Localization in Wireless Sensor Networks

Roudy Dagher, Roberto Quilez

Chapter 9

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Roudy Dagher[1,2] and Roberto Quilez[2]

[1] *Etineo, France,* [2] *Inria Lille – Nord Europe, France*

Abstract *With the proliferation of Wireless Sensor Networks (WSN) applications, knowing the node current location have become a crucial requirement. Location awareness enables various applications from object tracking to event monitoring, and also supports core network services such as: routing, topology control, coverage, boundary detection and clustering. Therefore, WSN localization have become an important area that attracted significant research interest. In the most common case, position related parameters are first extracted from the received measurements, and then used in a second step for estimating the position of the tracked node by means of a specific algorithm. From this perspective, this chapter is intended to provide an overview of the major localization techniques, in order to provide the reader with the necessary inputs to quickly understand the state-of-the-art and/or apply these techniques to localization problems such as robot networks. We first review the most common measurement techniques, and study their theoretical accuracy limits in terms of Cramer-Rao lower bounds. Secondly, we classify the main localization algorithms, taking those measurements as input in order to provide an estimated position of the tracked node(s).*

9.1 Introduction

Recent technological advances in micro-electronics, digital electronics and wireless communication, have made possible the development of low-cost, low-power, multi-functional and highly integrated sensor nodes that are able to communicate in a wireless ad-hoc fashion over short distances [3]. These tiny nodes, typically equipped with processing, sensing, power management and communication capabilities collaborate to form a Wireless Sensor Network (WSN). Sensed data is typically sent over the network, in a multi-hop manner, to a control center either directly or via a *base station/sink*. The main constraints in such networks are the limited amount of energy and computing resources of the nodes.

With the significant development and deployment of WSN, associating the sensed data with its physical location becomes a crucial requirement. Knowing the node's location enables a myriad of location-based applications such as object tracking, environment monitoring, intrusion detection, and habitat monitoring [73] [25]. Location estimation also supports core network services such as: routing, topology control, coverage, boundary detection and clustering [43].

Localization is defined as the process of obtaining a node location with respect to a set of known reference positions. It is also referred to as *location estimation* or *positioning*. Nodes at reference positions are called *anchor nodes*[1], and nodes with unknown positions are called *tracked nodes*[2]. Based on reference positions of a few anchor nodes in the network, and inter-node measurements such as range and connectivity, localization algorithms estimate the position of a tracked node in the network. Depending on targeted applications, the coordinate system may be global or local (e.g. habitat monitoring).

In the most common case, position related parameters are first extracted from the received measurements, and then used in a second step for estimating the position of the tracked node by means of a specific approach: fingerprinting, geometric or statistical methods [20]. The used technique highly depends on the application's requirements and challenges:

- *Environment*
  The environment where a WSN is deployed may be challenging, as localization performance is affected by multipath and non-line-of-sight

[1] also referred to as reference node, beacon device, base station, etc.
[2] also referred to as non-anchor node, target node, blindfolded device, mobile station etc.

(NLOS) propagation. Environment variability is typically due to: presence of obstacles, metallic environments acting as wave guides, interference, etc.

- *Complexity*
  In the context of WSN, nodes are typically battery-powered with limited computing power and memory. Therefore, it may not be feasible to implement complex localization algorithms. However, in some cases the base station may have advanced computing capabilities and act as a localization server for the application.
- *Accuracy*
  Coarse-grained accuracy of several meters may be sufficient for patient tracking inside a hospital, and may be addressed by simple low-cost Zigbee-based solutions [17]. Conversely, fine-grained accuracy usually requires specialized hardware such as Ultra-wideband (UWB) [22].
- *Scalability*
  Scalability of a localization algorithm determines how well it accommodates as the number of nodes increases and the coverage area is expanded. This metric is very important in dense networks.
- *Latency*
  Depending on the tracked object dynamics, the latency to determine its location might be a big concern. It should be considered with respect to other layers such as the Medium Access Control (MAC) layer for channel access latency.
- *Dependability*
  The system should be able to keep operating even if some anchor nodes are faulty. This is referred to as system *fault tolerance*. In [51] a WSN localization system with error detection/correction is presented. Another issue to consider is *network lifetime*, mainly in the case of battery-powered nodes.

In brief, the choice of a sensor network localization technique often involves a trade-off among the above-listed constraints in order to suit the requirements of the targeted application(s). In essence, these challenges make localization in wireless sensor networks unique and intriguing. This chapter is intended to provide an overview of the major techniques that have been widely used for WSNs localization system. Based on the referenced material, a special effort has been made to broadly classify the different localization aspects in order to provide a starting block for this topic. The remainder of this chapter is organized as depicted in Fig. 9.1. First section

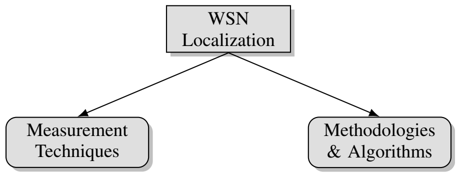


Fig. 9.1 Chapter overview.

presents the measurement techniques and their theoretical accuracy limits in terms of Cramer-Rao lower bounds. The second section covers the localization theory, strategies and algorithms taking those measurements as input in order to provide an estimated position of the tracked node(s).

9.2 Measurement Techniques

Position related parameters estimation is the first step of WSN localization. This estimation often relies on physical measurements, depending on the available hardware capabilities. On the other hand, network related measurements such as hop count, or neighborhood information can lead to coarse-grained localization that may be sufficient in dense networks. Figure 9.2 gives an overview of these measurement techniques. It is the type of measurements employed and the corresponding precision that fundamentally determine the estimation accuracy of a localization system and the localization algorithm being implemented by this system.

In the case of physical measurements, a result of estimation theory can be used to bound the localization error: the *Cramer-Rao lower bound* (CRLB) [30, 82]. This theoretical bound gives the best performance that can be achieved by an unbiased location estimator. If $\hat{\theta}$ is an unbiased estimator of an unknown parameter θ, then its covariance matrix $Cov(\hat{\theta})$ is bounded by the CRLB as

$$Cov(\hat{\theta}) \geq \Big\{ - E\big[\nabla_\theta(\nabla_\theta \ln f(X|\theta))\big]\Big\}^{-1} \tag{9.1}$$

where X is the random observation vector with probability density function $f(X|\theta))$, $E[\cdot]$ indicates the expected value, and ∇_θ is the gradient operator with respect to θ. Note that the CRLB is independent of the estimation method, and only depends on the statistical model of the observations. Therefore, the CRLB can serve as a benchmark for localization algorithms.

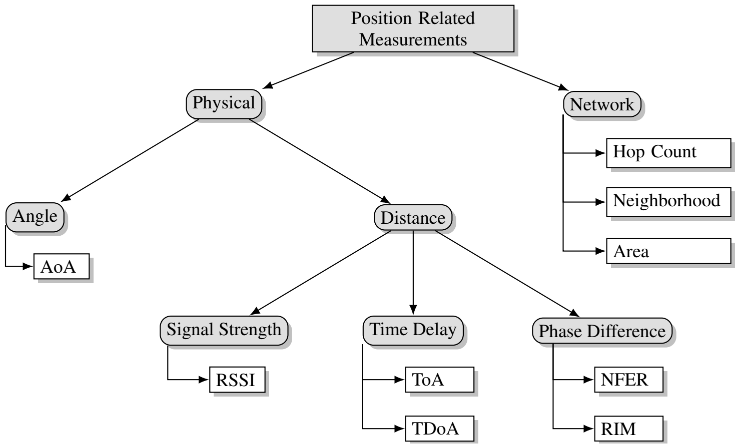


Fig. 9.2 Measurement techniques overview.

In the case where θ is scalar, the CRLB in Eq. (9.1) becomes

$$\sigma_{\hat{\theta}}^2 \geq \frac{1}{-E\left[\frac{\partial^2 \ln f(X|\theta)}{\partial \theta^2}\right]} = \frac{1}{-\int_{\mathbb{R}} \frac{\partial^2 \ln f(X|\theta)}{\partial \theta^2} f(X|\theta) \mathrm{d}X}. \tag{9.2}$$

9.2.1 *Physical measurements*

Physical measurements can be broadly classified into three categories according to the measurement type: angle measurements, distance related measurements and network related measurements.

9.2.1.1 *Angle measurements*

The angle or bearing relative to reference nodes is measured by estimating the angle of arrival (AoA) parameter between the tracked node and reference nodes. Given the angle measurements, the location of the tracked node may be determined by triangulation[3] [67]. The AoA measurement is commonly made available by the use of directional antennas or antenna arrays[4], by measuring the phase difference between the signal received by adjacent antenna elements $\Delta\Phi = 2\pi\frac{\Delta \sin \alpha}{\lambda}$ with Δ the inter-element spacing of the N_a elements antenna array, and λ the wavelength. In order to

[3]The use of triangulation to estimate distances goes back to antiquity: Thales similar triangles to estimate the height of the pyramids, distances to ships at sea as seen from a cliff, etc.

[4]Another technique uses receiver antenna's amplitude response [47].

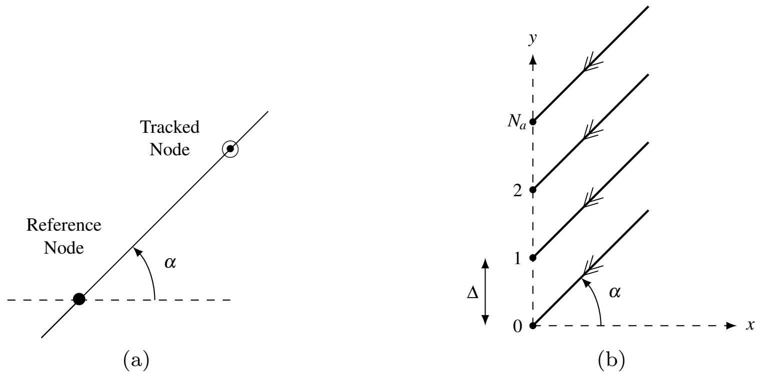


Fig. 9.3 AoA measurement with antenna array. (a) AoA definition. (b) Antenna Array.

understand the accuracy limit with this type of measurements, consider a uniform antenna as in Fig. 9.3. Under the assumption that the signal, with effective bandwidth β, arrives at the speed of light c at each antenna element via a single path, with the same signal to noise ratio (SNR) for all elements, the CRLB for the variance of an unbiased AoA estimate α can be expressed by [22]:

$$\sqrt{Var\{\hat{\alpha}\}} \geq \frac{\sqrt{3}c}{\sqrt{2}\pi\sqrt{\mathrm{SNR}}\beta\sqrt{N_a\left(N_a^2-1\right)}\Delta\cos\alpha}. \tag{9.3}$$

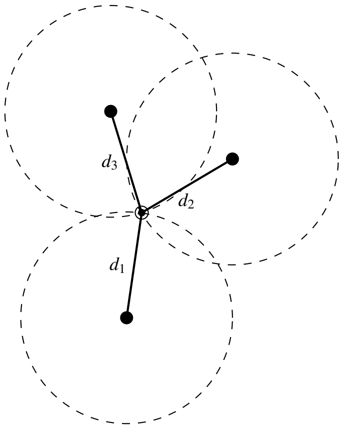
Equation (9.3) states that the AoA accuracy increases with the SNR, effective bandwidth, and the array size. Finally, the best accuracy is obtained when the signal direction and the antenna line are perpendicular i.e., $\alpha = 0$.

