# Robust Optimization for Energy-aware Routing with Redundancy Elimination

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# *Robust Optimization for Energy-aware Routing with Redundancy Elimination*[†]

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La gestion efficace de la consommation de l'énergie des réseaux de télécommunications est de nos jours un sujet d'une très grande importance. Plusieurs études ont réussi à prouver que le routage basé sur la consommation d'énergie réduit considérablement la consommation totale d'énergie du réseau. Nous avons, dans cet article, combiné cette technique à celle de l'élimination de redondance de trafic, pour diminuer davantage l'énergie consommée par un réseau coeur. Nous avons considéré une formulation robuste de ce problème dans le cas où il existe une incertitude autant au niveau de la valeur du volume de trafic que de celui du taux de redondance. Nous proposons, pour résoudre ce problème, un modèle de programmation linéaire en nombres entiers, un algorithme exact et une heuristique qui nous permettent des économies d'énergie allant de 16% à 28% comparé à la méthode classique de routage basé sur l'énergie.

**Keywords:** Energy-aware Routing, Redundancy Elimination, Green Networking, Robust Network Optimization

## 1 Introduction

The majority of the energy consumption of backbone networks, or more precisely IP routers, is due to the number of active elements such as ports, line cards, base chassis while the traffic load has only a marginal influence [CMN11]. Following this observation, people have proposed energy-aware routing (EAR) aiming at minimizing the number of used links while all the traffic demands are routed without any overloaded links [CMN11, GMPR12].

The redundancy elimination (RE) technique is an active research domain in order to reduce link load for backbone networks [ZA13]. It consists in splitting packets into small chunks, each being indexed with a small key. Then, keys are substituted to chunks in traffic flows, and the original data are recovered on downstream routers. However, RE has a drawback since it increases energy consumption of routers [GMPR12]. To find a good trade-off, in our previous work, we have proposed GreenRE - a model that combines EAR and RE to increase energy efficiency for backbone network [GMPR12]. In the GreenRE model, each of the demand has a static traffic volume and is associated with a constant factor of redundant traffic. To handle future changes and avoiding overloaded links, the peak volumes of traffic demand and the lowest RE rates are used as the worst case realization. Such assumption clearly leads to inefficient usage of network resources and poor energy savings. To alleviate this limitation of the GreenRE model, the uncertainty on traffic volumes and RE rates has to be precisely modeled and taken into account in the optimization process.

In mathematical literature, the technology-independent $\Gamma$-robustness has been introduced in [BS03] and then successfully applied to various network design problems [KKR13, CKPT13]. This approach is based on an observation that in real traffic traces, only few demands are simultaneously at their peaks. So, the authors considered a parameter $\Gamma > 0$ so that at most $\Gamma$ demands deviate simultaneously from their nominal traffic volumes. In summary, we make the following contributions :

- We apply the idea of $\Gamma$-robustness to our problem and formally define the Robust-GreenRE model using mixed integer linear programming.
- Since EAR is NP-hard problem [CMN11], we propose effective heuristic algorithm for large instances.

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– By simulation, we show the energy savings offered by our methods on backbone networks with real-life data traffic traces and compression rate fluctuations.

## 2 Energy Savings with Redundancy Elimination

### 2.1 GreenRE model

The GreenRE problem is defined on a directed graph $G = (V, E)$ where $V$ is a set of routers and $E$ represents a set of links. In this network, any physical link between two routers is a bi-directional link. We use the notation $\{uv\}$ to denote a physical link (without direction) and $uv$ as an arc with direction from $u$ to $v$. A link $\{uv\}$ is considered to be active if there is data going through at least one of its directions. Each active link $\{uv\}$ and router $u$ is respectively associated with a power consumption value $PE_{\{uv\}} =$ 200 Watts [CMN11] and $PN_u = 30$ Watts [GMPR12]. We are given a set $\mathcal{D} = \{(s,t) \in V \times V : s \neq t\}$ representing the traffic demands, where $D^{st}$ denotes the volume of demand from $s$ to $t$. We denote $\gamma^{st}$ as percentage of unique (non redundant) traffic. Then, we reformulate the GreenRE model as follows :

