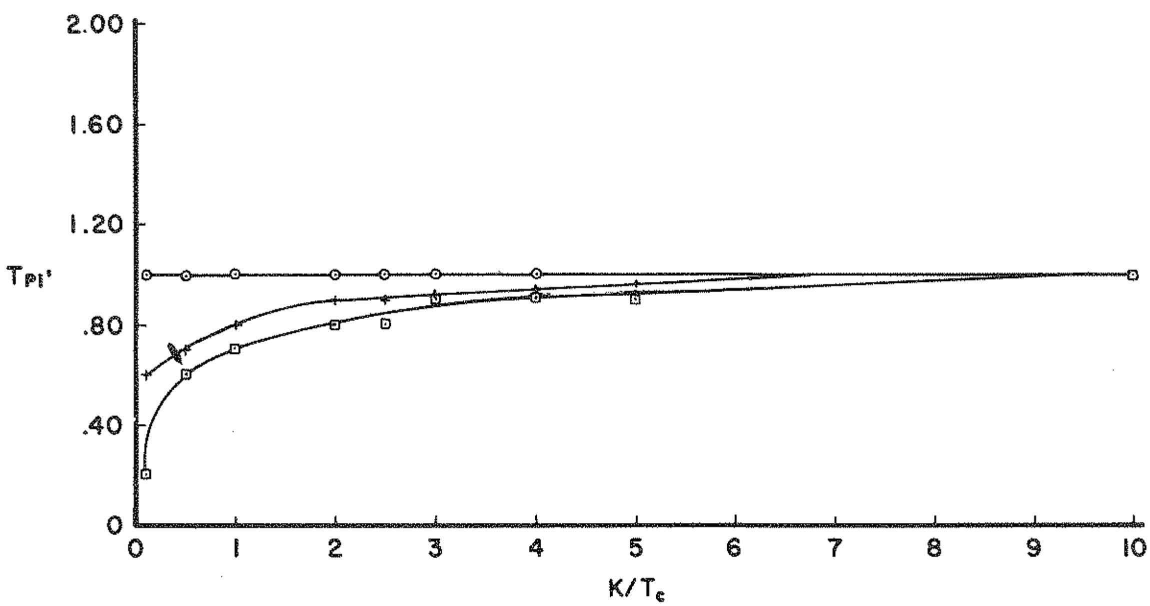


FIGURE 32. VARIATION OF TIME TO PEAK OF DIMENSIONLESS IUH WITH K/T_c

were further used. Hence, for each storm the particular value of T_c obtained for that storm was used to compute the instantaneous unit hydrographs for all the above mentioned shapes of time-area-concentration diagrams.

Regeneration

In order to test the regeneration performance of the conceptual model, the instantaneous unit hydrographs derived for all four cases mentioned above were then used successively with Eq. 7 to compute the corresponding outflow hydrographs for the excess rainfall of the storm from which the values of T_c and K were derived. The outflow hydrographs thus obtained for the four cases were compared with the observed hydrograph. For each case, the statistical measures defined by Eqs. 45, 46 and 47 were also computed. The results showed that 1) the peak discharges of the regenerated hydrographs in all four cases were very nearly the same, 2) the computed peak discharges of the regenerated hydrographs obtained by using T_c values which were computed by using Snyder's definition were less by 50 to 100 percent when compared to the peak discharges of the observed hydrographs. Thus, the IUH derived by using the value of T_c obtained by adopting Snyder's definition did not satisfactorily regenerate the hydrograph of the storm from which it was derived.

The above results are subject to the assumption that the storage coefficient K of the linear reservoir is assumed to be equal to the time lag T_4, although as mentioned earlier, the optimum value of K was observed to be slightly different from T_4. However, the assumption is valid as the storage coefficient is theoretically equal to the time lag T_4 as given by Eq. 56.

Thus these results indicated that an alternate approach for a better estimation of T_c was necessary. As the conceptual model consists of two parameters, T_c and K, and the value of K can be obtained by Eq. 56, only the parameter T_c remains to be determined such that the conceptual model has better regeneration performance. Hence, for each storm, an "optimum value" of T_c was computed according to the criterion that the sum of the squares of the deviation of the observed and the computed hydrograph ordinates is minimized. It was observed that the optimum value of T_c also varied from storm to storm in any watershed. The relation between the optimum value of T_c thus obtained and the value of T_c computed according to Snyder's definition is shown in Fig. 33. The results of regeneration obtained by using optimum values of T_c are presented in Table 11 and Fig. 34. As expected, when the optimum value of T_c was used, the regeneration was better than that obtained by using the value of T_c from Snyder's method.

Fourier Transform Method

The Fourier Transform method which was discussed earlier has been used to evaluate the response function for a given storm and this response function, in turn is used to regenerate the outflow hydrograph. The method of analysis used, which was developed by Blank and Delleur,[32] is given below.

Let $Y(\omega)$, $X(\omega)$ and $H(\omega)$ represent the Fourier Transform of the output $Y(t)$, the input $X(t)$ and the unit impulse response function $h(t)$

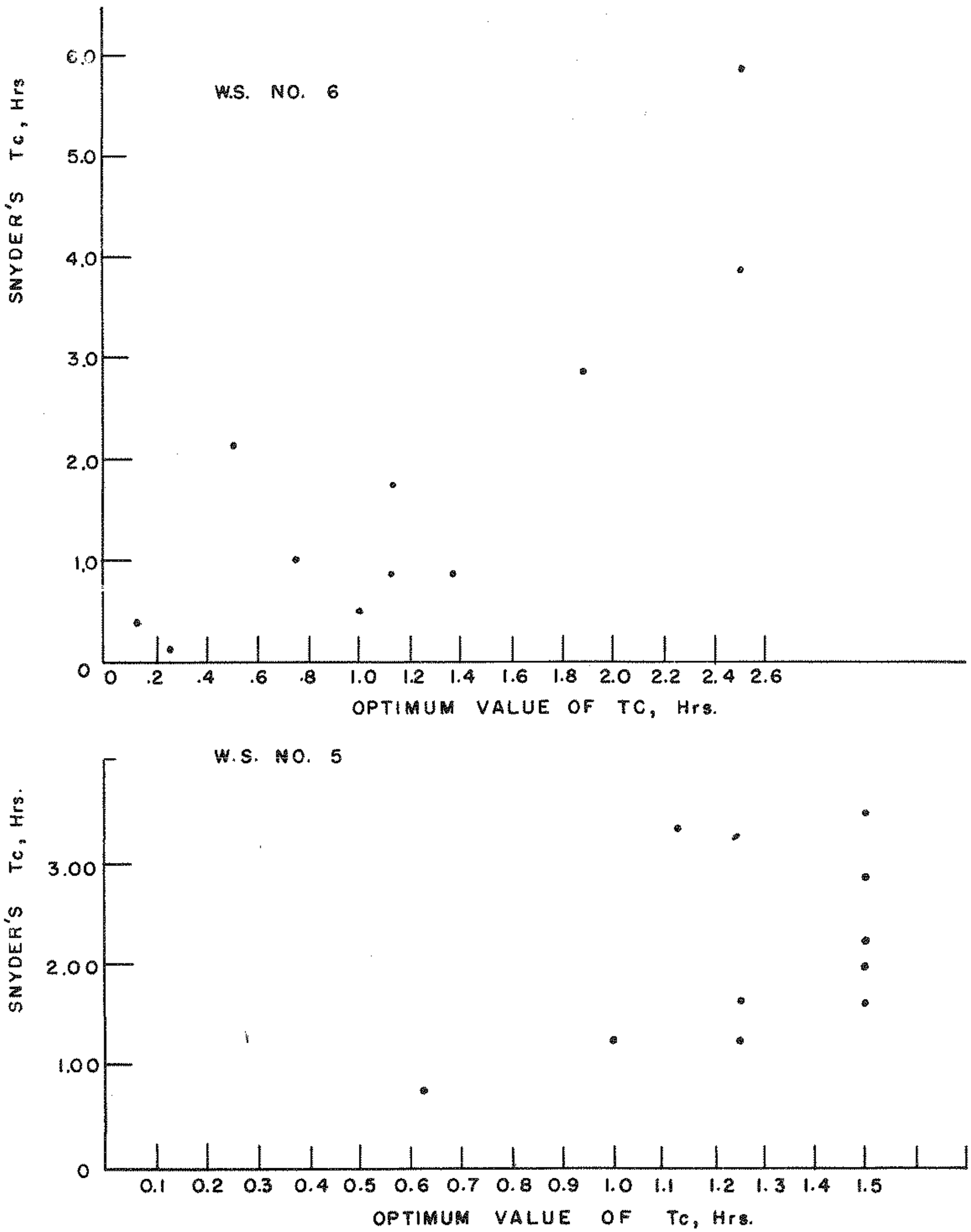


FIGURE 33. COMPARISON OF VALUES OF T_c BY SNYDER'S METHOD WITH OPTIMUM VALUES OF T_c

TABLE II. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING SNYDER'S T_C AND THE OPTIMUM T_C IN THE LINEAR-CHANNEL LINEAR-RESERVOIR MODEL

	OBSERVED	SNYDER'S T_C	OPTIMUM T_C	
APRIL22,1963. PLEASASENTRUN-A				ARLINGTON
STORAGE COEFFICIENT	3.910	3.910	3.910	
TRAVEL TIME		2.880	1.500	
PEAK DISCHARGE	182.675	88.037	99.262	
TIME TO PEAK	3.750	3.500	2.500	
SIG.ERROR SQ.	0.000	97440.019	149193.692	
INTEG.SQ.ERROR	0.000	7.313	9.049	
CORR. COEFF.	1.000	.915	.736	
SPE.COR.COEFFT.		.888	.823	
JULY12,1964 PLEASANTRUN-A				ARLINGTON
STORAGE COEFFICIENT	2.584	2.584	2.584	
TRAVEL TIME		1.250	1.000	
PEAK DISCHARGE	80.438	48.317	50.140	
TIME TO PEAK	1.625	1.625	1.375	
SIG.ERROR SQ.	0.000	7894.171	6449.426	
INTEG.SQ.ERROR	0.000	6.665	6.025	
CORR. COEFF.	1.000	.916	.937	
SPE.COR.COEFFT.		.927	.941	
NOV.8,1966 PLEASANTRUN- A				ARLINGTON
STORAGE COEFFICIENT	3.446	3.446	3.446	
TRAVEL TIME		2.000	1.500	
PEAK DISCHARGE	324.738	136.515	144.435	
TIME TO PEAK	2.250	2.500	2.125	
SIG.ERROR SQ.	0.000	174141.272	175035.973	
INTEG.SQ.ERROR	0.000	7.871	7.891	
CORR. COEFF.	1.000	.860	.850	
SPE.COR.COEFFT.		.879	.879	
MAY22,1965 PLEASANTRUN-B				BROOKVILLE
STORAGE COEFFICIENT	3.963	3.963	3.963	
TRAVEL TIME		.875	1.125	
PEAK DISCHARGE	122.131	47.275	46.216	
TIME TO PEAK	1.500	1.250	1.500	
SIG.ERROR SQ.	0.000	25746.953	24531.546	
INTEG.SQ.ERROR	0.000	9.336	9.113	
CORR. COEFF.	1.000	.771	.789	
SPE.COR.COEFFT.		.827	.836	
JUNE4,1963 PLEASANTRUN-B				BROOKVILLE
STORAGE COEFFICIENT	2.930	2.930	2.930	
TRAVEL TIME		1.000	.750	
PEAK DISCHARGE	293.848	193.470	201.559	
TIME TO PEAK	1.250	1.375	1.125	
SIG.ERROR SQ.	0.000	85336.767	63564.739	
INTEG.SQ.ERROR	0.000	5.417	4.675	
CORR. COEFF.	1.000	.927	.949	
SPE.COR.COEFFT.		.943	.958	
JULY25,1964 PLEASANTRUN-B				BROOKVILLE
STORAGE COEFFICIENT	1.433	1.433	1.433	
TRAVEL TIME		1.125	1.750	
PEAK DISCHARGE	979.782	597.304	507.924	
TIME TO PEAK	1.500	1.375	1.875	
SIG.ERROR SQ.	0.000	950942.339	1722009.866	
INTEG.SQ.ERROR	0.000	9.731	13.095	
CORR. COEFF.	1.000	.945	.767	
SPE.COR.COEFFT.		.924	.858	

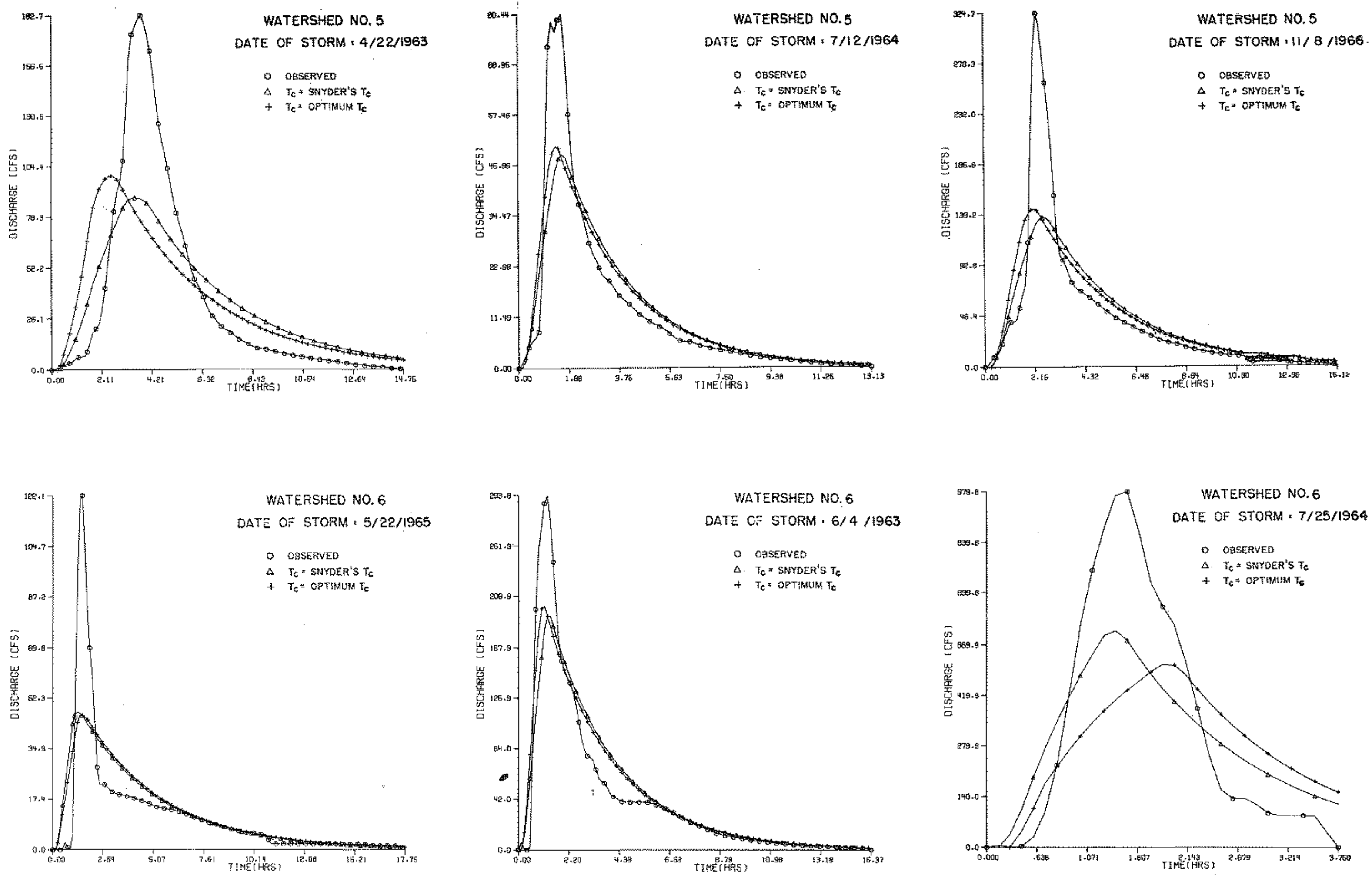


FIGURE 34. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE LINEAR-CHANNEL LINEAR-RESERVOIR MODEL

respectively, where Y(t), X(t) and h(t) are all positive for every value of $t > 0$.

By definition,

$$Y(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t}\, Y(t)dt$$

$$= \int_{-\infty}^{\infty} [\cos \omega t - j \sin \omega t]\, Y(t)dt$$

$$= \int_{-\infty}^{\infty} Y(t) \cos \omega t \, dt - j \int_{-\infty}^{\infty} Y(t) \sin \omega t \, dt$$

$$= Y_r(\omega) + j\, Y_j(\omega) \tag{80}$$

Similarly,

$$X(\omega) = X_r(\omega) + j\, X_j(\omega) \tag{81}$$

and

$$H(\omega) = H_r(\omega) + j\, H_j(\omega) \tag{82}$$

where the subscripts r and j in Eqs. 80, 81 and 82 represent real and imaginary parts.

The unit impulse response function h(t) can be obtained by taking the inverse Fourier Transform of Eq. 82.

Hence,

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t}\, H(\omega)d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} [\cos\omega t + j \sin\omega t][H_r(\omega) + j\, H_j(\omega)]d\omega$$

$$= \frac{1}{2\pi} \Big[\int_{-\infty}^{\infty} H_r(\omega)\cos\omega t \, d\omega + j \int_{-\infty}^{\infty} H_j(\omega)\cos\omega t \, d\omega$$

$$+ j \int_{-\infty}^{\infty} H_r(\omega)\sin\omega t \, d\omega - \int_{-\infty}^{\infty} H_j(\omega)\sin\omega t \, d\omega \Big] \tag{83}$$

The second and the third terms of the right hand side of Eq. 83 are odd functions and hence the Eq. 83 reduces to

$$h(t) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} H_r(\omega)\cos\omega t\, d\omega - \int_{-\infty}^{\infty} H_j(\omega)\sin\omega t\, d\omega\right] \tag{84}$$

Thus to obtain the response function h(t), it is required to evaluate $H_r(\omega)$ and $H_j(\omega)$.

Consider Eq. 26

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \left(\frac{Y_r(\omega) + j\,Y_j(\omega)}{X_r(\omega) + j\,X_j(\omega)}\right) \times \left(\frac{X_r(\omega) - j\,X_j(\omega)}{X_r(\omega) - j\,X_j(\omega)}\right)$$

$$= \left[\frac{[Y_r(\omega)\,X_r(\omega) + Y_j(\omega)\,X_j(\omega)]}{[X_r(\omega)]^2 + [X_j(\omega)]^2}\right]$$

$$- j\left[\frac{[Y_r(\omega)\,X_j(\omega) - Y_j(\omega)\,X_r(\omega)]}{[X_r(\omega)]^2 + [X_j(\omega)]^2}\right]$$

$$= H_r(\omega) + j\,H_j(\omega) \tag{85}$$

$$H_r(\omega) = \frac{Y_r(\omega)\,X_j(\omega) + Y_j(\omega)\,X_r(\omega)}{[X_r(\omega)]^2 + [X_j(\omega)]^2}$$

and

$$H_j(\omega) = \frac{Y_j(\omega)\,X_r(\omega) - Y_r(\omega)\,X_j(\omega)}{[X_r(\omega)]^2 + [X_j(\omega)]^2} \tag{86}$$

Each term on the right hand side of Eqs. 85 and 86 can be evaluated by using Eq. 23 and consequently h(t) can be evaluated from Eq. 84. The IUH so obtained can be used in Eq. 7 to regenerate the outflow hydrograph. Typical results of regeneration in which the above described method is used are presented in Fig. 35.

Evaluation of the Methods of Analysis

The regeneration performance of five linear models, four of which are conceptual models and the fifth, which is the Fourier transform method of evaluating the kernel function, have been tested. For some of the conceptual models the parameters were estimated by optimizing

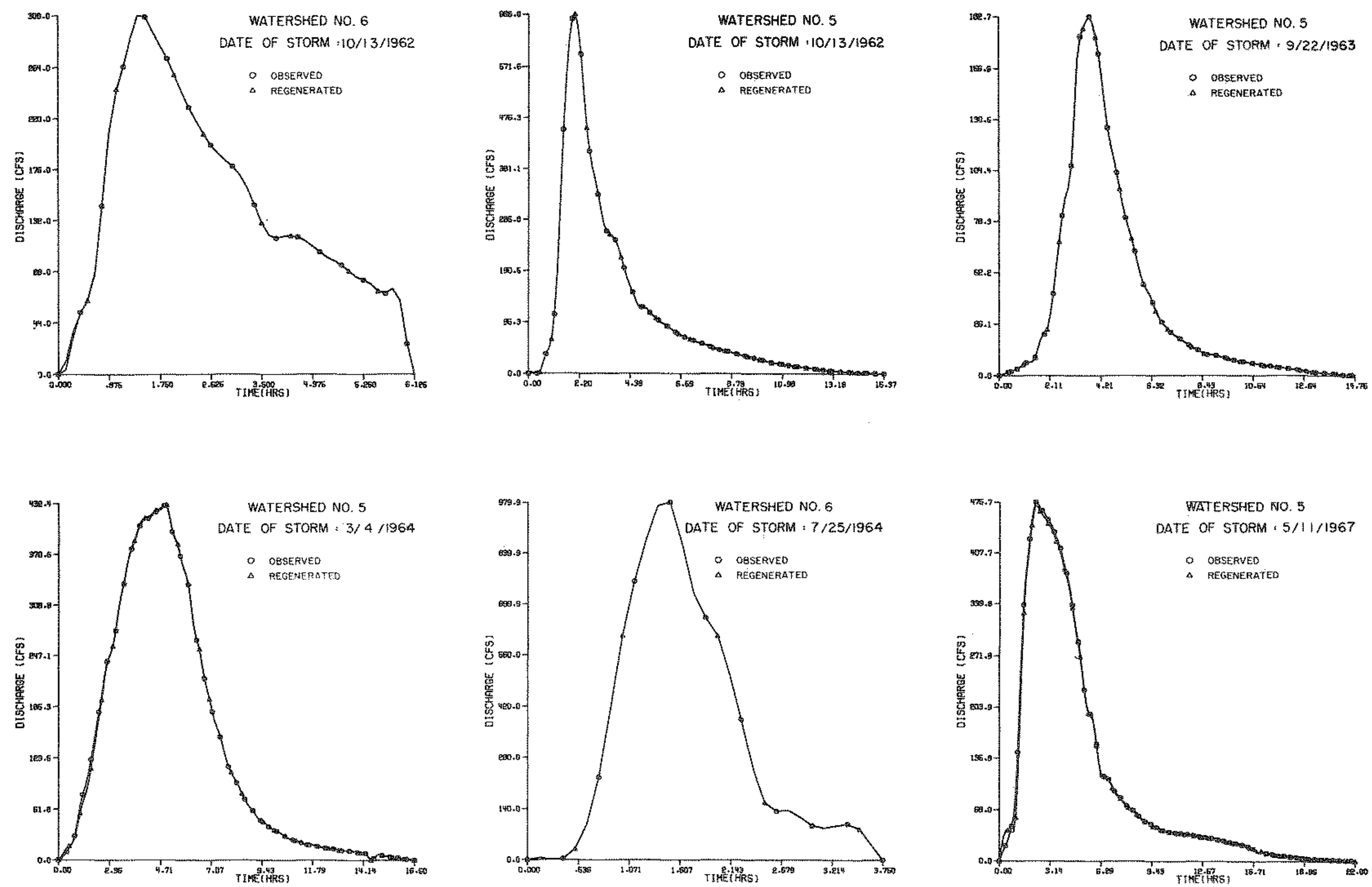


FIGURE 35. TYPICAL RESULTS OF REGENERATION OBTAINED BY USING THE FOURIER TRANSFORM METHOD

them according to different criteria, and the optimized parameters were used in regeneration. A summary of the analyses conducted is presented in Table 12. The computer programs used for the analyses are given in Appendix B of the thesis. The statistical parameters which are defined by Eqs. 45, 46, and 47 were computed in order to obtain a quantitative measure of the regeneration performance. Based on the ratings assigned to these statistical parameters (Appendix A-1), the regeneration performance of the various methods were classified and typical results for some of the watersheds are shown in Table 13. Table 13 should be interpreted as indicated below. The rows in Table 13 refer to the ratings mentioned in Appendix A-1. The columns refer to the different models, and to the different methods used for regeneration (Table 12), and also to the statistical measures used to compare the results of regeneration. The numbers in Table 13 refer to the number of storms which satisfy the criterion to qualify for ratings such as E., V.G., etc., which are based on statistical measures such as the linear correlation coefficient R, and so on. For example, for watershed No. 1 when the single linear reservoir model (method 1) was used for regeneration, when the criterion used for comparison between the observed and regenerated hydrographs was R, seven storms deserved the rating E, whereas thirteen storms deserved the same rating E, when the criterion used for comparison was ISE. The coefficient of linear correlation R between the observed and regenerated hydrographs was greater than 0.99 for seven storms whereas the ISE was less than 3% for thirteen storms. The total number of storms satisfying a given rating or a rating superior to it can be obtained by summing up the number of storms above the particular

Table 12. Summary of Methods of Analysis

No.	Model	Method	Criterion
1	Single Linear Reservoir Model	1	$K = T_4$. Time Lag Value computed from excess rainfall and direct runoff data is used as the storage coefficient K.
		2	$K = K_1$, where K_1 is the storage coefficient which yields the minimum value of the quantity $\sum_{i=1}^{N} (Q_O(i) - Q_C(i))^2$.
		3	$K = K_2$, where K_2 is the storage coefficient which yields the minimum value of $\sum_{i=1}^{n} \left[\left(\frac{Q_{po} - Q_{pc}}{Q_{po}} \right)^2 + \left(\frac{T_{po} - T_{pc}}{T_{po}} \right)^2 \right]$
2	Nash Model	1	Values of parameters n and K_N were estimated by using the equations: $M_1 = n\,K_N = M_{1Q} - M_{1I}$ $M_2 = n(n+1)K_N^2 = M_{2Q} - M_{2I} - 2n\,K_N\,M_{1I}$
3	Double Routing Method	1	$K_D = K_{D1}$. K_{D1} is half of the recession constant obtained by using the equation, $\ln \frac{Q_1}{Q_2} = \frac{t_2 - t_1}{K}$ in the region closest to the peak of the observed direct runoff hydrograph.

Table 12 (Continued)

		2	$K_D = K_{D2}$ where K_{D2} is half of the recession constant obtained by using the equation, $\ln \frac{Q_1}{Q_2} = \frac{t_1-t_2}{K}$ in the region closest to the end of observed direct runoff hydrograph.
		3	$K_D = K_{D3}$ where K_{D3} is half of the recession constant obtained by using the equation, $\ln \frac{Q_1}{Q_2} = \frac{t_1-t_2}{K}$ in the region midway between the peak and the end of the observed direct runoff hydrograph.
		4	$K_D = K_{D4}$ where K_{D4} is the arithmetic average of K_{D1}, K_{D2} and K_{D3} obtained as indicated above. $K_{D4} = \frac{1}{3} (K_{D1} + K_{D2} + K_{D4})$
		5	$K_D = T_4/2$. Storage coefficient K_D is assumed as half of the time lag T_4, where $T_4 = T_Q-T_I$ (Eq. 56).
4-a	Linear-Channel Linear-Reservoir Model	1	Time-area-concentration diagram is assumed to be rectangular in shape.
		2	Time-area-concentration diagram is assumed to be equilateral triangle in shape.

Table 12 (Continued)

		3	Time-area-concentration diagram is assumed to be a triangle with apex at the left end of the base of the triangle.
		4	Time-area-concentration diagram is assumed to be a triangle with apex at the right end of the base of the triangle.
4-b		1	Travel time T_c is optimized $(K = T_2)$ to give the minimum value of the quantity $\sum_{i=1}^{N} (Q_O(i)-Q_C(i))^2$; for method 1, above.
5	Fourier Transform Method	1	The response function (IUH) is obtained by using Fourier Transforms of excess rainfall and direct runoff.

Table 13. Evaluation of Various Methods of Analysis

Watershed No.	Rating**	Single Linear Reservoir Model						Double Routing Model			Nash Model			Linear Channel-Reservoir Model			Fourier Transform Method		
	*	Method 1			Method 2			Method 5						Method 2					
		R	R_s	ISE	R	R_s	ISE	R	R_s	ISE	R	R_s	ISE	R	R_s	ISE	R	R_s	ISE
1	E	7	9	13	6	9	10	3	2	5	7	6	8	5	5	7	14	14	14
	V.G.	7	6	2	8	6	5	6	6	7	4	4	2	5	5	3	0	0	1
	G	1	1	1	1	0	1	3	4	4	1	0	3	4	2	2	1	1	0
	F	1	0	0	1	1	0	3	3	0	0	0	3	0	0	3	0	0	0
	P	0	0	0	0	0	0	1	1	0	4	6	0	2	4	1	1	1	1
2	E	1	2	6	1	1	5	1	0	1	1	0	2	1	0	1	9	9	10
	V.G.	3	3	3	3	4	2	1	2	2	2	2	1	2	3	3	2	2	1
	G	2	2	2	2	2	3	2	1	3	1	4	4	2	3	3	0	0	0
	F	1	1	0	1	1	1	5	4	4	4	4	3	4	4	4	0	0	0
	P	4	3	0	4	3	0	2	4	1	3	1	1	2	1	0	0	0	0

*For explanation of the methods, see Table 12.