For a detailed overview on AoA measurements, refer to [47].

9.2.1.2 *Distance measurements*

By using the distance of the tracked node to several reference nodes, the position of the tracked node can be computed using the multilateration method [73].

In order to estimate the distance, several ranging techniques have been developed [47]. Among them, the Received Signal Strength (RSS) based, and the time based ranging are the most popular. A less popular technique, is the Near Field Electromagnetic Ranging (NFER) that exploits specific near-field properties of radio waves for ranging purposes. Yet another less adopted technique, the Radio Interferometric Positioning (RIPS) that applies *interferometry* to radio waves in WSN.



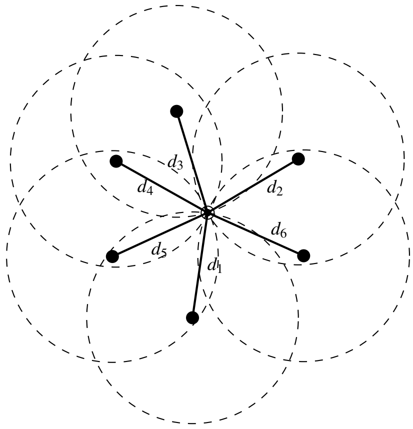


Fig. 9.4 Distance measurements and multilateration (Ranging circles). (a) Trilateration. (b) Multilateration.

Received Signal Strength This technique is based on the received signal strength indicator (RSSI), a standard feature found in most of the current off-the-shelf devices. The most typical RSS systems are based on propagation loss equations [66], given by the following log-normal shadowing model[5]

$$P_r(d)[dBm] = P_0(d_0)[dBm] - 10n_p log_{10}\frac{d}{d_0} + X_\sigma \tag{9.4}$$

where:

- $P_r(d)[dBm]$: the received power in dB milliwatts at distance d from the transmitter.
- $P_0(d_0)[dBm]$: the reference power in dB milliwatts at a reference distance d_0 from the transmitter.
- n_p : the path loss exponent that measures the rate at which the received signal strength decays with distance. Example of values: 2.0 in free space, 1.6 to 1.8 inside a building [66].
- X_σ : a zero-mean normal variable, with standard deviation σ, that accounts the shadowing effect. Example of σ values: 0 dB in free space, and 5.8 dB inside a building [66].

Note that many other models have also been proposed in the literature [66], but the log-normal model is the most popular one due to its simplicity.

[5] $P[mW] = 10^{P[dBm]/10}$ and $P[dBm] = 10\log_{10}(P[mW]/1mW)$.

From the channel model in Eq. (9.4), we can derive the distribution of the RSS-based range measurements

$$\hat{d} = d \cdot 10^{\frac{X_\sigma}{10n_p}} = 10^{\log_{10}(d) + \frac{X_\sigma}{10n_p}} = e^{(\ln 10)\cdot[\log_{10}(d) + \frac{X_\sigma}{10n_p}]} = e^{(\ln d) + \frac{\ln 10}{10n_p} X_\sigma}. \tag{9.5}$$

Therefore RSS-based range measurements are distributed according to a log-normal distribution

$$\hat{d} \sim \ln \mathcal{N}(\ln d, \frac{\sigma \ln 10}{10n_p})\cdot \tag{9.6}$$

Which yields to the unbiased range estimator from the RSS measurements [47]:

$$\hat{d} = d_0 \left(\frac{P_r(d)[mW]}{P_0(d_0)[mW]} \right)^{-1/n_p} e^{-\frac{\sigma^2}{2\eta^2 n_p^2}} \quad ,\text{with } \eta = \frac{10}{\ln 10}\cdot \tag{9.7}$$

The CRLB of an unbiased range estimator is derived in [64]

$$\sqrt{Var\{\hat{d}\}} \geq \frac{(\ln 10)\sigma}{10n_p} \cdot d. \tag{9.8}$$

Equation (9.8) shows that the ranging accuracy depends on the channel parameters n_p and σ, that are environment dependent. Also it deteriorates as the distance between the transmitter and the receiver increases. Thus, in order to maintain the estimation error of less than δd, the tracked node has to be within the range of [64]

$$r_0 = \frac{10}{\ln 10} \cdot \frac{\sigma}{n_p} \cdot \delta d. \tag{9.9}$$

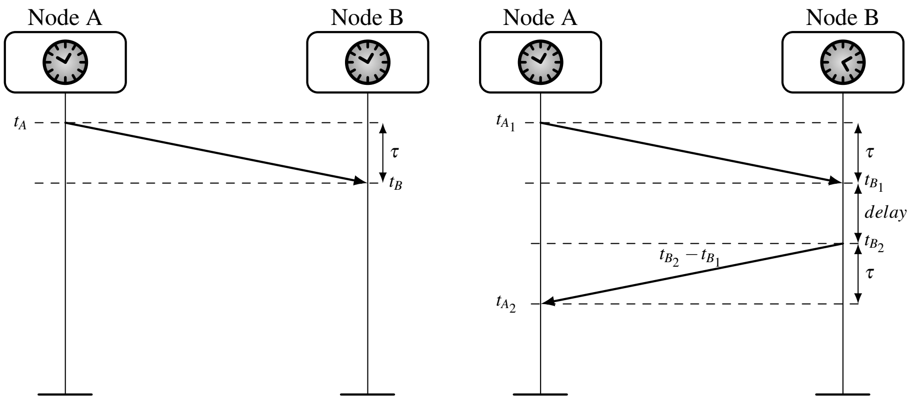
Finally, the result in Eq. (9.8) is generalized by [59], in the case of N reference nodes and one tracked node in the 2D case:

$$\sqrt{Var\{\hat{d}\}} \geq \frac{(\ln 10)\sigma}{10n_p} \cdot \left(\sqrt{\frac{\sum_{i=1}^{N} d_i^{-2}}{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\frac{d_{\perp ij} d_{ij}}{d_i^2 d_j^2} \right)}} \right) \tag{9.10}$$

where d_i is the distance between reference and tracked nodes, d_{ij} is the distance between reference nodes i and j, and $d_{\perp ij}$ j is the shortest distance from the tracked node to the line segment connecting nodes i and j. This result highlights the impact of the geometric distribution of the reference nodes on the localization accuracy.

Propagation Time Distance between neighboring nodes can be estimated using propagation time measurements. Namely, Time of Arrival (ToA) methods are used when reference and tracked nodes are synchronized, or Time Difference of Arrival (TDoA) that only requires synchronization between reference nodes. Time delay measurements commonly use generalized cross-correlation or matched filter receivers [22, 35].

Time of Arrival (ToA) There are two categories of ToA-based distance measurements: *one-way* propagation time, and *round-trip* propagation time measurements (cf. Fig. 9.5). The *one-way* propagation time



(a) (b)

Fig. 9.5 Propagation time. (a) One way. (b) Round trip.

measurements measure the difference between sending time at transmitter node (t_A) and receiving time at receiver node (t_B). This delay is related to the inter-node distance by $\tau = d/c$. The main drawback of this technique, is that the local time of the transmitter and the receiver should be accurately synchronized. Assuming Line Of Sight (LOS) conditions and Gaussian noise at receiver level, the CRLB for one-way propagation time ranging is given by [45, 64]

$$\sqrt{Var\{\hat{d}\}} \geq \frac{c}{2\sqrt{2}\pi} \cdot \frac{1}{\beta} \cdot \frac{1}{\sqrt{SNR}}. \tag{9.11}$$

Equation (9.11) shows that, unlike RSS-based distance estimation, the accuracy can be improved by increasing the effective signal bandwidth β and/or the SNR. However, in practice, the transmitter-receiver synchronization requirement increases the cost and the complexity to the system.

In order to cope with synchronization constraint, the *round-trip* propagation time measurements measure the difference between sending time t_{A_1} at transmitter node and the time when the signal is echoed back by the receiver node t_{A_2}. That is

$$\textit{round-trip} = 2 \cdot \tau = (t_{A_2} - t_{A_1}) - (t_{B_2} - t_{B_1}).$$

Note that synchronization between nodes A and B is no longer required since the same clock is used to compute the round-trip propagation time.

In practice, the internal delay for node B to echo the signal, should be considered with care in order to avoid jitter introducing meters of errors on distance measurements. For instance, a hardware implementation with *a priori* calibration is a good option.

Assuming LOS conditions, Gaussian noise at receiver level, and no changes in the bandwidth or SNR conditions, the CRLB for *round-trip* propagation time ranging is given by [45]

$$\sqrt{Var\{\hat{d}\}} \geq \frac{1}{2}\left(\frac{c}{2\sqrt{2}\pi} \cdot \frac{1}{\beta} \cdot \frac{1}{\sqrt{SNR}}\right). \tag{9.12}$$

Equation (9.12) shows that the ranging accuracy for *round-trip* propagation time is twice better than the *one-way* scenario.

With recent advances in radio technology, the UWB signals are used for accurate time-based ranging, even in challenging environments, at short distances with very low energy levels [22, 23]. Nanotron Technologies Gmbh have designed the Chirp Spread Spectrum (CSS) modulation technique [24] to cope with multi-path effects, operating in the 2.44 GHz band, with approximately 60MHz effective bandwidth. In order to avoid synchronization and eliminate clock drift and offset, Nanotron Technologies Gmbh have also developed SDS-TWR[6], an extension of the *round-trip* propagation time ranging presented earlier. The CSS modulation confers to SDS-TWR its robustness to multi-path effects. Furthermore, time-based ranging have been standardized with release of the 802.15.4a standard, that adopted both UWB and CSS signaling [29, 70].

