Active Ground Vehicle Control with use of a Super-Twisting Algorithm in the Presence of Parameter Variations

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***Abstract*— In this paper, we consider a vehicle equipped with active front steer and rear torque vectoring. While the former adds an incremental steer angle to the driver's input, the latter imposes a torque by means of the rear axle. The active front steer control is actuated through the front tires, while the rear torque vectoring can be actuated through the rear tires. A nonlinear controller using the super–twisting algorithm is designed in order to track in finite time lateral and yaw angular velocity references. We consider one estimation method, given in the literature, to estimate the tire–road coefficient, and we design a dynamic controller to estimate the perturbing terms due to the vehicle's mass and inertia variations, and due to the variation of the tire parameters. Comparisons with a simple PI–based controller are done, and some simulation results highlight the advantages of the proposed controller.**

I. Introduction

Active control actions are important tools for increasing the vehicle safety. These actions impose forces and torques to the vehicle in order to track a (feasible and safe) reference behavior, reducing the number of accidents. This was made possible mainly thanks to the wide use of electronic devices, which allow the control of a number of functions. In this paper we consider a rear–wheel drive vehicle equipped with Active Front Steering (AFS) and Rear Torque Vectoring (RTV) devices. The AFS provides an additional steering angle over the driver steering angle, while the RTV gives an asymmetric left/right wheel torque on the rear axle. The tire road friction coefficient estimation is crucial to have an efficient vehicle control [2], [9], [22], [33], [34]. Unfortunately, parametric estimation is not always accurate or even possible and, therefore, proposed controllers must have some robustness properties with respect to those uncertainties. Various type of tyre models can be considered here, see e.g. [11], [28] and reference therein. For the sake of clearness, in the following we will consider the popular Pacejka's model [29], although the same approach applies to other models, such as the Burckhard's one [11], [36].

In this paper, the tire–road friction coefficient and the parameter of the Pacejka's formula are assumed not perfectly known, as well as the vehicle's mass and inertia. Hence, it is important to design a controller which is robust with respect to those parametric uncertainties. In this regards, the sliding mode techniques may provide effective solutions. A formal method to design first order sliding mode controllers was given in [15], [38]. High Order Sliding Mode (HOSM) controllers were introduced in [17]. Since this seminal work, many papers have dealt with such technique [16], [4], [20], [19], [26], [21], [27]. HOSM techniques have been used first for smooth control systems and, more recently, for finite time differentiators. In [1] an interesting comparison of HOSM differentiator and high gain observer with respect to noise has been done. Finally, in [23] HOSM is used to solve vehicle control problems.

We use the sliding mode techniques to estimate the terms arising from parameter variations. More precisely, in this work it is proposed a second order sliding mode algorithm, the well–known Super–Twisting (ST) algorithm, for tracking a reference trajectory in the presence of parameter variations. In [22], [6], [8] an estimation of the product between the tire–road friction coefficient and the tire stiffness coefficient was proposed. In this paper we consider this parameter estimated, and we want to estimate the perturbing terms arising from the variation of the remaining parameters, namely those appearing in the tire model and the vehicle's mass and inertia. We will show that the proposed algorithm provides good results with respect to measurement noise and parametric uncertainties. To test the obtained results, we will compare the ST–based controller with a simpler one, based on a PI.

The paper is organized as follows. In Section II, the mathematical model of a vehicle is recalled, and the control problem is set. In Section III, a simple nonlinear controller is recalled. In Section IV, a ST–based controller is presented, and some observations on the transient behavior are given. In Section V, the proposed controller is tested with simulations. Some comments conclude the paper.

II. Mathematical model and problem formulation

We consider a vehicle equipped with AFS, which add an incremental steer angle δ_c on top to the driver's input δ_d, and RTV, which imposes a torque M_z by means of the rear axle. Since, we consider a rear–wheel drive vehicle, these two actions are decoupled. In fact, the AFS control is actuated through the front tires, while the RTV is actuated through the rear tires.

The complex vehicle model, having 6 degrees of freedom, is approximated for the control purposes by a simple "bicycle" model, representing the essential dynamics of interest [7]

$$\begin{aligned} m(\dot v_x - v_y\omega_z) &= 0\\ m(\dot v_y + v_x\omega_z) &= \mu(F_{yf} + F_{yr})\\ J_z\dot\omega_z &= \mu(F_{yf}l_f - F_{yr}l_r) + M_z \end{aligned} \tag{1}$$

where m, J_z are the vehicle mass and inertia momentum, l_f, l_r are the front and rear vehicle length, v_x, v_y are the longitudinal, lateral velocities of the vehicle center of mass,

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and ω_z is the yaw rate. Moreover, μ is the maximum tire–road friction coefficient, M_z is the RTV moment, F_{yf}, F_{yr} are the tire front and rear lateral forces, normalized with respect to μ. This model is considered acceptable under the following simplifying hypotheses

i. motion on a horizontal plane;
ii. constant longitudinal velocity;
iii. absence of shaking/pitch and roll (stiff suspensions) motions;
iv. rigidity of the steering system;
v. negligible mass of wheels;
vi. small front wheel angles (less than 10°);
vii. negligible aerodynamic resistance and absence of lateral wind;
viii. constant tire vertical loads;
ix. ideal active actuators (saturation have been taken into account in simulations).