**For explanation of ratings E, V.G., G, F, and P, see Appendix A-1

R: linear correlation coefficient (Eq. 45)

R_s: special correlation coefficient (Eq. 46)

ISE: integral square error (Eq. 47)

Table 13 (Continued)

	E	0	0	0	0	0	0	0	1	4	1	3	4	0	0	2	9	9	10
	V.G.	1	4	7	0	2	4	5	5	5	5	5	5	5	1	3	2	2	1
5	G	2	3	3	1	2	4	2	4	3	2	2	3	1	3	2	0	0	0
	F	2	1	2	2	2	4	3	2	0	2	1	0	1	1	0	0	0	1
	P	7	4	0	9	6	0	0	0	0	2	1	0	0	2	0	1	1	0
	E	0	0	0	0	0	1	0	1	2	2	3	3	1	1	2	11	11	11
	V.G.	2	2	3	1	2	2	2	2	3	2	2	4	5	4	5	1	1	1
6	G	0	4	5	1	0	4	3	3	4	4	4	3	2	4	4	0	0	0
	F	3	2	3	1	2	5	1	3	3	1	0	2	1	1	1	0	0	0
	P	7	4	0	9	8	0	6	0	0	3	3	0	3	2	0	0	0	0
	E	0	0	0	0	0	0	0	0	3	0	0	5	0	0	1	8	8	9
	V.G.	0	1	4	0	0	3	3	3	3	5	5	0	5	4	3	1	1	0
7	G	2	2	1	2	3	3	0	3	2	1	1	2	0	1	4	0	0	0
	F	1	1	4	0	1	3	2	0	1	0	0	1	4	1	1	0	0	0
	P	6	5	0	7	5	0	4	3	0	3	3	1	0	3	0	0	0	0

Table 13 (Continued)

11	E	0	1	1	0	0	0	1	1	0	1	1	1	0	0	0	15	15	15
	V.G.	2	3	2	2	2	2	5	6	7	3	7	6	6	8	8	0	0	0
	G	7	7	8	2	6	7	4	6	4	7	5	6	5	5	5	2	2	2
	F	2	2	6	7	2	8	4	3	6	1	1	4	2	2	3	0	0	0
	P	6	4	0	6	7	0	3	1	0	5	3	0	4	4	1	0	0	0
12	E	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	11	11	11
	V.G.	3	3	3	3	2	1	1	2	1	3	4	3	2	4	3	2	2	1
	G	3	4	3	1	3	4	7	7	7	4	4	3	7	8	5	0	1	1
	F	2	3	8	1	1	8	4	3	6	0	0	7	2	0	6	1	0	1
	P	5	3	0	8	7	1	2	2	0	7	6	1	3	2	0	0	0	0
13	E	2	2	3	2	3	3	2	2	2	4	4	6	0	3	3	5	5	5
	V.G.	2	2	2	2	1	1	4	4	5	2	2	0	4	2	3	2	2	2
	G	1	2	2	1	1	2	1	2	1	1	1	1	2	2	2	0	0	0
	F	1	0	1	0	0	2	1	0	0	0	0	1	2	1	0	1	1	1
	P	2	2	0	3	3	0	0	0	0	1	1	0	0	0	0	0	0	0

rating. For example, for the Nash model used in watershed No. 2, the total number of storms satisfying the criterion F or above was 8 when the statistical parameter used for comparison was R, whereas it was 10 when the statistical measures R_s and ISE were used, which means that there were 8 storms for which the R was greater than 0.85 whereas there were 10 storms for which the ISE was less than 25%. From the results presented in Table 13 the following general observations can be made:

1) For most of the data tested, the Fourier transform method has obviously the best regeneration performance. 2) The single linear reservoir model, in general, has a satisfactory regeneration performance for small watersheds such as the West Lafayette watersheds. 3) For other watersheds, the Nash model has a relatively better regeneration performance in comparison with the other models.

The Fourier transform method is perhaps the most general method of obtaining the response function (IUH) of linear systems, whereas use of conceptual models to represent linear systems is neither unique nor general. As confirmed by regeneration, the Fourier transform method yields the best results. However, for the data tested, the IUH obtained by the Fourier transform method exhibited high frequency oscillations. Similar behavior has been reported originally by Blank and Delleur.[32] The oscillatory kernel function (IUH) may result from errors in hydrologic data.[32] Attempts to relate the magnitude of the peak, the time to peak, of the IUH to either the rainfall characteristics or the physiographic characteristics of the watershed were not successful. Due to the inability to obtain such relationships which are necessary for prediction, attention was concentrated mainly on conceptual models.

Another interesting fact that emerged from the study of the response functions obtained by the Fourier transform method was that for small watersheds, they were of the exponential decay type. The instantaneous unit hydrographs which result from using a single linear reservoir model, are, by definition, of the exponential type also (Eq. 16). This qualitative concurrence of the response functions obtained by the general method and by using the single linear reservoir model substantiates the use of the single linear reservoir model in the analysis of small watershed data. Some of the typical instantaneous unit hydrographs computed by using Ross Ade upper watershed data by the above mentioned methods are shown in Fig. 36. Consequently for the following study of effects of urbanization on runoff from small watersheds (less than 5 sq. miles), the single linear reservoir model was selected for analysis.

For larger watersheds, the choice of a particular model which can be used in further analysis was not so easy. By an examination of the regeneration performance it became obvious that, if at all possible, use of response functions obtained by the Fourier transform method would be the best choice. However, due to the previously mentioned difficulties experienced in relating the characteristics of the response function, the next best choice, the Nash model, was used in the further analysis.

Generally, the parameters of the Nash model or any other conceptual model, may be estimated by using the response functions obtained by a general method such as the Fourier transform method. This approach might lead to rather involved computations if the conceptual

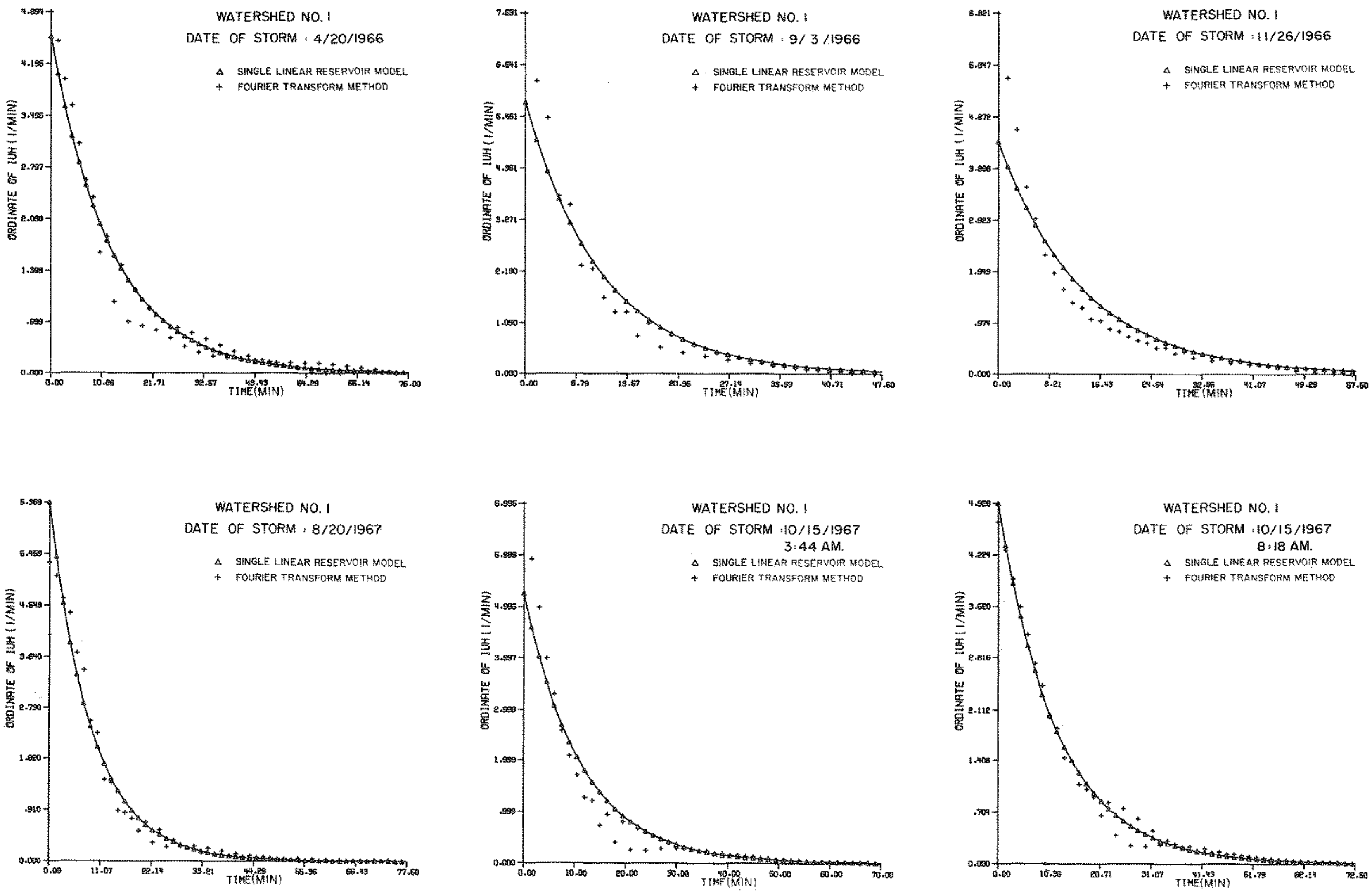


FIGURE 36. COMPARISON BETWEEN INSTATANEOUS UNIT HYDROGRAPHS OBTAINED BY THE FOURIER TRANSFORM METHOD WITH THOSE OBTAINED BY SINGLE LINEAR RESERVOIR MODEL

model were to have a large number of different parameters. However, for the two-parameter Nash model the following simple relationships can be applied to the kernel function, obtained by the Fourier transform method, to evaluate the parameters n and K_N.

$$\text{First moment of kernel function} = n\,K_N \tag{58}$$

$$\text{Second moment of kernel function} = n(n+1)\,K_N^2 \tag{59}$$

An alternative method of evaluating the parameters n and K_N directly from excess rainfall-direct runoff data is by using Eqs. 60 and 61. The latter method was the one which was used for all the analysis reported in the present study. For the data from two watersheds Eqs. 58 and 59 were used to compute n and K_N and these values were compared with those obtained by using Eqs. 60 and 61 (Table 14). Not surprisingly, the values of n and K_N obtained by these two methods agree well.

The regeneration obtained by using Nash model, however, was not as good as the regeneration obtained by the Fourier method. This difference is expected because the runoff regenerated by the Fourier transform method should be equal to the output which was used for the kernel function evaluation within the accuracy of the calculations. That is, the error or difference between the given and regenerated output is due to accumulated errors due to the evaluation of the integrals by finite series and to the truncation of the limits.

The ordinates of IUH obtained by Nash's model on the other hand, are described by a two parameter function which does not agree with every point of the Fourier IUH. However, because of the relatively good performance of the Nash model it was selected for further analysis of data from large watersheds (of area greater than 5 square miles).

Table 14. Computation of Nash Model Parameters

No.	Watershed No.	Date of Storm	Fourier Transform Method			Nash Model				Time Lag
			First Moment of IUH	Second Moment of IUH	Area of IUH In.	Eqs. 58 and 59 n	Eqs. 58 and 59 K_N Hrs.	Eqs. 60 and 61 n	Eqs. 60 and 61 K_N Hrs.	T_4 Hrs.
	5									
1		5/ 1/62	4.60	34.00	0.995	1.65	2.79	1.65	2.78	4.60
2		7/14/62	4.93	57.50	1.286	0.73	6.73	2.24	1.37	3.06
3		4/22/62	3.91	18.87	1.000	4.25	0.92	4.19	0.93	3.91
4		1/ 6/62	6.55	51.92	0.996	4.75	1.38	4.26	1.64	6.97
5		10/13/62	2.11	6.31	0.998	2.40	0.88	2.43	0.88	2.14
6		11/10/62	4.52	37.53	1.028	1.20	3.78	1.38	2.82	3.90
7		11/ 8/66	3.50	18.71	1.003	1.90	1.85	1.92	1.81	3.48
8		5/11/67	4.02	28.90	1.016	1.26	3.18	1.36	2.88	3.91
9		3/ 4/64	3.75	18.88	0.994	2.91	1.29	2.83	1.34	3.77
10		7/12/64	2.69	12.60	1.008	1.35	1.99	1.48	1.75	2.59
	6									
1		10/ 5/67	6.14	46.77	0.999	5.39	1.14	5.35	1.15	6.15
2		1/ 1/66	6.00	54.73	0.979	1.93	3.12	2.07	2.79	5.77
3		9/20/66	2.66	12.82	0.999	1.23	2.16	1.23	2.16	2.66
4		5/22/65	4.14	30.25	1.000	1.30	3.18	1.30	3.18	4.14
5		6/20/64	4.49	41.53	1.381	0.94	4.76	0.82	3.09	2.53
6		7/25/64	1.44	2.43	1.000	5.76	0.25	5.70	0.25	1.44
7		8/11/64	2.82	14.37	0.999	1.24	2.28	1.24	2.27	2.82
8		4/30/62	3.78	22.90	1.003	1.66	2.28	1.79	2.06	3.68
9		10/13/62	3.16	15.54	1.000	1.80	1.76	1.79	1.76	3.16
10		6/ 4/63	2.93	15.56	1.000	1.23	2.38	1.23	2.38	2.93
11		7/20/63	2.48	14.02	1.006	0.78	3.17	0.76	3.27	2.50

CHAPTER V

RUNOFF PREDICTION BY USING THE SELECTED CONCEPTUAL MODELS

General

The single linear reservoir model and the Nash model were selected to simulate the rainfall-runoff process in urbanized watersheds. The selection of these models was based on their relatively superior regeneration performance and the convenience with which the parameters of these models can be related to the physiographic characteristics of the watershed and to the storm characteristics. These conceptual models are also used to investigate the effects of urbanization on runoff. In order to determine the time distribution of runoff due to future rainfall events, the parameters of these models must either be known or they must be estimated by using some known information. If these parameters, which are determined from 'historic' or observed data of rainfall and runoff, are essentially constant for a watershed, then they can be readily used to estimate the runoff due to future storm events. However, the values of the parameters of these models vary for different storms on a watershed (Ch. IV). The variation in the values of the parameters of the model may be due to any one or to a combination of the following factors: 1) inappropriate choice of the conceptual model itself, 2) the rainfall-runoff process may be a time varying phenomenon whereas the models selected are time invariant models, and 3) the relationship between the time lag T_4 and these variables was investigated. The physiographic characteristics which were considered in the study are, the area of the watershed, A, (in sq. miles), the length of the main stream, L, (in miles), the mean basin slope, $\overline{S}$, expressed as feet per mile, and the percentage of impervious area in the watershed, U. The storm characteristics which were considered are the volume of excess rainfall, P_E, and the duration of excess rainfall, T_R. Other factors such as the moisture content in the watershed before the occurrence of the storm, could not be considered because reliable quantitative data were not available. Factors such as aerial distribution of rainfall, distribution of impervious areas in the watershed were not considered because of the assumption of lumped linear system models. Thus the time lag T_4 was expressed in the following form:

$$T_4 = f_1 \ (A, L, \overline{S}, U, P_E, T_R) \tag{87}$$

The variable U, which represents the percentage of impervious area in the watershed, was transformed to a new variable (1+U) and was designated as "urbanization factor". In defining the urbanization factor, U is used as the ratio of impervious area to total area expressed in decimals. The urbanization factor has a range from 1.0 to 2.0 where the values 1.0 and 2.0 correspond respectively to rural and impervious watersheds, with some intermediate value for most of the urban watersheds. The urbanization factor is more suitable for regression analysis especially when logarithmic transformation of variables is involved. Also, a single power function relationship for T_4 in which the urbanization factor is included, can be used for both urban as well as rural watersheds. When the variable U is replaced by the new variable (1+U), Eq. 87 takes the form

$$T_4 = f_2 \ (A, L, \overline{S}, (1+U), P_E, T_R) \tag{88}$$

It was recognized that the time lag varies with the storm characteristics. However; previous investigators[45,50] used the average value of T_4 as the representative value of the time lag for a watershed for regeneration. If the average value $\overline{T}_4$ is satisfactory for prediction, in spite of the variation of time lag, it is very convenient to use such a constant value for prediction. The following preliminary study was conducted to investigate the accuracy of such prediction as no previous study yielded the necessary information.

The values of T_4 for all the storms on the 13 watersheds (Table 2) were computed by using the data of excess rainfall and of direct runoff. The arithmetic average of the time lag values obtained for each watershed was taken as the average time lag $\overline{T}_4$ for that watershed. In addition, published values of the average time lag and of the physiographic characteristics of fourteen more watersheds which are of comparable size (less than 20 sq. miles), were also used in this analysis to obtain a higher degree of freedom for the regression analysis. Values of the average time lag and the physiographic characteristics used in this analysis are presented in Table 15.

The graphical correlation method was adopted to investigate the dependence of $\overline{T}_4$ on the physiographic characteristics of the watershed. It was observed that the variation in slope $\overline{S}$ did not significantly influence the value of $\overline{T}_4$. Further, the variables A and L, as reported by several previous investigators,[66,92] are strongly correlated. This fact is valid for the present data also (Fig. 37), in which A and L are

TABLE 15. DATA USED IN THE AVERAGE TIME LAG INVESTIGATION

Watershed No.	Name of Watershed	Area of Watershed A (Sq. Miles)	Length of Main Stream L (Miles)	Mean Slope $\overline{S}$ (Ft/Mile)	Urbanization Factor (1+U)	Average Time Lag $\overline{T}_4$ (Hrs)	Source of Data
1	Ross Ade (upper), Lafayette, Ind.	.0455	.6613	112.0	1.380	0.210	
2	Ross Ade (lower), Lafayette, Ind.	.6125	2.177	112.0	1.374	0.610	
3	Purdue Swine Farm (upper), Lafayette, Ind.	0.278	0.644	3.900	1.213	0.300	
4	Purdue Swine Farm (lower), Lafayette, Ind.	.4562	1.070	3.400	1.133	0.400	Computed
5	Pleasant Run - Arlington, Indianapolis, Ind.	7.580	3.822	12.67	1.105	3.570	
6	Pleasant Run - Brookville, Indianapolis, Ind.	10.10	5.644	14.26	1.155	3.720	
7	Little Eagle Creek, Indianapolis, Ind.	19.31	11.10	23.23	1.021	9.340	as part of
8	Lawrence Creek, Indianapolis, Ind.	2.860	1.705	32.21	1.000	2.050	
9	Bear Creek, Near Trevlac, Ind.	7.090	3.864	39.07	1.000	5.440	the present
10	Bean Blossom Creek, Near Bean Blossom, Ind.	14.60	6.440	32.74	1.000	6.560	
11	Waller Creek - 38th St., Austin, Texas.	2.310	4.370	47.00	1.270	1.440	
12	Waller Creek - 23rd St., Austin, Texas.	4.130	5.230	47.00	1.370	1.580	study
13	Wilbarger Creek, Near Austin, Texas.	4.610	3.167	45.94	1.000	2.890	
14	Boneyard Creek, Champaign-Urbana, Illinois	4.450	2.840	9.504	1.370	1.152	Ref. No. 78
15	17th St., N. W. Parkway, Louisville, Ky.	0.220	0.928	20.06	1.830	0.380	
16	N. W. Trunk at Schawne Park, Louisville, Ky.	1.900	3.030	6.340	1.500	0.600	
17	W. Outfall near W. Parkway, Louisville, Ky.	2.770	4.170	4.700	1.700	0.833	
18	S. Outfall near State Fair Grounds, Louisville, Ky.	6.430	6.440	7.230	1.480	1.100	
19	S. W. Outfall, Cane Road, Louisville, Ky.	7.520	6.420	7.760	1.330	0.830	
20	Piney Branch at Vienna, Va.	0.290	0.500	86.50	1.300	0.180	Ref. No. 62
21	Little Pimmit Run tributary at Arlington, Va.	0.410	0.900	98.50	1.280	0.320	
22	Long Branch at Arlington, Va.	0.940	2.100	81.20	1.300	0.500	
23	Tripps Run tributary near Falls Church, Va.	0.500	1.100	102.0	1.250	0.320	
24	Holmes Run at Alexandria, Va.	18.90	10.70	31.30	1.120	5.800	
25	Back Lick Run at Springfield, Va.	2.020	2.300	50.30	1.150	1.100	
26	Turkey-Cock Run at Alexandria, Va.	2.260	2.800	78.20	1.080	1.300	
27	Freeman Airfield, Seymour, Ind.	.0128	.1051	30.62	2.000	0.100	Ref. No. 66

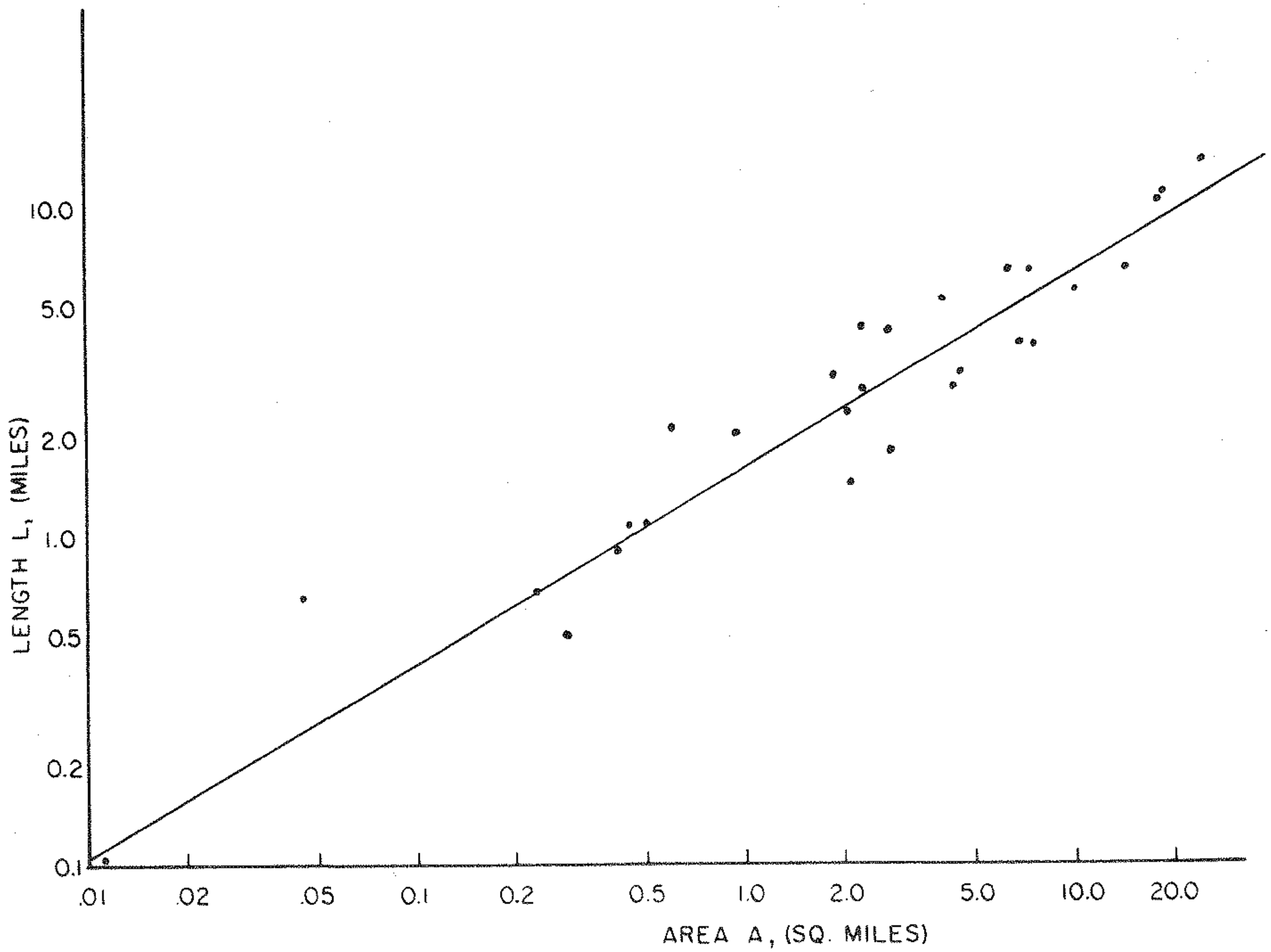


FIGURE 37. RELATIONSHIP OF WATERSHED AREAS WITH STREAM LENGTHS

related with a correlation coefficient (R) of 0.950. Hence, the average time lag can be expressed either as a function of the variables A and (1+U) or the variables L and (1+U). As the area A is easier to determine than the length L, the average time lag was related to the variables A and (1+U). The graphical correlation for $\overline{T}_4$ with the variables A and (1+U) is presented in Fig. 38. The linear correlation coefficient (R) between the observed and the estimated values of $\overline{T}_4$, obtained by using Fig. 38, is 0.983.

Analytical relationships for $\overline{T}_4$ as a function of physiographic characteristics were also determined by using the multiple regression analysis. (A brief description of the technique is presented in Appendix A-2.) An equation of the form

$$\overline{T}_4 = C_0 \; A^{C_1} \; L^{C_2} \; \overline{S}^{C_3} \; (1+U)^{C_4} \qquad (89)$$

was fitted by the method of multiple regression analysis to the values presented in Table 15. The results are presented in Table 16. Equations 90, 91 and 92 represent the expressions for $\overline{T}_4$ obtained by regression analysis with gradual deletion of the variables that were not significant at the 0.01% level. The linear correlation coefficient R, the standard error of estimate and the coefficient of determination (R^2) associated with each of the Eqs. 90, 91 and 92 are presented in Table 16.