Finally, other time-based techniques have been developed, such as the *Cricket* system [63] that combines RF communication and ultra-sound ranging. The idea behind the cricket system is to take benefit from the fact that, in the air, the speed of the sound is much smaller than the speed of the light. The radio link is only used to synchronize the ultra-sound microphones. Other techniques use audible sound for time of arrival measurements such as *Beepbeep* [60] and *Beep* [46]. Finally, the *Lighthouse* approach uses laser beams to measure the distance between tiny dust nodes and the lighthouse, a modified base station device [68].

Time Difference of Arrival (TDoA) Another type of distance-based measurements is the Time Difference of Arrival (TDoA), that consists of measuring the difference between the arrival times between two signals traveling between the target and the tracked nodes. For illustrating the TDoA principle, consider the case of one tracked node and two reference

[6]Symetrical Double-Sided Two Way Ranging

nodes. In this case, the position of the tracked node is on a hyperbola, with foci the reference nodes, as illustrated in Fig. 9.6. With more than two reference nodes, the estimated position is at the intersection of the hyperbolas whose foci are at the locations of the reference nodes pairs.

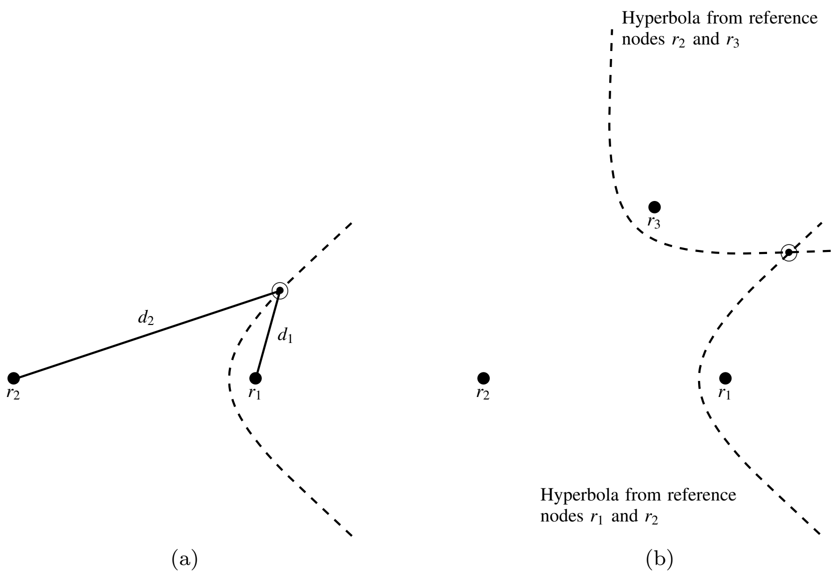


Fig. 9.6 TDOA measurements. (a) With two reference nodes. (b) With three reference nodes.

One approach for measuring TDoA is to first measure the ToA for each signal between each (reference node, tracked node) pairs as follows

$$\begin{cases} \hat{\tau}_1 = \tau_1 + \text{offset}_1 & (9.13) \\ \hat{\tau}_2 = \tau_2 + \text{offset}_2. & (9.14) \end{cases}$$

Since the reference nodes are synchronized, $\text{offset}_1 = \text{offset}_2$ which leads to the TDoA estimate

$$\hat{\tau}_{TDoA} = \hat{\tau}_1 - \hat{\tau}_2 = \frac{d_1 - d_2}{c}. \tag{9.15}$$

The corresponding CRLB expression can be deduced from the ToA case, which shows that its accuracy increases with effective bandwidth and/or SNR [23]. It is also proved in [64] that TDoA approach cannot perform better than ToA approach in terms of accuracy limits.

Finally, another approach for measuring TDoA is the generalized cross correlation method, given by

$$\hat{\tau}_{TDoA} = arg \max_{\tau} \left| \int_0^T r_1(t) r_2(t+\tau) dt \right| \tag{9.16}$$

where $r_i(t)$ is the signal between the tracked node and the ith reference node, and T the observation interval. Refer to [22] and [47] for more details and discussions.

Phase Difference Measurements The position of the tracked node can also be estimated using phase difference measurements. The below presented techniques exploit fundamental laws of physics to determine ranging information. Namely, Near field electromagnetic ranging (NFER) exploits near field phase behavior discovered by Heinrich Hertz [75], and Radio Interferometric Measurements (RIM) is inspired by the interferometric positioning in the optical regime developed thirty years earlier [14, 48].

Near Field Electromagnetic Ranging (NFER) Near field electromagnetic ranging (NFER) uses the near field phase relationship of electric and magnetic components to infer range estimate [74, 76].

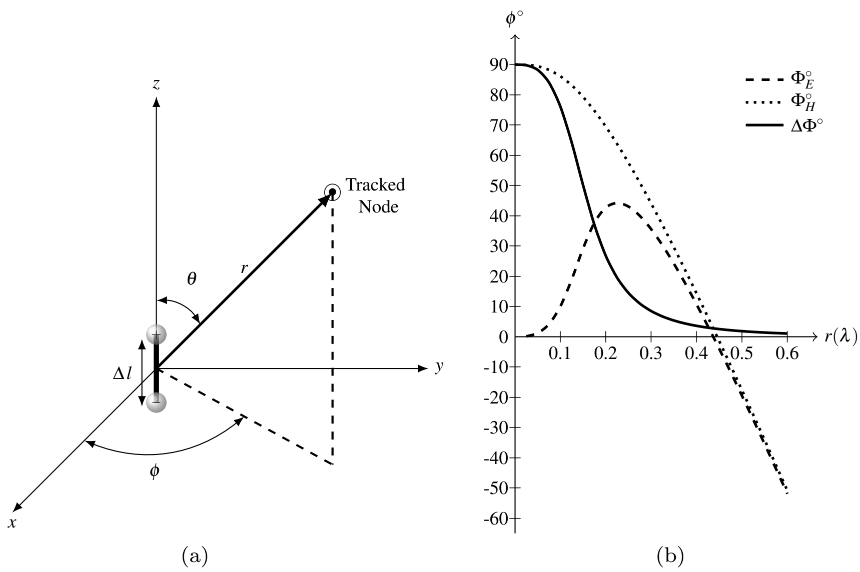


Fig. 9.7 Near field electromagnetic ranging (NFER). (a) Spherical coordinates, Hertzian dipole. (b) Phase difference versus range.

Consider a Hertzian dipole of length Δl, carrying a uniform current, $I = I_0 \cos(\omega t)$ as in Fig. 9.7. In spherical coordinates (r, θ, ϕ), the electric field component E_θ and magnetic field component H_ϕ are given by [32, 75]

$$\begin{cases} E_\theta = \dfrac{I_0 \Delta l \sin\theta}{4\pi\omega\epsilon_0} \left[\dfrac{\beta}{r^2}\cos(\omega t - \beta r) + \left(\dfrac{1}{r^3} - \dfrac{\beta^2}{r}\right)\sin(\omega t - \beta r)\right] & (9.17) \\ \\ H_\phi = \dfrac{I_0 \Delta l \sin\theta}{4\pi\omega} \left[\dfrac{1}{r^2}\cos(\omega t - \beta r) - \dfrac{\beta}{r}\sin(\omega t - \beta r)\right] & (9.18) \end{cases}$$

where $\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$, c is the speed of light, λ is the wavelength, and ϵ is the permittivity of free space. From Eq. (9.17) and Eq. (9.18), we can compute the phase difference between E_θ and H_ϕ, which leads to [32, 75]

$$\Delta\phi = \Phi_E - \Phi_H = \cot^{-1}\left(\frac{\omega r}{c} - \frac{c}{\omega r}\right) - \cot^{-1}\frac{\omega r}{c}. \tag{9.19}$$

The plot of $\Delta\phi$ in Fig. 9.7 clearly shows a direct mapping between $\Delta\phi$ and the range r. By using the relationship $\cot(a-b) = -\frac{1+\cot a \cot b}{\cot a - \cot b}$, the range can be related to the phase by [76]:

$$r = \frac{\lambda}{2\pi}\sqrt[3]{\cot\Delta\phi}. \tag{9.20}$$

The CRLB of NFER is derived in [32] for both ranging and 2D-localization. For the ranging case, the CRLB is given by

$$\sqrt{Var\{\hat{d}\}} = b\left(\frac{c^6 + d^6\omega^6}{3\,c^3 d^2 \omega^3}\right) \tag{9.21}$$

with:

$$b^2 = \frac{U^2}{U^3 + V^2(1-U)},\ U = \frac{SNR_E + SNR_H}{2},\ \text{and}\ V = \frac{SNR_E - SNR_H}{2}.$$

Equation (9.21) shows that accuracy is dependent on real location, the used frequency, and the signal-to-noise ratio. In the 2D case, the CRLB additionally shows the impact of the geometrical conditioning, and that the optimal accuracy cannot be achieved by using only one frequency. Kim *et al.* have presented a scheme using multiple frequencies in order to improve accuracy [31]. In the case of the NFER, CRLB as derived in [32] becomes a tool for choosing the optimal frequency for a given coverage area.

Radio Interferometric Measurements (RIM) Radio Interferometric Ranging, exploits radio frequency interference of two waves emitted from two reference nodes at slightly different frequencies, in order to obtain the ranging information for localization by measuring the relative phase offset. In [48] authors describe the Radio-Interferometric Positioning (RIPS) localization method and provide a mean for measuring the phase difference indirectly using RSSI. In order to illustrate the principle of RIPS, consider three reference nodes A,B,C and one tracked node D as in Fig. 9.8.