This model is very used in the literature and in the industry applications [7] since, despite its simplicity, it well captures the real vehicle major characteristics, such as the steady state and dynamic responses of the yaw rate, lateral acceleration, lateral velocity.

The front/rear lateral forces

$$F_{yf} = F_{yf}(\alpha_f), \quad F_{yr} = F_{yr}(\alpha_r) \tag{2}$$

depend on the front/rear tire slip angles (rad)

$$\alpha_f = \delta - \frac{v_y + l_f\omega_z}{v_x}, \quad \alpha_r = -\frac{v_y - l_r\omega_z}{v_x}$$

with $\delta = \delta_d + \delta_c$ the road wheel angle (rad), sum of the driver angle δ_d (rad) and the AFS angle δ_c (rad). The driver angle δ_d is assumed at least continuously differentiable with respect to time.

Various type of tyre models can be considered, see e.g. [14], [28] and reference therein. For the sake of clearness, here the popular Pacejka model is considered [29], although the same approach applies to other models (e.g. the Burckhard model, just to mention one, see [14])

$$\begin{aligned} F_{yj}(\alpha_j) &= D_j\varphi_j(\alpha_j) \\ \varphi_j(\alpha_j) &= \sin\Big[C_j \arctan\Big(B_j\alpha_j \\ &\qquad - E_j\big(B_j\alpha_j - \arctan(B_j\alpha_j)\big)\Big)\Big] \end{aligned} \tag{3}$$

$j = f, r$, with $\varphi_j(\alpha_j)$ the normalized lateral force, and B_j the stiffness factor, C_j the shape factor, D_j the peak factor, and E_j the curvature factor (see [29] for further details). These experimental parameters are often not perfectly known in practice, and are of high importance for the determination of the correct control action to be exerted. Usually, they will be approximated by their nominal values B_{j0}, C_{j0}, D_{j0}, E_{j0}, frequently given by the manufacturer or obtained by parametric identification. The nominal normalized lateral force is hence

$$\begin{aligned} \varphi_{j0}(\alpha_j) = \sin\Big[C_{j0} \arctan\Big(&B_{j0}\alpha_j \\ &- E_{j0}\big(B_{j0}\alpha_j - \arctan(B_{j0}\alpha_j)\big)\Big)\Big] \end{aligned}$$

$j = f, r$. It is clear that this approximation impacts the controller value and, therefore, the real trajectory. Hence, a more accurate determination of these tire parameters is desirable, and in this paper this aspect will be considered.

The normalized tire characteristic φ_j is an odd function of the slip angle α_j which, in an interval $[-\alpha_{j,\max}, \alpha_{j,\max}]$, increases linearly with α_j from the minimum $\varphi_j(-\alpha_{j,\max}) = -\varphi_{j,\max}$ until a maximum $\varphi_j(\alpha_{j,\max}) = \varphi_{j,\max}$, while decreases for $\alpha_j < -\alpha_{j,\max}$ and $\alpha_j > \alpha_{j,\max}$, with asymptotic values for high (negative and positive) slip angles. Since between the minimum and the maximum the function φ_f is invertible, for the sake of simplicity one considers as AFS control the difference [7]

$$\Delta_c = \varphi_f(\alpha_f) - \varphi_f(\alpha_{f0}), \quad \alpha_{f0} = \delta_d - \frac{v_y + l_f\omega_z}{v_x}. \tag{4}$$

Once a control value $\bar{\Delta}_c$ is computed, the real control δ_c can be determined as follows

$$\delta_c = \begin{cases} -\delta_d + \dfrac{v_y + l_f\omega_z}{v_x} + \varphi_f^{-1}(\varphi^\circ) & \text{if } |\varphi^\circ| \le \varphi_{f,\max} \\ -\delta_d + \dfrac{v_y + l_f\omega_z}{v_x} + \alpha_{f,\max} & \text{otherwise} \end{cases} \tag{5}$$

with $\varphi^\circ = \bar{\Delta}_c + \varphi_f(\alpha_{f0})$ the value assumed by $\varphi_f(\alpha_f)$, namely inverting the function φ_f up to the tire maximum point $\alpha_{f,\max}$, and saturating the inverse function elsewhere. This allows avoiding the mathematical complications arising from the fact that the input δ_c appears inside a (possibly quite complicated) function. In any case, with (5) the real input δ_c can be easily retrieved.