$$\min \quad \sum_{\{uv\}\in E} PE_{\{uv\}} x_{\{uv\}} + \sum_{u\in V} PN_u w_u \tag{1}$$

$$\text{s.t.} \sum_{v\in N(u)} \left(f^{st}_{vu} + g^{st}_{vu} - f^{st}_{uv} - g^{st}_{uv}\right) = \begin{cases} -1 & \text{if } u = s, \\ 1 & \text{if } u = t, \\ 0 & \text{otherwise} \end{cases} \qquad \forall u \in V,\ (s,t) \in \mathcal{D} \tag{2}$$

$$\sum_{(s,t)\in\mathcal{D}} D^{st}\left(f^{st}_{uv} + \gamma^{st} g^{st}_{uv}\right) \le \mu C_{uv} x_{\{uv\}} \qquad \forall \{uv\} \in E \tag{3}$$

$$\sum_{v\in N(u)} \left(g^{st}_{uv} - g^{st}_{vu}\right) \le w_u \qquad \forall u \in V,\ (s,t) \in \mathcal{D} \tag{4}$$

$$\sum_{v\in N(u)} \left(g^{st}_{vu} - g^{st}_{uv}\right) \le w_u \qquad \forall u \in V,\ (s,t) \in \mathcal{D} \tag{5}$$

$$0 \le f^{st}_{uv},\ g^{st}_{uv} \le 1 \qquad \forall (u,v) \in E,\ (s,t) \in \mathcal{D} \tag{6}$$

$$x_{\{uv\}},\ w_u \in \{0,1\} \qquad \forall \{uv\} \in E,\ u \in V \tag{7}$$

We use binary variables $x_{\{uv\}}$ and $w_u$ to denote respectively activated links and RE-routers. $N(u)$ is the set of neighbors of $u$ in the graph $G$. Variables $f^{st}_{uv}$ and $g^{st}_{uv}$, $\forall \{uv\} \in E, (s,t) \in \mathcal{D}$ denote the fraction of normal and compressed flows (s, t) on link (u, v). The objective function (1) is to minimize the power consumption of the network. Equations (2) establish flow conservation constraints. We use constraints (3), where $C_{uv}$ is link capacity and $\mu$ denotes the maximum link utilization, to limit flows on a link to its available capacity. Constraints (4) and (5) are used to determine whether RE service is enabled on router $u$ or not. We refer the readers to our research report [CKP14] for more explanation on the formulation.

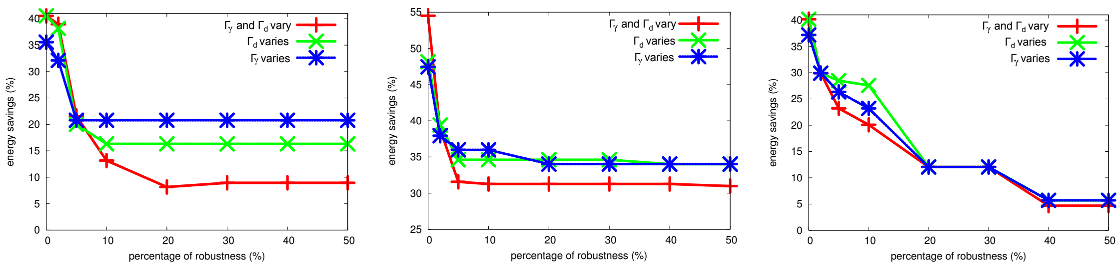
Although the GreenRE model is already a complex task, it does not take the fluctuation in real-life traffic into account. Hence, a Robust-GreenRE model should be proposed to address this issue by taking both traffic demand and redundancy rate uncertainty into account while satisfying the capacity constraints (3).