Let A and B transmit at the same time, two unmodulated sine waves at two close frequencies f_a,f_b. The resulting interference is a beat signal with a beat frequency $|f_a - f_b|$. The reference node C and the tracked node D, acting as receivers, will receive the beat signal with a phase difference depending on the distance difference between the quartet (A, B, C, D)

$$\Delta\Phi = \frac{2\pi}{\lambda} \left(d_{AD} - d_{BD} - d_{BC} - d_{AC}\right) \bmod 2\pi \tag{9.22}$$

where $\lambda = \frac{c}{(f_a+f_b)/2}$, and d_{XY} is the euclidean distance between nodes X and Y. Which yields to the definition of the *q-range* measurement, defined as

$$q_{ABCD} = d_{AD} - d_{BD} - d_{BC} - d_{AC}. \tag{9.23}$$

Note that a single RIPS measurement given by Eq. (9.23), places the tracked point on a hyperbola branch (cf. Fig. 9.9) whose foci are the transmitter pair (A, B). The relative phase offset in Eq. (9.22) can then be rearranged as follows [37]

$$\Delta\phi = \frac{\Delta\Phi}{2\pi}\lambda = q_{ABCD} \bmod \lambda. \tag{9.24}$$

Note that several measurements are required at different transmit frequency pairs (f_a, f_b) in order to resolve the phase ambiguity (due to mod 2π), therefore *the q-range* can be found by solving the following system of n equations

$$\Delta\phi_i = q_{ABCD} \bmod \lambda_i. \tag{9.25}$$

In order to find the position of the tracked node, consider the case where the transmitter pair is (A, C) and the receiver pair (B, D). The associated *q-range* is

$$q_{ACBD} = d_{AD} - d_{CD} - d_{BC} - d_{AB}. \tag{9.26}$$

The position of the tracked node can be obtained by solving the system of equations obtained from Eq. (9.26) and Eq. (9.26). Geometrically, this corresponds to the intersections of the two ranging hyperbolas.

RIPS trilateration is discussed in detail in [37]. For a complete review of the interferometry in WSN refer to [71].

In [84] authors present an analytical study of the impact of RIPS measurement noise on the localization error. It appears that the localization error is small if the tracked node is located inside the triangle $\triangle ABC$, and generally increases at a steady exponential rate as the tracked node moves away from the triangle, unless it is close to L_{ABC}. Where L_{ABC} is the union of the three lines representing the extensions of the sides of the triangle $\triangle ABC$, but not including the interiors of the edges of $\triangle ABC$.

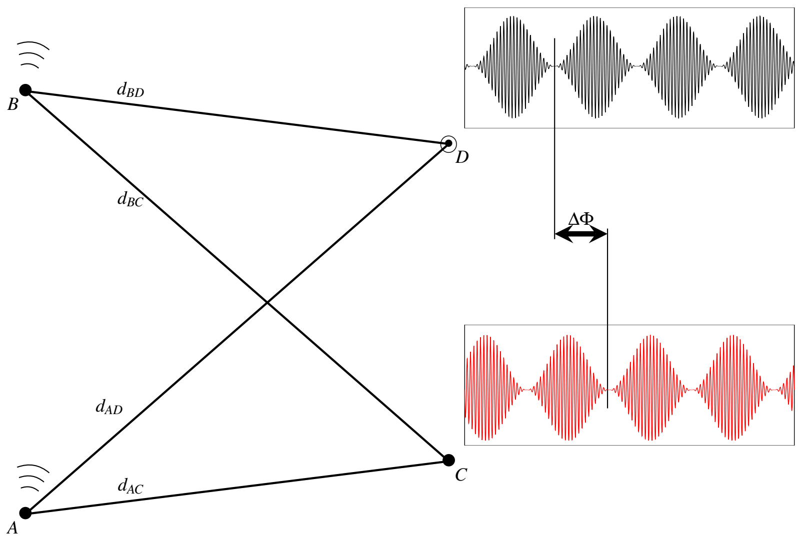


Fig. 9.8 RIPS measurement technique.

9.2.2 *Network related measurements*

Network connectivity measurements are possibly the simplest measurements. The tracked node location can be inferred by analyzing its neighboring reference nodes in terms of connectivity, radio coverage area, and neighborhood proximity [43]. This kind of measurements are very cost effective and straightforward in large-scale networks.

In connectivity measurements, a node measures the number of nodes in its transmission range. This measurement defines a proximity constraint between these two nodes, which can be exploited for localization [9, 15]. For

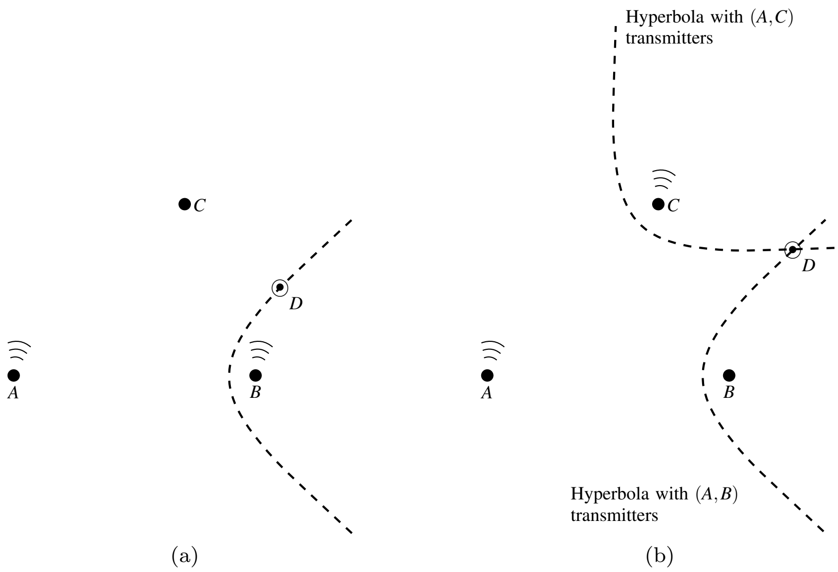


Fig. 9.9 RIPS localization rounds. (a) First Round. (b) Second Round.

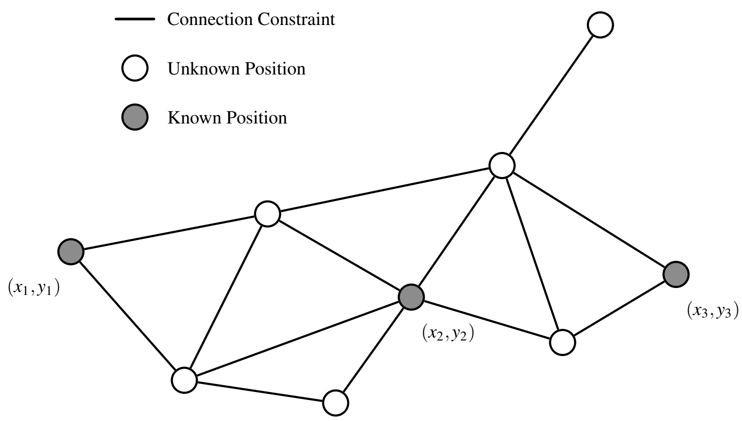


Fig. 9.10 Graph illustrating connectivity constraints.

instance, when a tracked node detects three neighboring reference nodes, it can assume to be close to these nodes and estimate its location as the centroid of the three reference nodes [26].

9.3 Localization Theory and Algorithms

In this section, we give a brief introduction to some fundamental theories in sensor network localization, and a set of major sensor network localization algorithms are discussed.

The objective behind a positioning methodology is to determinate the *location information* of a number of nodes. Location information can be interpreted as any form of location indicator such as exact location, the deployment area or the location distribution. As we have seen in Sec. 9.2, parameters extracted from signals traveling between the nodes will allow to establish pairwise spatial relationships (angle, distance or proximity). Localization algorithms will take those parameters as inputs to estimate the position of the target nodes according to a certain strategy.

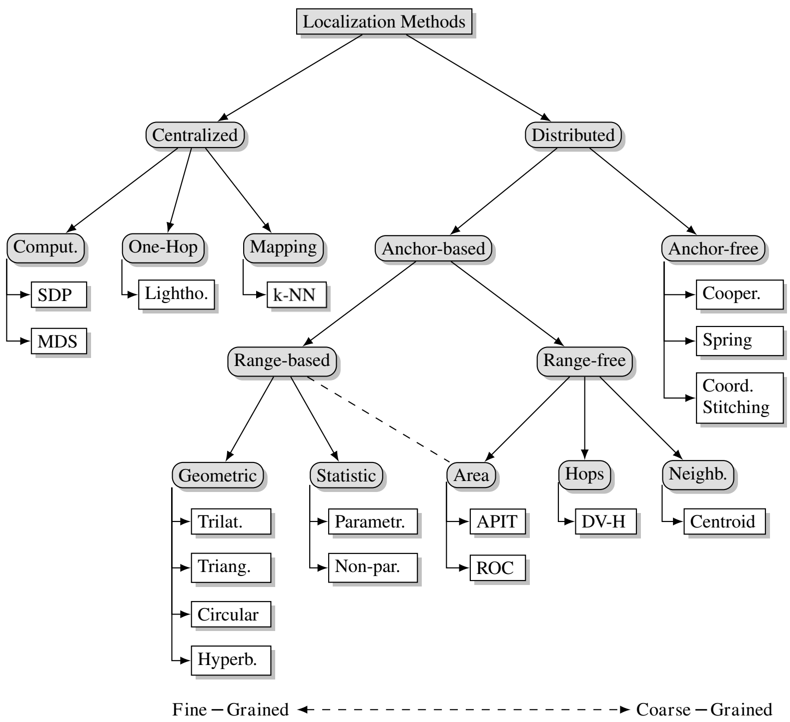


Fig. 9.11 Location methods overview.

There exist well organized surveys in the literature that propose different classifications of the localization systems based on different criteria at different hierarchical levels such as the number of hops, the presence of reference nodes (anchors) or the computational organization [4, 18, 20, 47, 56, 58, 73, 83]. The purpose of this section is not to provide definitive and exhaustive taxonomy of the different localization methods. Instead, this section is intended to present a comprehensive introduction to some of today's most popular localization methods. A special effort has been made to conciliate approaches from different authors to render a general schema. In the Fig. 9.11, a set of the most representative localization methods are displayed within some of the implementation choices associated to them.

We have first categorized the localization algorithms into two main types; namely centralized and distributed, based on the direct dependency on a centralized resource.

9.3.1 *Centralized methods*

Centralized localization methods present a direct dependency on a centralized resource. This resource can be some information previously collected (*mapping*), a central machine with powerful computational capabilities (*centralized computing*) or some kind of *one hop* location reference (landmark, satellite...) providing this centralized service.