With the convention (4), equations (1) can be rewritten in the form

$$\begin{aligned} \dot{v}_x &= v_y\omega_z \\ \dot{v}_y &= -v_x\omega_z + \frac{1}{m}\Big(\theta_f\varphi_f(\alpha_{f0}) + \theta_r\varphi_r(\alpha_r)\Big) + \frac{\theta_f}{m}\Delta_c \\ \dot{\omega}_z &= \frac{1}{J_z}\Big(\theta_f\varphi_f(\alpha_{f0})l_f - \theta_r\varphi_r(\alpha_r)l_r\Big) + \frac{\theta_f l_f}{J_z}\Delta_c + \frac{1}{J_z}M_z \end{aligned} \tag{6}$$

where

$$\theta_f = \mu D_f, \qquad \theta_r = \mu D_r \tag{7}$$

are the products of the tire–road friction coefficient with the tire stiffness coefficients.

A classical *control problem* is to determine Δ_c, M_z so that the following tracking errors [6], [8]

$$e_{v_y} = v_y - v_{y,\text{ref}}, \qquad e_{\omega_z} = \omega_z - \omega_{z,\text{ref}} \tag{8}$$

tend to zero in the presence of variations of the Pacejka's parameters B_j, C_j, E_j, $j = f, r$ and of m, J_z. Here, $v_{y,\text{ref}}$, $\omega_{z,\text{ref}}$ are acceptable signals of the lateral and yaw velocities. To generate these signals, it is possible to introduce the dynamics of a "reference vehicle", with $v_{x,\text{ref}} = v_x$ and

$$\begin{aligned} \dot{v}_{y,\text{ref}} &= -v_x\omega_{z,\text{ref}} + \frac{1}{m}\Big(\theta_f\varphi_{f,\text{ref}}(\alpha_{f0,\text{ref}}) \\ &\qquad\qquad + \theta_r\varphi_{r,\text{ref}}(\alpha_{r,\text{ref}})\Big) \\ \dot{\omega}_{z,\text{ref}} &= \frac{1}{J_z}\Big(\theta_f\varphi_{f,\text{ref}}(\alpha_{f0,\text{ref}})l_f - \theta_r\varphi_{r,\text{ref}}(\alpha_{r,\text{ref}})l_r\Big). \end{aligned} \tag{9}$$

Usually $F_{yf,\text{ref}}$, $F_{yr,\text{ref}}$ can be taken to resemble the real characteristics, eliminating the "decreasing part" of the curves which could give unexpected behaviors for the driver. Hence, they can be chosen as

$$\begin{aligned} F_{yf,\text{ref}}(\alpha_{f0,\text{ref}}) &= D_f\varphi_{f,\text{ref}}(\alpha_{f0,\text{ref}}) \\ F_{yr,\text{ref}}(\alpha_{r,\text{ref}}) &= D_r\varphi_{r,\text{ref}}(\alpha_{r,\text{ref}}) \end{aligned}$$

with $\varphi_{j,\text{ref}}$ strictly increasing functions.

Note in (9) the presence of the tire–road friction coefficient μ, through θ_f, θ_r, also used in (6). In fact, to avoid to impose behaviors which could results to be impossible to track, due to the finite lateral force that can be exerted by the tires, the "reference" coefficients θ_f, θ_r should be equal to the real one. As explained in the introduction, we assume that this estimation problem has been already solved (e.g. as in [8]).

III. Recalls of Nonlinear Global Stabilization

In this section, following [8], a simple nonlinear controller is presented. From (8), (6), (9) the tracking error dynamics are

$$\begin{aligned} \dot{e}_{v_y} &= -v_x e_{\omega_z} + \frac{1}{m}(\theta_f e_f + \theta_r e_r) + \frac{\theta_f}{m}\Delta_c \\ \dot{e}_{\omega_z} &= \frac{1}{J_z}(\theta_f l_f e_f - \theta_r l_r e_r) + \frac{\theta_f l_f}{J_z}\Delta_c + \frac{1}{J_z}M_z \end{aligned} \tag{10}$$

with

$$\begin{aligned} e_f(\alpha_{f0}, \alpha_{f0,\mathrm{ref}}) &= \varphi_f(\alpha_{f0}) - \varphi_{f,\mathrm{ref}}(\alpha_{f0,\mathrm{ref}}) \\ e_r(\alpha_r, \alpha_{r,\mathrm{ref}}) &= \varphi_r(\alpha_r) - \varphi_{r,\mathrm{ref}}(\alpha_{r,\mathrm{ref}}). \end{aligned} \tag{11}$$