### 2.2 Robust-GreenRE Model

The idea of robustness is that we should reserve some space in the link capacity to accommodate the fluctuation in the traffic volumes and RE rates. To do so, we define a function $\delta(f, g, \Gamma_d, \Gamma_\gamma)$ such that :

$$\sum_{(s,t)\in\mathcal{D}} \overline{D}^{st}\left(f^{st}_{uv} + \overline{\gamma}^{st} g^{st}_{uv}\right) + \delta(f, g, \Gamma_d, \Gamma_\gamma) \le \mu C_{uv} x_{uv} \qquad \forall uv \in E \tag{3'}$$

$\Gamma_d$ and $\Gamma_\gamma$ denote respectively the number of demands that deviate from their volumes and RE rates. Traffic volume $D^{st}$ and RE rate $\gamma^{st}$ variate respectively in range $[\overline{D}^{st}, \overline{D}^{st} + \widehat{D}^{st}]$ and $[\overline{\gamma}^{st}, \overline{\gamma}^{st} + \widehat{\gamma}^{st}]$. Let us call respectively $Q$ and $Q'$ the sets of variated demands and variated RE rates. We use the notations $Q_d = Q \backslash Q'$,




(a) Abilene (b) Geant (c) Germany

FIGURE 1: Energy savings vs. robustness for Abilene, Geant and Germany network

$Q_\gamma = Q'\backslash Q$ and $Q_{d\gamma} = Q \cap Q'$ as independent sets such that : $Q_{d\gamma}$ contains demands in which both traffic volumes and RE rates can deviate, $Q_d$ (resp. $Q_\gamma$) contains demands in which only traffic volumes (resp. RE rates) can deviate from their nominal values. Then the worst case scenario when considering fluctuation on an arc $uv$ is given by :

$$\sum_{(s,t)\in\mathcal{D}} \overline{D}^{st} f^{st}_{uv} + \max_{Q\subseteq\mathcal{D}} \Big\{ \sum_{(s,t)\in Q} \widehat{D}^{st} f^{st}_{uv} \Big\} + \sum_{(s,t)\in\mathcal{D}} \overline{D}^{st}\overline{\gamma}^{st} g^{st}_{uv} + \max_{Q_\gamma = Q'\backslash Q} \Big\{ \sum_{(s,t)\in Q_\gamma} \overline{D}^{st}\widehat{\gamma}^{st} g^{st}_{uv} \Big\} +$$

$$\max_{Q_{d\gamma}=Q\cap Q'} \Big\{ \sum_{(s,t)\in Q_{d\gamma}} (\widehat{D}^{st}\widehat{\gamma}^{st} + \widehat{D}^{st}\overline{\gamma}^{st} + \overline{D}^{st}\widehat{\gamma}^{st}) g^{st}_{uv} \Big\} + \max_{Q_d = Q\backslash Q'} \Big\{ \sum_{(s,t)\in Q_d} \widehat{D}^{st}\overline{\gamma}^{st} g^{st}_{uv} \Big\} \leq \mu C_{uv} x_{uv} \ \forall uv \in E \quad (3")$$

Obviously, Constraints (3') and (3") are equivalent if $\delta(f,g,\Gamma_d,\Gamma_\gamma)$ is the maximum part of Constraint (3"). Constraint (3") can be rewritten as a set of many constraints corresponding to all possible sets $Q_d$, $Q_\gamma$ and $Q_{d\gamma}$, but the resulting model has an exponential number of constraints. To overcome this difficulty, we thus propose three methods (we refer the readers to our research report for more details [CKP14]) :

– *Compact formulation :* the maximum parts of Constraint (3") can be formulated using ILP (called the primal problem). Using duality theorem, we formulate the dual problem based on the primal one. Finally, the robust capacity constraint (3") can be reformulated in a compact form by integrating the dual formulation into the deterministic MILP model (1)– (7). This method provides a lower bound on the optimal solution.

– *Constraint generation :* the main idea is to generate iteratively subsets of traffic demands (called $S_i$ at step $i$) that can deviate from their traffic volumes and/or RE rates. We start the algorithm by solving the model (1)– (7) to find a feasible solution. Then based on $f^{st}_e$ and $g^{st}_e$ from the feasible solution, we solve the primal problem (in the *compact formulation*) to find a subset $S_i$ that causes violation on capacity constraints (due to traffic volumes and RE rates fluctuation of demands in the subset $S_i$). Then, we add new capacity constraints corresponding to $S_i$ into the model (1)– (7) and repeat the process until no more violation is found. This method gives bounds and can find an exact solution but requires long time of execution due to an exponential number of constraints.