9.3.1.1 *Centralized computing*

Centralized computing basically migrates inter-node ranging information and connectivity data to a sufficiently powerful central base station to be processed. All other nodes in the network only gather the location related information, such as RSS, and send it to the central processing location. The base station calculates the estimated location of all the tracked nodes and communicates it back if requested.

The advantage of centralized processing is minimizing the required capabilities (e.g., processing power and memory space) of the nodes, excepting the central location processing node. This benefit however comes with a communication cost, creating high traffic levels and increasing latency since all the nodes must communicate with a single central receiver to determine their location. The high level of traffic can cause bottlenecks in the network and limit the location update rate. The latency and traffic problems get worse increasing the size of the network. Therefore the centralized processing method is more suitable for small network or a network where the location update rate is low.

Two representative proposals in this category are *Semidefinite Programming (SDP)* techniques [15] and *Multi-Dimensional Scaling (MDS)* [78].

Semidefinite Programming (SDP) Many localization problems can be formulated as a convex optimization problem. They can be solved using *linear* and *semidefinite programming (SDP)* techniques [15]. SDP is a generalization of linear programming and has the following form:

$$\begin{array}{ll} \text{Minimize} & c^T x \\ \text{Subject to:} & F(x) = F_0 + x_1 F_1 + \ldots + x_N F_N \leq 0 \\ & Ax < B \\ & F_k = F_k^T \end{array} \tag{9.27}$$

where $x = [x_1 y_1, x_2 y_2, \ldots, x_m y_m, x_{m+1} y_{m+1}, \ldots, x_N y_N]^T$. The first m entries are fixed and correspond to the reference nodes positions and the remaining $n - m$ positions are computed by the algorithm. The objective is to find a possible position for each target node when a the position of a set of reference nodes is given. Proximity constraints imposed by known connections can be represented as *linear matrix inequalities (LMIs)*. In the case of nodes communicating within a perfect circle, the estimated region is convex and can not be described by linear equations but as an LMI. For a maximum radio range R_{max}, if two nodes, with positions x_i and x_j, are in communication, their separation must be less than R_{max}, i.e., it exists a proximity constraint between them. This can be represented as a *radial constraint* and expressed as a LMI: $\|x_j - x_i\| \leq R_{max}$.

The advantage of this method is that it is simple to model hardware that provides ranges or angles and simple connectivity. SDP simply finds the intersection of the constrains. There are efficient computational methods available for most of convex programming problems. However, this estimation methodology requires centralized computation. To solve the optimization problem, each node must report its connectivity to a central computer. This approach also requires to handle large data structures and lacks of scalability because of its complexity. The relevant operation for radial data is $O(n^3)$ where n is the number of convex constraints.

Multi-Dimensional Scaling (MDS-MAP) *Multi-dimensional Scaling (MDS)* [78] is often used as part of information visualization techniques for exploring similarities or dissimilarities. It displays the structure of distance-like data as a geometric picture.

The typical goal of MDS is to create a configuration of n points in one, two or three dimensions. Only the inter-point *distances* are known. MDS enables the reconstruction of the relative positions of the point based on the pairwise distances.

Typical procedure of MDS involves three stages:

- First, compute the shortest path between all pairs of nodes. The shortest path distances are used to construct the distance matrix for MDS. The (i, j) entry represents the distance along the shortest path from the node i to j. If only connectivity information is available, that distance will be the number of hops. However, it can also incorporate distance information between neighboring nodes when it is available.
- In a second stage, classical MDS is applied to the distance matrix to obtain estimated relative node positions.
- Finally , the relative positions are transformed to absolute positions with the help of some number of fixed anchor nodes.

The strength of MDS-MAP is that it can be used when there are few or no anchor nodes. It can use both connectivity and distance measurement ranging techniques and provides both absolute and relative positioning. However, the main problem with MDS is its poor asymptotic performance, which is $O(n3)$.

More detailed work based on MDS can be found in [1].

9.3.1.2 *One-Hop positioning*

This kind of positioning methods require *line of sight (LOS)*, i.e., direct contact, between the reference position, the landmark, and the node to locate. The most representative solution of this class is the GPS, where receiver has to have a clear line of sight to the satellites to operate.

An interesting approach to estimate distance between an optical receiver and transmitter is the *Lighthouse* [68] location system. It is a laser-based solution which allows nodes to autonomously estimate their location with high accuracy without additional infrastructure components besides a base station device. The transmitter consists on a parallel optical beam rotating at a constant speed. The receiver is equipped with an optical sensor and a clock. Measuring the time it *sees* the beam (t_{beam}), the distance (d), from the base station can be calculated when the rotational speed (or the time it takes for a complete rotation, (t_{turn}) and the width of the beam (b), are known:

$$d = \frac{b}{2\sin(\pi t_{beam}/t_{turn})}. \tag{9.28}$$

As a result we have a simple ranging system where all the potential positions of the receiver form a cylinder with radius d centered at the lighthouse rotation axis. Using three such lighthouses in different placements, the location of the node can be inferred with trilateration principles. Nevertheless, the original proposal aimed to have a unique base station. For that purpose, distances are measured in the three axes of the space using mutually perpendicular rotation axes in single positioning device. A major advantage of this system is that the optical receiver can be very small in cost and size. However the transmitter may be large and expensive and the LOS requirement remains a big handicap in many practical cases.

9.3.1.3 *Profiling techniques*

In Sec. 9.2 we have seen different ways to estimate distances between nodes. Localization algorithms can then be applied to these distances to obtain the estimated position of the tracked node. Nevertheless, wireless sensor network environments, and specially indoor environments, are often complicated to model and their model parameters determination is also a difficult task. Such a challenging scenario can be overcome using another approach, namely *profiling-based techniques* [5] [36, 61, 69].

The main idea behind these localization techniques, also referred to as *mapping* or *fingerprinting*, is to determine a regression scheme based on a set of training data and then to estimate the position of a given node according to that regression function. They work by first constructing a kind of map of the signal parameters behavior for each anchor node over a coverage area. In addition to anchor nodes, a collection of n sample points with a priori chosen positions must be defined to collect that training data. At each location, $l_i = (x_i, y_i)^T$, a vector of signal parameters m_i is obtained. Typically the m_{ij} entry corresponds to the value of the *signal strength* from/at the anchor j when the anchor node is at location l_i although other signal parameters may be used.

These *training data* can then be expressed as:

$$\tau = \{(m_1, l_1), (m_2, l_2), \ldots, (m_n, l_n)\}\,. \tag{9.29}$$

For the training set given in Eq. (9.29), a *position estimation rule* must then be determined, i.e., a pattern matching algorithm or regression function to estimate the location l of a given target node based on a parameter vector m related to the target node. Some common mapping techniques used in location estimation include *k-nearest neighbors (k-NN)* estimation, *support vector regression (SVR)* and neural networks [16, 21, 39, 49, 55]. As in [20] we develop *k-NN* in order to provide an intuition of a simple mapping-based position estimation.

k-nearest neighbors (k-NN) In its simplest version *1-NN* determines the estimated position of a target node at the location l_j in the training set τ that has the associated vector m_j with the shortest Euclidean distance to the measured parameter vector m:

$$j = \arg_{i \in 1,\ldots,n} \min \parallel m - m_i \parallel . \tag{9.30}$$

In general, *k-NN* determines the position of the target node with the help of the k parameters vectors in τ that have the smallest distances to the given parameter vector m. The position $\hat{l}$ is then estimated by the weighted sum of the positions corresponding to those k nearest parameter vectors:

$$\hat{l} = \sum_{i=1}^{k} w_i(m) l_i, \tag{9.31}$$

where w_i is the weighting factor associated to the ith reference location. Various weights can be used as studied in [49]. For instance, in the uniform weighted scheme, the sample mean of position is used:

$$\hat{l} = \frac{1}{k} \sum_{i=1}^{k} l_i. \tag{9.32}$$

The main advantage of mapping techniques is that they have a certain degree of inherent robustness. They can provide very accurate position estimation in challenging environments with multipath and non-line of sight propagation. However, the main disadvantage is the requirement that the training database should be large enough and representative of the current environment for accurate position estimation. The database should be updated frequently enough so that channel characteristics in the training and position estimation phases do not differ significantly. Such an update requirement can be very costly for positioning systems operation in dynamic environments, such an outdoor positioning system.

9.3.2 *Distributed algorithms*

One way to overcome the traffic bottleneck of the centralized processing method is to divide the network into sections and allocate a node capable of executing the positioning algorithm to each section. An alternative approach, is to distribute the location-estimation task among almost all the nodes in the network. In this way, there is no centralized location processing node and each node determines its own location by communication only with nearby anchor nodes and potentially other tracked nodes. In a fully distributed processing method, all the nodes must satisfy certain processing capabilities and memory space requirements. One of the advantages of distributed processing is relatively uniform packet traffic, which makes it easy to expand the traffic of the network.

Distributed algorithms can be classified according to whether they use pre-configured reference positions (anchor-based vs anchor-free) or the granularity of the measured employed, i.e., whether they make some kind of range (distance or angle) correlation (range-based vs range-free).

9.3.2.1 *Anchor-based techniques*

Anchor-based algorithms assume that a certain number of nodes, referred to as *anchors* or *beacons*, know their own position through manual configuration or an external positioning system such as GPS. Tracked nodes' location is then determined by referring to that reference positions with the help of inter-sensor measurements such us the ones we have seen in Sec. 9.2.

Depending on the measurement techniques employed, anchor-based algorithms can be classified [43] from fine-grained to coarse-grained into several categories such as: *location*, *distance*, *angle*, *area*, *hop-count* and *neighborhood* (see Fig. 9.11). This classification allows us to broadly distinguish localization algorithms between *range-based* and *range-free* [26]. Range-based approaches rely on signal features such as signal strength, time of flight or angle of arrival for calculating relative distances or angles. In contrast, range-free methods do not try to estimate direct point-to-point distance from the received signal parameters; they use topological information (connectivity, signal comparison ...) rather than ranging.

Range-based localization techniques Range-based localization applies geometric techniques to estimate the position of a target node. It uses a set of position related parameters in a number of reference nodes to describe geometric figures that are supposed to intersect in the point of interest.

Geometric methods As we have seen in Sec. 9.2, some measurements parameters can define a geometric figure of uncertainly around the anchor node. The position of the target node can be estimated as the intersection of those figures. The received signal strength or the time of arrival of the signal determine a circumference by translating that parameter to physical distance. Using *trilateration* the estimated location for the target node is given by the intersection of three circles from three different references, (x_i, y_i) (see Fig. 9.12(a)). In the case of the time difference of arrival, the geometric figure corresponds to an hyperbola and an intersection point can also result as an estimation. On the other hand, the angle of arrival measure defines a straight line passing through both, the target and the reference nodes In this case, two parameters are enough to calculate the estimated position via *triangulation*.