The following PI–based controller

$$\begin{aligned} \dot{I}_v &= e_{v_y} \\ \dot{I}_\omega &= e_{\omega_z} \\ \Delta_c^\circ &= -\frac{m}{\theta_f}(k_{11}e_{v_y} + k_{10}I_v) + \frac{m v_x}{\theta_f}e_{\omega_z} - e_f - \frac{\theta_r}{\theta_f}e_r \\ M_z^\circ &= -J_z(k_{21}e_{\omega_z} + k_{20}I_\omega) - (\theta_f l_f e_f - \theta_r l_r e_r) - \theta_f l_f \Delta_c^\circ \\ &= m l_f \lambda_{22} e_{v_y} - J_z(k_{21}e_{\omega_z} + k_{20}I_\omega) - m v_x l_f e_{\omega_z} \\ &\qquad + (\theta_f l_f + \theta_r l_r)e_r \end{aligned} \tag{12}$$

with $k_{10}, k_{11}, k_{20}, k_{21} > 0$, ensures the global exponential closed–loop stability of the error dynamics

$$\begin{aligned} \ddot{e}_{v_y} + k_{11}\dot{e}_{v_y} + k_{10}e_{v_y} &= 0 \\ \ddot{e}_{\omega_z} + k_{21}\dot{e}_{\omega_z} + k_{20}e_{\omega_z} &= 0. \end{aligned}$$

When the estimation $\hat{\theta}_f, \hat{\theta}_r$ are available for the parameters θ_f, θ_r, the control (12) can be substituted by

$$\begin{aligned} \dot{I}_v &= e_{v_y} \\ \dot{I}_\omega &= e_{\omega_z} \\ \hat{\Delta}_c^\circ &= -\frac{m_0}{\hat{\theta}_f}(k_{11}e_{v_y} + k_{10}I_v) + \frac{m_0 v_x}{\hat{\theta}_f}e_{\omega_z} - e_{f0} - \frac{\hat{\theta}_r}{\hat{\theta}_f}e_{r0} \\ \hat{M}_z^\circ &= m_0 l_f \lambda_{22} e_{v_y} - J_{z0}(k_{21}e_{\omega_z} + k_{21}I_\omega) - m_0 v_x l_f e_{\omega_z} \\ &\qquad + (\hat{\theta}_f l_f + \hat{\theta}_r l_r)e_{r0} \\ e_{f0} &= \varphi_{f0}(\alpha_{f0}) - \varphi_{f,\mathrm{ref}}(\alpha_{f0,\mathrm{ref}}) \\ e_{r0} &= \varphi_{r0}(\alpha_r) - \varphi_{r,\mathrm{ref}}(\alpha_{r,\mathrm{ref}}) \end{aligned} \tag{13}$$

where the nominal values m_0, J_{z0}, B_{j0}, C_{j0}, E_{j0}, are considered for m, J_z, B_j, C_j, E_j, $j = f, r$.

Moreover, using the estimations $\hat{\theta}_f, \hat{\theta}_r$ and the nominal parameter values, the following reference generator with estimated parameters can be considered

$$\begin{aligned} \dot{v}_{y,\mathrm{ref}} &= -v_x\omega_{z,\mathrm{ref}} + \frac{1}{m_0}\Big(\hat{\theta}_f\varphi_{f,\mathrm{ref}}(\alpha_{f0,\mathrm{ref}}) \\ &\qquad + \hat{\theta}_r\varphi_{r,\mathrm{ref}}(\alpha_{r,\mathrm{ref}})\Big) \quad (14) \\ \dot{\omega}_{z,\mathrm{ref}} &= \frac{1}{J_{z0}}\Big(\hat{\theta}_f\varphi_{f,\mathrm{ref}}(\alpha_{f0,\mathrm{ref}})l_f - \hat{\theta}_r\varphi_{r,\mathrm{ref}}(\alpha_{r,\mathrm{ref}})l_r\Big). \end{aligned}$$

IV. Stabilization via Super–Twisting Controller

In this section we will deal with perturbative terms appearing in the vehicle's dynamics, due to the variations of the parameters m, J_z, B_j, C_j, E_j, $j = f, r$. In the following we use a ST controller [20] in order to estimate these perturbative terms. It is worth noting that the main advantage of the ST technique is that it ensures finite time convergence and robustness properties, as well as regular (at least continuous) inputs.

Remark 1: It is worth noting that the same approach can be used to consider error in the identification of l_r, l_f. Anyway, these parameters are not subject to variations. In order to keep the following passages simple, we do not consider such identification errors. ◇

A. The Super–Twisting Algorithm

The ST algorithm, introduced in [25] and widely used for control, observation and robust exact differentiation, is a second order sliding modes algorithm. Considering the system $\dot{x}_1 = u$, $x_1, u \in \mathbb{R}$, one can introduce a dynamic feedback so that the feedback system is eventually given by

$$\begin{aligned} \dot{x}_1 &= -\lambda_1|x_1|^{1/2}\operatorname{sign}(x_1) + x_2 \\ \dot{x}_2 &= -\lambda_2\operatorname{sign}(x_1) \end{aligned} \tag{15}$$

where $x_2 \in \mathbb{R}$ gives the controller dynamics, and $\lambda_1, \lambda_2 > 0$ are gains. The origin $(x_1, x_2) = (0, 0)$ is finite–time stable, namely it is reached in finite time [25]. Making use of (15), one can consider the following controller