It may be noted that for the data tested, inclusion of the stream length L and the slope $\overline{S}$ does not significantly improve the correlation. As the area A and the stream length L are by themselves highly correlated, inclusion of both A and L does not add any significant new information to equation 89. Explanation for the ineffectiveness of $\overline{S}$

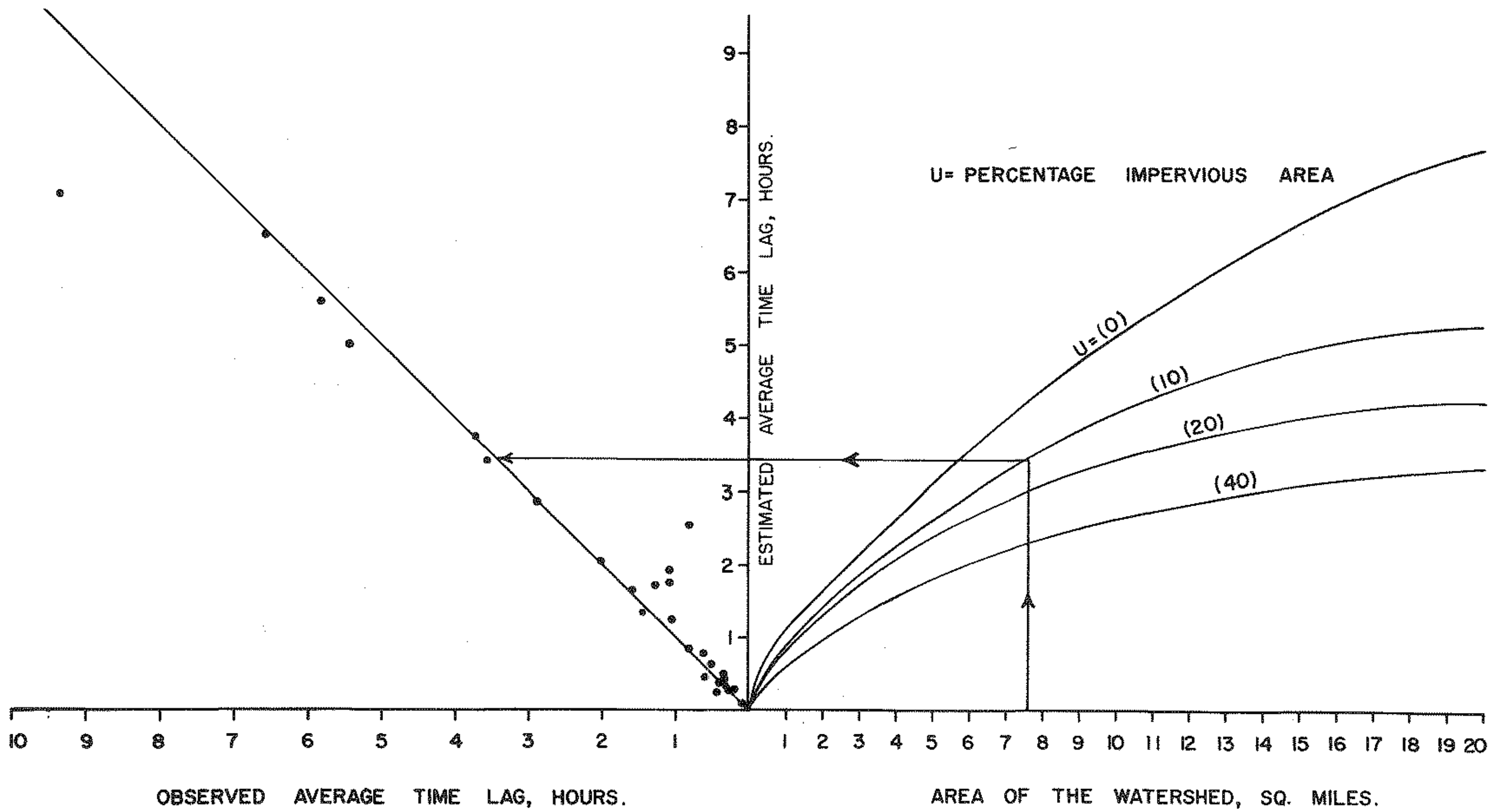


FIGURE 38. GRAPHICAL CORRELATION OF $\overline{T}_4$ WITH AREA AND URBANIZATION FACTOR

Table 16. Results of Multiple Correlation Between $\overline{T}_4$ and A, (1+U), L, $\overline{S}$

Equation *	Equation Number	Correlation Coefficient R	Standard Error of Estimate (Hrs)	Coefficient of Determination (R^2)
$\overline{T}_4 = 0.78\ A^{0.496}\ L^{0.073}\ \overline{S}^{(0.075)}\ (1+U)^{-1.289}$	(90)	0.930	0.481	0.864
$\overline{T}_4 = 0.780\ A^{0.542}\ \overline{S}^{(-.081)}\ (1+U)^{-1.210}$	(91)	0.929	0.471	0.864
$\overline{T}_4 = 0.803\ A^{0.512}\ (1+U)^{-1.433}$	(92)	0.927	0.469	0.859

* All numbers were rounded off to the third decimal place.

in estimating $\overline{T}_4$ is not so straightforward. It is generally thought that the mean basin slope $\overline{S}$, is one of the significant factors which influence the velocity of flow and hence the discharge in the stream, thus influencing the time lag. This is particularly the case for rural watersheds as reported by several investigators[60,61,64] in their studies of dependence of average time lag on $\overline{S}$. However, Wu[93] reported that for small rural watersheds in Hawaii with areas less than 20 sq. miles, the average values of the time lag $\overline{T}_2$ are a function of area A only and the mean basin slope $\overline{S}$ does not influence $\overline{T}_2$ significantly. Wu's analysis was conducted by using data from small, rural watersheds in Hawaii, where the watersheds are generally steep and infiltration rates are high and these characteristics might have influenced the watershed response. For urban watersheds, on the other hand, because of the channel improvement and storm sewer constructions, the direct influence of the mean basin slope $\overline{S}$ on $\overline{T}_4$ is considerably reduced. In sewered areas, the slope of the sewer lines will be kept constant and the irregularities in the terrain will be negotiated by providing drops at manholes. The actual velocity in the sewer is not affected by the slope of the terrain. The velocity of flow in the sewers varies within narrow limits and can be considered as constant. Hence it can be concluded that the value of $\overline{T}_4$ is mainly a function of area A of the watershed or length of the stream L, which are closely related, and of the urbanization factor (1+U). Consequently, the average value of time lag $\overline{T}_4$, can be estimated by using Eq. 92 or Fig. 38. As mentioned earlier, Eq. 92 and Fig. 38 can be used for both rural as well as urban watersheds.

When the parameter K of the single linear reservoir model was assumed to be the average value of $\bar{T}_4$, the prediction obtained proved unsatisfactory. In order to illustrate this point, the results of prediction for six storms on the Ross Ade upper watershed, obtained by assuming the average value $\bar{T}_4$ as the storage coefficient K, are shown in Table 17 and Fig. 39. The observed hydrograph and the hydrograph regenerated by using the value of T_4 as the storage coefficient K, for the particular storm, are also presented in Table 17 and Fig. 39 for comparison. Thus, although the value of $\bar{T}_4$ can be estimated with considerable accuracy by using Fig. 38 or Eq. 92, it cannot be used as a representative value of time lag for all the storms of the watershed for predicting the runoff.

As the prediction of runoff by using $\bar{T}_4$ was very unsatisfactory, it was proposed to estimate the values of T_4 for individual storms as a function of the physiographic characteristics of the watershed, and the storm characteristics. In the preliminary analysis mentioned earlier, it was observed that the variable $\bar{S}$ was not significant in influencing the value of $\bar{T}_4$. Hence the variable $\bar{S}$ was not considered for further analysis. Although there is a strong correlation between the variables A and L, both the following two relationships were separately investigated:

$$T_4 = f_3\ (A,\ (1+U),\ P_E,\ T_R) \tag{93}$$

and

$$T_4 = f_4\ (L,\ (1+U),\ P_E,\ T_R) \tag{94}$$

Because the data of rainfall and runoff for some of the watersheds were not readily available, the regression analysis for Eqs. 93 and 94 was limited to the data from eleven watersheds (mentioned in Table 2) only.

TABLE 17. TYPICAL RESULTS OF PREDICTIONS OBTAINED USING THE AVERAGE TIME LAG (T_4)

	OBSERVED	REGENERATED	PREDICTED
APRIL 13 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	.27	.28	.16
TIME TO PEAK	6.50	6.50	7.00
STORAGE COEFFT.	5.44	5.44	12.60
SIG.ERROR SQ.		.01	.14
INTEG.SQ.ERROR.		1.89	7.06
CORR.COEFF.		.99	.93
SPE.COR.COEFF.		.99	.92
APRIL 20 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	1.07	.80	.82
TIME TO PEAK	8.00	9.00	9.00
STORAGE COEFFT.	12.94	12.94	12.60
SIG.ERROR SQ.		.82	.79
INTEG.SQ.ERROR.		2.67	2.61
CORR.COEFF.		.97	.97
SPE.COR.COEFF.		.98	.98
MAY 12 , 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	.22	.28	.26
TIME TO PEAK	9.50	9.00	9.50
STORAGE COEFFT.	11.37	11.37	12.60
SIG.ERROR SQ.		.10	.06
INTEG.SQ.ERROR.		3.12	2.52
CORR.COEFF.		.94	.95
SPE.COR.COEFF.		.97	.98
MAY 21 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	.89	.83	1.03
TIME TO PEAK	22.50	19.00	18.00
STORAGE COEFFT.	20.07	20.07	12.60
SIG.ERROR SQ.		2.73	9.26
INTEG.SQ.ERROR.		2.86	5.26
CORR.COEFF.		.89	.76
SPE.COR.COEFF.		.96	.85
JULY 5 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	14.73	10.93	8.02
TIME TO PEAK	9.50	7.50	9.50
STORAGE COEFFT.	7.45	7.45	12.60
SIG.ERROR SQ.		181.50	493.30
INTEG.SQ.ERROR.		4.09	6.74
CORR.COEFF.		.96	.93
SPE.COR.COEFF.		.97	.92
JULY 28 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	12.85	14.62	20.53
TIME TO PEAK	22.50	13.00	10.50
STORAGE COEFFT.	21.00	21.00	12.60
SIG.ERROR SQ.		761.25	3193.84
INTEG.SQ.ERROR.		2.76	5.65
CORR.COEFF.		.92	.81
SPE.COR.COEFF.		.96	.81

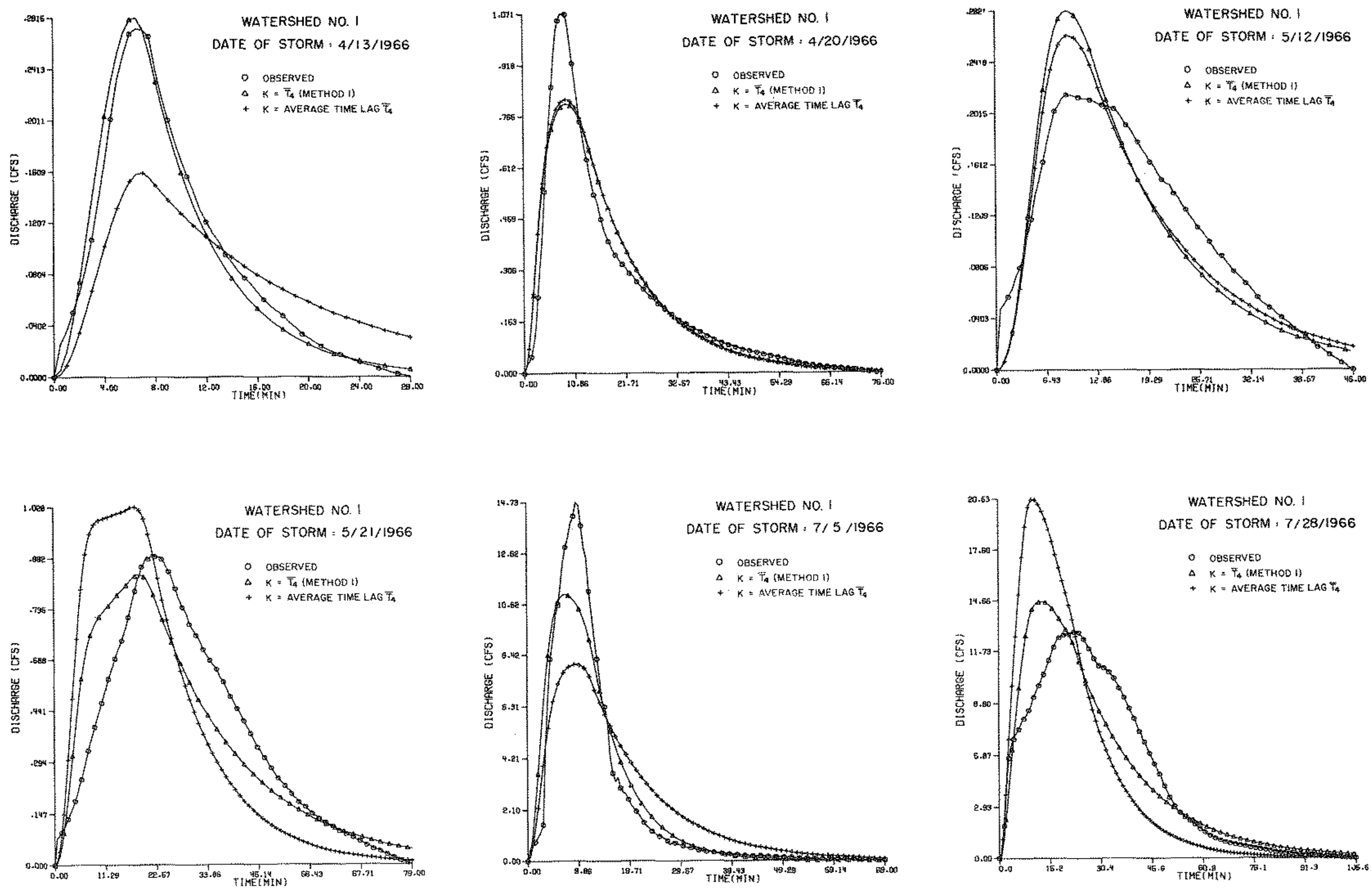


FIGURE 39. TYPICAL RESULTS OF PREDICTION OBTAINED BY USING $\overline{T}_4$

TABLE 18. DATA USED FOR ESTIMATION OF T_4, K_1, K_2, K_N, Q_P, AND T_P

	AREA(SQ.M)	U(%)	R_E(INCHES)	T_R(HOURS)	K_N(HOURS)	Q_p(cfs)	T_p(HOURS)	K_1(HOURS)	T_4(HOURS)
WATERSHED	.046	38.000	.012	.167	.152	1.360	.142	.192	.187
	.046	38.000	.009	.250	.127	.678	.233	.292	.250
No.1	.046	38.000	.015	.242	.206	1.070	.133	.192	.216
	.046	38.000	.006	.150	.138	.414	.183	.300	.225
	.046	38.000	.093	.210	.119	14.730	.158	.117	.124
	.046	38.000	.036	.250	.156	2.960	.217	.192	.169
	.046	38.000	.179	.210	.169	20.040	.067	.183	.171
	.046	38.000	.006	.250	.165	.387	.250	.317	.274
	.046	38.000	.029	.334	.165	1.897	.308	.250	.219
	.046	38.000	.013	.375	.177	.951	.317	.225	.207
	.046	38.000	.025	.100	.215	2.641	.133	.203	.209
	.046	38.000	.006	.167	.156	.507	.100	.250	.226
	.046	38.000	.013	.192	.175	1.980	.125	.158	.187
	.046	38.000	.010	.192	.226	1.394	.100	.142	.184
	.046	38.000	.162	.334	.146	16.700	.125	.092	.108
	.046	38.000	.011	.142	.162	.808	.183	.317	.278
	.046	38.000	.050	.292	.161	6.430	.200	.117	.153
	.046	38.000	.074	.217	.186	9.270	.092	.142	.133
	.046	38.000	.020	.142	.254	1.655	.092	.242	.243
	.046	38.000	.011	.125	.212	1.470	.083	.158	.187
	.046	38.000	.015	.125	.196	1.818	.092	.192	.199
WATERSHED	.613	37.400	.026	.175	.187	22.610	.350	.576	.434
	.613	37.400	.022	.092	.515	14.730	.175	.514	.528
	.613	37.400	.027	.300	.255	23.920	.258	.297	.311
No.2	.613	37.400	.024	.134	.495	14.000	.250	.641	.598
	.613	37.400	.037	.258	.319	17.260	.525	.707	.581
	.613	37.400	.035	.425	.464	13.930	.550	.998	.836
	.613	37.400	.016	.134	.180	10.190	.533	.892	.602
	.613	37.400	.018	.134	.429	7.860	.441	1.300	.994
	.613	37.400	.022	.117	.531	11.930	.325	.730	.660
	.613	37.400	.024	.092	.273	13.640	.500	.803	.589
WATERSHED	7.580	10.500	.160	2.080	2.933	214.500	2.125	3.380	4.060
	7.580	10.500	.141	1.170	1.866	224.800	2.250	4.680	3.610
	7.580	10.500	.486	1.950	2.991	475.700	2.250	4.620	4.070
No.5	7.580	10.500	.471	2.660	1.387	430.300	4.875	5.070	3.920
	7.580	10.500	.635	.780	1.815	86.440	1.625	3.250	2.690
	7.580	10.500	.253	.780	1.520	462.900	1.875	3.380	2.590
	7.580	10.500	.464	2.210	1.968	517.700	3.250	3.770	3.400
	7.580	10.500	.509	1.690	2.892	453.300	2.875	5.070	4.780
	7.580	10.500	.252	5.590	1.420	348.600	4.750	2.730	3.180
	7.580	10.500	.113	1.690	.975	182.700	3.750	5.070	4.070
	7.580	10.500	.317	3.510	1.701	224.300	7.000	7.250	7.250
	7.580	10.500	.191	1.690	.914	308.000	1.375	2.860	2.220
	7.580	10.500	.414	1.690	5.187	389.600	2.750	5.070	5.770
WATERSHED	10.100	15.500	.073	1.170	1.196	110.500	5.500	5.070	6.400
	10.100	15.500	.147	4.030	2.902	143.100	5.750	5.070	6.000
	10.100	15.500	.115	1.820	2.243	324.100	1.250	2.730	2.770
No.6	10.100	15.500	.035	.910	3.311	122.100	1.500	4.680	4.300
	10.100	15.500	.282	2.470	1.963	662.500	2.375	3.380	3.290
	10.100	15.500	.091	1.690	3.210	330.200	1.500	1.560	2.630
	10.100	15.500	.200	.650	.262	580.000	1.400	2.470	1.490
	10.100	15.500	.251	.780	2.365	604.100	1.250	3.120	2.930
	10.100	15.500	.315	2.210	2.143	470.400	3.875	4.680	3.830
	10.100	15.500	.269	1.690	1.834	667.700	2.000	3.900	3.290
	10.100	15.500	.108	.650	2.477	294.000	1.250	3.250	3.050
	10.100	15.500	.402	2.080	3.402	740.500	1.625	2.080	2.600
WATERSHED	19.310	2.100	.357	6.000	3.507	313.100	18.850	9.750	26.090
	19.310	2.100	.887	4.750	4.903	1114.000	9.250	9.750	8.460
	19.310	2.100	1.114	9.250	3.687	966.600	20.000	9.280	15.060
No.7	19.310	2.100	.764	11.250	10.000	319.200	7.750	7.000	8.690
	19.310	2.100	1.032	8.250	6.582	976.500	13.000	9.750	11.740
	19.310	2.100	.197	8.250	3.175	169.200	16.250	9.750	14.060
	19.310	2.100	.359	6.250	7.166	378.200	7.250	9.750	9.540
	19.310	2.100	.064	1.250	1.178	137.500	14.250	9.750	17.030
WATERSHED	2.860	0.000	.740	2.000	1.253	416.900	2.500	5.000	3.910
	2.860	0.000	.369	1.500	1.411	374.200	1.000	1.750	1.740
	2.860	0.000	.357	1.500	2.383	271.100	1.500	3.000	3.170
No.8	2.860	0.000	2.155	4.000	.326	1343.000	4.500	5.000	3.500
	2.860	0.000	.636	1.750	.889	849.600	1.000	.500	.410
	2.860	0.000	.369	1.500	2.155	356.600	1.500	1.500	1.870
	2.860	0.000	.078	1.250	.124	245.200	.750	.250	.430
	2.860	0.000	.945	2.500	1.453	725.200	2.500	4.000	3.150
	2.860	0.000	.473	2.000	1.590	519.700	2.000	1.500	1.630
	2.860	0.000	.384	1.500	.787	330.200	1.500	1.500	1.580
	2.860	0.000	.118	1.000	.228	267.000	.750	.500	.500
	2.860	0.000	.265	2.000	1.002	317.200	2.000	2.500	2.050
WATERSHED	7.000	0.000	.394	9.000	5.223	177.200	9.500	10.600	10.470
	7.000	0.000	.281	7.500	2.511	255.400	7.500	6.000	6.030
	7.000	0.000	.251	4.000	1.624	195.100	1.750	7.000	5.370
No.9	7.000	0.000	.459	3.500	3.721	271.100	5.500	8.000	6.860
	7.000	0.000	.766	2.500	3.874	430.200	2.500	7.500	6.600
WATERSHED	14.600	0.000	.273	13.500	2.717	361.700	14.500	9.500	9.110
	14.600	0.000	.363	4.500	2.049	622.600	7.500	10.500	7.540
	14.600	0.000	.387	3.000	2.287	748.400	5.500	8.500	6.230
No.10	14.600	0.000	.212	4.000	1.753	413.600	6.600	11.000	7.750
	14.600	0.000	.239	4.000	2.116	760.500	6.000	5.500	4.600
	14.600	0.000	.613	10.000	1.413	725.300	10.000	8.500	7.190
	14.600	0.000	.665	2.500	.681	363.300	2.000	1.250	1.300
WATERSHED	2.310	27.000	.270	1.690	1.333	175.200	1.020	2.660	2.400
	2.310	27.000	.298	1.690	2.170	215.200	1.170	1.690	1.910
	2.310	27.000	.159	1.170	.314	151.200	1.300	2.340	1.500
No.11	2.310	27.000	.174	.390	.897	312.500	.780	1.690	1.310
	2.310	27.000	.470	.910	.661	463.700	1.430	2.470	1.810
	2.310	27.000	.342	1.300	1.035	494.000	1.040	.780	1.040
	2.310	27.000	.269	2.210	1.230	165.700	1.820	1.450	2.030
	2.310	27.000	.190	.910	1.560	235.700	.260	1.040	1.270
	2.310	27.000	.119	1.040	.313	111.500	5.070	5.070	5.550
	2.310	27.000	.079	1.040	.740	76.080	1.630	2.340	1.620
	2.310	27.000	.738	1.170	.434	645.200	1.170	.910	.900
	2.310	27.000	.669	2.080	.390	613.900	1.330	.650	.520
	2.310	27.000	.512	1.170	.745	462.100	1.430	1.430	1.250
	2.310	27.000	.308	2.990	.740	263.300	1.300	1.300	1.175
	2.310	27.000	1.405	3.640	.220	1366.000	2.090	2.340	1.910
	2.310	27.000	.776	2.600	2.012	603.700	1.040	.780	1.030
	2.310	27.000	.759	2.600	.600	781.000	2.470	1.690	1.500
WATERSHED	4.130	37.000	.367	1.690	1.993	636.700	1.600	1.560	1.910
	4.130	37.000	.433	1.300	.599	1296.000	1.000	.910	.990
	4.130	37.000	.200	.910	1.142	505.500	.500	.760	.930
No.12	4.130	37.000	.503	1.820	.453	1059.000	1.500	1.560	1.420
	4.130	37.000	.274	.910	.509	1019.000	.500	.520	.580
	4.130	37.000	.161	1.040	1.256	270.400	1.750	2.080	1.640
	4.130	37.000	1.096	2.080	1.053	2602.000	.275	.260	.360
	4.130	37.000	.201	.780	1.021	468.000	1.250	1.560	1.200
	4.130	37.000	.459	1.170	.913	418.000	.875	1.560	1.360
	4.130	37.000	.452	2.990	.630	1034.000	.750	1.300	1.080
	4.130	37.000	1.450	2.600	1.252	2909.000	.625	.650	.660
	4.130	37.000	.196	1.430	.740	631.300	1.000	.780	.740
WATERSHED	4.610	0.000	.436	1.950	.994	668.700	1.000	1.170	1.110
	4.610	0.000	1.093	1.950	.698	1719.000	1.250	1.300	1.140
	4.610	0.000	.727	1.300	1.301	952.000	1.250	1.430	1.370
No.13	4.610	0.000	1.560	2.860	.573	1697.000	2.125	1.170	1.100
	4.610	0.000	.499	3.640	1.184	462.000	1.250	1.170	1.000
	4.610	0.000	1.120	7.280	1.037	530.600	6.625	4.550	4.560
	4.610	0.000	1.152	5.590	.814	733.400	5.500	5.070	5.040
	4.610	0.000	.374	1.620	1.541	465.800	2.000	3.770	3.000

Values of the variables used in the analysis for a total of 125 storms are presented in Table 18. Equations of the form

$$T_4 = C_0 \; A^{C_1} \; (1+U)^{C_2} \; P_E^{\;C_3} \; T_R^{\;C_4}$$

and

$$T_4 = C_0 \; L^{C_1} \; (1+U)^{C_2} \; P_E^{\;C_3} \; T_R^{\;C_4}$$

for the relationships 93 and 94 respectively, were fitted to the data by the multiple regression analysis the results of which are shown in Table 19.

Table 19. Results of Multiple Correlation Between T_4 and A, (1+U), P_E and T_R

Equation*	Eq.	Corr. Coefft. (R)	Standard Error (Hrs.)	Coefft. of Determination (R^2)
$T_4 = 0.831 \; A^{0.458} \; (1+U)^{-1.66} \; P_E^{\;-0.267} \; T_R^{\;0.371}$	(95)	0.923	0.506	0.851
$T_4 = 0.731 \; L^{0.943} \; (1+U)^{-4.303} \; P_E^{\;-2.114} \; T_R^{\;0.238}$	(96)	0.933	0.473	0.871

* All numbers were rounded off to the third decimal place.

The correlation coefficients for Eqs. 95 and 96 are nearly the same. As area A of the watershed is easier to determine than the stream length L, Eq. 95 is more convenient to use. The graphical solution of Eq. 95 is presented in Fig. 40 and the method of using the diagram is indicated by arrowmarks.

Prediction of Runoff by Using Single Linear Reservoir Model

Although the parameter K of the single linear reservoir model is theoretically the same as the time lag T_4, optimization of the parameter

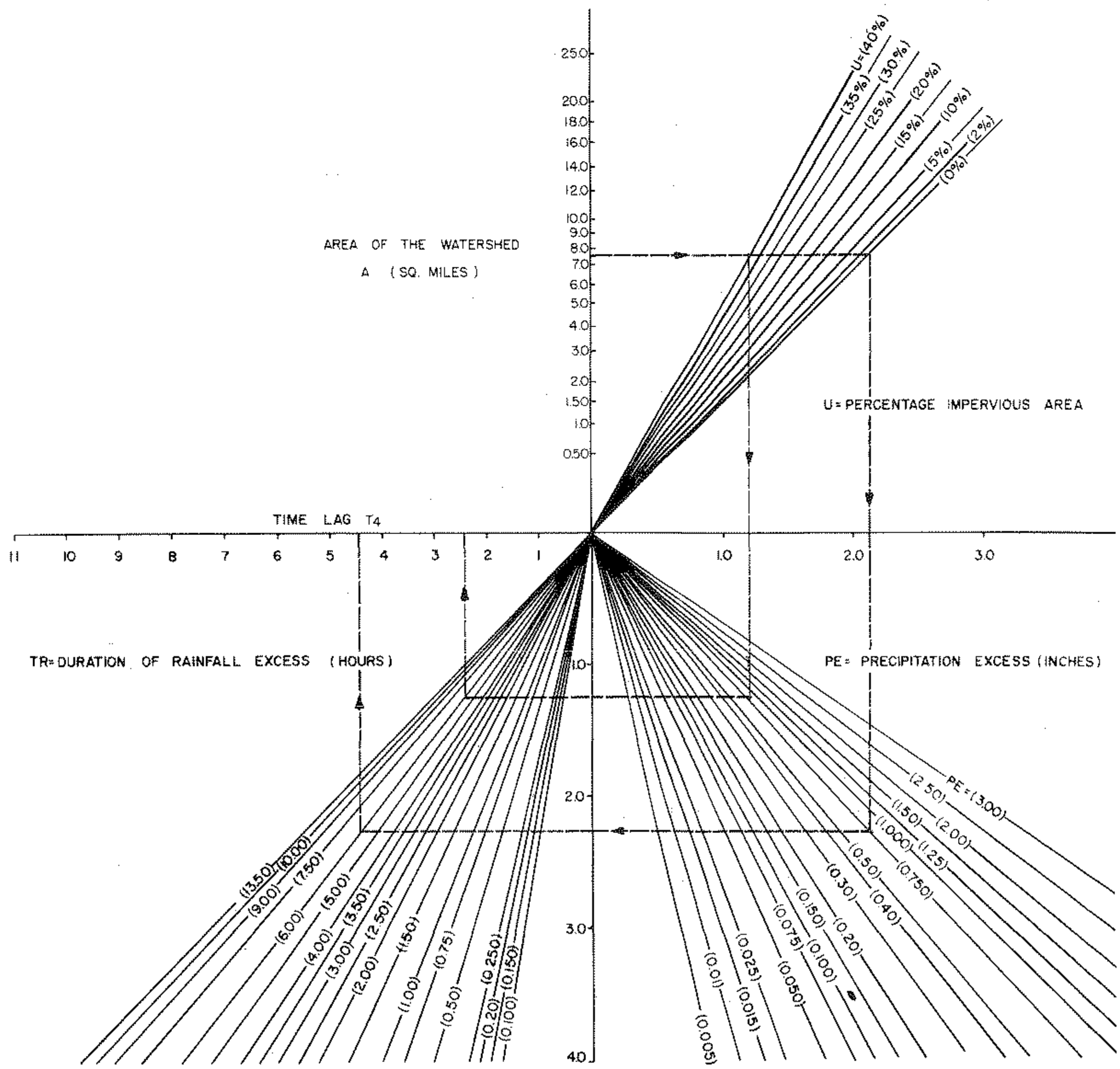


FIGURE 40. GRAPHICAL SOLUTION OF EQ. 95 FOR ESTIMATION OF T_4

K was required to improve regeneration. Consequently, it was decided to estimate the values of K_1 and K_2 and then use them to obtain the corresponding IUH. As the coefficient K is obtained by time lag considerations, and then optimized to obtain K_1 and K_2, the variation in K_1 and K_2 can be analyzed in a manner similar to the analysis of time lag T_4. Consequently, K_1 and K_2 were postulated to be related to the storm characteristics and physiographic characteristics in the following form:

$$K_1 = f_5 \ (A, \ (1+U), \ P_E, \ T_R) \qquad (97)$$

$$K_2 = f_6 \ (A, \ (1+U), \ P_E, \ T_R) \qquad (98)$$

Equations of the form

$$K_1 = C_0 \ A^{C_1} \ (1+U)^{C_2} \ P_E^{\ C_3} \ T_R^{\ C_4}$$

$$K_2 = C_0 \ A^{C_1} \ (1+U)^{C_2} \ P_E^{\ C_3} \ T_R^{\ C_4}$$

for the relationships 97 and 98 respectively were fitted to the data by multiple regression analysis. The resulting equations for the optimum values of the storage coefficients K_1 and K_2 and the associated statistics such as the correlation coefficient etc., are presented in Table 20.