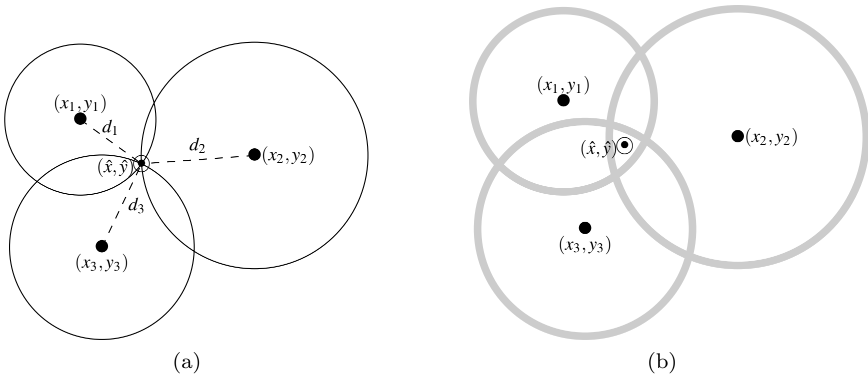


Fig. 9.12 Location estimation using Trilateration. (a) Ideal case. (b) Including range error.

Unfortunately, in a practical implementation, the ranging measurements contain noises. The presence of fading and shadowing may lead this method to produce no results at all. Error in distance estimation can prevent the bearing lines to have a common intersection point (see Fig. 9.12(b)). Thus, an optimization algorithm must be applied to choose an estimated location according to some criteria. Two sample algorithms are the circular and the hyperbolic positioning algorithms. The former optimizes directly the error associated to distances while the latter makes distance-difference optimization.

Circular positioning algorithm The *Circular Positioning* algorithm [40] adopts the criterion of minimizing the sum square error ε. This error can be expressed as:

$$\varepsilon = \sum_{i=1}^{n} \left(\sqrt{(x_i - x)^2 + (y_i - y)^2} - d_i \right)^2, \tag{9.33}$$

where (x_i, y_i) is the position of each reference node. The position (x, y) of the node that minimizes that error can be calculated using the steepest descend method defined by:

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_{k+1} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}_k - \alpha \begin{bmatrix} \frac{\delta\varepsilon}{\delta x} \\ \frac{\delta\varepsilon}{\delta y} \end{bmatrix}_{x=\hat{x}_k, y=\hat{y}_k} . \tag{9.34}$$

This method requires an initial location to begin the iteration, which can be the midpoint of the reference positions under consideration.

Hyperbolic positioning algorithm The *Hyperbolic Positioning* algorithm [40] also referred to as *Linear Least Squared errors (LLS)* [18] does not minimize directly the sum of the squared errors of the erroneous distance estimations to reference positions as in the previous case. Instead it minimizes a linear function of it by subtracting two distances estimations i.e., it minimizes the sum of the distance to the hyperbolas resulting from the subtraction.

Considering n reference positions we can write the distance estimations $(d_{i=1\ldots n})$ to the target node as:

$$d_i^2 = (x - x_i)^2 + (y - y_i)^2. \tag{9.35}$$

To solve the previous system of equations a linearization is performed by subtracting the location of the first reference from all other equations [54]. The resulting system of equations can be expressed in the form $Ax = b$ as:

$$\begin{bmatrix} 2x_1 - 2x_2 & 2y_1 - 2y_2 \\ 2x_1 - 2x_3 & 2y_1 - 2y_3 \\ \cdots & \cdots \\ 2x_1 - 2x_n & 2y_1 - 2y_n \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} d_2^2 - d_1^2 + x_1^2 - x_2^2 + y_1^2 - y_2^2 \\ d_3^2 - d_1^2 + x_1^2 - x_3^2 + y_1^2 - y_3^2 \\ \cdots \\ d_n^2 - d_1^2 + x_1^2 - x_n^2 + y_1^2 - y_n^2 \end{bmatrix} . \tag{9.36}$$

Therefore, the estimated position of the target node can be calculated as the least squares solution of this equation given by:

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = (A^T A)^{-1}(A^T b). \tag{9.37}$$

Both, circular and hyperbolic algorithms, give the same weight to the different distance estimations. Nevertheless, as we have seen in Sec. 9.2, measurements such as received signal strength (RSS) do not depend linearly on the distance between the nodes. From Eq. (9.4) it can be deduced that the same error in the RSS measurement will produce larger errors in the distance estimation if the distance between the nodes is higher. That is, the accuracy of the distance estimations depends on the distance itself. The use of weighted techniques to improve the accuracy of the hyperbolic and circular positioning algorithms respectively has been proposed [80]. They give more weight to those measurements corresponding to short distances, which accuracy is expected to be greater.

Statistical location techniques Unlike the geometric techniques, the statistical approach presents a theoretical framework for position estimation for multiple measurement parameters with or without the presence or noise.

In order to formulate this generic framework, consider the following model for each of the N estimated parameters:

$$z_i = f_i(x,y) + \eta_i, \tag{9.38}$$

where η_i is the noise at the corresponding estimation and $f_i(x,y)$ is the real value of the signal parameter at the position (x,y).

As we have seen in Sec. 9.2 for ToA/RSS, AoA and TDoA, $f_i(x,y)$ can be expressed as:

$$f_i(x,y) = \begin{cases} \sqrt{(x-x_i)^2 + (y-y_i)^2} & \text{ToA/RSS} \\ \tan^{-1}\left(\frac{(y-y_i)}{(x-x_i)}\right) & \text{AoA} \\ \sqrt{(x-x_i)^2 + (y-y_i)^2} - \sqrt{(x-x_0)^2 + (y-y_0)^2} & \text{TDoA} \end{cases} \tag{9.39}$$

In the case that the probability density function of the noise η is known for a set of parameters, *parametric* approaches such as *Bayesian* and *Maximum Likelihood (ML)* can be used. Those techniques are studied in detail in [20]. In the absence of that information *non-parametric* methods must be used. Actually, *profiling techniques*, such as *k-NN*, *SVR* and *neural networks* approaches referred in Sec. 9.3.1.3, are examples of non-parametric estimators since they do not make any assumption concerning the density of probability function of the noise.

Range-free localization techniques Cost and hardware limitations in wireless sensor nodes often prevent the use of range-based localization schemes. For some applications coarse accuracy is sufficient and range-free solutions have been revealed as a valid cost-effective alternative. Range-free based solutions do not try to estimate absolute distances among nodes using any signal feature such as signal strength, angle of arrival or time or flight. They, alternatively, use coverage range, i.e., connectivity, or comparative features between signals. Three approaches can be distinguished according to the *granurality* of the measurement employed: area, hop-count and neighborhood based.

Area based Signals coming from beacon nodes can define coverage areas described by geometric shapes. The area based location estimation method will compute the intersection of those coverage areas and will give the centroid of this region as the resulting location estimation for the tracked node. For instance, if a tracked node receives a signal from another anchor node, a circular region, centered in that anchor node and radius its maximum coverage distance, is delimited. When several reference nodes can be listened, the overlapped area of those circles will determine an estimated location for the tracked node (see Fig. 9.13(a)). This can be extended to other scenarios. For example when angular sector can be determined for the incoming signal from the beacon nodes or when lower coverage bounds are also available to describe different geometric figures (see Fig. 9.13(b)). More detailed work about localization based on connectivity-induced constraints can be found in [15].

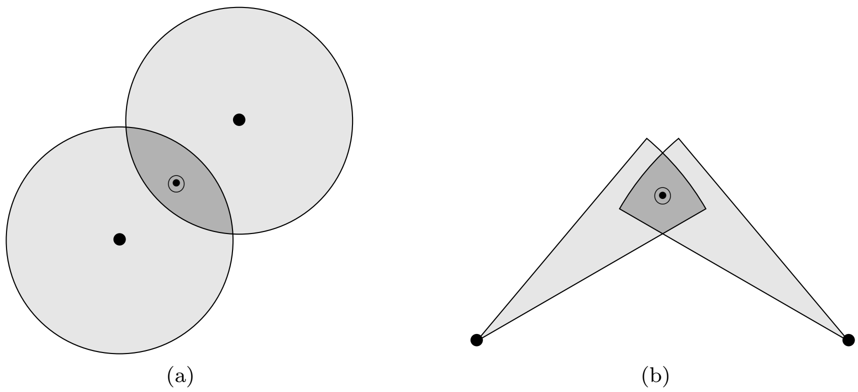


Fig. 9.13 Area measurements. (a) Circles overlapping. (b) Sectors overlapping.

One popular area-based range-free location estimation scheme is *APIT* [26]. The *APIT* algorithm isolates the environment into triangles (see Fig. 9.14). The vertexes of these triangular regions are anchors nodes that the tracked node can hear. The presence inside or outside those triangular regions allows to narrow down the area in which the tracked could reside. The estimated position is obtained from the centroid of the area provided by the intersection of the reference triangles that contains the tracked node.

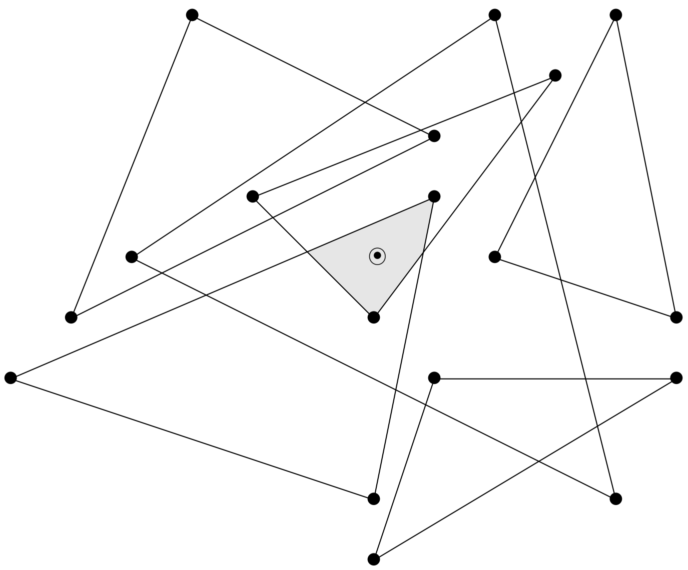


Fig. 9.14 APIT algorithm overview.