$$\begin{aligned} \dot{\chi}_1 &= -\lambda_{12}\operatorname{sign}(e_{v_y}) \\ \dot{\chi}_2 &= -\lambda_{22}\operatorname{sign}(e_{\omega_z}) \\ \Delta_c &= \frac{m}{\theta_f}\Big(-\lambda_{11}|e_{v_y}|^{1/2}\operatorname{sign}(e_{v_y}) + \chi_1\Big) + \frac{m v_x}{\theta_f}e_{\omega_z} \\ &\quad - e_f - \frac{\theta_r}{\theta_f}e_r \\ M_z &= J_z\Big(-\lambda_{21}|e_{\omega_z}|^{1/2}\operatorname{sign}(e_{\omega_z}) + \chi_2\Big) \\ &\quad - (\theta_f l_f e_f - \theta_r l_r e_r) - \theta_f l_f \Delta_c \\ &= J_z\Big(-\lambda_{21}|e_{\omega_z}|^{1/2}\operatorname{sign}(e_{\omega_z}) + \chi_2\Big) \\ &\quad - m l_f\Big(-\lambda_{11}|e_{v_y}|^{1/2}\operatorname{sign}(e_{v_y}) + \chi_1\Big) \\ &\quad - m v_x l_f e_{\omega_z} + \theta_r(l_f + l_r)e_r \end{aligned} \tag{16}$$

with $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ chosen to ensure finite time convergence of the resulting closed–loop error vehicle dynamics (10), (16) become

$$\begin{aligned} \dot{e}_{v_y} &= -\lambda_{11}|e_{v_y}|^{1/2}\operatorname{sign}(e_{v_y}) + \chi_1, \ \dot{\chi}_1 = -\lambda_{12}\operatorname{sign}(e_{v_y}) \\ \dot{e}_{\omega_z} &= -\lambda_{21}|e_{\omega_z}|^{1/2}\operatorname{sign}(e_{\omega_z}) + \chi_2, \ \dot{\chi}_2 = -\lambda_{22}\operatorname{sign}(e_{\omega_z}) \end{aligned} \tag{17}$$

whose structure is that of equations (15). As a consequence, the origin of the error dynamics is finite–time stable [25].

Remark 2: The terms in (16)

$$\dot{\chi}_1 = -\lambda_{12}\operatorname{sign}(e_{v_y}), \quad \dot{\chi}_2 = -\lambda_{22}\operatorname{sign}(e_{\omega_z})$$

are the integral actions on the sign of the errors e_{v_y}, e_{ω_z}, respectively. ◇

Remark 3: The PI and ST–based controllers have the same dimension. ◇

Analogously to (13), when θ_r, θ_f are estimated and the nominal parameter values are considered, we obtain the following controller

$$
\begin{aligned}
\dot{\chi}_1 &= -\lambda_{12}\,\mathrm{sign}(e_{v_y}) \\
\dot{\chi}_2 &= -\lambda_{22}\,\mathrm{sign}(e_{\omega_z}) \\
\hat{\Delta}_c &= \frac{m_0}{\hat{\theta}_f}\Big(-\lambda_{11}|e_{v_y}|^{1/2}\,\mathrm{sign}(e_{v_y}) + \chi_1\Big) \\
&\quad + \frac{m_0 v_x}{\hat{\theta}_f} e_{\omega_z} - e_{f0} - \frac{\hat{\theta}_r}{\hat{\theta}_f} e_{r0} \\
\hat{M}_z &= J_{z0}\Big(-\lambda_{21}|e_{\omega_z}|^{1/2}\,\mathrm{sign}(e_{\omega_z}) + \chi_2\Big) \\
&\quad - (\hat{\theta}_f l_f e_{f0} - \hat{\theta}_r l_r e_{r0}) - \hat{\theta}_f l_f \hat{\Delta}_c \\
&= J_{z0}\Big(-\lambda_{21}|e_{\omega_z}|^{1/2}\,\mathrm{sign}(e_{\omega_z}) + \chi_2\Big) \\
&\quad - m_0 l_f\Big(-\lambda_{11}|e_{v_y}|^{1/2}\,\mathrm{sign}(e_{v_y}) + \chi_1\Big) \\
&\quad - m_0 v_x l_f e_{\omega_z} + \hat{\theta}_r (l_f + l_r) e_r
\end{aligned}
\tag{18}
$$

with e_{f0}, e_{r0} as in (13), while the reference generator is as in (14).