The graphical solution of Eq. 99 is presented in Fig. 41. Thus, the optimum values of the storage coefficient K which are to be used in the single linear reservoir model can be estimated by using Eq. 99 and 100 respectively. The optimized values of the storage coefficients K_1 and K_2 were based respectively on the criterion of minimizing the sum of the squares of the differences of the ordinates of the observed and of the regenerated hydrographs, and on the criterion of minimizing the

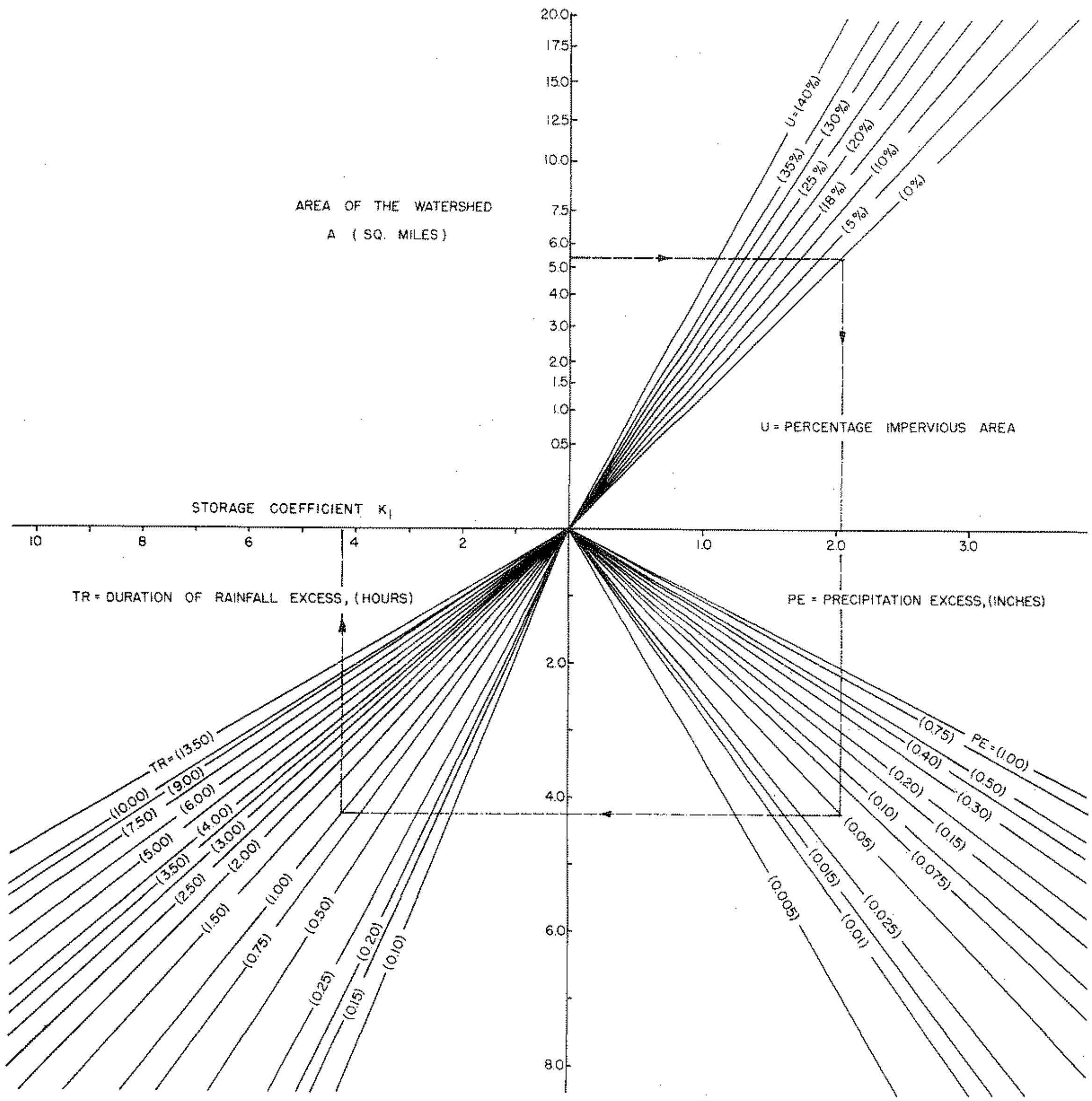


FIGURE 41. GRAPHICAL SOLUTION OF EQ.99 FOR ESTIMATION OF K_1

deviation between the peak discharges of the observed and the regenerated hydrographs.

Table 20. Results of Multiple Correlation Analysis for the Parameters K_1 and K_2

Equation*	Eq.	Corr. Coefft. (R)	Standard Error (Hrs.)	Coefft. of Determination (R^2)
$K_1 = 0.887\ A^{0.490}\ (1+U)^{-1.683}\ P_E^{-0.24}\ T_R^{0.294}$	(99)	0.909	0.555	0.827
$K_2 = 0.788\ A^{0.409}\ (1+U)^{-2.06}\ P_E^{-0.15}\ T_R^{0.156}$	(100)	0.857	0.614	0.735

* All numbers were rounded off to the third decimal place.

The corresponding parameters K_1 or K_2 are used to obtain the IUH which in turn can be used in Eq. 7 along with the excess rainfall to predict the direct runoff. Data of all the storms on all the watersheds smaller than 5 square miles (watersheds 1, 2, 3, 4, 8, 11, 12, 13) were used in the analysis. The prediction obtained by using the estimated values of K_1 agree well with the regeneration obtained by using the optimum values of K_1. Typical results of prediction by using the estimated value of K_1 are presented in Table 21 and Fig. 42. The observed and the regenerated hydrographs obtained by using the optimum values of K_1 are presented in Fig. 42 to provide comparison.

Prediction of Runoff by Using Nash Model

As mentioned in Chapter IV, the Nash model was found to yield comparatively good regeneration for watersheds larger than 5 square miles

TABLE 21. PREDICTION PERFORMANCE OF THE SINGLE LINEAR RESERVOIR MODEL

	OBSERVED	REGENERATED	PREDICTED
MARCH 22 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	1.36	1.21	1.18
TIME TO PEAK	8.50	8.50	8.50
STORAGE COEFFT.	11.30	11.50	11.82
SIG.ERROR SQ.		.54	.55
INTEG.SQ.ERROR.		1.82	1.84
CORR.COEFF.		.98	.98
SPE.COR.COEFF.		.99	.99
NOVEMBER 26 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	.51	.51	.54
TIME TO PEAK	6.00	6.00	5.50
STORAGE COEFFT.	13.57	15.00	13.92
SIG.ERROR SQ.		.04	.06
INTEG.SQ.ERROR.		1.03	1.23
CORR.COEFF.		.99	.99
SPE.COR.COEFF.		1.00	1.00
AUG. 10 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	.95	.91	.87
TIME TO PEAK	19.00	19.00	19.50
STORAGE COEFFT.	12.41	13.50	14.54
SIG.ERROR SQ.		.35	.39
INTEG.SQ.ERROR.		1.30	1.37
CORR.COEFF.		.99	.99
SPE.COR.COEFF.		.99	.99
JULY 11 1966 ROSS ADE(UPPER)			
PEAK DISCHARGE	2.96	3.20	3.44
TIME TO PEAK	13.00	12.00	12.00
STORAGE COEFFT.	11.32	11.50	10.10
SIG.ERROR SQ.		3.05	5.08
INTEG.SQ.ERROR.		1.37	1.77
CORR.COEFF.		.99	.99
SPE.COR.COEFF.		.99	.99
OCT 15,1967 3.44 AM.ROSS ADE(UPPER)			
PEAK DISCHARGE	1.47	1.46	1.37
TIME TO PEAK	5.00	4.00	4.00
STORAGE COEFFT.	11.49	9.00	9.79
SIG.ERROR SQ.		.96	.77
INTEG.SQ.ERROR.		2.47	2.21
CORR.COEFF.		.98	.98
SPE.COR.COEFF.		.98	.99
OCT. 15, 1967 8.18 AMROSS ADE(UPPER)			
PEAK DISCHARGE	1.81	1.66	1.84
TIME TO PEAK	5.50	4.00	4.00
STORAGE COEFFT.	12.23	11.50	10.11
SIG.ERROR SQ.		1.05	1.55
INTEG.SQ.ERROR.		1.89	2.29
CORR.COEFF.		.99	.98
SPE.COR.COEFF.		.99	.99

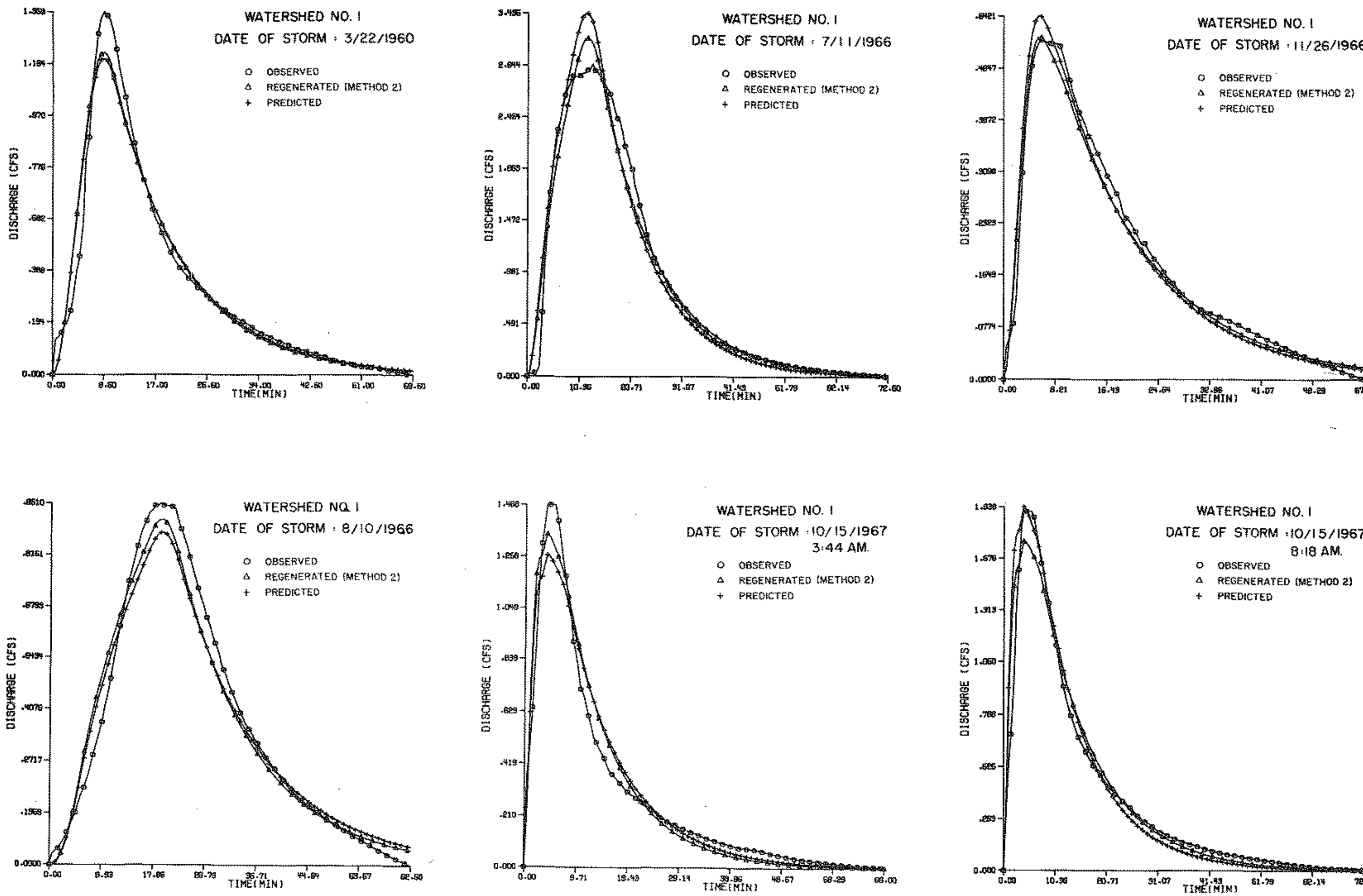


FIGURE 42. TYPICAL RESULTS OF PREDICTION OBTAINED BY USING THE SINGLE LINEAR RESERVOIR MODEL

and smaller than 20 squares miles, and hence was selected to predict runoff from rainfall on such watersheds. In order to use the Nash model for prediction purposes, it is necessary to estimate the values of the parameters n and K_N.

The parameter K_N is the storage coefficient of each of the linear reservoirs in the cascade of linear reservoirs of the Nash model, and hence has the units of time. The parameter n is the number of linear reservoirs. Further, the two parameters n and K_N are related to the time lag T_4 by the relationship

$$n\ K_N = T_4 \tag{58}$$

If either one of the two parameters n or K_N is estimated from the physiographic characteristics of the watershed and from the rainfall characteristics, then the other can be computed by using Eq. 58. Also, from Eq. 58 the parameter K_N can be considered as a multiple of the time lag T_4 and hence it can be anticipated that K_N behaves in a manner similar to that of the time lag T_4. Consequently, the following relationship for the parameter K_N was investigated:

$$K_N = f_7\ (A,\ (1+U),\ P_E,\ T_R) \tag{101}$$

A power function equation of the form

$$K_N = C_0\ A^{C_1}\ (1+U)^{C_2}\ P_E^{C_3}\ T_R^{C_4} \tag{102}$$

for the relationship 101 was fitted by the multiple regression analysis. The resulting equation is

$$K_N = 0.575\ A^{0.389}\ (1+U)^{-0.622}\ P_E^{-0.106}\ T_R^{0.222} \tag{103}$$

The correlation coefficient for the Eq. 103 is 0.852 whereas the standard error of estimate and the coefficient of determination are 0.586 and 0.725 respectively. The graphical solution of Eq. 103 is presented in Fig. 43. Thus, the value of the parameter K_N of the Nash model can be estimated either by using Eq. 103 or Fig. 43. As the time lag T_4 can be estimated by using Eq. 95, the parameter n can be computed by using Eq. 58. Once the parameters n and K_N are estimated, the corresponding IUH can be defined by using Eq. 21, which can then be used in Eq. 7 along with the excess rainfall to predict the direct runoff. Data of all the storms on all the watersheds larger than 5 square miles (watersheds 5, 6, 7, 9 and 10) were used for predictions. Although the prediction obtained was not very good, the results compared reasonably well with those of regeneration obtained by using the same data. The observed hydrograph, the regenerated hydrograph, and the predicted hydrograph obtained by using the estimated values of n and K_N, for some of the storms on watersheds 5 and 6 are shown in Table 22 and Fig. 44.

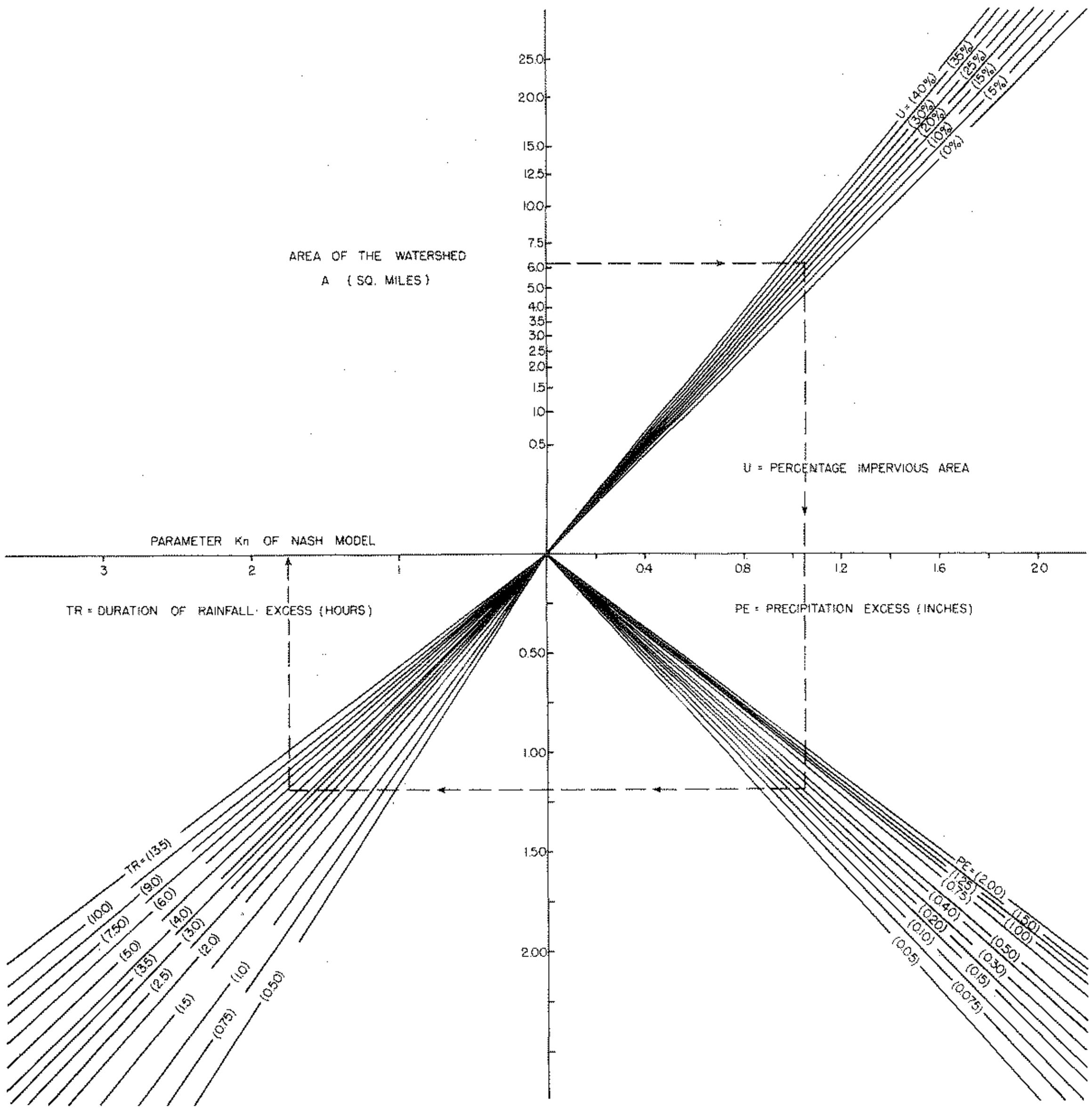


FIGURE 43. GRAPHICAL SOLUTION OF EQ. 103 FOR ESTIMATION OF K_N

TABLE 22. PREDICTION PERFORMANCE OF NASH MODEL

	OBSERVED	REGENERATED	PREDICTED	
APRIL 30,1962 PLEASANTRUN-A				ARLINGTON
TIME LAG	3.26	3.26	2.97	
NASH K		1.69	1.53	
NASH N		1.73	1.94	
PEAK DISCHARGE	517.66	398.96	439.17	
TIME TO PEAK	3.25	2.63	2.63	
SIG.ERROR SQ.		361108.36	251108.04	
INTEG.SQ.ERROR		3.96	3.31	
CORR.COEFFT.		.96	.97	
SPE.COR.COEF.		.96	.98	
MAY1 1962 PLEASANTRUN-A				ARLINGTON
TIME LAG	4.52	4.52	2.51	
NASH K		2.69	1.40	
NASH N		1.68	1.79	
PEAK DISCHARGE	453.23	375.04	660.02	
TIME TO PEAK	2.88	2.50	1.88	
SIG.ERROR SQ.		118299.54	1643973.94	
INTEG.SQ.ERROR		1.76	6.83	
CORR.COEFFT.		.98	.89	
SPE.COR.COEF.		.99	.81	
APRIL30,1962 PLEASANTRUN-B				BROOKVILLE
TIME LAG	3.83	3.83	3.37	
NASH K		2.14	1.72	
NASH N		1.79	1.96	
PEAK DISCHARGE	470.40	373.26	423.78	
TIME TO PEAK	4.03	2.86	2.86	
SIG.ERROR SQ.		513182.86	392133.39	
INTEG.SQ.ERROR		3.81	3.65	
CORR.COEFFT.		.91	.93	
SPE.COR.COEF.		.94	.96	
JULY20,1963 PLEASANTRUN-B				BROOKVILLE
TIME LAG	2.60	2.60	3.08	
NASH K		3.40	1.65	
NASH N		.76	1.87	
PEAK DISCHARGE	740.52	765.57	581.40	
TIME TO PEAK	1.69	1.95	2.60	
SIG.ERROR SQ.		308918.20	2324130.86	
INTEG.SQ.ERROR		2.56	7.58	
CORR.COEFFT.		.98	.78	
SPE.COR.COEF.		.98	.85	
AUG.11,1964 PLEASANTRUN-B				BROOKVILLE
TIME LAG	2.91	2.91	2.33	
NASH K		2.33	1.36	
NASH N		1.25	1.71	
PEAK DISCHARGE	504.09	426.47	504.42	
TIME TO PEAK	1.30	.91	1.30	
SIG.ERROR SQ.		78883.22	79154.31	
INTEG.SQ.ERROR		2.22	2.25	
CORR.COEFFT.		.99	.99	
SPE.COR.COEF.		.99	.99	
JAN1,1966. PLEASANTRUN-B				BROOKVILLE
TIME LAG	5.85	5.85	5.25	
NASH K		2.74	2.14	
NASH N		2.14	2.45	
PEAK DISCHARGE	143.09	112.15	126.61	
TIME TO PEAK	5.98	5.33	5.33	
SIG.ERROR SQ.		12030.22	11399.96	
INTEG.SQ.ERROR		1.52	1.48	
CORR.COEFFT.		.98	.99	
SPE.COR.COEF.		.99	.99	

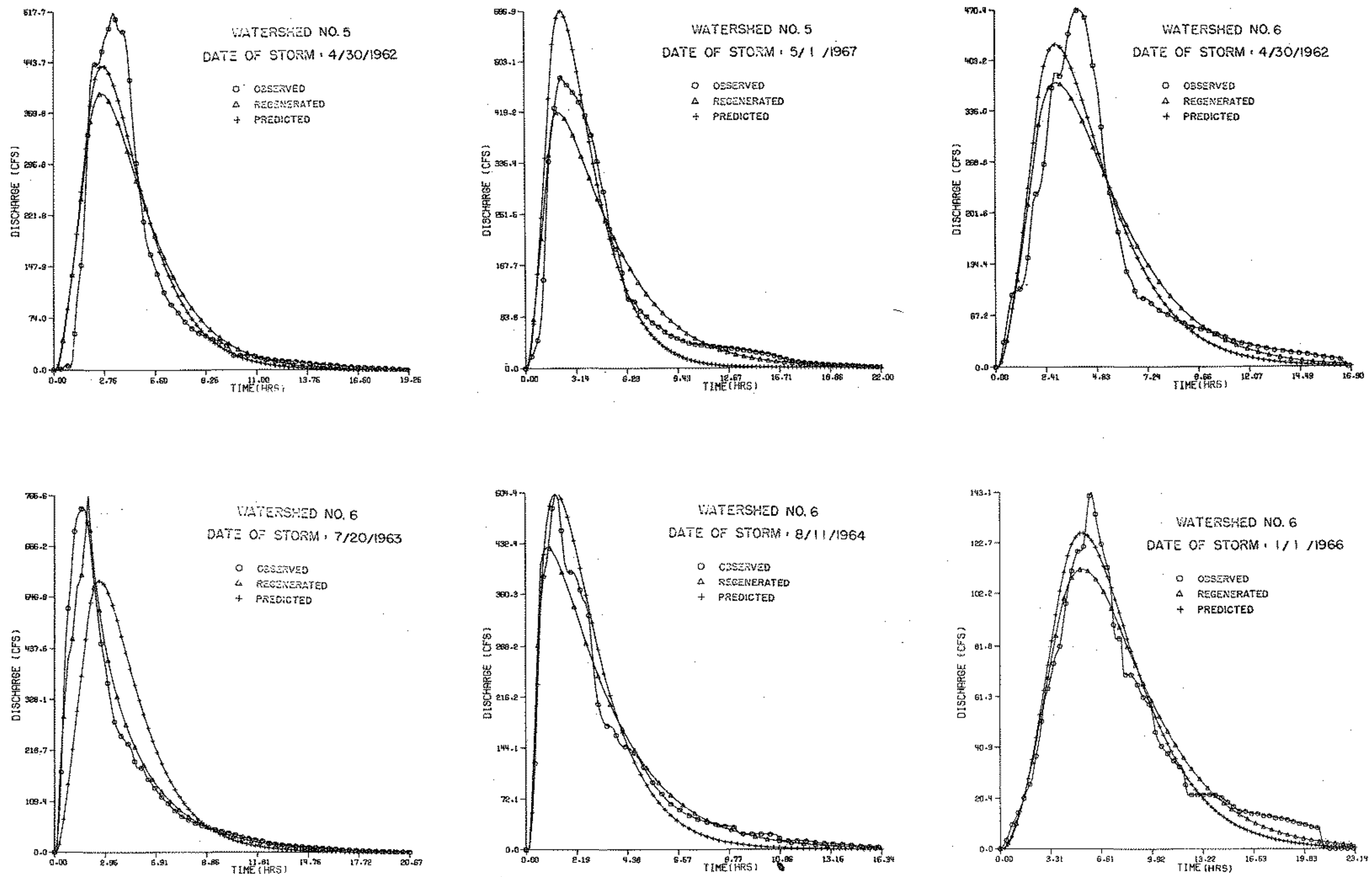


FIGURE 44. RESULTS OF PREDICTION OBTAINED BY USING NASH MODEL.

CHAPTER VI

EFFECTS OF URBANIZATION ON RUNOFF

Some of the hydrologic characteristics which are affected by urbanization are: the volumes of annual, monthly and daily runoff, the peak rate of runoff, the shapes of the hydrograph, the unit hydrograph and the IUH, the frequency of peak discharge, the infiltration rate, the base flow, and the time lag, etc. However, the main emphasis in the present investigation has been on the changes caused by urbanization on the characteristics of the runoff hydrograph and of the instantaneous unit hydrograph. More specifically, the changes in the time lag, in the shapes of the IUH, and of the direct runoff hydrograph; the changes in the peak discharge, in the time to peak discharge of direct runoff hydrograph, and the changes in the frequency of peak discharges, are first discussed qualitatively. The changes in these characteristics as functions of only the urbanization factor are discussed next, with specific reference to the Ross Ade upper watershed, less than 5 sq. miles, and the Pleasant Run watershed at Arlington, larger than 5 sq. miles.

The discussion of time lag, peak discharge, time to peak discharge and the parameters of the conceptual models, is based on linear regression equations which relate these quantities to the physiographic characteristics of the watershed and to the storm characteristics (Chapter V). However, the regression equations were developed by using the limited data of only 125 storms on 11 watersheds and hence they do not cover the entire range of all the variables. Although extrapolation of the results obtained by using these equations may lead to erroneous results, certain general trends are obvious. It is hoped that as more data from urbanized watersheds become available, the trends indicated by this study can be investigated further.

Time Lag

Time lag is related to the physiographic characteristics of the watershed including the urbanization factor and the storm characteristics as given by Eq. 95,

$$T_4 = 0.831\ A^{0.458}\ (1+U)^{-1.662}\ P_E^{-0.267}\ T_R^{0.371} \tag{95}$$

The time lag T_4, as indicated by Eq. 95 increases with the watershed area and with the duration of the excess rainfall; it decreases with an increase in the impervious area, and in the volume of excess rainfall. Further, the rates of change of time lag T_4 with respect to the individual variables can be investigated by considering the partial derivatives of T_4 with respect to each of the variables, which are presented in Table 23.

The rate of change in time lag is influenced most by the urbanization factor, followed by volume and duration of excess rainfall and the area of the watershed. Although all these variables decrease the rate of change of time lag, increase beyond a certain range in any of these four variables does not significantly influence the rate of change of time lag (Table 23).

Table 23. Partial Derivatives of Time Lag T_4

$\frac{\partial T_4}{\partial A}$	= 0.458	$A^{-0.542}$	$[0.831\ (1+U)^{-1.662}$	$P_E^{-0.267}$	$T_R^{0.371}]$
$\frac{\partial T_4}{\partial (1+U)}$	= (-1.662)	$(1+U)^{-2.662}$	$[0.831\ A^{0.458}$	$P_E^{-0.267}$	$T_R^{0.371}]$
$\frac{\partial T_4}{\partial P_E}$	= (-0.267)	$P_E^{-1.267}$	$[0.831\ A^{0.458}$	$(1+U)^{-1.662}$	$T_R^{0.371}]$
$\frac{\partial T_4}{\partial T_R}$	= 0.371	$T_R^{-0.629}$	$[0.831\ A^{0.458}$	$(1+U)^{-1.662}$	$P_E^{-0.267}]$

With specific reference to the urbanization factor, the time lag decreases with an increase in the urbanization factor up to a certain value of (1+U), beyond which the increase of the urbanization factor does not result in a significant reduction of the time lag. This can also be seen from Fig. 40.