Another interesting approach is the *Ring Overlapping Circles (ROC)* [42] algorithm (see Fig. 9.15). Each anchor broadcasts beacon messages that will be received by both, neighboring anchors and the tracked node. By comparing the received signal strength by those anchors to the one received by the tracked node, the region where the tracked node lies within can be determined (in light gray in Fig. 9.15). This ring area is centered in the beacon anchor node and has as higher bound a circle with radius equal to the distance to the anchor which received signal strength is immediately inferior to the one received by the tracked node. The lower bound of the ring is delimited by a circle with radius equal to the distance to the anchor node with received signal strength immediately superior. The process is repeated by each anchor node resulting in several overlapping rings. Finally, the center of gravity of the overlapped area (in dark gray in Fig. 9.15) is reported as estimated position.

Two key assumptions are made by the ROC algorithm. Firstly, the received signal strength decreases monotonically with the distance, so we can conclude that a node that receives a higher signal strength is closer. Secondly, the antennas are supposed to be isotropic. Nevertheless, the algorithm is proclaimed to be resilient to irregular radio propagation patterns

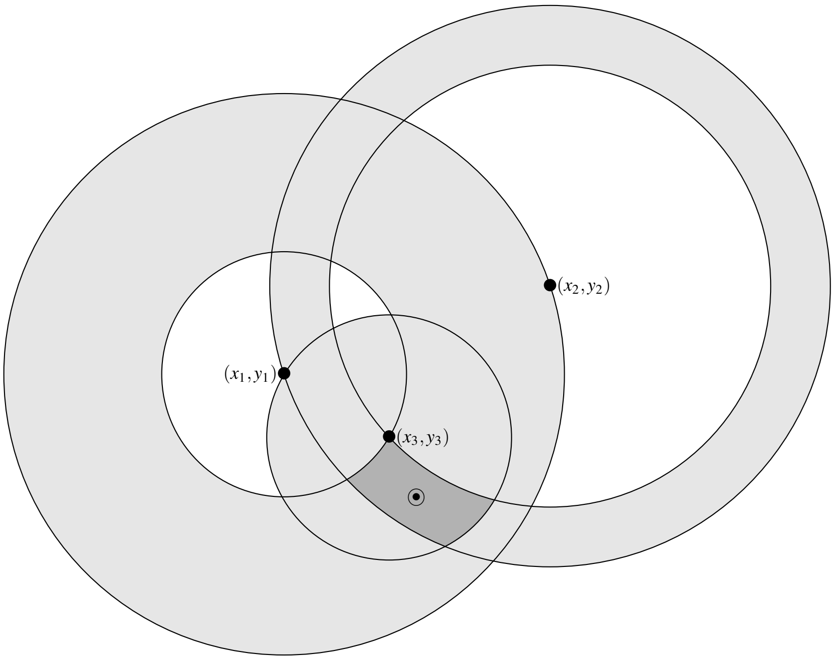


Fig. 9.15 Ring Overlapping Circles algorithm with 3 anchors.

and capable to achieve better performance than APIT with less communication overhead [41].

Hop counter If the maximum radio range among nodes is well-known, their distance from each other can be determined to be inferior to that range with high probability. *DV-HOP* [57] algorithm uses this connectivity measurements to determine the location of a node. All the anchor nodes will broadcast a beacon message that will be propagated through the network. This message includes the anchor node location and a *hop-counter* that will be incremented at every hop. Each anchor node keeps the minimum hop-counter value per anchor. This procedure enables all the nodes in the network (including anchors) to get the shortest distance (least number of hops) to anchors. To translate hop-count to physical distance, an anchor i with position (x_i, y_i) estimates the average single hop distance h_i with the following formula:

$$h_i = \frac{\sum \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{h_{ij}}, \qquad (9.40)$$

where h_{ij} is the minimum number of hops to another anchor node j with position (x_j, y_j). This estimated hop size is then propagated to nearby nodes. Finally, once the distance estimation is made to at least three anchors, triangulation is used to report the estimated position. The main advantages of this algorithm are its simplicity and the fact that it does not depend on measurement error. The more anchors can be heard, the more precise the localization is. The main drawback is that it will only work for isotropic networks. When an obstacle prevents an edge from appearing in the connectivity graph the hop-counter methodology can lead to an inaccurate location estimation. In Fig. 9.16 we can see how the number of hops between node A and node C are equal to the hop count between node B and node D due to the presence of an obstacle, although the later are physically closer.

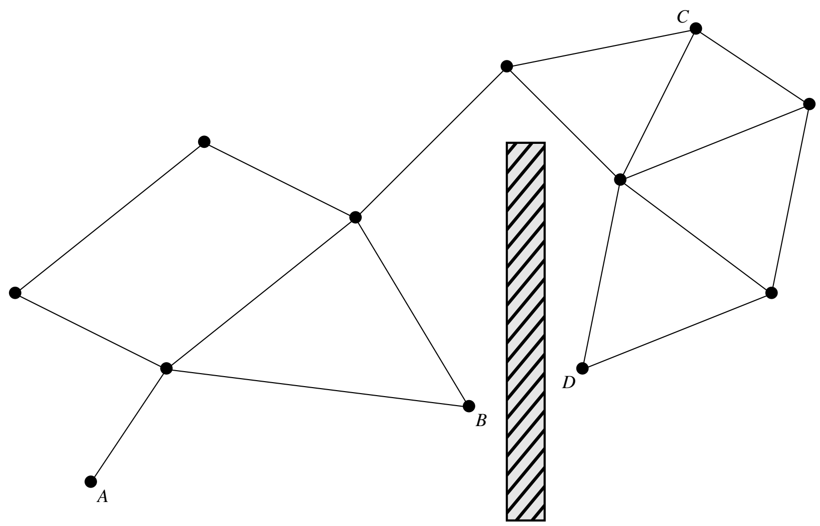


Fig. 9.16 Hop-counter with obstacle, example.

The *DV-Distance* algorithm is presented together with DV-hop proposing a similar method but distances between neighboring nodes are used instead of hops. Many other modifications of this algorithm to improve performance under certain network conditions can be found in literature [65, 77].

The *Amorphous* algorithm [53] proposes a different approach to DV-Hop to calculate the average single hop distance. It uses the density of the network, n_{local}, to correct the average hop distance estimation, d_{hop}, with the help of the Kleinrock and Silvester formula [34] for a maximum radio range R:

$$d_{hop} = R\left(1 + e^{-n_{local}} - \int_{-1}^{1} e^{\frac{-n_{local}}{\pi}(\arccos t - t\sqrt{1-t^2})} dt\right). \qquad (9.41)$$

Neighborhood measurements One of the simplest coarse-grained localization methods is using the connectivity measurement, which is more robust to unpredictable environments, for neighbor proximity. The only decision to make is whether a node is within the range of another. Reference nodes can be deployed through the localization area determining non-overlapping regions. When a tracked node receives a beacon from an anchor, it will consider that reference position as its own position.

In the case of anchors (reference positions) with overlapping regions of coverage, *Centroid Location (CL)* [9] can be used. The tracked node can listen to a given subset of anchor beacons containing their reference positions (x_i, y_i) to infer its proximity to them. The node will calculate its estimated position using the following centroid formula:

$$(\hat{x}, \hat{y}) = \left(\frac{x_1 + ... + x_N}{N}, \frac{y_1 + ... + y_N}{N}\right). \qquad (9.42)$$

The same authors have also proposed a reduction of the estimation error placing additional anchors using a novel density adaptive algorithm, *HEAP* [10].

Another way to ensure a localization improvement is including weights when averaging the coordinates of the beacon nodes. This is the *Weighted Centroid Location (WCL)* [8] algorithm.

The weight is a function depending on the distance and the environment conditions so different weights may be used. Small distances to neighboring anchors lead to a higher weight than to remote anchors. To calculate the approximated position of a tracked node i, every reference location j, from the n anchor nodes in range, obtains a weight w_{ij} depending on the distance:

$$(\hat{x_i}, \hat{y_i}) = \frac{\sum_{j=1}^{n} (w_{ij} \cdot (x_j, y_j))}{\sum_{j=1}^{n} w_{ij}}. \qquad (9.43)$$

To determine the associated weight to a reference either the *link quality indication (LQI)* or *Received Signal Strength indicator (RSSI)* could be used [7]. Nevertheless, in the LQI case, if all the references in range provide relative high values the influence of one anchor's LQI becomes relative low. The *Adaptative WCL (AWCL)* [6] algorithm proposes to compensate high

LQI values giving more influence to the differences between the LQIs instead of the nominal values. It reduces measured LQI values of each reference in range by a part q of the lowest LQI (Eq. (9.44)),

$$(\hat{x_i}, \hat{y_i}) = \frac{\sum_{j=1}^{n} ((LQI_{ij} - q \cdot min(LQI_{1...n}) \cdot (x_j, y_j))}{\sum_{j=1}^{n} (LQI_{ij} - q \cdot min(LQI_{1...n}))}. \tag{9.44}$$

A *Selective Adaptive Weighed Centroid Localization (ASWCL)* [19] approach has also been proposed to improve the accuracy by adapting the weights according to their statistical distribution.

9.3.2.2 *Anchor-free techniques*

Anchor-based algorithms have some limitations because they need another positioning scheme to place the beacon nodes. In some cases, the environment may prevent the use of such positioning system (e.g., GPS and indoor locations) so pre-configured anchors providing known reference positions are not available. In addition, the practice reveals that a large number of beacons must be deployed to provide an acceptable positioning error [11]. They require a deployment effort and they may not scale well. In contrast, anchor-free algorithms are able to determine each node relative coordinates using local distance information and without relying on beacons that are aware of their positions. Note that no absolute positions are obtained, but this is a fundamental limitation of the problem statement and not part of the algorithm itself. The relative coordinate space should be able to be translated to any other global coordinate system easily. The centralized MDS algorithm (See 9.3.1.1) is a sample of anchor-free algorithm that can obtain final absolute positions with the help of an additional step involving three or more beacons. Some popular distributed anchor-free approaches are *relaxation-based* algorithms and *coordinates stitching.*

Relaxation-based algorithms These approaches are coarse grained localization methods with a refinement phase where typically each node corrects its position to optimize a local error metric. We will briefly introduce two of the most popular relaxation-based approaches.