B. Transient Behavior

The aim of this section is to highlight some difficulties that could arise during the transient, due to the fact that the estimation of θ_f, θ_r can not be instantaneous and/or perfect. For, let us rewrite the error dynamics considering (6), (14) and the control (18)

$$
\begin{aligned}
\dot{e}_{v_y} &= \frac{\theta_f}{\theta_f - \tilde{\theta}_f}\frac{m_0}{m}\Big(-\lambda_{11}|e_{v_y}|^{1/2}\,\mathrm{sign}(e_{v_y}) + \chi_1\Big) + E_1 \\
\dot{\chi}_1 &= -\lambda_{12}\,\mathrm{sign}(e_{v_y}) \\
\dot{e}_{\omega_z} &= \frac{J_{z0}}{J_z}\Big(-\lambda_{21}|e_{\omega_z}|^{1/2}\,\mathrm{sign}(e_{\omega_z}) + \chi_2\Big) + E_2 \\
\dot{\chi}_2 &= -\lambda_{22}\,\mathrm{sign}(e_{\omega_z}) \\
E_1 &= -v_x e_{\omega_z} + \frac{1}{m}\Big(\theta_f \varphi_f(\alpha_{f0}) + \theta_r \varphi_r(\alpha_r)\Big) \\
&\quad + \frac{\theta_f}{m}\Big(\frac{m_0 v_x}{\theta_f - \tilde{\theta}_f} e_{\omega_z} - e_{f0} - \frac{\theta_r - \tilde{\theta}_r}{\theta_f - \tilde{\theta}_f} e_{r0}\Big) \\
&\quad - \frac{1}{m_0}\Big((\theta_f - \tilde{\theta}_f)\varphi_{f,\mathrm{ref}}(\alpha_{f0,\mathrm{ref}}) \\
&\qquad + (\theta_r - \tilde{\theta}_r)\varphi_{r,\mathrm{ref}}(\alpha_{r,\mathrm{ref}})\Big) \\
E_2 &= \frac{1}{J_z}\Big(\theta_f \varphi_f(\alpha_{f0}) l_f - \theta_r \varphi_r(\alpha_r) l_r\Big) \\
&\quad - \frac{1}{J_z}\Big((\theta_f - \tilde{\theta}_f) l_f e_{f0} - (\theta_r - \tilde{\theta}_r) l_r e_{r0}\Big) + \frac{\tilde{\theta}_f l_f}{J_z}\hat{\Delta}_c \\
&\quad - \frac{1}{J_{z0}}\Big((\theta_f - \tilde{\theta}_f)\varphi_{f,\mathrm{ref}}(\alpha_{f0,\mathrm{ref}}) l_f \\
&\qquad - (\theta_r - \tilde{\theta}_r)\varphi_{r,\mathrm{ref}}(\alpha_{r,\mathrm{ref}}) l_r\Big)
\end{aligned}
$$

where $\tilde{\theta}_f = \theta_f - \hat{\theta}_f$, $\tilde{\theta}_r = \theta_r - \hat{\theta}_r$. Since E_1, E_2 depend on $\tilde{\theta}_f$, $\tilde{\theta}_r$, it is clear that these errors E_1, E_2 influence the vehicle's controlled dynamics. It is also worth noting that, in practice, the signals $\hat{\theta}_f, \hat{\theta}_r$ are filtered in order to obtain smoother signals. This smoothing introduces a delayed time response in the estimation. Since the controller gains have to be high, this estimation could determine a deterioration of the transient in the tracking. Therefore, attention has to be posed to the correct choice of these gains and of the parameters used in the filter.

V. Simulation results

The control law (13), (14), (16), has been applied to a vehicle characterized by the (real) parameters $m = 1480$ kg, $J_z = 2386$ kg m^2, $l_f = 1.17$, $l_r = 1.43$ m, $B_f = 1.81$, $C_f = 7.2$, $D_f = 8854$ N m, $E_f = 0$, $B_r = 1.68$, $C_r = 11$, $D_r = 8394$ N m, $E_r = 0$.

A challenging test maneuvers has been considered, given by a step steer of $\delta_{d,sw} = +100°$ of the steering wheel at $t = 0.5$ s, followed by a step steer of $\delta_{d,sw} = -100°$ at $t = 2.5$ s, and finally $\delta_{d,sw} = 0$ at $t = 4.5$ s. The ratio between the steering wheel angle $\delta_{d,sw}$ and δ_d is 16. This maneuver is performed at 27 m/s (97.2 km/h). To make the maneuver more challenging, at $t = 3.5$ s there is a change of the friction from $\mu = 0.9$ (dry road) to $\mu = 0.4$ (iced road). A random variation of 5% is superimposed to these values.

A saturation of 3° is considered for δ_c, and a saturation of 8000 N m is considered for M_z.

To test the robustness properties of the controller, we consider a parameter variation with respect to the nominal ones $m_0 = 0.81\, m$, $J_{z0} = 0.92\, J_z$, $l_{f0} = l_f$, $l_{r0} = l_r$, $B_{f0} = 1.1\, B_f$, $C_{f0} = 1.1\, C_f$, $D_{f0} = D_f$, $E_{f0} = E_f$, $B_{r0} = 0.8\, B_r$, $C_{r0} = 0.8\, C_r$, $D_{r0} = D_r$, $E_{r0} = E_r$.