– *Heuristic algorithm :* the algorithm works via two steps. In the first step, the heuristic assumes that all routers are RE-routers (all traffic flows can be compressed) and tries to find feasible solution minimizing the number of active links. Then, in the second step, based on the solution found in the first step, we try to disable RE service on as many routers as possible to save energy.

## 3 Computational Evaluation

We solved the Robust-GreenRE model with IBM ILOG Cplex 12.4 solver. All computations were carried out on a computer equipped with a 2.7 Ghz CPU and 8 GB RAM. We consider real-life traffic traces collected from the SNDlib : Abilene, Geant and Germany50 networks [OWPT10]. The readers can find more simulation scenarios in the research report [CKP14].

**Energy savings vs. robustness :** Fig. 1 shows the trade-off between energy savings and the level of robustness regarding the parameters $(\Gamma_d, \Gamma_\gamma)$. We consider three test cases (1) both $\Gamma_d$ and $\Gamma_\gamma$, (2) only $\Gamma_\gamma$

and (3) only $\Gamma_d$ vary their values. In the Case 1, both $\Gamma_d$ and $\Gamma_\gamma$ vary with the same value of robustness. In Case 2 (resp. Case 3), while $\Gamma_\gamma$ (resp. $\Gamma_d$) varies, $\Gamma_d$ (resp. $\Gamma_\gamma$) is set to 2% of the total demands. In all the three networks, the solutions do not change when $\Gamma_d, \Gamma_\gamma \geq \frac{|\mathcal{D}|}{2}$ where $|\mathcal{D}|$ is the total number of demands. It is noted that when $\Gamma_d = \Gamma_\gamma = |\mathcal{D}|$, the Robust-GreenRE model becomes the GreenRE as it consists the worst case realization of demands and RE rates. Indeed, large values of $\Gamma$ reduces the interest for robust optimization. More precisely, when $\Gamma_d, \Gamma_\gamma \geq 30\%$, energy savings offered by the Robust-GreenRE model are almost the same as the GreenRE model, while when $\Gamma_d, \Gamma_\gamma \leq 20\%$ the Robust-GreenRE model allows for significant energy savings. It is because when the values of $\Gamma_d, \Gamma_\gamma$ covers all of these dominating demands, increasing $\Gamma_d, \Gamma_\gamma$ does not affect the routing solution and the percentage of energy savings remains stable.

**Robust-GreenRE vs. GreenRE vs. Classical EAR :** In Fig. 2, we compare the Robust-GreenRE model with the GreenRE and the classical EAR models. As observation in real traffic traces [CKP14], only few demands are at peak values simultaneously, so we choose small values of $\Gamma_d$ and $\Gamma_\gamma$ ($2\% - 5\%$) in the coparison. Since the GreenRE model does not take into account RE rate deviation, we set $\gamma^{st} = 0.8$ (20% of traffic is redundant). Furthermore, since traffic volume variations are not handled by GreenRE and EAR models, all demands are at peak. We observe that the lowest energy savings are achieved by EAR and GreenRE models. As expected, the Robust-GreenRE model outperforms the other models and allows for $16 - 28\%$ additional energy savings in all cases.

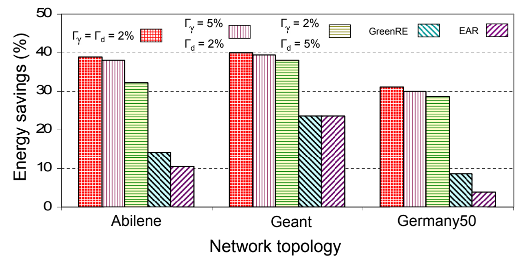



FIGURE 2: Robust-GreenRE vs. GreenRE vs. EAR.

# 4 Conclusion

In this paper, we formally defined the Robust-GreenRE problem. Taking into account the uncertainties of traffic volumes and redundancy elimination rates, this model provides a more accurate evaluation of energy savings for backbone networks. Based on real-life traffic traces, we have shown a significant improvement of energy savings compared to other models.

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