The Response of the Watershed

As mentioned earlier, the response of the watershed to a storm can be characterized by the IUH, or by the unit hydrograph of a specified duration. Several investigators[66,69] of the effects of urbanization on runoff have considered a single unit hydrograph to be "representative" of the watershed response. Changes in the shapes of these "representative" unit hydrographs for different watersheds with various degrees of urbanization are then studied.

As pointed out earlier,[76,90,91] there is no unique unit hydrograph representative of any watershed, and consequently, the changes in the shape of the unit hydrograph due to increased urbanization alone cannot be easily isolated. The IUH also varies from storm to storm on a watershed and hence there is no unique IUH which is representative of the watershed response. However, the parameters of the IUH can be explicitly related to the urbanization factor and the storm characteristics. Hence the effects of urbanization on the watershed response can be evaluated separately from those of the changes in the storm characteristics. This is the approach adopted in this study.

The IUH of the single linear reservoir model, which has been selected to simulate the rainfall-runoff process on urban watersheds of area less than 5 sq. miles, is given by

$$h(t) = \frac{1}{K} e^{-t/K} \qquad (16)$$

The IUH [h(t)] is obviously a function of (1/K), with a maximum value at time t = 0. The storage constant K is theoretically equal to the time lag T_4, and consequently h(t) is a function of the time lag T_4, which has been related to the physiographic characteristics, including urbanization factor of the watershed, and to the storm characteristics as in Eq. 95. In Eq. 95, the urbanization factor is inversely related to the time lag and hence to the storage constant K. Therefore, any increase in the urbanization factor results in a decrease of T_4 and hence of K, and thus the magnitude of the peak of the IUH will be increased. The time to peak of IUH of the single linear reservoir model is always zero regardless of the changes in urbanization factor.

The IUH of the Nash model, which has been selected to simulate the rainfall-runoff process in watersheds of area larger than 5 sq. miles and less than 20 sq. miles, is given by

$$h(t) = \frac{1}{K_N} (t/K_N)^{n-1} \frac{e^{-(t/K_N)}}{\Gamma(n-1)} \tag{21}$$

The parameters n and K_N in Eq. 21 are related to time lag as

$$n\ K_N = T_4 \tag{56}$$

The parameters K_N and the time lag T_4 (in Eq. 56) are estimated by using Eqs. 103 and 95 and the value of the parameter n, applicable to the particular storm is evaluated by using Eq. 56. Eqs. 95 and 103 also indicate that with all other conditions remaining the same, an increase in the urbanization factor causes a decrease in the values of T_4, K_N and of n.

The changes in the magnitude of the peak and the time to peak of the IUH of the Nash model can be interpreted better by using the following expressions:

$$T_{PI} = (n-1)K_N \tag{104}$$

$$Q_{PI} = \frac{1}{K_N} \left(\frac{t}{K_N}\right)^{n-1} \frac{e^{-(t/K_N)}}{\Gamma(n-1)} \Bigg|_{t=T_{PI}} \tag{105}$$

With an increase in the urbanization factor, the values of K_N and n decrease, which in turn increase the factor $\frac{1}{K_N} (t/K_N)^{n-1} \frac{1}{\Gamma(n-1)}\Big|_{t=T_{PI}}$ but decrease the factor e^{-t/K_N} in Eq. 105. However, it should be noted that the product of $K_N{}^n$ (n!) decreases at a faster rate than e^{-t/K_N} alone and hence the magnitude of peak of h(t) increases with an increase in the urbanization factor. Further, the time to peak of the IUH of the Nash model decreases with an increase in the urbanization factor.

Thus for both conceptual models used in the present study the magnitude of the peak of the IUH increases and the time to peak of IUH for the Nash model decreases due to an increase in the urbanization factor.

Peak Discharge

As the magnitude of the peak of the IUH was observed to increase with the urbanization factor, it can be inferred as a general result that the magnitude of the peak discharge of the runoff hydrograph increases with increased urbanization factor. Several investigators[67,69] have related either the magnitude of the unit hydrograph peak discharge or the unit hydrograph peak discharge per square mile, to the area of the watershed and to the time lag T_2 which was assumed to be unique for any watershed. As the time lag itself varied from storm to storm on any watershed, any relationship developed for Q_p must involve physiographic and storm characteristics. Further, such a relationship for Q_p will be useful for practical designs where only the magnitude of peak discharge is required. Hence, the following analysis was conducted. An equation of the form

$$Q_p = C_0 \, A^{C_1} \, (1+U)^{C_2} \, P_E^{C_3} \, T_R^{C_4}$$

was fitted to the data of peak discharge of the observed direct runoff hydrographs, the physiographic and storm characteristics by the multiple regression analysis. The resulting equation is

$$Q_p = 484.1 \, A^{0.723} \, (1+U)^{1.516} \, P_E^{1.113} \, T_R^{-0.403} \qquad (106)$$

The correlation coefficient, the standard error of estimate and the coefficient of determination for Eq. 106 are respectively 0.9844, 0.4025(cfs) and 0.969.

The magnitude of the peak discharge Q_p, as suggested by Eq. 106, increases with an increase in the area of the watershed, in the urbanization factor, or in the volume of excess rainfall, and decreases with an increase in the duration of excess rainfall. Further, the rates of change of Q_p with the individual variables can be interpreted by considering the partial derivatives of Q_p with respect to each of the variables, which are presented in Table 24.

Table 24. Partial Derivatives of Peak Discharge Q_p

$\frac{\partial Q_p}{\partial A}$	$= 0.723$	$A^{-0.267}$	$[484.1$	$(1+U)^{1.516}$	$P_E^{1.113}$	$T_R^{-0.403}]$
$\frac{\partial Q_p}{\partial (1+U)}$	$= 1.516$	$(1+U)^{0.516}$	$[484.1$	$A^{0.723}$	$P_E^{1.113}$	$T_R^{-0.403}]$
$\frac{\partial Q_p}{\partial P_E}$	$= 1.113$	$P_E^{0.113}$	$[484.1$	$A^{0.723}$	$(1+U)^{1.516}$	$T_R^{-0.403}]$
$\frac{\partial Q_p}{\partial T_R}$	$= (-0.403)$	$T_R^{-1.403}$	$[484.1$	$A^{0.723}$	$(1+U)^{1.516}$	$P_E^{1.113}]$

The rate of change of peak discharge increases with increase in urbanization factor, and in the volume of excess rainfall and decreases with the increase in area of the watershed and in duration of excess rainfall. The increase and the rate of increase of Q_p is influenced most by the urbanization factor. Hence, any increase in the urbanization factor directly results in increase in peak discharge Q_p. For example, for a watershed of area 10.1 square miles (watershed 6), and for an excess rainfall of 0.30 inches occurring in one hour, an increase in the urbanization factor from 1.0 to 1.20 results in an increase in peak discharge by 30 percent, and an increase in the urbanization factor from 1.0 to 1.40, results in an increase in peak discharge by 68 percent. A graphical solution of Eq. 106 is shown in Fig. 45.

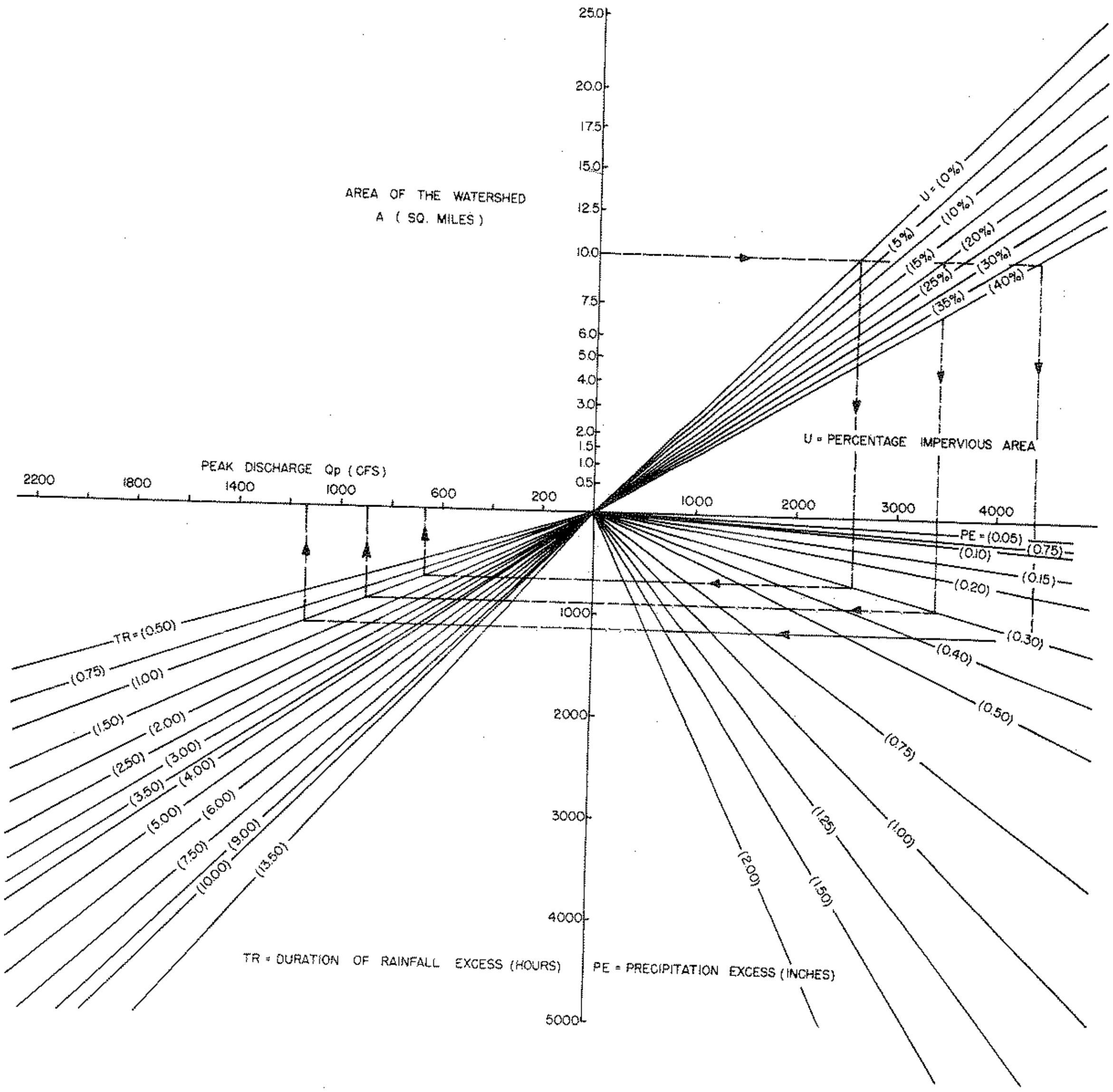


FIGURE 45. GRAPHICAL SOLUTION OF EQ. 106 FOR ESTIMATION OF Q_P

Time to Peak Discharge

Just as the peak discharge Q_p can be estimated from Eq. 106 the time to peak discharge T_p can also be related to the physiographic and storm characteristics. The linear regression analysis applied to the values of observed time to peak and the physiographic and the storm characteristics, yielded the equation

$$T_p = 0.775\ A^{0.323}\ (1+U)^{-1.285}\ P_E^{-0.195}\ T_R^{0.634} \qquad (107)$$

The correlation coefficient, standard error of estimate and the coefficient of determination for Eq. 107 are respectively 0.93, 0.509 (hrs), and 0.865. The time to peak discharge T_p increases with an increase in the area of watershed, and in the duration of excess rainfall, whereas it decreases with an increase in the urbanization factor and in the volume of excess rainfall. A study of the partial derivatives of T_p with respect to the variables A, (1+U), P_E and T_R, indicates decreasing rates of change of T_p with an increase of all four parameters. The rate of decrease of T_p with respect to the increase in the urbanization factor is the highest compared to its variation due to change in parameters A, P_E and T_R. Thus the urbanization factor influences the time to peak discharge T_p and the value of T_p decreases with an increase in the urbanization factor. For example, for a watershed of area 10.1 square miles (watershed 6), and for an excess rainfall of 0.3 inches occurring in one hour, an increase in the urbanization factor from 1.0 to 1.20 results in a decrease in the time to peak discharge by 23 percent, and an increase in the urbanization factor from 1.0 to 1.40 results in the decrease of time to peak by 39 percent. A graphical solution of Eq. 107 is presented in Fig. 46.

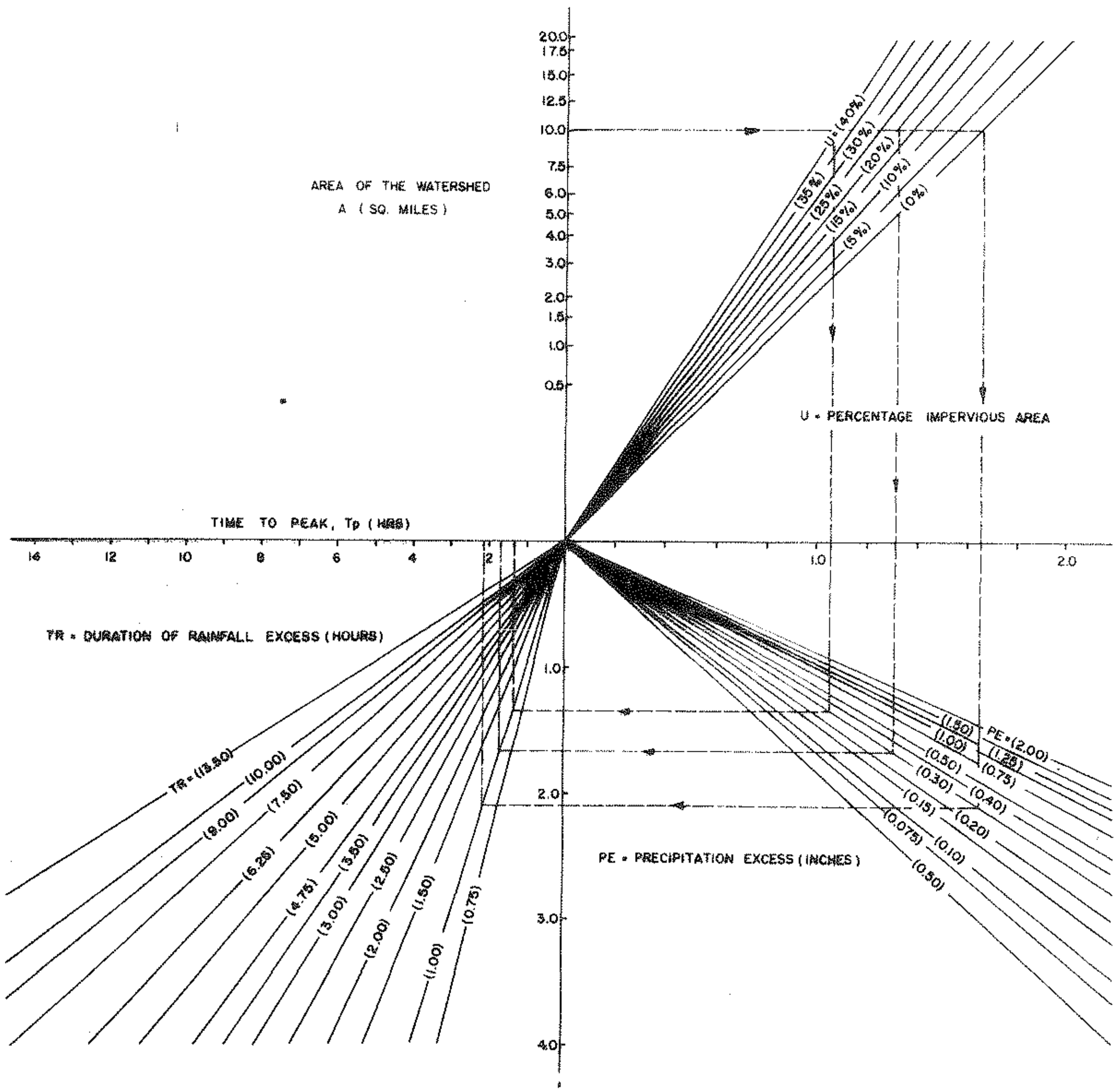


FIGURE 46. GRAPHICAL SOLUTION OF EQ. 107 FOR ESTIMATION OF Tp

Frequency of Peak Discharge

Effect of urbanization on magnitudes and frequency of flooding is an important consideration in planning land use and development. The information of the expected frequency and of the magnitude of floods is essential to provide adequate protection to life and property. A number of methods are available for the estimation of the magnitudes and frequencies of floods expected from watersheds in their rural conditions. However, as a watershed changes from rural to suburban or urban conditions, the methods to evaluate changes in magnitudes and frequencies of floods have not yet been well developed. Procedures to estimate the magnitudes and frequencies of floods in urban areas have been developed and are being used by U.S. Geological Survey.[62,94,95,96] Principally, these procedures are based on the regional flood frequency curves obtained from data on rural watersheds which are located in the same region of the urban watersheds. In order to extend the flood frequency relationships to urban watersheds, certain assumptions such as the equivalence of rainfall and runoff frequencies for completely impervious areas, had to be made. These assumptions may not be universally valid. Using the regional flood frequency curves obtained from the rural watershed data and some assumptions as mentioned above, flood frequency curves for urban watersheds have been developed. Most of the results of these procedures are qualitative. Although these approaches appear to be a first step in this direction, accurate methods of obtaining flood frequency curves for urban watersheds are yet to be developed.

The West Lafayette watersheds could not be used in the investigation of the effect of urbanization on the frequency of floods because of the lack of long term records, instead three watersheds (Nos. 5, 7, and 8) near Indianapolis, Indiana were selected for this purpose. The values of the urbanization factor for watersheds Nos. 5, 7, and 8 are respectively 1.10, 1.02 and 1.00. The length of record of floods on these watersheds ranges from 9 to 12 years. The frequency analysis was conducted for the annual peak floods only. The recurrence interval T_{Re} was computed by the formula[97]

$$T_{Re} = (n+1)/m$$

where n = number of years of record

m = the rank of the magnitude of the flood with the highest as 1.

The annual peak floods were plotted against the recurrence interval T_{Re}, on extremal probability paper. Straight lines were fitted to the points by using the general equation for hydrologic frequency analysis.[98] The flood frequency curves thus obtained are presented in Fig. 47.

The average annual flood is defined as the flood which has a recurrence interval of 2.33 years. The average annual floods for each of the three watersheds were estimated from Fig. 47. A plot of the average annual flood against the drainage area of the watershed is presented in Fig. 48, which may be used to estimate the average annual flood for any gaged or ungaged watershed situated near Indianapolis. The dimensionless flood frequency curves were then developed by dividing the ordinates of the flood frequency curves (Fig. 47) by the value of the average annual flood. The dimensionless flood frequency curves were plotted as shown in Fig. 49 for recurrence intervals greater than 2.33

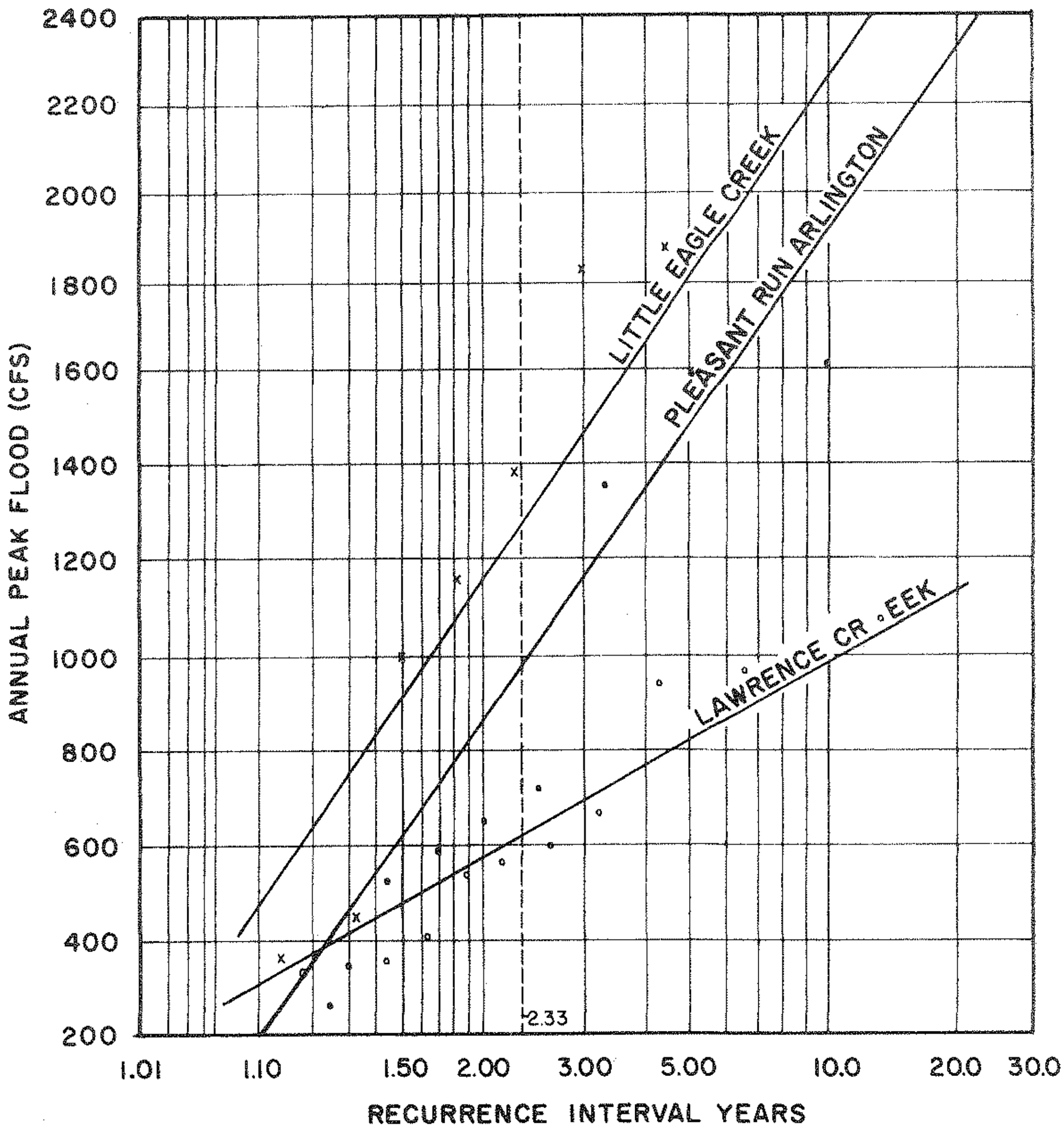


FIGURE 47. FLOOD FREQUENCY CURVES FOR WATERSHEDS NEAR INDIANAPOLIS, INDIANA.

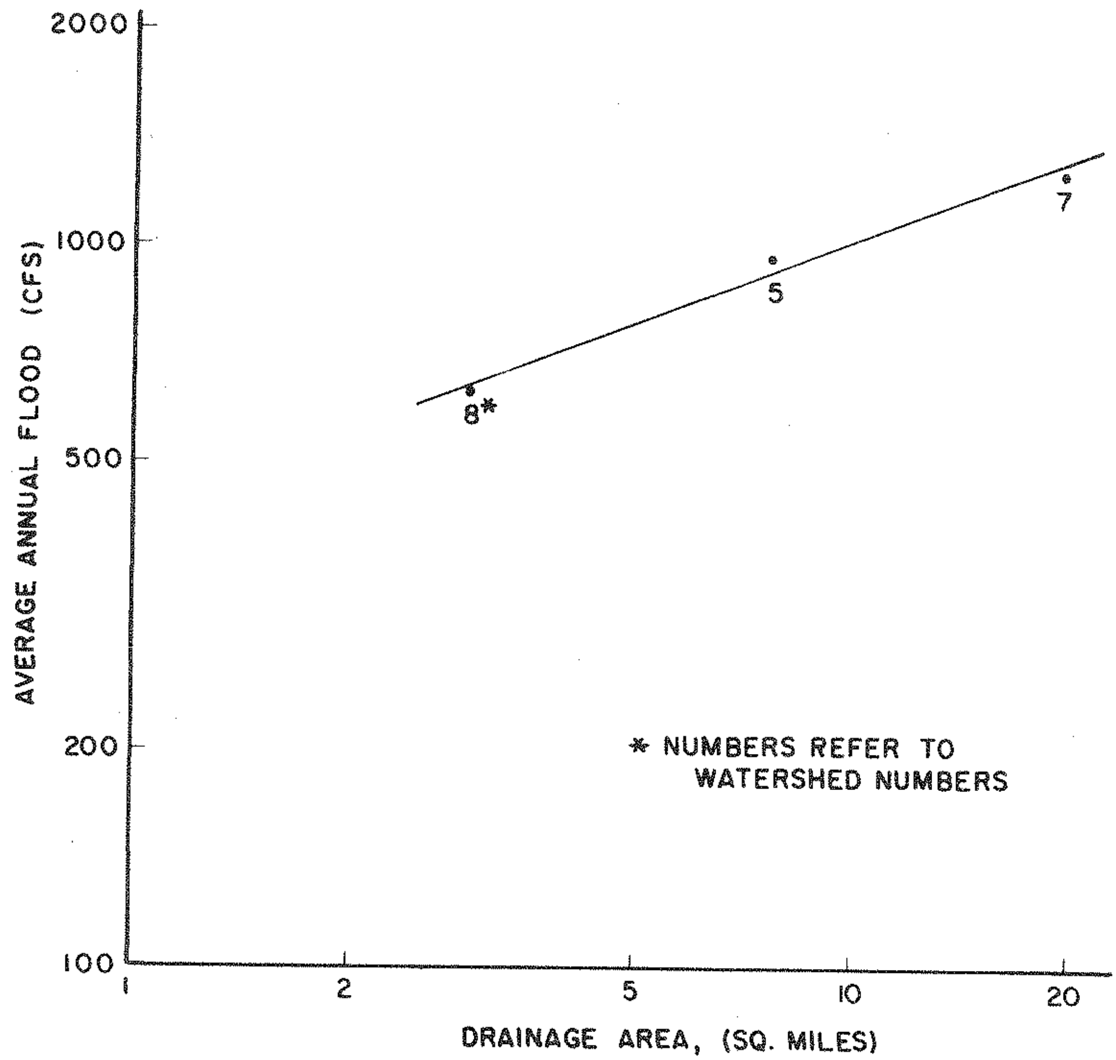


FIGURE 48. REGIONAL CURVE FOR THE RELATIONSHIP OF THE AVERAGE ANNUAL FLOOD TO THE AREA OF THE WATERSHED (INDIANAPOLIS AREA)

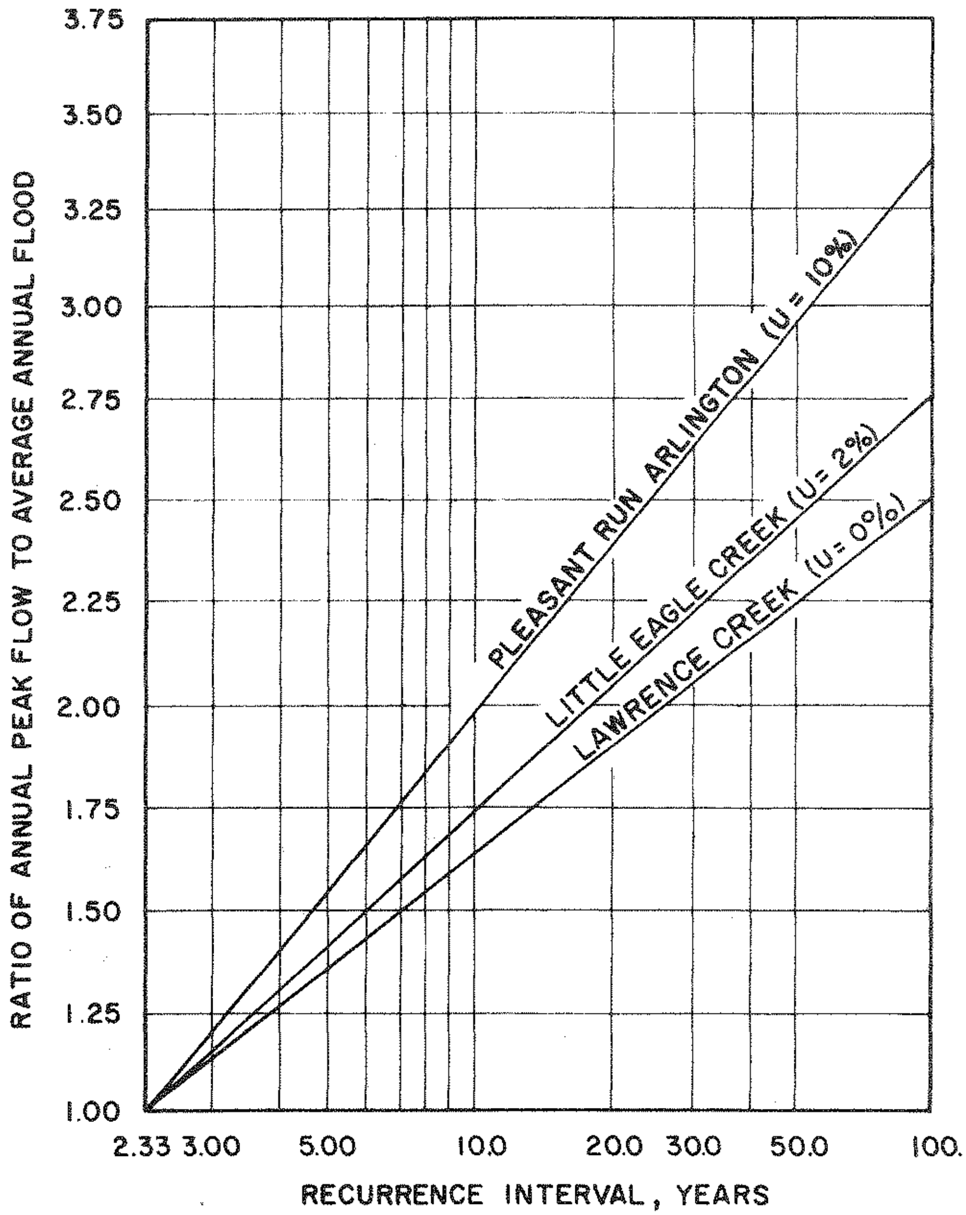


FIGURE 49. DIMENSIONLESS FLOOD FREQUENCY CURVES FOR WATERSHEDS NEAR INDIANAPOLIS, INDIANA.

years. The ordinates of the dimensionless flood frequency curves are listed in the last column of Table 25. From Fig. 49, it may be observed that as the urbanization factor increases, the magnitude of the flood of any frequency ($1/T_{Re}$) increases and that the frequency ($1/T_{Re}$) of the flood of any magnitude increases. Although these trends are clear, generalized conclusions cannot be drawn because of the limited range of data used. However, the method of analysis, being general, can be used for any other region.