In the following we compare the ST–based controller (16) with the PI–based controller (12). The gains of the ST–based controller (16) have been fixed equal to $\lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22} = 150$, while the controller gains for the PI–based controller (12) have been chosen equal to $k_{10} = k_{20} = 22.5$, $k_{11} = k_{21} = 18$.

These values ensure comparable results for the ST and PI–based controllers in the case of absence of parameter variations. Since the function $\mathrm{sign}(\cdot)$ is discontinuous in zero, and therefore very sensible to noise and quantization errors, we have considered the approximation $\mathrm{sign}(x) \approx 2\arctan(100x)/\pi$. A consequence of this approximation is the loss of the finite time convergence property.

In the following simulations, we consider first the case in which m, J_z, B_j, C_j, D_j, $j = f, r$, are equal to the nominal values. With both the ST and PI–based controller (16), (12) the references are well tracked, see Figs. 1 and 2. As can be noted, the absolute tracking errors are small for both controllers, see Figs. 3, 4.

When dealing with parameter variations, the tracking behavior is better with the ST–based controller, due to its robustness with respect to matching perturbations. As shown in Figs. 5, 6, both controllers show good results. Nevertheless, we can note in Figs. 7, 8 that the ST–based controller ensures a smaller tracking error. This is highlited by smaller peaks (see Figs. 7.b, 8.b), noticeable after the instants $t = 0.5$ s, $t = 2.5$ s and $t = 4.5$ s in which the maneuver imposes sharp changes.

Conclusions

In this paper a nonlinear controller using a ST estimator has been designed. A comparison between this controller and a simple PI–based controller has been done on a challenging maneuver, showing better results. An interesting property of the proposed ST–based controller is its robustness against parameter variations.

Future work will entail the extension of the proposed method to the estimation of all the parameters appearing in the model, and the comparison of the proposed controllers in the case of larger parameter variations.

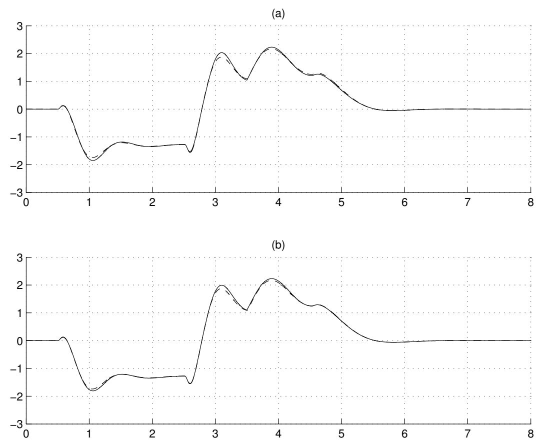

Fig. 1. Nominal parameters. v_y (solid) and $v_{y,\text{ref}}$ (dashed) [m/s vs s]: a) PI–based controller; b) ST–based controller.

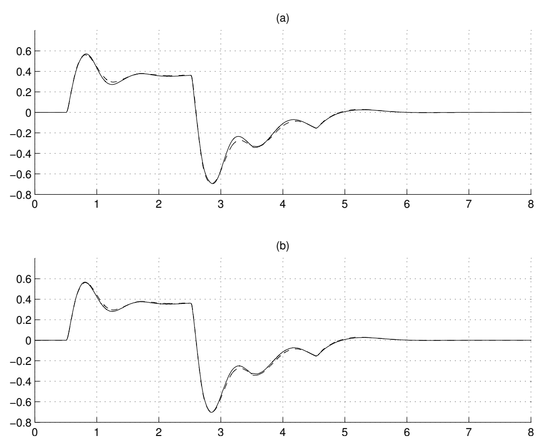

Fig. 2. Nominal parameters. ω_z (solid) and $\omega_{z,\text{ref}}$ (dashed) [rad/s vs s]: a) PI–based controller; b) ST–based controller.

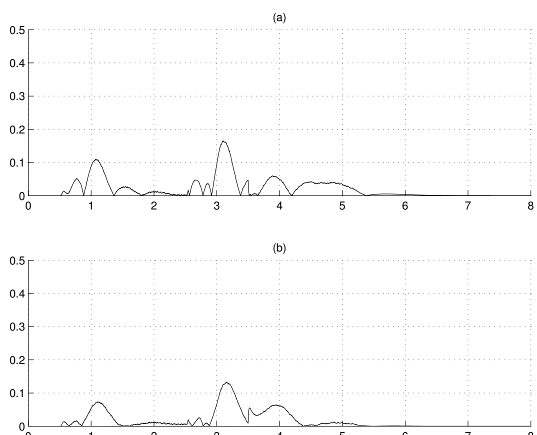

Fig. 3. Nominal parameters. Absolute tracking error $|v_y - v_{y,\text{ref}}|$ [m/s versus s]: a) PI–based controller; b) ST–based controller.