Cooperative ranging In the *cooperative ranging* methodologies, every single node plays the same role, and repeatedly and concurrently executes the following functions:

- Receive ranging and location information from neighboring nodes.
- Solve a local localization problem.

- Transmit the obtained results to the neighboring nodes.

After some repetitive iterations the system will converge to a global solution.

The local localization problem is revolved by making assumptions when necessary and compensating the error through corrections and redundant calculations as more information becomes available. These assumptions are needed at first in order to deal with the under-determined set of equations presented by the first few nodes. The *Assumption Based Coordinate (ABC)* algorithm [72] propose the following procedure from the perspective of a node n_0:

- The node n_0 is located at the position $(0, 0, 0)$.
- The fist node to establish communication, n_1, is placed at $(r_{10}, 0, 0)$ where r_{10} is the estimated distance from some signal parameter.
- The location of the next node n_2, (x_2, y_2, z_2), is determined using the estimated distance to both n_0 and n_1 and assuming that $y_2 > 0$ and $z_2 = 0$,

$$\begin{aligned} x_2 &= \tfrac{r_{01}^2 + r_{02}^2 + r_{12}^2}{2r_{01}} \\ y_2 &= \sqrt{r_{02}^2 + x_2^2} \\ z_2 &= 0. \end{aligned} \tag{9.45}$$

- Next location n_3 (x_3, y_3, z_3) is obtained with the only assumption that the square involved in finding z_3 is positive,

$$\begin{aligned} x_3 &= \tfrac{r_{01}^2 + r_{03}^2 + r_{13}^2}{2r_{01}} \\ y_3 &= \tfrac{r_{03}^2 - r_{23}^2 + x_2^2 + y_2^2 - 2x_2x_3}{2y_2} \\ z_3 &= \sqrt{r_{03}^2 + x_3^2 + y_3^2}\,. \end{aligned} \tag{9.46}$$

From this point forth, the system of equations used to solve for further nodes is no longer under-determined, and so the standard algorithm can be employed for each new node. Under ideal conditions, this algorithm thus far will produce a topologically correct map with a random orientation relative to a global coordinate system.

The main advantage of this approach is that global resources for a centralized computing are not required. Nevertheless, the convergence of the system may take some time and nodes with high mobility may be hard to cover.

Spring Model The *AFL (Anchor-Free Localization)* [62] algorithm, also referred to as *Spring Model*, describes a fully decentralized algorithm where nodes start from a random initial coordinate assignment and converge to a consistent solution using only local node interactions. The key idea in AFL is fold-freedom, where nodes first configure into a topology that resembles a scaled and unfolded version of the true configuration, and then run a force-based relaxation procedure.

The AFL algorithm proceeds in two phases:

- The first phase is a heuristic that produces a fold-free graph embedding which *looks similar* to the original embedding. Five reference nodes are chosen, one in the center n_0, and four in the periphery, n_1, n_2, n_3 and n_4, where the couples (n_1, n_2) and (n_3, n_4) are roughly perpendicular to each other. The choice of these nodes is performed using a hop-count approximation to distance (e.g., the first peripheral node is selected maximizing the number of hops to the initial node, $\max h_{0,1}$). Finally a node n_5 is selected and supposed to be centered by minimizing the distance in hops between n_1 and n_2 ($\min |h_{1,5} - h_{2,5}|$) and the distance between n_3 and n_4 ($\min |h_{3,5} - h_{4,5}|$) for contender nodes. Now, for all nodes n_i, the heuristics approximate the polar coordinates using the maximum radio range, R, as follows:
$$\begin{aligned} \rho_i &= h_{i,5}R \\ \theta_i &= tan^{-1}\left[\frac{(h_{1,i}-h_{2,i})}{(h_{3,i}-h_{4,i})}\right]. \end{aligned} \tag{9.47}$$
- The second phase uses a mass-spring based optimization to correct and balance localized errors. It runs concurrently at each node. At any time any node n_i has a current estimated position $\hat{p}_i$ that periodically sends to its neighbors. Using these positions, the distance $\hat{d}_{ij}$ to each neighbor n_j is estimated. Also knowing the *measured* distance r_{ij} to n_j, a force $\vec{F}_{ij}$ in the direction $\vec{v}_{ij}$ (unit vector from $\hat{p}_i$ to $\hat{p}_j$), is given by Eq. (9.48),
$$\vec{F}_{ij} = \vec{v}_{ij}(\hat{d}_{ij} - r_{ij}). \tag{9.48}$$
The resultant energy E_i of node i due to the difference of the measured and the estimated distances between nodes, can be expressed in terms of the square of the magnitude of the forces $\vec{F}_{ij}$ as Eq. (9.49),
$$E_i = \sum_j E_{ij} = \sum_j (\hat{d}_{ij} - r_{ij})^2. \tag{9.49}$$

The main advantage of relaxation based algorithms is that they are fully distributed and concurrent and they operate without anchors nodes. Nevertheless, while the computational is modest and local, it is unclear how these algorithms scale to much larger networks [4]. Furthermore, there are no provable means to avoid local minima, which could be even worse at larger scales. Traditionally, local minima have been avoided by starting the optimization process at a favorable position, but another alternative would be to use optimization techniques such as simulated annealing [33].

Coordinate system stitching Some methods focus on fusing the precision of centralized schemes with the computational advantages of distributed systems as we have seen in Sec. 9.3.2.2. Another approach with the same goal that has received some attention [12, 50, 52, 57] is *Coordinate system stitching*. Coordinates system stitching works as follows:

- First, it localizes clusters in the network. They normally are overlapping regions composed by a single node and their one-hop neighbors.
- Then, it refines the localization of the clusters with an optional local map for each cluster placing cluster nodes in a relative coordinate system.
- Finally, it merges those cluster regions computing coordinate transformations between these local coordinate systems.

The fist two steps may be slightly different depending on the algorithm, while the last third step is usually the same. In [50] sub-regions are formed using one-hop neighbors. Then, local maps are computed by choosing three nodes to define a relative coordinate system and using multilateration to iteratively add additional nodes to the map, resulting in a *multilateration sub-tree*.

More robust local maps can be obtained according to [52]. Instead of using three arbitrary nodes to define a map, *robust quadrilaterals* are used, considering a robust quad as a fully-connected set of four nodes where each sub-triangle is also *robust*. A robust sub-triangle with a shortest side of length b and a smallest angle θ must accomplish Eq. (9.50),

$$b \sin^2 \theta > d_{min}, \tag{9.50}$$

where d_{min} is a predetermined constant based on the average measured error. The idea is that the points of a robust quad can be placed correctly with respect to each other. Once an initial robust quad has been chosen, any node that connects to three of the four points in the initial quad can be added using multilateration. This preserves the probabilistic guarantees provided by the initial robust quad, since the node form a new robust quad with the points from the original. By induction, any number of nodes can be added to the local map, as long as each node has a range to three members of the map. These local maps or *clusters*, are now ready to be stitched together.

Coordinates system stitching techniques are quite interesting since they are inherently distributed and they enable the use of sophisticated local maps algorithms. Nevertheless, registering local maps iteratively, can lead to error propagation and perhaps unacceptable error rates as the network grows. In addition, the algorithm may converge slowly since a single coordinate system must propagate from its source to the entire network. Furthermore, these techniques are prone to leave *orphan nodes* because, either they could not be added to the local map, or their local map failed to overlap with neighboring local maps.

9.4 Other Issues in Localization

In this section we outline some aspects involved in the localization theory of wireless ad-hoc and sensor networks that have not been covered in previous sections such as hybrid solutions, mobility and the application of the graph theory.

9.4.1 *Graph theory and localizability*

A fundamental question in the wireless sensor network (WSN) localization is whether a solution to the localization problem is unique. The network, with the given set of anchors, non-anchors and inter-sensor measurements, is said to be *uniquely localizable* if there is a unique set of locations consistent with the given data. Graph theory has been found to be particularly useful for solving the above problem of unique localization. Graph theory also forms the basics of many localization algorithms, especially for the category of distance based localization problem, although it has been used to other types of measurements as well.

A graphical mode for distance based localization problem can be built by representing each sensor in the network uniquely by a vertex. An edge exits between two devices if the distance between the corresponding sensors is known. Note that there is always a vertex between two anchors since the distance can be obtained form their known locations. The obtained graph $G(V, E)$, where V is the set of wireless communication devices and E the set of edges, is called the underlying graph of the sensor network. Details of the graph theoretical representations of the WSN and their use in localization can be found in [28, 43].

9.4.2 *Hybrid schemes*

Hybrid schemes simply combine two or more existing techniques to achieve a better performance such as using both multidimensional scaling (MDS) and proximity based maps (PDS) [13]. Initially, some anchors are deployed (primary anchors). In the first phase some sensors are selected as secondary anchors which are localized thought MDS (Sec. 9.3.1.1). Nodes which are neither primary nor secondary are called normal sensors. In a second phase those normal sensors are localized through proximity distance mapping. Other examples of hybrid schemes are the use of MDS and Adhoc positioning system (APS) [2] and stochastic approaches based on the combination of deductive and inductive methods [44].

9.4.3 *Mobility*

Mobility of sensors nodes obviously have an impact on the localization process. The uncertainty of the node movement may lead to increase the difficulty of the localization task. Nevertheless, in some cases, statistical approaches having capabilities to handle uncertainty of node movements, can tackle localization of mobile sensor nodes. The *sequential Monte Carlo localization (MCL)* method [27] exploits mobility to improve the accuracy and the precision of the localization. The simultaneous localization and tracking scheme based on Laplace method *(LaSLAT)* [81] employs Bayesian filters to accomplish the task of localizing mobile nodes, in which location estimates are iteratively updated given batches of new measurements. Empirical studies have shown that LaSLAT can tolerate noisy range measurements and achieve satisfactory location accuracy.

The localization of static sensors using one mobile anchor equipped with GPS has also been proposed [79]. The mobile anchor periodically transmits a beacon message including its latest position while traversing the area where static sensor nodes are deployed. Upon receiving the beacon packets, a static sensor determines its location relative to the anchor according to the received signal strength (RSS) of the beacon packet through Bayesian inference. The *on beacon mobility scheduling* is also subject of study [38] in order to determine the best beacon trajectory so that each sensor receives sufficient beacon signals with minimum delay.

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