The dimensionless flood frequency curves obtained by the method of analysis outlined above, can also be used for ungaged watersheds of the same region. With the value of the drainage area, the average annual flood can be obtained from Fig. 48. Then the frequency of a specified flood or the magnitude of a flood of a specified frequency can be estimated from Fig. 49 by using the appropriate curve corresponding to the urbanization factor applicable to the watershed under consideration. Although the results presented have a range of urbanization factor of only 1.00 to 1.10, it is hoped that as more data on urban watersheds become available, these trends can be further investigated.

Effects of Urbanization on Time Distribution of Runoff

Some of the characteristics of runoff such as the peak discharge, time to peak and time lag have been shown to be related to physiographic characteristics of the watershed including the urbanization factor, and the storm characteristics. Besides the values of the peak discharge and the time to peak discharge, the time distribution of runoff and its changes due to urbanization should be known for proper design of

Table 25. Computations for Flood Frequency Curves

		Computations for Fig. 47				Computations for Fig. 49		
Watershed	No.	Date of Flood	Magnitude of Flood Peak Q_{AP} cfs	Rank m	Return Period $T_{Re} = \frac{n+1}{m}$	Recurrence Interval, T_{Re} Years	Magnitude of Flood Peak Q_{AP}^* cfs	$\frac{Q_{AP}^*}{AAF^{**}}$
Pleasant Run	1	7.13.60	535	7	1.43	1.10	212.0	0.22
at	2	4.25.61	1360	3	3.33	1.50	616.0	0.64
Arlington Avenue	3	3.21.62	650	5	2.00	2.00	875.4	0.91
	4	3. 4.63	1610	1	10.00	2.33	962.0	1.00
	5	3. 9.64	1590	2	5.00	3.00	1193.0	1.24
	6	2. 9.65	700	4	2.50	4.00	1365.0	1.42
	7	2.10.66	262	8	1.25	5.00	1509.0	1.57
	8	12. 8.67	590	6	1.66	10.00	1884.0	1.96
	9	4. 4.68	1360	3	3.33	20.00	2292.0	2.39
Little Eagle	1	6.23.60	460	7	1.29	1.10	90.0	0.07
Creek at	2	4.25.61	1940	1	9.00	1.50	904.0	0.72
Speedway	3	1.26.62	992	6	1.50	2.00	1165.0	0.93
	4	3. 4.63	1830	3	3.00	2.33	1252.0	1.00
	5	4.21.64	1880	2	4.50	3.00	1484.0	1.17
	6	2.10.65	1160	5	1.80	4.00	1659.0	1.32
	7	2.10.66	361	8	1.125	5.00	1803.0	1.44
	8	12. 8.66	1390	4	2.25	10.00	2182.0	1.74

Table 25 (Continued)

						20.00	2580.0	2.06
Lawrence Creek	1	6.12.52	418	8	1.625	1.10	316.0	0.50
Benjamin	2	5.22.53	355	10	1.30	1.50	484.0	0.77
Harrison	3	4. 5.54	340	11	1.18	2.00	593.0	0.93
	4	6.10.58	1080	1	13.00	2.33	629.0	1.00
	5	1.21.59	940	3	4.33	3.00	725.0	1.15
	6	8. 3.60	550	7	1.86	4.00	797.0	1.26
	7	4.25.61	970	2	6.50	5.00	829.0	1.32
	8	4.30.62	600	5	2.60	10.00	993.0	1.58
	9	3. 4.63	678	4	3.25	20.00	1183.3	1.88
	10	3. 8.64	678	4	3.25			
	11	4.18.65	360	9	1.44			
	12	12. 8.66	578	6	2.16			

*Values obtained after fitting the straight line on the extremal probability paper.

**Value of annual flood that corresponds to a recurrence interval of 2.33 years.

the urban drainage facilities. Consequently, the conceptual models which were chosen to analyze the effects of urbanization - the single linear reservoir model and the Nash model - were used to demonstrate the effects of changes in the urbanization factor only.

As shown earlier, the parameters of the conceptual models can be related to the physiographic characteristics including the urbanization factor and the storm characteristics (Eqs. 95, 99, 103). Hence the IUH corresponding to these conceptual models mentioned above and the direct runoff hydrographs are also affected by not only the urbanization factor but also by the other factors such as storm characteristics. As a result of these effects, it is not possible to clearly demonstrate the effect of only the urbanization factor on runoff by using the observed data. Consequently, in order to investigate the specific effects of the increase in percentage of impervious areas in the watershed, numerical experiments were conducted in which all other factors such as storm characteristics were assumed to remain constant and the urbanization factor alone was varied to represent various percentages of impervious areas in the watershed. The urbanization factor was varied between 1.0 and 1.80 at increments of 0.10 to correspond to 10 percent increase in impervious areas.

Four different storms on each watershed were used for this investigation. For each watershed the maximum and the minimum values of the storm characteristics P_E and T_R were selected (Table 26) from among the available data and the four possible combinations of these characteristics were formulated to obtain the four hypothetical storms. These four combinations which represent the following cases designated as A, B, C and D are: A) P_E maximum and T_R maximum, B) P_E maximum and T_R minimum, C) P_E minimum and T_R maximum, and D) P_E minimum and T_R minimum. For each of the selected storms two distributions of rainfall, 1) the uniform or the rectangular, and 2) the isosceles triangular distributions were considered. Thus in all, 72 combinations of the urbanization factor, the volume of excess rainfall and the duration of excess rainfall were used on each watershed. The extreme values of the storm characteristics selected are presented in Table 26.

Table 26. Storm Characteristics Used in the Numerical Experiment

Watershed No.	Volume of Excess Rainfall (P_E) (in.)		Duration of Excess Rainfall (T_R) (Hrs.)	
	Maximum	Minimum	Maximum	Minimum
1	0.20	0.01	0.50	0.10
5	0.80	0.20	6.00	0.50

In each case, the parameters of the conceptual model were estimated using the appropriate value of the urbanization factor (1+U), and Eqs. 95, 99, and 103 and the outflow hydrograph was predicted by using the conceptual model. The hydrologic factors such as time lag, peak discharge, time to peak discharge, peak of the IUH, percentage increase in peak discharge, and the percentage decrease in time to peak and percentage decrease of time lag, referred to the corresponding values of the rural condition, are presented in Tables 27 and 28.

The instantaneous unit hydrographs and the corresponding predicted runoff hydrographs for all the above mentioned 8 cases of storms on watersheds 1 and 5, and for urbanization factors of 1.0, 1.20, 1.40,

TABLE 27. RESULTS OBTAINED BY USING THE SINGLE LINEAR RESERVOIR MODEL TO DEMONSTRATE THE EFFECTS OF URBANIZATION ON RUNOFF FROM WATERSHED NO. 1.

1) RECTANGULAR EXCESS RAINFALL DISTRIBUTION

STORM CHARACTERISTICS ARE PE=0.200 TR=30.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	14.44	12.32	10.66	9.34	8.25	7.36	6.61	5.98	5.43
STORAGE COEFFT K1	14.09	12.01	10.37	9.06	8.00	7.12	6.39	5.77	5.24
PEAK OF IUH (QPI)	.07	.08	.09	.11	.12	.14	.15	.17	.18
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	10.24	10.63	10.90	11.08	11.19	11.25	11.27	11.26	11.24
TIME TO PEAK(TP)	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	16.97	34.95	53.88	73.68	94.37	115.76	137.95	160.85
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	3.81	6.47	8.23	9.30	9.86	10.07	10.03	9.82
PERCENT.DECR.OF TP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

STORM CHARACTERISTICS ARE PE=0.200 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	7.94	6.78	5.87	5.14	4.54	4.05	3.64	3.29	2.99
STORAGE COEFFT K1	8.76	7.44	6.46	5.65	4.98	4.44	3.99	3.60	3.27
PEAK OF IUH (QPI)	.11	.13	.15	.17	.19	.21	.24	.26	.28
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	28.44	31.56	34.41	37.03	39.38	41.47	43.28	44.86	46.19
TIME TO PEAK(TP)	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	16.82	34.56	53.16	72.44	92.50	113.23	134.58	156.51
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	10.98	20.99	30.20	38.47	45.74	52.18	57.66	62.27
PERCENT.DECR.OF TP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

STORM CHARACTERISTICS ARE PE= .010 TR=30.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	32.14	27.43	23.74	20.78	18.37	16.39	14.71	13.30	12.10
STORAGE COEFFT K1	28.93	24.64	21.28	18.60	16.42	14.62	13.12	11.86	10.76
PEAK OF IUH (QPI)	.04	.04	.05	.05	.06	.07	.07	.08	.09
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	.39	.42	.45	.47	.49	.51	.52	.53	.54
TIME TO PEAK(TP)	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	15.52	32.43	50.59	69.89	90.22	111.51	133.69	156.71
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	7.32	14.86	20.11	25.36	29.81	33.69	36.66	38.81
PERCENT.DECR.OF TP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

STORM CHARACTERISTICS ARE PE= .010 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	17.60	15.09	13.06	11.43	10.11	9.01	8.09	7.32	6.65
STORAGE COEFFT K1	18.02	15.35	13.26	11.59	10.23	9.11	8.17	7.38	6.70
PEAK OF IUH (QPI)	.05	.06	.07	.08	.10	.11	.12	.13	.14
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	.83	.94	1.06	1.17	1.28	1.39	1.49	1.59	1.68
TIME TO PEAK(TP)	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	16.83	34.79	53.75	73.66	94.46	116.09	138.52	161.72
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	13.70	27.47	41.31	54.49	67.48	80.01	91.94	103.38
PERCENT.DECR.OF TP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

2) ISOSCELES TRIANGULAR EXCESS RAINFALL DISTRIBUTION

STORM CHARACTERISTICS ARE PE=0.200 TR=30.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	14.44	12.32	10.66	9.34	8.25	7.36	6.61	5.98	5.43
STORAGE COEFFT K1	14.09	12.01	10.37	9.06	8.00	7.12	6.39	5.77	5.24
PEAK OF IUH (QPI)	.07	.08	.09	.11	.12	.14	.15	.17	.18
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	12.24	13.16	14.02	14.76	15.42	16.00	16.50	16.94	17.32
TIME TO PEAK(TP)	22.00	21.50	21.00	20.50	20.00	19.50	19.00	19.00	18.50
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	16.97	34.95	53.88	73.68	94.37	115.76	137.95	160.85
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	7.52	14.53	20.62	25.99	30.72	34.83	38.38	41.55
PERCENT.DECR.OF TP	0.00	2.27	4.55	6.82	9.09	11.36	13.64	13.64	15.91

STORM CHARACTERISTICS ARE PE= 0.200 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	7.94	6.78	5.87	5.14	4.54	4.05	3.64	3.29	2.99
STORAGE COEFFT K1	8.76	7.44	6.46	5.65	4.98	4.44	3.99	3.60	3.27
PEAK OF IUH (QPI)	.11	.13	.15	.17	.19	.21	.24	.26	.28
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	29.36	32.98	36.38	39.64	42.54	45.56	48.36	51.07	53.55
TIME TO PEAK(TP)	5.00	5.00	5.00	5.00	5.00	5.00	4.50	4.50	4.50
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	16.82	34.56	53.16	72.44	92.50	113.23	134.58	156.51
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	12.32	24.13	35.29	45.60	55.27	65.02	74.26	82.39
PERCENT.DECR.OF TP	0.00	0.00	0.00	0.00	0.00	0.00	10.00	10.00	10.00

STORM CHARACTERISTICS ARE PE= .010 TR=30.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	32.14	27.43	23.74	20.78	18.37	16.39	14.71	13.30	12.10
STORAGE COEFFT K1	28.93	24.64	21.28	18.60	16.42	14.62	13.12	11.86	10.76
PEAK OF IUH (QPI)	.04	.04	.05	.05	.06	.07	.07	.08	.09
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	.42	.46	.49	.53	.57	.60	.63	.66	.69
TIME TO PEAK(TP)	25.00	24.50	23.50	23.00	22.50	22.00	22.00	21.50	21.00
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	15.52	32.43	50.59	69.89	90.22	111.51	133.69	156.71
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	9.27	18.47	27.43	36.00	44.12	51.78	58.93	65.57
PERCENT.DECR.OF TP	0.00	2.00	6.00	8.00	10.00	12.00	12.00	14.00	16.00

STORM CHARACTERISTICS ARE PE= .010 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (MIN)	17.68	15.09	13.06	11.43	10.11	9.01	8.09	7.32	6.65
STORAGE COEFFT K1	18.02	15.35	13.26	11.59	10.23	9.11	8.17	7.38	6.70
PEAK OF IUH (QPI)	.05	.06	.07	.08	.10	.11	.12	.13	.14
TIME OF IUH PEAK,TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PEAK DISCHARGE (QP)	.84	.96	1.07	1.19	1.31	1.43	1.54	1.66	1.77
TIME TO PEAK(TP)	5.50	5.50	5.50	5.50	5.50	5.00	5.00	5.00	5.00
PERCENT.INCR.OF T4	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT.INCR.OF QPI	0.00	16.83	34.79	53.75	73.66	94.46	116.09	138.52	161.72
PERCENT.DECR.OF TPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PERCENT.INCR.OF QP	0.00	14.16	28.38	42.63	56.72	70.51	84.74	98.74	112.24
PERCENT.DECR.OF TP	0.00	0.00	0.00	0.00	0.00	9.09	9.09	9.09	9.09

TABLE 28. RESULTS OBTAINED BY USING THE NASH MODEL TO DEMONSTRATE THE EFFECTS OF URBANIZATION ON RUNOFF FROM WATERSHED NO. 5

1) RECTANGULAR EXCESS RAINFALL DISTRIBUTION

STORM CHARACTERISTICS ARE PE= .800 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	4.34	3.70	3.20	2.80	2.48	2.21	1.99	1.80	1.63
STORAGE COEFFT KN	1.92	1.81	1.72	1.63	1.56	1.49	1.44	1.38	1.33
PEAK OF IUH (UPI)	.17	.20	.23	.26	.29	.33	.37	.42	.48
TIME OF IUH PEAK,TPI	2.38	1.88	1.50	1.13	.88	.75	.50	.38	.25
PEAK DISCHARGE (QP)	516.16	550.60	577.74	597.46	612.66	623.36	630.96	636.40	640.33
TIME TO PEAK(TP)	6.50	6.25	6.13	6.00	6.00	6.00	6.00	6.00	6.00
PERCENT,INCR.OF TA	-0.00	14.66	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	15.60	31.30	49.06	68.71	90.67	115.13	143.40	175.63
PERCENT,DECR.OF TPI	-0.00	21.05	36.84	52.63	63.16	68.42	76.95	84.21	89.47
PERCENT,INCR.OF QP	0.00	6.67	11.93	15.75	18.69	20.77	22.24	23.29	24.05
PERCENT,DECR.OF TP	-0.00	3.85	5.77	7.69	7.69	7.69	7.69	7.69	7.69

STORM CHARACTERISTICS ARE PE= .800 TR= .50

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	1.72	1.47	1.27	1.11	.99	.88	.79	.71	.65
STORAGE COEFFT KN	1.11	1.05	.99	.94	.90	.86	.83	.80	.77
PEAK OF IUH (UPI)	.42	.50	.60	.72	.89	1.05	1.21	1.36	1.50
TIME OF IUH PEAK,TPI	.63	.38	.25	.13	.13	.13	.13	.13	.13
PEAK DISCHARGE (QP)	1626.94	1923.69	2267.13	2605.89	2906.39	3170.11	3400.28	3606.92	3849.10
TIME TO PEAK(TP)	.88	.75	.63	.63	.63	.63	.63	.50	.50
PERCENT,INCR.OF TA	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	18.94	42.41	70.59	109.26	147.60	184.96	220.95	255.32
PERCENT,DECR.OF TPI	-0.00	40.00	60.00	80.00	80.00	80.00	80.00	80.00	80.00
PERCENT,INCR.OF QP	0.00	18.24	39.37	60.70	78.67	94.89	109.04	121.70	136.63
PERCENT,DECR.OF TP	-0.00	14.29	28.57	28.57	28.57	28.57	28.57	42.86	42.86

STORM CHARACTERISTICS ARE PE= .200 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	6.28	5.36	4.64	4.06	3.59	3.20	2.88	2.60	2.36
STORAGE COEFFT KN	2.23	2.10	1.99	1.89	1.81	1.73	1.66	1.60	1.55
PEAK OF IUH (UPI)	.13	.14	.16	.18	.20	.23	.25	.28	.31
TIME OF IUH PEAK,TPI	4.00	3.25	2.63	2.13	1.75	1.50	1.25	1.00	.88
PEAK DISCHARGE (QP)	106.57	116.27	124.97	132.60	139.03	144.35	148.36	151.73	154.28
TIME TO PEAK(TP)	7.75	7.13	6.75	6.38	6.25	6.13	6.00	6.00	6.00
PERCENT,INCR.OF TA	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	13.93	28.73	44.47	61.21	79.34	98.71	119.77	142.33
PERCENT,DECR.OF TPI	-0.00	18.75	34.38	46.88	56.25	62.50	68.75	75.00	78.13
PERCENT,INCR.OF QP	0.00	9.16	17.26	24.42	30.45	35.44	39.21	42.37	44.76
PERCENT,DECR.OF TP	-0.00	8.06	12.90	17.74	19.35	20.97	22.58	22.58	22.58

STORM CHARACTERISTICS ARE PE= .200 TR= .50

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	2.50	2.13	1.84	1.61	1.43	1.27	1.14	1.03	.94
STORAGE COEFFT KN	1.28	1.21	1.15	1.09	1.04	1.00	.96	.92	.89
PEAK OF IUH (UPI)	.29	.34	.39	.45	.52	.61	.70	.83	.96
TIME OF IUH PEAK,TPI	1.25	.88	.75	.50	.38	.25	.25	.13	.13
PEAK DISCHARGE (QP)	285.27	329.96	380.26	434.86	496.83	568.74	636.87	698.87	764.84
TIME TO PEAK(TP)	1.50	1.25	1.00	.88	.75	.63	.63	.63	.63
PERCENT,INCR.OF TA	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	16.10	34.09	54.84	78.63	106.33	137.69	181.67	226.13
PERCENT,DECR.OF TPI	-0.00	30.00	40.00	60.00	70.00	80.00	80.00	90.00	90.00
PERCENT,INCR.OF QP	0.00	15.67	33.30	52.44	74.16	99.37	123.25	144.98	168.11
PERCENT,DECR.OF TP	-0.00	16.67	33.33	41.67	50.00	58.33	58.33	58.33	58.33

2) ISOSCELES TRIANGULAR EXCESS RAINFALL DISTRIBUTION

STORM CHARACTERISTICS ARE PE= .800 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	4.34	3.70	3.20	2.80	2.48	2.21	1.99	1.80	1.63
STORAGE COEFFT KN	1.92	1.81	1.72	1.63	1.56	1.49	1.44	1.38	1.33
PEAK OF IUH (UPI)	.17	.20	.23	.26	.29	.33	.37	.42	.48
TIME OF IUH PEAK,TPI	2.38	1.88	1.50	1.13	.88	.75	.50	.38	.25
PEAK DISCHARGE (QP)	583.43	637.47	686.08	729.40	768.51	803.38	835.31	863.99	889.69
TIME TO PEAK(TP)	6.00	5.63	5.25	5.00	4.75	4.50	4.38	4.25	4.13
PERCENT,INCR.OF TA	-0.00	14.66	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	15.60	31.30	49.06	68.71	90.67	115.13	143.40	175.63
PERCENT,DECR.OF TPI	-0.00	21.05	36.84	52.63	63.16	68.42	78.95	84.21	89.47
PERCENT,INCR.OF QP	0.00	9.26	17.59	25.02	31.72	37.70	43.17	48.09	52.53
PERCENT,DECR.OF TP	-0.00	6.25	12.50	16.67	20.83	25.00	27.08	29.17	31.25

STORM CHARACTERISTICS ARE PE= .800 TR= .50

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	1.72	1.47	1.27	1.11	.99	.88	.79	.71	.65
STORAGE COEFFT KN	1.11	1.05	.99	.94	.90	.86	.83	.80	.77
PEAK OF IUH (UPI)	.42	.50	.60	.72	.89	1.05	1.21	1.36	1.50
TIME OF IUH PEAK,TPI	.63	.38	.25	.13	.13	.13	.13	.13	.13
PEAK DISCHARGE (QP)	1646.13	1951.57	2313.89	2771.20	3203.85	3603.90	3970.25	4303.88	4606.90
TIME TO PEAK(TP)	.88	.75	.63	.50	.50	.50	.50	.50	.50
PERCENT,INCR.OF TA	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	18.94	42.41	70.59	109.26	147.60	184.96	220.95	255.32
PERCENT,DECR.OF TPI	-0.00	40.00	60.00	80.00	80.00	80.00	80.00	80.00	80.00
PERCENT,INCR.OF QP	0.00	18.56	40.57	68.35	94.63	118.93	141.19	161.45	179.86
PERCENT,DECR.OF TP	-0.00	14.29	28.57	42.86	42.86	42.86	42.86	42.86	42.86

STORM CHARACTERISTICS ARE PE= .200 TR= 6.00

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	6.28	5.36	4.64	4.06	3.59	3.20	2.88	2.60	2.36
STORAGE COEFFT KN	2.23	2.10	1.99	1.89	1.81	1.73	1.66	1.60	1.55
PEAK OF IUH (UPI)	.13	.14	.16	.18	.20	.23	.25	.28	.31
TIME OF IUH PEAK,TPI	4.00	3.25	2.63	2.13	1.75	1.50	1.25	1.00	.88
PEAK DISCHARGE (QP)	114.69	127.47	139.78	151.24	161.75	171.33	180.12	188.10	195.56
TIME TO PEAK(TP)	7.38	6.75	6.13	5.75	5.50	5.25	5.00	4.88	4.63
PERCENT,INCR.OF TA	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	13.93	28.73	44.47	61.31	79.34	98.71	119.77	142.33
PERCENT,DECR.OF TPI	-0.00	18.75	34.38	46.88	56.25	62.50	68.75	75.00	78.13
PERCENT,INCR.OF QP	0.00	11.14	21.87	31.86	41.03	49.38	57.04	64.00	70.51
PERCENT,DECR.OF TP	-0.00	8.47	16.95	22.03	25.42	28.81	32.20	33.90	37.29

STORM CHARACTERISTICS ARE PE= .200 TR= .50

URBANIZATION FACTOR	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
TIME LAG (HRS)	2.50	2.13	1.84	1.61	1.43	1.27	1.14	1.03	.94
STORAGE COEFFT KN	1.28	1.21	1.15	1.09	1.04	1.00	.96	.92	.89
PEAK OF IUH (UPI)	.29	.34	.39	.45	.52	.61	.70	.83	.96
TIME OF IUH PEAK,TPI	1.25	.88	.75	.50	.38	.25	.25	.13	.13
PEAK DISCHARGE (QP)	286.64	332.28	383.63	441.47	507.74	581.47	673.77	761.77	844.70
TIME TO PEAK(TP)	1.50	1.13	1.00	.75	.63	.50	.50	.50	.50
PERCENT,INCR.OF TA	-0.00	14.65	26.15	35.35	42.84	49.04	54.22	58.61	62.36
PERCENT,INCR.OF QPI	0.00	16.10	34.09	54.84	78.63	106.33	137.69	181.67	226.13
PERCENT,DECR.OF TPI	-0.00	30.00	40.00	60.00	70.00	80.00	80.00	90.00	90.00
PERCENT,INCR.OF QP	0.00	15.92	33.84	54.02	77.13	102.86	135.06	165.76	194.69
PERCENT,DECR.OF TP	-0.00	25.00	33.33	50.00	58.33	66.67	66.67	66.67	66.67

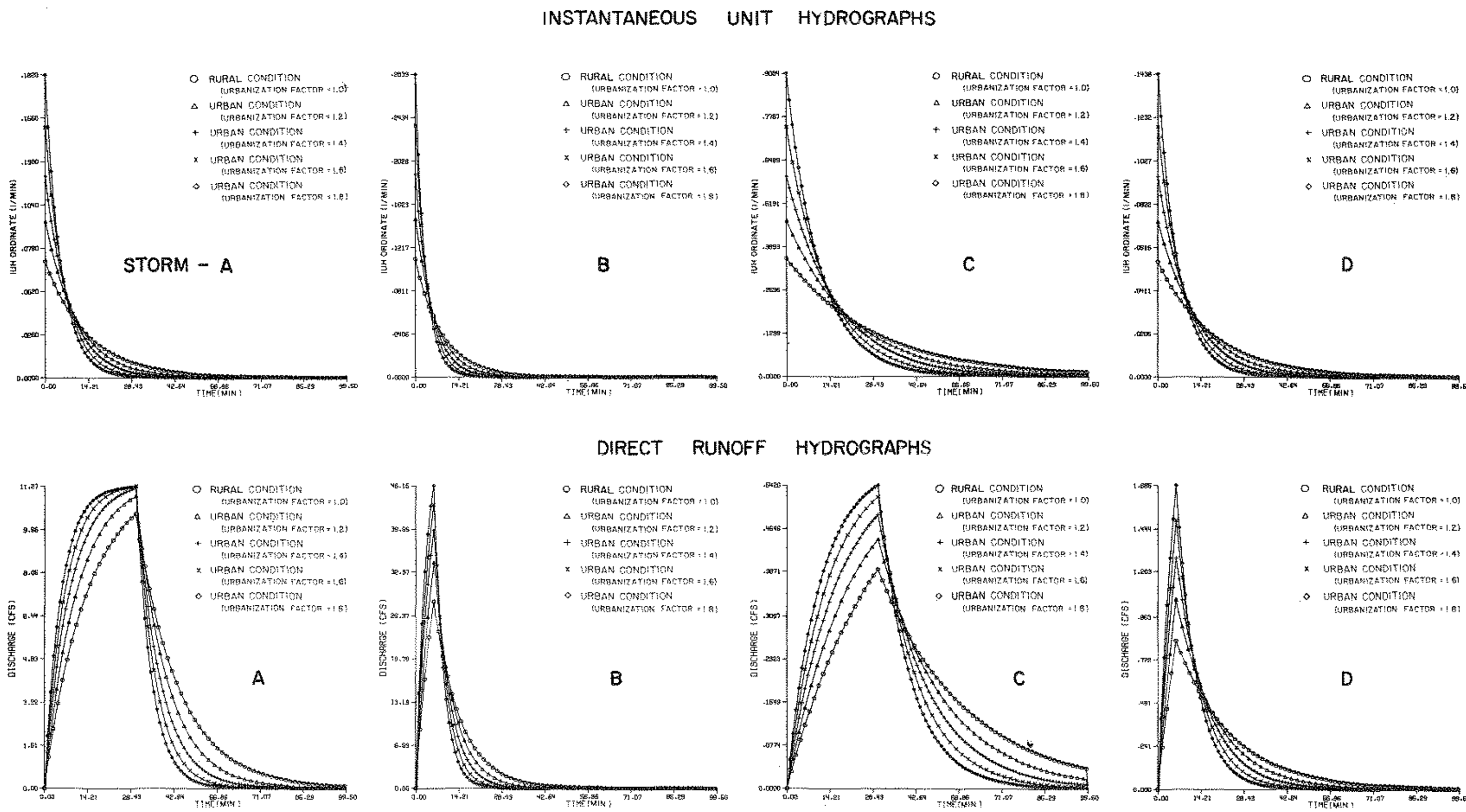


FIGURE 50. THE EFFECTS OF URBANIZATION ON THE SHAPES OF THE IUH AND ON THE DIRECT RUNOFF HYDROGRAPHS (THE SINGLE LINEAR RESERVOIR MODEL – RECTANGULAR EXCESS RAINFALL DISTRIBUTION) WATERSHED I.