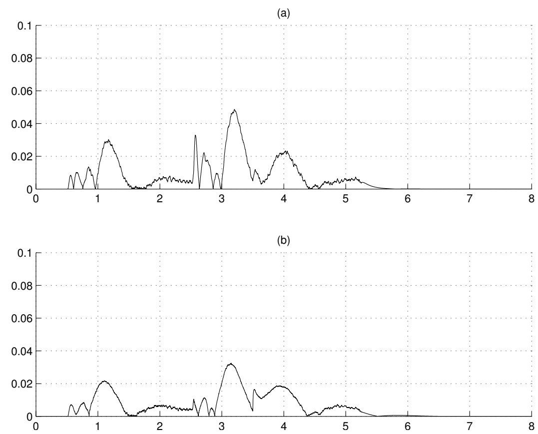

Fig. 4. Nominal parameters. Absolute tracking error $|\omega_z - \omega_{z,\text{ref}}|$ [rad/s vs s]: a) PI–based controller; b) ST–based controller.

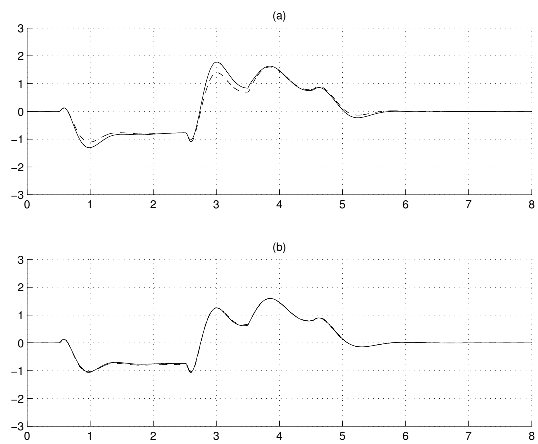

Fig. 5. Real parameters. v_y (solid) and $v_{y,\text{ref}}$ (dashed) [m/s vs s]: a) PI–based controller; b) ST–based controller.

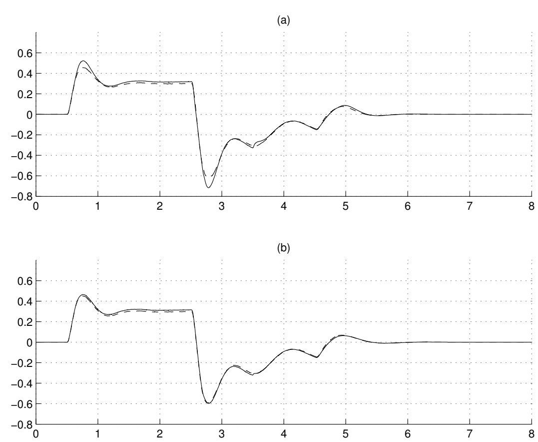

Fig. 6. Real parameters. ω_z (solid) and $\omega_{z,\text{ref}}$ (dashed) [rad/s vs s]: a) PI–based controller; b) ST–based controller.

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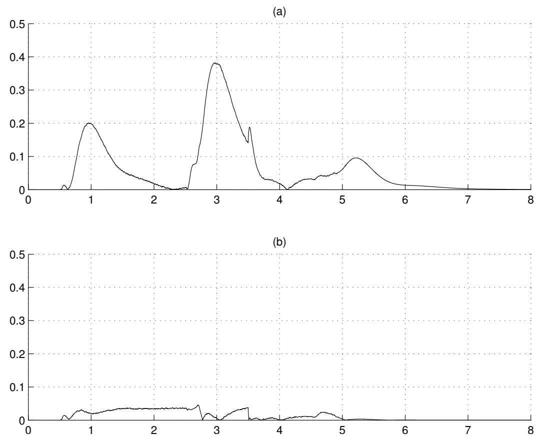

Fig. 7. Real parameters. Absolute tracking error $|v_y - v_{y,\mathrm{ref}}|$ [m/s vs s]: a) PI–based controller; b) ST–based controller.

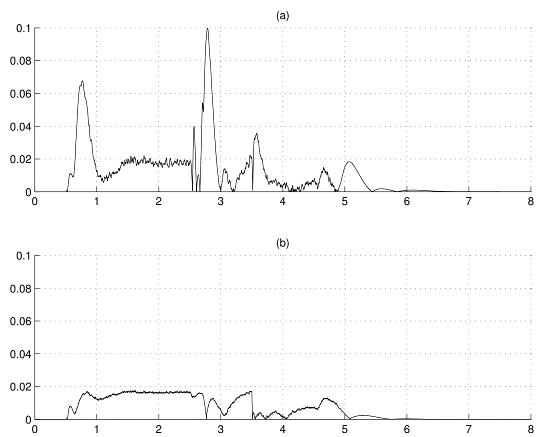

Fig. 8. Real parameters. Absolute tracking error $|\omega_z - \omega_{z,\mathrm{ref}}|$ [rad/s vs s]: a) PI–based controller; b) ST–based controller.

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