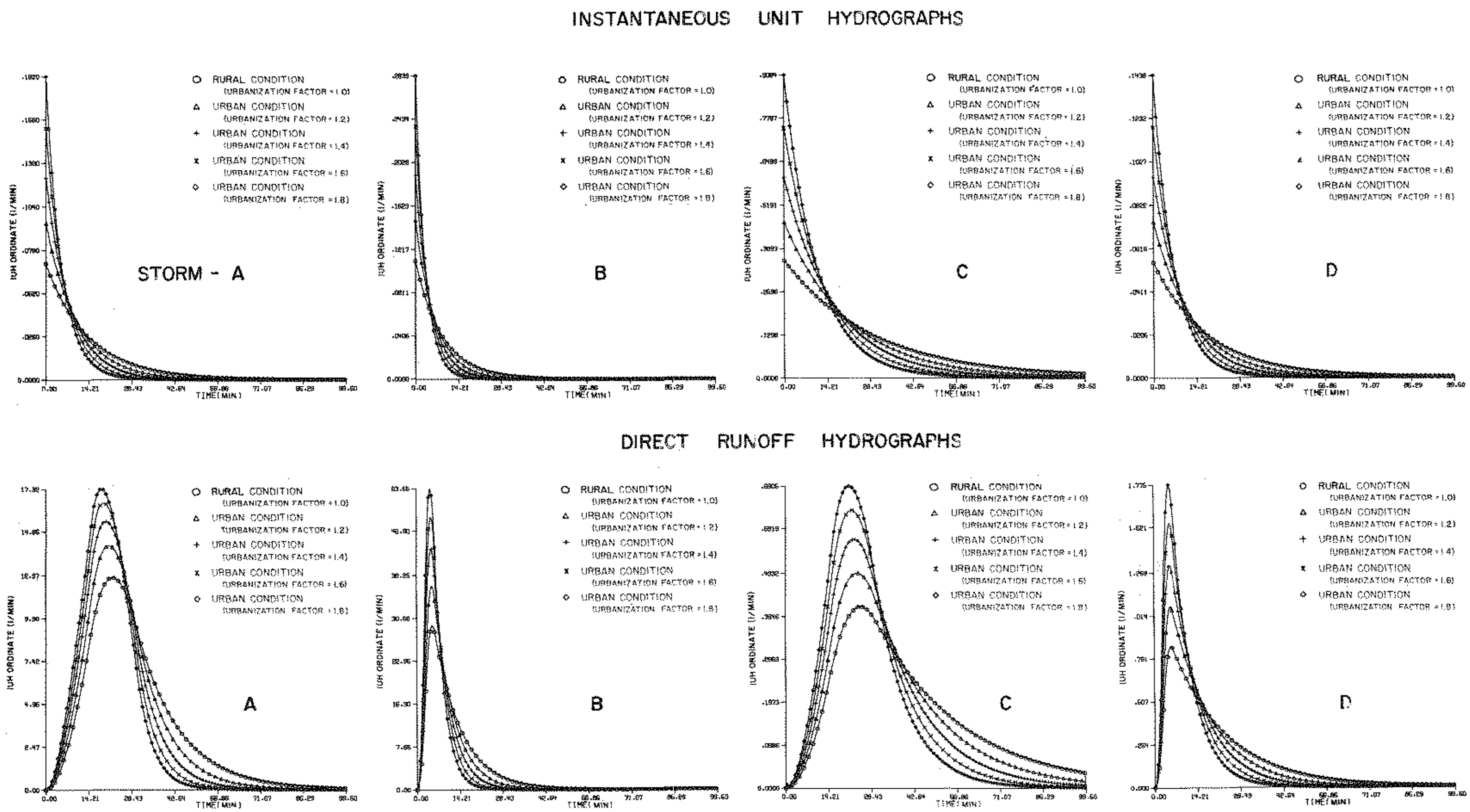


FIGURE 51. THE EFFECTS OF URBANIZATION ON THE SHAPES OF THE IUH AND ON THE DIRECT RUNOFF HYDROGRAPHS (THE SINGLE LINEAR RESERVOIR MODEL– ISOSCELES TRIANGULAR EXCESS RAINFALL DISTRIBUTION) WATERSHED I.

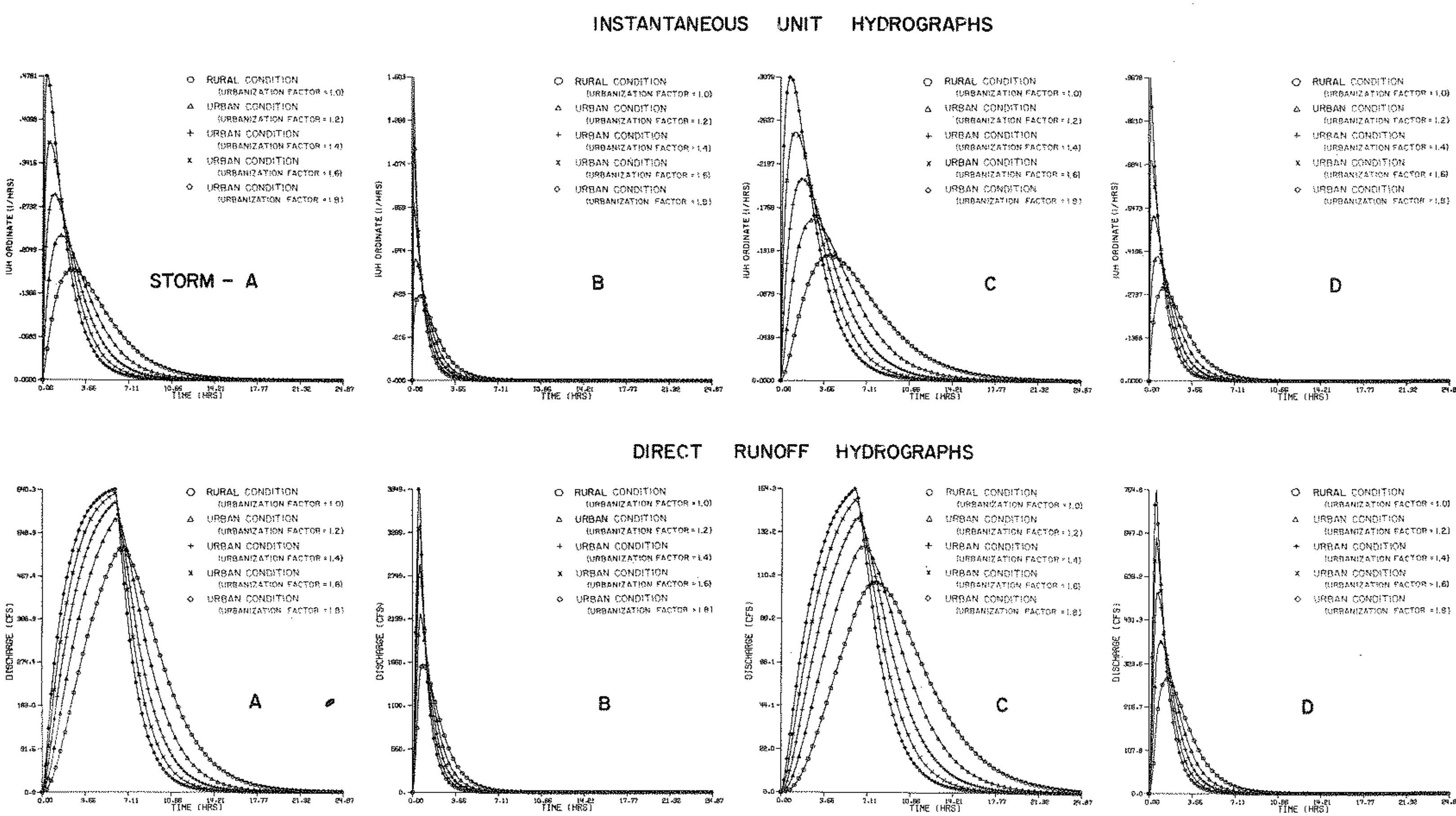


FIGURE 52. THE EFFECTS OF URBANIZATION ON THE SHAPES OF THE IUH AND ON THE DIRECT RUNOFF HYDROGRAPHS (THE NASH MODEL– RECTANGULAR EXCESS RAINFALL DISTRIBUTION) WATERSHED 5.

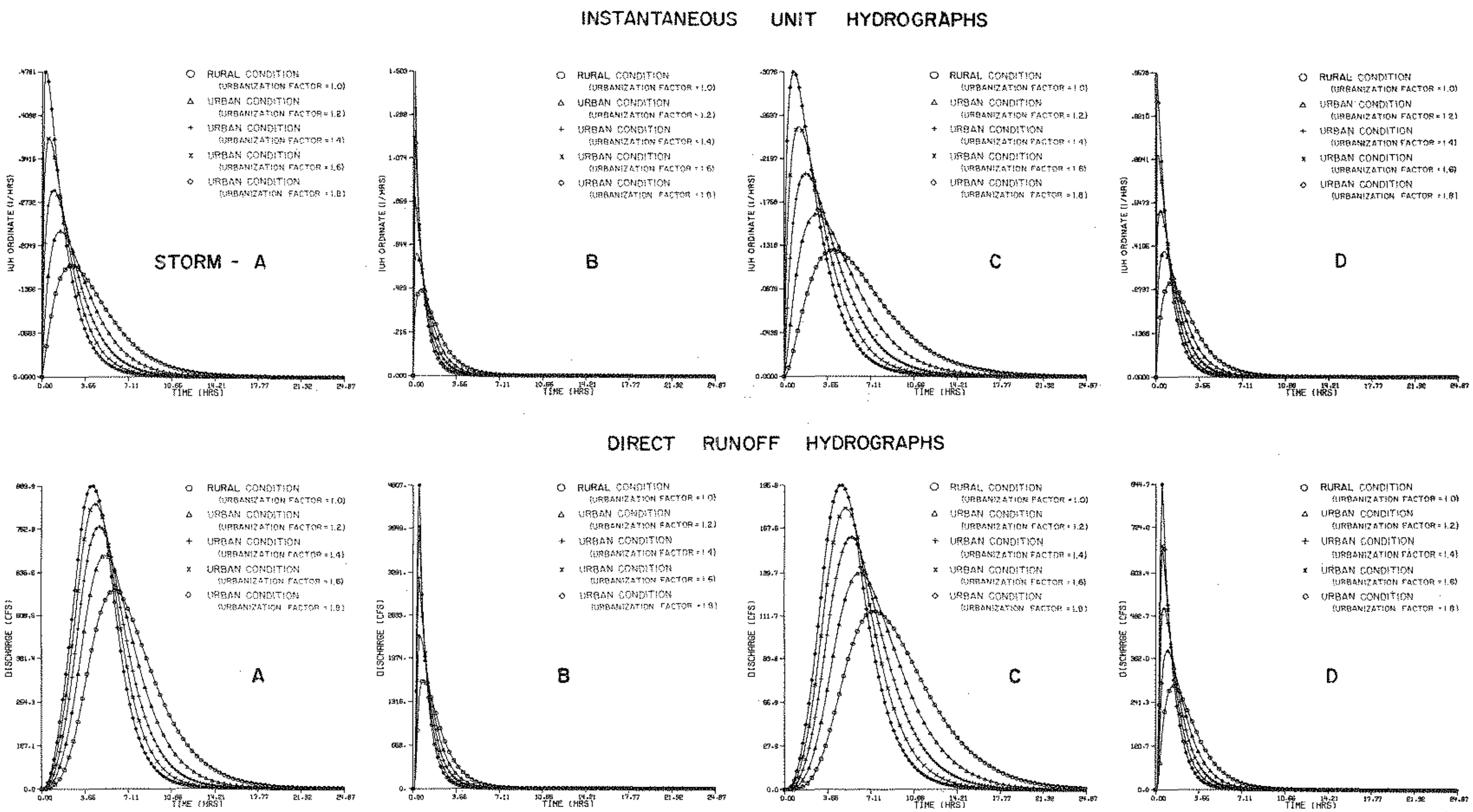


FIGURE 53. THE EFFECTS OF URBANIZATION ON THE SHAPES OF THE IUH AND THE DIRECT RUNOFF HYDROGRAPHS (THE NASH MODEL- ISOSCELES TRIANGULAR EXCESS RAINFALL DISTRIBUTION) WATERSHED 5.

1.60 and 1.80 are presented in Figs. 50, 51, 52 and 53. Besides the increase in the magnitudes of the peaks of the instantaneous unit hydrographs and of the corresponding runoff hydrographs with an increase in urbanization factor, figures 50 to 53 also show that the time distribution of the instantaneous unit hydrographs and of the runoff hydrographs change significantly and that the effective base time decreases with an increase in the urbanization factor.

The time lag T_4, the parameter K_1, the single linear reservoir model and the peak value of the IUH are presented as a function of the urbanization factor in Fig. 54 for watershed 1. The parameters K_N and n of the Nash model, the peak value and time to peak of the IUH are presented as a function of the urbanization factor in Figs. 55 and 56 for watershed 5. Peak discharge and time to peak discharge variations with urbanization factor are shown in Fig. 57 for watershed 5. The results presented for values of urbanization factor beyond 1.40 are obtained by extrapolation of the prediction equations (Eqs. 95, 99, and 103) and hence they should be viewed with caution.

From the above results, it can be observed that for an increase in urbanization factor from 1.0 to 1.40, the time lag decreases by about 43%, the peak discharge increases by about 90% and the time to peak discharge decreases by about 56%. Besides these changes in time lag etc., the time distribution of runoff is also significantly altered as shown in Figs. 50, 51, 52 and 53.

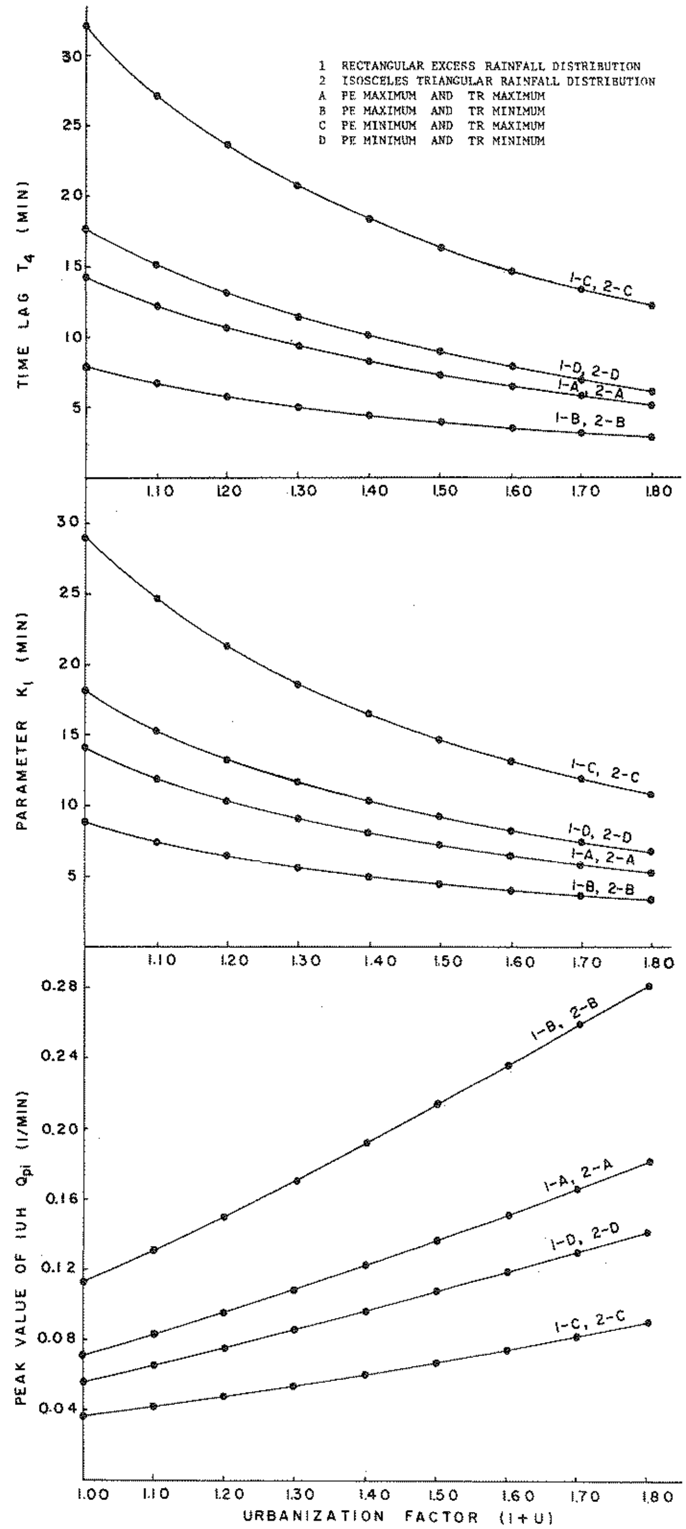


FIGURE 54. VARIATION OF THE TIME LAG T_4, OF THE STORAGE COEFFICIENT K_1, OF THE PEAK VALUE OF THE IUH Q_{pi} WITH THE URBANIZATION FACTOR

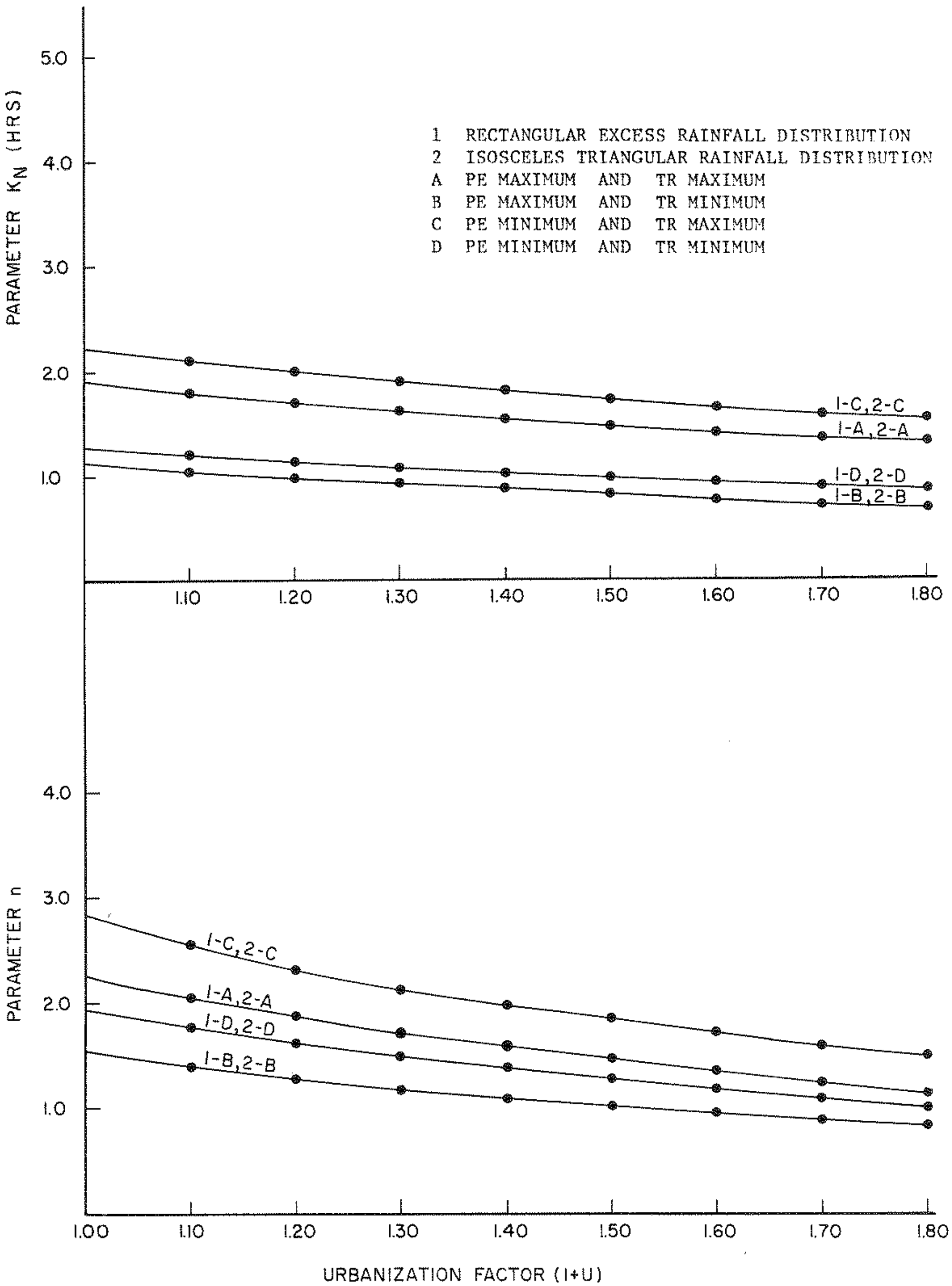


FIGURE 55. VARIATION OF PARAMETERS K_N AND n WITH THE URBANIZATION FACTOR.

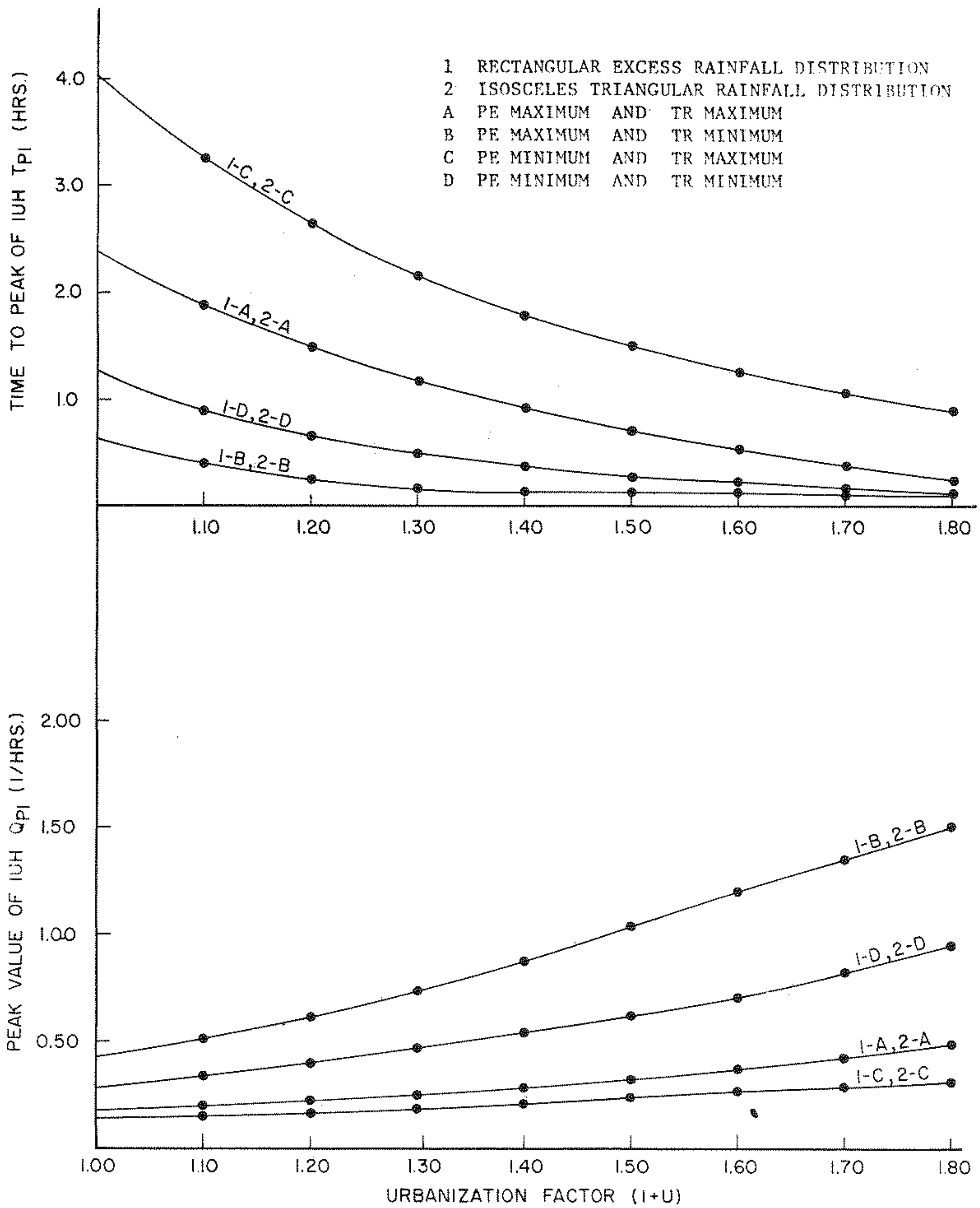


FIGURE 56. VARIATION OF PEAK VALUE AND TIME TO PEAK OF IUH WITH THE URBANIZATION FACTOR.

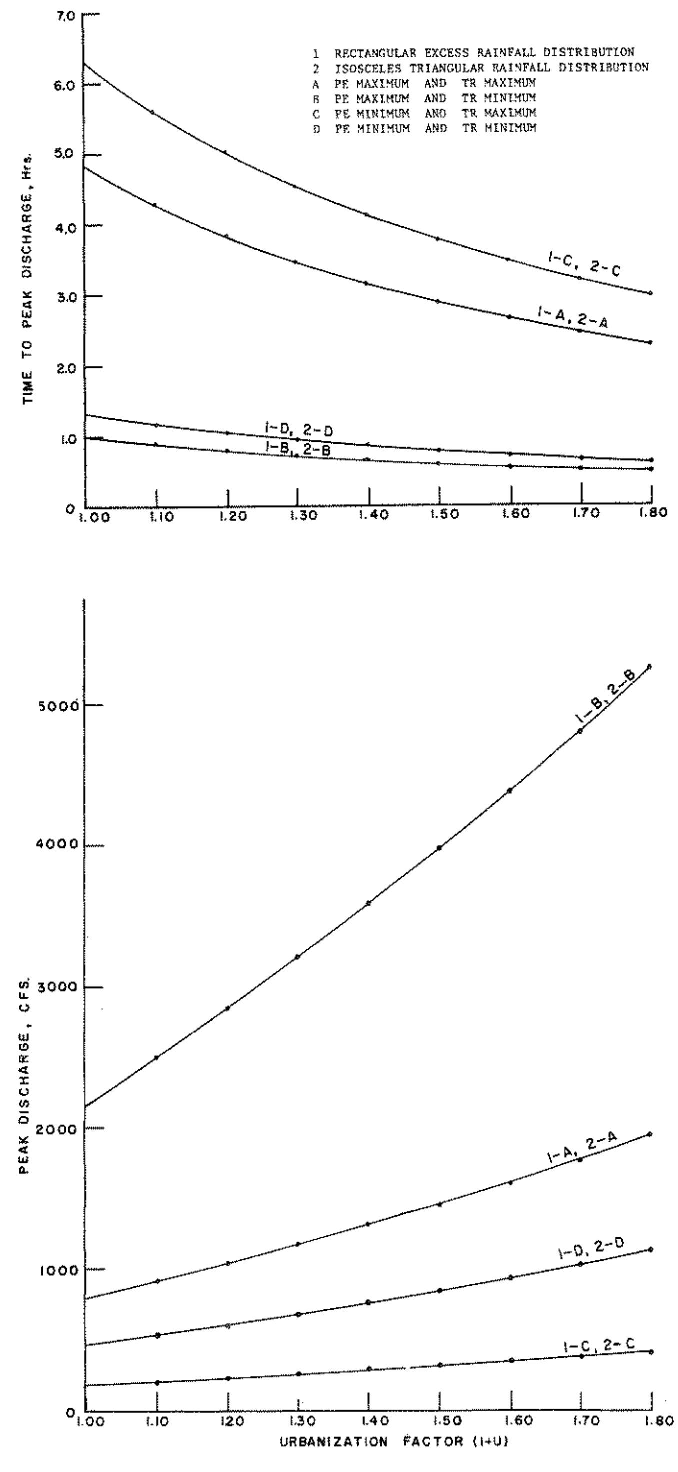


FIGURE 57. VARIATION OF Q_p AND T_p WITH THE URBANIZATION FACTOR (WATERSHED 5)

CHAPTER VII

DISCUSSION AND CONCLUSIONS

Data

Records with good definition, reliability, and of sufficient length are not found for most of the urban watersheds. This situation, however, has been recently recognized and perhaps will be corrected in the near future.[99] The problem of sparsity of data is compounded if the effects of urbanization are to be studied comprehensively.

In general the response of urbanized watersheds to rainfall is faster than that of the rural watersheds and hence the time scales of the data recording machines have to be enlarged to obtain better definition of time distribution of runoff. Available data do not usually have good time definition. In order to obtain better time definition, certain improvements have been made upon the commercially available standard instrumentation, and the improved instruments are used in the gaging stations located in West Lafayette area. As a result of these improvements and modifications, the cumulative rainfall depth records are magnified by a factor of 8. Both the rainfall and discharge are recorded on the same chart which moves at a speed of 1/10 of an inch per minute so that rainfall and discharge values can be read at intervals of 15 seconds. By recording both rainfall and discharge on the same chart, errors due to nonsynchronization of rainfall and runoff data are reduced to a minimum.

Although the location of a single raingage in the watershed and in the same spot as the stream gaging station might not accurately indicate the time of commencement of rainfall in larger watersheds, no such problem has been encountered so far in West Lafayette watersheds. This is probably due to the small areas of these watersheds.

In the present study, paved roads, parking lots, building roofs, foot paths and side walks were considered in a single unit for the definition of urbanization factor. No attempts were made to distinguish among the relative effectiveness of different types of built-up areas in their contribution to changes in watershed response. Thus the urbanization factor may be considered only as an index of the urban development of an area. It is conceivable that the different types of built-up areas may affect the watershed response to different degrees, although, with the present state of knowledge in hydrology it is hard to distinguish such effects. On the other hand, there is the possibility that these differences caused by the various types of built-up areas might be insignificant.

A factor which might be of greater interest and significance than the types of built-up areas is the concentration of these areas in any watershed. If the built-up area is located far from the gaging station its effects on watershed response would be different than if it is located closer to the gaging station. A study of the distribution of these areas and their effect on watershed response may be conducted, at least initially, along the lines of the Area-Stream factor correlation study conducted by the T.V.A.[100]

Occasionally, some difficulties such as the Parshall flumes getting choked up by dirt, the rainfall receiver being filled with dirt by children, failure of electrical clocks, rare and intense storms flooding the instrument house, etc., have been encountered in the operation of gaging stations. These problems are being rectified as far as possible.

Although a few programs for collection of accurate rainfall-runoff data from urban watersheds are underway, hardly any data about soil moisture, infiltration, evaporation and spatial distribution of rainfall in urban watersheds are available. In order to remedy this situation to a certain extent, recently, after the present investigation was well underway, instrumentation to measure rates of evaporation, temperature changes and wind velocities has been set up at one of the West Lafayette stations (Chap. III). Soil moisture data are being collected by the Purdue Agronomy Department from the O'Niel and agronomy farms which are located in the Lafayette area. It is planned to use all this information in the next investigation which will be an extension of the present study. With regard to the spatial variation in rainfall and its possible effects on the response from West Lafayette watersheds, there is only a single raingage in each of the watersheds and consequently the spatial variation of rainfall on any one watershed cannot be estimated. Analysis of records of rainfall from all the four West Lafayette stations should yield the necessary spatial distribution information. However, in view of the small areas of these watersheds, and as the lumped linear systems analysis methods were used in the present study, the spatial distribution of storms was not considered.

Base Flow Separation and Determination of Excess Rainfall

All the existing methods of base flow separation can only be considered as approximate. The problems involved in base flow separation have been rather extensively discussed, most recently by Hall.[84] Although the method of base flow separation adopted in the present study has been known to be approximate, no further work was conducted to study the effects of different methods of base flow separation on the performance of system models used. Such a study would perhaps be worthwhile though not for small, urbanized, sewered watersheds, as base flow may be of no importance in sewered areas.

In a similar vein, the determination of excess rainfall used in the present study has also been approximate. In order to use better methods to determine the excess rainfall, more accurate data about the moisture and temperature conditions in the watershed at the inception of rainfall, variation of the moisture content of the soil as the storm progressed, etc., are necessary. Due to lack of such information the approximate method of obtaining excess rainfall was used. Although these approximations involved in base flow separation and excess rainfall determination were recognized, the same method of base flow separation and excess rainfall determination were used for all the storms for the sake of consistency. Errors involved in these approximations may be small because, as reported by Willeke,[45] the methods of separation of base flow and determination of excess rainfall might not influence the performance of the conceptual models, although the analysis by Fourier transform method seems to be very sensitive to errors in the data.[32]

The hydrograph, as is well known, is a combination of base flow, interflow and surface runoff. The linear system analysis methods must be applied, strictly speaking, to only the surface runoff and the excess rainfall causing the surface runoff. Separation of interflow and surface runoff is a subjective process and the increase in accuracy involved in working only with surface runoff may not be greater.[101] Consequently, no separation of interflow and surface runoff was made, but their combination, called as direct runoff was used in the study. However, some of the variation in K_D can perhaps be explained by contributions of three different types of flow to the recession limb of the hydrograph.

Methods of Analysis

There is an abundance of models available for analysis of the rainfall-runoff process, although the performance of very few of these has been tested under a variety of conditions. Consequently, as no information with regard to the superiority of any particular model was available, four conceptual models along with the general method of obtaining the kernel function by using the Fourier transforms were initially studied. The criterion used for the selection of models to study the effects of urbanization was the regeneration performance of the models. However, the prediction performance of any model is as important as the regeneration performance and hence the models selected for studying the effects of urbanization were also tested for satisfactory prediction performance (Ch. V).

Among the various methods tested, (Table 12) the regeneration performance of the kernel function obtained by the Fourier transform method was obviously the best of all the methods tested for all watersheds (Table 13). However, these response functions exhibited high frequency oscillations in most cases. Attempts to relate the time to peak, magnitude of the peak of the response function to either the geophysical characteristics of the watershed or the storm characteristics were not successful. Consequently attention was concentrated on the use of conceptual models. Nevertheless, the Fourier transform method was useful in the identification of the system models.

The response functions for small urban watersheds (less than 5 sq. miles), were found to be of an exponential decay type, similar in shape to the instantaneous unit hydrographs obtained by using the single linear reservoir model (Fig. 36). This qualitative resemblance of response functions suggests that the use of single linear reservoir model for small (less than 5 sq. miles) urban watersheds might be accurate enough. For watersheds larger than 5 sq. miles the kernel function exhibited the familiar bell shape, thereby suggesting the use of a different model such as Nash's. Using the first and second moments of the response function obtained by the Fourier transform method, the parameters of the Nash model could be accurately estimated (Ch. IV). This observation is of course in accordance with Nash's theory. The same technique might perhaps be useful for estimation of the parameters of any other linear model, provided the parameters of the model are not numerous.

Small Watersheds

When the storage coefficient is assumed to be equal to the time lag the single linear reservoir model yielded good regeneration performance. The regeneration performance was further improved by optimizing the value of the storage coefficient according to two specified criteria (Table 12). The corresponding optimum values of the storage coefficient K_1 and K_2 bear definite relationships with the time lag T_4 (Fig. 23), and they can also be accurately estimated by knowing the physiographic characteristics of the watershed and the storm characteristics (Eqs. 99 and 100). The prediction performance of the single linear reservoir model was quite satisfactory when the estimated value of the parameter K_1 was used for prediction, and compared well with the regeneration performance obtained by using the corresponding optimum value of K_1 (Fig. 42 and Table 21). The estimation of the parameters K_1 and K_2 may perhaps be improved by considering the factors such as antecedent moisture content in the soil prior to the occurrence of the storm, etc., which were not considered in the present study.

As previously mentioned, there is a qualitative similarity between the response functions obtained by the Fourier transform method and the IUH obtained by the single linear reservoir model for the data from watersheds less than 5 sq. miles. This resemblance was apparent for both rural and urban watersheds. Also, the parameter K of the single linear reservoir model itself has a very good regeneration performance which can be improved by optimizing the parameter K. Both the parameter K, and its optimized versions can be accurately estimated by using the physiographic and storm characteristics. Further, for small watersheds the storage-discharge relationship can be considered as being linear (Fig. 22). All these observations strengthen the view that for analysis of rainfall-runoff process on small urban watersheds the single linear reservoir model is quite adequate. This model has been extensively used so far[44,45,46,etc.] in urban hydrology without being supported by a detailed analysis such as that reported in the present study.

Large Watersheds

The regeneration performance of the single linear reservoir model became progressively poorer as the watershed areas increased, whereas the Fourier transform method was very satisfactory for large watersheds also, in spite of the variation in the response function from storm to storm. However, both the above methods were abandoned, the first one because of lack of accuracy, and the second one because of the high frequency oscillations and difficulties experienced in estimating the appropriate response functions.

Of the other models tested, in the double routing method there was considerable ambiguity involved in the selection of an appropriate value of the storage coefficient K_D. There is no unique value of K_D that can be obtained by considering any portion of the recession curve, although the model itself is based on the existence of such a unique value of K_D which could be obtained by considering any part of the recession limb. For larger watersheds, such as those considered by Holtan and Overton,[26,27] the recession limb might perhaps behave like an exponential decay curve and consequently the model might be satisfactory. However, for watersheds of smaller area such as those considered in the present study the exponential decay part starts late in the recession curve. Actually three different regions, corresponding to surface runoff, interflow, and base flow exist in the recession limb and three entirely different values of K_D can consequently be obtained (Fig. 27, Table 7). When these different values of K_D and their average were used in the model, the regeneration performance was unsatisfactory. When the values of K_D were considered to be one half the value of the time lag, a relatively better regeneration was obtained although it was still poor (Figs. 28, 29 and 30).

Apart from this difficulty in the selection of the proper value of K_D, the double routing method has another disadvantage. The double routing method, as the discussion in Chapter IV shows, is nothing but a special case of the Nash model, with n being equal to 2, although this fact is not referred to in the original work.[26,27] Whereas in the Nash model, the values of both n and K can be determined to give a better regeneration, in the double routing method this freedom is lost without any advantages gained. As Nash model was also considered for analysis, and as the double routing method does have the above mentioned difficulties it was not pursued further.

In the analysis using the linear-channel linear-reservoir model the parameter T_c was evaluated by the two methods suggested by Snyder and Clark. The parameter T_c was then used along with time lag T_4 as the storage coefficient of the linear reservoir and tested for regeneration, which was not satisfactory. Although the results were slightly improved when the parameter T_c was optimized such that the sum of the squares of the differences between the corresponding ordinates of the observed and the computed hydrographs was minimized, the regeneration was not as good as the regeneration obtained by using the Nash model. Consequently, the single linear-channel linear-reservoir model was not used in further analysis.

Among the four conceptual models considered for analysis of data from large watersheds, the Nash model yielded relatively better regeneration performance (Table 13). The parameters of the model could also be accurately estimated by using the physiographic and storm characteristics (Eqs. 56, 95 and 103). The prediction performance of the Nash model compared favorably with its regeneration performance. In view of all these considerations the Nash model was selected for further analysis.

In conclusion, it is obvious that the estimation of the response function by the Fourier transform method and its use in prediction is very promising. If some of the disadvantages of this method, which have been previously mentioned, were to be eliminated, it would perhaps be a very powerful method in hydrology. On the other hand, the nonlinear behavior of larger watersheds might be so pronounced that no linear model, however sophisticated it might be, could give accurate results. Another aspect of the problem, not considered in the present study, is the moisture content of the watershed prior to the storm inception, which obviously affects the watershed response. However, analysis of these factors was not undertaken as part of the present study due to lack of time. Some of these aspects will be investigated in a continuation study of the present work. Hence, the single linear reservoir model and the Nash model were selected for the analysis of effects of urbanization on runoff.

Time Lag

Just as the proper choice of a model is important, choice of a proper definition of time lag also deserves careful consideration. The time lag in linear system analysis is defined by the distance (Table 1) between centers of mass of input and output functions. Time lag T_4 is relatively insensitive to errors made in base flow separation (T_1 and T_2 are susceptible to such errors), to other computational errors such as the determination of the point of inflexion on the recession limb of the hydrograph, which is required to evaluate T_C. Furthermore, certain definitions of time lag may give spurious relationships. For example, Askew[101] has noted that the time lag T_4 in his analysis was not related to the temporal variation of rainfall, (and this was confirmed by the present study also), whereas Minshall[76] found such a relationship to exist. This relationship may exist only for the particular definition of time lag T_2 used by Minshall and may not be general. Considering all these factors, T_4 was selected as the time lag to be used in the present study.

An important conclusion about the time lag is that an average, representative value of time lag for a watershed, whether it be a small watershed or a large watershed, leads to very erroneous prediction performance. The time lag values in the present study deviated as much as ±50% from the mean (Table 18) and the prediction of runoff by using the average value will be unsatisfactory (Table 17 and Fig. 39). This observation of variation in time lag which was first made more than 30

The analysis indicated that in general, as a result of urbanization of a watershed, the time lag and the time to peak discharge decrease whereas the magnitude of the peak discharge and the frequency of peak discharge increase. Quantitatively, for a change in urbanization factor from 1.0 to 1.50, the time lag decreases by about 50%, the time to peak discharge decreases by about 66%, and the peak discharge increases by about 84%, and the peak discharge of IUH increases by about 115%, (Figs. 54, 56 and 57). The decrease in time lag reflects the change of shape of the IUH and consequently that of the runoff hydrograph (Figs. 50, 51, 52 and 53). A comparison of the results of the present study with those obtained by the other investigators is presented in Table 30. However, as mentioned earlier (Chapter II) an accurate comparison of qualitative results obtained by various investigators is not possible because of the differences in quantifying the urban development as well as the differences in the definitions of the parameters such as time lag, etc.

Changes in factors such as the value of daily, monthly or yearly runoff, base flow, infiltration, were not evaluated in the present study because of a lack of a continuous record of data for long periods of time. A preliminary investigation of available data indicated that either the length of the record was insufficient to make meaningful analyses, or when longer records were available they were usually incomplete.

Conclusions

1. Adequate accurate data of rainfall, runoff, soil moisture condition, temperature and evaporation are not available for analysis of the

Table 30. Comparison of the Results of the Present Study With Those Obtained by Other Investigators

Investigator	Stage of Urbanization (U) of Watershed	Change in Time Lag Compared to Rural Conditions	Change in Peak Discharge Compared to Rural Conditions
Carter (60)	Partially sewered	-66%	---
	Completely sewered	-85%	up to +80%
Wiitala (63)	U = 25%	-70%	more than +100%
Eagleson (47)	U = 33% to 83%	-49% to 86%	---
Vansickle (65)	Rural to completely sewered	-67% to 92%	200% to 500%
Espey, et.al. (66)	U = 27% **	-47%	+ 6%
	U = 37% **	-46%	+ 51%
Riley, et.al. (69)	U = 27% **	- 7%	+ 5%
	U = 37% **	-10%	+ 29%
Stall and Smith (75)	U = 38.1% **	-80%	+100%
Leopold (95)	U = 20%	---	+ 50%
	U = 50%	---	+170%
present study*	watershed 5, U = 30%	-35%	+15% to + 68%
	watershed 5, U = 50%	-49%	+20% to +118%
	watershed 1, U = 30%	-35%	+ 8% to + 42%
	watershed 1, U = 50%	-49%	+10% to + 70%

* See Table Nos. 28 and 29.

** These values are obtained by analysis of unit hydrographs.

rainfall-runoff process in urban watersheds. A well-integrated nation-wide program to remedy this situation would not only further the understanding of urban rainfall-runoff process, but also would affect considerable monetary savings for the agencies which are involved in urban drainage development programs.

2. The linear system methods can be usefully employed in the analysis of effects of urbanization on runoff. The methods of linear system analysis are relatively more accurate for small watersheds (up to about 5 sq. miles).

3. There is neither a unique unit hydrograph nor an instantaneous unit hydrograph applicable for any watershed. Rainfall-runoff process on small, urbanized watersheds can be modelled by using a linear time invariant model, although the parameter variation with storms must be considered. However, for watersheds larger than those used in the present study, the nonlinearities may be strong enough as to require the use of nonlinear models.

4. The Fourier transform method gave the best regeneration performance. Because of certain disadvantages it could not be used directly to examine the effects of urbanization on runoff.

5. For small watersheds (less than 5 sq. miles), the response function computed by the Fourier transform method and the instantaneous unit hydrograph obtained by the single linear reservoir model were similar. This similarity supports the use of the single linear reservoir model in small urban watersheds.

6. The regeneration performance of the single linear reservoir model for small watershed data was good. The parameter K in the single

linear reservoir model can also be estimated accurately and used for prediction.

7. Although the storage coefficient K of the single linear reservoir model is theoretically the same as the time lag T_4, the results of the regeneration and prediction can be improved by optimizing the value of the storage coefficient according to prescribed criteria (Chap. IV). In view of these observations, the use of the single linear reservoir model is recommended for modelling the rainfall-runoff process in small urban watersheds.

8. For watersheds of area larger than five square miles, of all the conceptual models tested, the Nash model gave the best regeneration performance and hence was selected to study the effects of urbanization on runoff on large watersheds. The double routing model, which is a special case of Nash model, and the linear-reservoir linear-channel model and the single linear reservoir model were considered to be unsatisfactory.

9. For the data tested, the hypothesis of determining the recession constant K and its use in routing in the double routing method was found to be invalid. This was due to the variation in the value of K with different parts of the recession curve.

10. Use of an average, representative value of time lag for a watershed leads to erroneous prediction performance. The time lag values for any watershed vary from storm to storm and are strongly correlated with some of the physiographic characteristics including urbanization factor, and also with the storm characteristics.

11. The analysis indicated that the time lag and the time to peak discharge decrease, whereas the magnitude of the peak discharge and the frequency of peak discharge increase with urbanization.

12. The variations in the IUH due to urbanization, as well as in the more important runoff characteristics can be quantitatively estimated by the proper equations developed.

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APPENDICES

APPENDIX A-1

STATISTICAL MEASURES

If a linear relationship between two variables X and Y is assumed, the corresponding linear correlation coefficient is defined by,

$$R = \frac{\sum_{i=1}^{N} X_i Y_i - \left(\sum_{i=1}^{N} X_i\right)\left(\sum_{i=1}^{N} Y_i\right)}{\left\{\left[N \sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2\right]\left[N \sum_{i=1}^{N} Y_i^2 - \left(\sum_{i=1}^{N} Y_i\right)^2\right]\right\}^{1/2}} \qquad \text{(A-1-1)}$$

where N is the number of observations of X and Y. The linear correlation coefficient (R) has the following properties:

i) $-1 \leq R \leq +1$

ii) The closer is the value of R to either +1 or -1, the better is the agreement between the two variables for the assumed linear relationship.

iii) A value of R closer to zero indicates that the two variables are uncorrelated.

Another measure of agreement between the known variable X and its estimated value Y can be defined in terms of the sum of the squares of their derivation, or

$$\sum_{i=1}^{N} (X_i - Y_i)^2$$

As $\sum_{i=1}^{N} (X_i - Y_i)^2$ is not negative,

$$\sum_{i=1}^{N} (X_i^2 + Y_i^2 - 2 X_i Y_i) \geq 0 \qquad \text{(A-1-2)}$$

$$\left(\sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} Y_i^2 - 2 \sum_{i=1}^{N} X_i Y_i \right) \geq 0$$

$$\left(\sum_{i=1}^{N} X_i^2 \right) \geq 2 \sum_{i=1}^{N} X_i Y_i - \sum_{i=1}^{N} Y_i^2$$

i.e.

$$1 \geq \frac{2 \sum_{i=1}^{N} X_i Y_i - \sum_{i=1}^{N} Y_i^2}{\sum_{i=1}^{N} X_i^2} \qquad \text{(A-1-3)}$$

$$\left[\frac{2 \sum_{i=1}^{N} X_i Y_i - \sum_{i=1}^{N} Y_i^2}{\sum_{i=1}^{N} X_i^2} \right] \leq 1 \qquad \text{(A-1-4)}$$

The expression on the left hand side of the inequality A-1-4 is referred to as a "Special Correlation Coefficient and is denoted by R_s.

$$R_s = \frac{2 \sum_{i=1}^{N} X_i Y_i - \sum_{i=1}^{N} Y_i^2}{\sum_{i=1}^{N} X_i^2} \qquad \text{(A-1-5)}$$

The Special Correlation Coefficient (R_s) has the following properties:

i) $R_s \leq +1$

ii) $R_s = +1$ if ($X_i = Y_i$ for $i=1, N$)

iii) $R_s = 0$ if $Y_i = 2 X_i$

By comparing the linear correlation coefficient R and the special correlation coefficient R_s, it may be observed that, 1) The Special Correlation Coefficient R_s is similar to the linear correlation coefficient

R, in that the closer is the value of R_s to +1 the better is the agreement between the observed and the estimated values, 2) the Special Correlation Coefficient does not have the property of invariance under change of scale and location, and 3) the distribution of R_s is unknown and hence the test of significance of the value of R_s cannot be performed. Thus, although R_s is not a correlation coefficient in the usual sense, it still can be used as a measure of agreement between the observed and estimated values of a variable.

The integral square error is another statistical measure which describes the agreement between the time distribution of the observed and the estimated values of a variable. The smaller is the value of the Integral Square Error (Eq. 47), the better is the agreement between the observed and the estimated values of a variable.

To compare the relative performance of each model, the ratings which have been assigned based on the values of these statistical measures are presented in Table A-1-1.

Table A-1-1. Ratings of the Statistical Measures

Correlation Coefficient (R)	Rating
$0.99 \leq R < 1.0$	Excellent
$0.95 \leq R < 0.99$	Very Good
$0.90 \leq R < 0.95$	Good
$0.85 \leq R < 0.90$	Fair
$0.00 \leq R < 0.85$	Poor
Special Correlation Coefficient (R_s)	
$0.99 \leq R_s < 1.0$	Excellent
$0.95 \leq R_s < 0.99$	Very Good
$0.90 \leq R_s < 0.95$	Good
$0.85 \leq R_s < 0.90$	Fair
$0.00 \leq R_s < 0.85$	Poor
Integral Square Error (ISE)	
$0\% < ISE \leq 3.0\%$	Excellent
$3.0\% < ISE \leq 6.0\%$	Very Good
$6.0\% < ISE \leq 10.0\%$	Good
$10.0\% < ISE \leq 25.0\%$	Fair
$25.0\% < ISE$	Poor

APPENDIX A-2

MULTIPLE REGRESSION ANALYSIS

Let a variable Y be a function of several independent variables $X_1, X_2, X_3, \ldots X_k$. The method of obtaining a relationship between Y and the variables $X_1, X_2, \ldots X_k$, such that the sum of the squares of the differences between the values of Y and its estimated values obtained by any assumed relationship is minimum, is called the multiple regression analysis. If the relationship assumed between Y and the independent variables $X_1, X_2, \ldots X_k$, is linear, then the regression analysis is said to be linear regression analysis. For example, let $\hat{Y}$ be the estimated value of Y obtained by assuming a linear relationship such as

$$\hat{Y} = C_0 + C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_kX_k \qquad \text{(A-2-1)}$$

where C_i, $(i=0, 1, 2, \ldots k)$ are the regression coefficients. Then, by multiple linear regression analysis it is desired to obtain values for the coefficients C_i, $(i=0, 1, 2, \ldots k)$. The method of least squares, by using which the quantity $\sum_{i=1}^{N} (Y(i)-\hat{Y}(i))^2$ is minimized, is employed to evaluate the coefficients C_i, $(i=0, 1, 2, \ldots k)$. Thus, it is required to minimize the quantity:

$$\phi^2 = \sum_{i=1}^{N} \left(Y(i)-\hat{Y}(i)\right)^2$$

$$= \sum_{i=1}^{N} \left[Y(i)-\left(C_0+C_1X_1(i)+C_2X_2(i)+\ldots+C_kX_k(i)\right)\right]^2 \qquad \text{(A-2-2)}$$

Taking the partial derivatives of ϕ with respect to the coefficients C_i,(i=0, 1, 2,...k) and equating the resulting expressions to zero, the following normal equations result:

$$\sum_{i=1}^{N} 2\left[Y(i) - \left(C_0+C_1X_1(i)+C_2X_2(i)+\ldots+C_kX_k(i)\right)\right](-1) = 0$$

$$\sum_{i=1}^{N} 2\left[Y(i) - \left(C_0+C_1X_1(i)+C_2X_2(i)+\ldots+C_kX_k(i)\right)\right](-X_1(i)) = 0 \quad \text{(A-2-3)}$$

$$\sum_{i=1}^{N} 2\left[Y(i) - \left(C_0+C_1X_1(i)+C_2X_2(i)+\ldots+C_kX_k(i)\right)\right](-X_k(i)) = 0$$

The solution of the set of these (k+1) simultaneous equations gives the values of the coefficients C_i,(i=0, 1, 2,...k). Thus the values of the coefficients C_0, C_1, C_2,...C_k of the assumed linear relationship (Eq. A-2-1) can be obtained.

If the assumed relationship (Eq. A-2-1) is nonlinear, then in some cases the relationship can be linearized. For example, let

$$\hat{Y} = C_0 X_1^{C_1} X_2^{C_2} X_3^{C_3} X_4^{C_4} \ldots X_k^{C_k} \quad \text{(A-2-4)}$$

A relationship of the form (A-2-4) is also referred to as a "power function model". Taking logarithms on either side of the Eq. A-2-4,

$$\ln \hat{Y} = \ln C_0+C_1 \ln X_1+C_2 \ln X_2+\ldots+C_k \ln X_k \quad \text{(A-2-5)}$$

Equation (A-2-5) is a linear equation of the form of Eq. A-2-1. Hence the method of linear regression can be applied to Eq. A-2-5, as illustrated above.

The results of the regression analysis are characterized by the following statistics:

1) Linear Correlation Coefficient (R)

$$R = \frac{\sum_{i=1}^{N} X(i)Y(i) - \sum_{i=1}^{N} X(i) \sum_{i=1}^{N} Y(i)}{\left[\left[N \sum_{i=1}^{N} X(i)^2 - \left(\sum_{i=1}^{N} X_i\right)^2\right]\left[N \sum_{i=1}^{N} (Y_i)^2 - \left(\sum_{i=1}^{N} Y(i)\right)^2\right]\right]^{1/2}}$$

2) Standard Error of Estimate: SER

$$SER = \frac{\sum_{i=1}^{N} \left(Y(i) - \hat{Y}(i)\right)^2}{N}$$

3) Coefficient of Determination = R^2

APPENDIX A-3

PREDICTION INTERVAL

(The following material follows closely from the Ref. 105 pp. 253-255)

It is often the case, that a confidence statement about the mean value is less important, whereas a probability statement about a future observation is relevant.

Let Y be the predicted value which corresponds to an observed value X, obtained by using the prediction equation of the form Y = A + BX, then the following statement can be made: The probability is $1 - \alpha$ that a future observation Y* corresponding to X* will lie in the "prediction interval"

$$A + BX^* \pm t_{\alpha/2;N-2}\, S_{Y/X} \sqrt{1 + \frac{1}{N} + \frac{(X^* - \bar{X})^2}{\sum_{i=1}^{N} (X_i - \bar{X})^2}}$$

where $t_{\alpha/2;N-2}$ is the t-distribution value, which can be obtained from standard books on statistics,

$S_{Y/X}$ is the standard error of estimate,

$\bar{X}$ is the mean value of X_i,

and N is the number of observations.

The expression for the prediction interval can be obtained from the distribution of the quantity $(Y^* - \hat{Y}^*)$, where Y* is the future observation which is assumed to be normally distributed with mean A + BX* and variance σ^2, whereas $\hat{Y}*$ is the point on the fitted line corresponding to $X = X*$, and $\hat{Y}*$ can be assumed to be normally distributed[105] with mean $A + BX*$ and variance

$$\gamma = \sigma^2 \left[\frac{1}{N} + \frac{(X* - \bar{X})^2}{\sum_{i=1}^{N} (X_i - \bar{X})^2} \right]$$

Further, $Y*$ and $\hat{Y}*$ can be considered to be independent because since $\hat{Y}*$ is derived from the N observed values while $Y*$ corresponds to a future observation. Hence $Y* - \hat{Y}*$ is normally distributed with mean zero and variance

$$\gamma' = \gamma + \sigma^2$$

Since $(N-2)\ S^2_{Y/X}/\sigma^2$ has a Chi-Square distribution with (N-2) degrees of freedom and is distributed independently of $Y* - \hat{Y}*$,

$$\frac{Y* - \hat{Y}*}{S_{Y/X}\sqrt{\gamma'}}$$

has a t-distribution with (N-2) degrees of freedom. It follows that

$$\text{Prob}\left(- t_{\alpha/2;N-2} \leq \frac{Y* - \hat{Y}*}{S_{Y/X}\sqrt{\gamma'}} \leq t_{\alpha/2;N-2} \right) = 1 - \alpha$$

or

$$\text{Prob}\left(A + BX* - t_{\alpha/2;N-2}\ S_{Y/X}\sqrt{\gamma'} \leq Y* \leq A + BX* + t_{\alpha/2;N-2}\ S_{Y/X}\sqrt{\gamma'} \right) = 1 - \alpha$$